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FURTHER REMARKS ON THE STABILITY OF BOILING HEAT TRANSFER

Report 58-5. Project 34

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Novak Zuber
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Work performed under Contract No. AT(11-1)-34

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The authors desire to express their appreciation to Professor J. W. Westwater of the University of Illinois for his kindness in making available his excellent moving pictures and still photographs. These photographs guided the analytical developments reported herewith and were instrumental in the development of the theory.

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ABSTRACT

An analytical expression is presented which permits the prediction of the maximum nucleate heat flux in pool boiling of saturated or subcooled liquids. The numerical values of the empirical constants which appear in the Kutateladze and Borishanskii criteria for the "burnout" heat flux are derived from the theory. An analytical expression for the empirical function which appears in the correlation of Griffith is also derived.

The hitherto unexplored features of transition boiling, i. e., the hydrodynamic instability, the well defined geometrical configuration and the frequency dependence are described, supported by experimental evidence, and used as the basis for the analytical work. The peak heat flux in transition boiling is shown to be limited by the combined effects of Taylor and Helmholtz instabilities; whereas the minimum transitional heat flux is limited by the effect of Taylor instability only. The analysis leads to the conclusion that because of the statistical nature of the disturbances and the "bandwidth" of the unstable wavelengths which govern the process, an inherent uncertainty exists in determining the exact value of the heat flux at "burnout". The width of this uncertainty range is $\pm$ 14%. The often noted poor reproducibility of experimental data on "burnout" can be inferred, therefore, from the analysis.

The reason analytical attacks upon the "burnout" problem, based upon considerations of bubble agitation and other nucleate boiling characteristics, have not been successful is discussed. The literature has shown conclusively that the surface conditions play the dominant role in determining the superheat accompanying a given heat flux. The analysis reveals, in addition, that even if the nucleating characteristics of the surface were known (which they are not) the problem could still not be solved from these considerations because the mechanism of the instability occurs in the fluid away from the surface.

The extension of the analysis to flow systems is briefly discussed. The location of the burnout point along a heated channel as predicted by the

idealized system, for uniform and nonuniform heat transfer rates is discussed also.

The detailed and general features of this simple idealized system appear to be in qualitative as well as quantitative agreement with published experimental data. Further experimental investigations are suggested.

NOMENCLATURE

a	=	thermal diffusivity
c	=	specific heat at constant pressure
f	=	frequency
g	=	acceleration due to gravity
h	=	enthalpy
k	=	thermal conductivity
L	=	latent heat of vaporization
m	=	wave number
n	=	angular frequency
P	=	pressure
ΔP	=	vapor pressure difference corresponding to the liquid superheat temperature difference $(T_w - T_s)$
P	=	critical thermodynamic pressure
q	=	heat flux
R	=	bubble radius
R_o	=	$2\ \sigma/\Delta P$ = radius of critical bubble
T_w	=	temperature of the heating surface and, consequently, of the superheated liquid in contact with it
T_s	=	saturation temperature corresponding to pressure on system
T_L	=	temperature of the bulk liquid
u_v	=	velocity of vapor
u_L	=	velocity of liquid
ρ	=	mass density
λ	=	wave length
σ	=	surface tension
τ	=	period
μ	=	viscosity

Subscripts

L	=	liquid
v	=	vapor

REGIMES OF BOILING HEAT TRANSFER

Developments in nuclear reactors and rocket engines where exceedingly high heat transfer rates occur in comparatively small areas, have focused attention on boiling as a mode of transferring heat at high flux densities. To attain these high heat transfer rates by forced convection would require excessively high velocities with resulting high pressure drops. With nucleate boiling, however, they can be reached at much lower bulk velocities. For this reason extensive experimental and theoretical studies of this phenomenon are conducted in this country and abroad.

The phenomenon is complex because three different regimes exist: nucleate, transition and film boiling. The change from one regime to another is accompanied by marked changes in the hydrodynamic and thermal state of the system. These regimes are illustrated in Figure 1. When the temperature of the heating surface is below the fluid saturation temperature heat is transferred by convection, forced or natural, depending on the system. This nonboiling region (AB) has been extensively investigated and equations have been derived which permit the prediction of heat transfer rates. Nucleate boiling (BC) starts when the temperature of the surface exceeds the saturation by a few degrees. Adjacent to the surface a thin layer of superheated liquid is formed in which bubbles nucleate and grow from some preferred spots on the surface. The thermal resistance of this superheated liquid film is greatly reduced by the agitation produced by the bubbles. An increase of the wall temperature is accompanied by a large increase of the bubble population causing in turn a sharp increase of the heat flux. However, as the temperature increases, bubbles become so numerous that their motions interact. Under these conditions the nucleate heat flux reaches its peak. If the temperature is further increased transition boiling begins. Westwater and Santangello[1]* have found that in this region (CD) no liquid-solid contact exists. The surface is blanketed by an unstable, irregular film of vapor which is in

* Superscript numbers refer to bibliography listed at the end of the report.

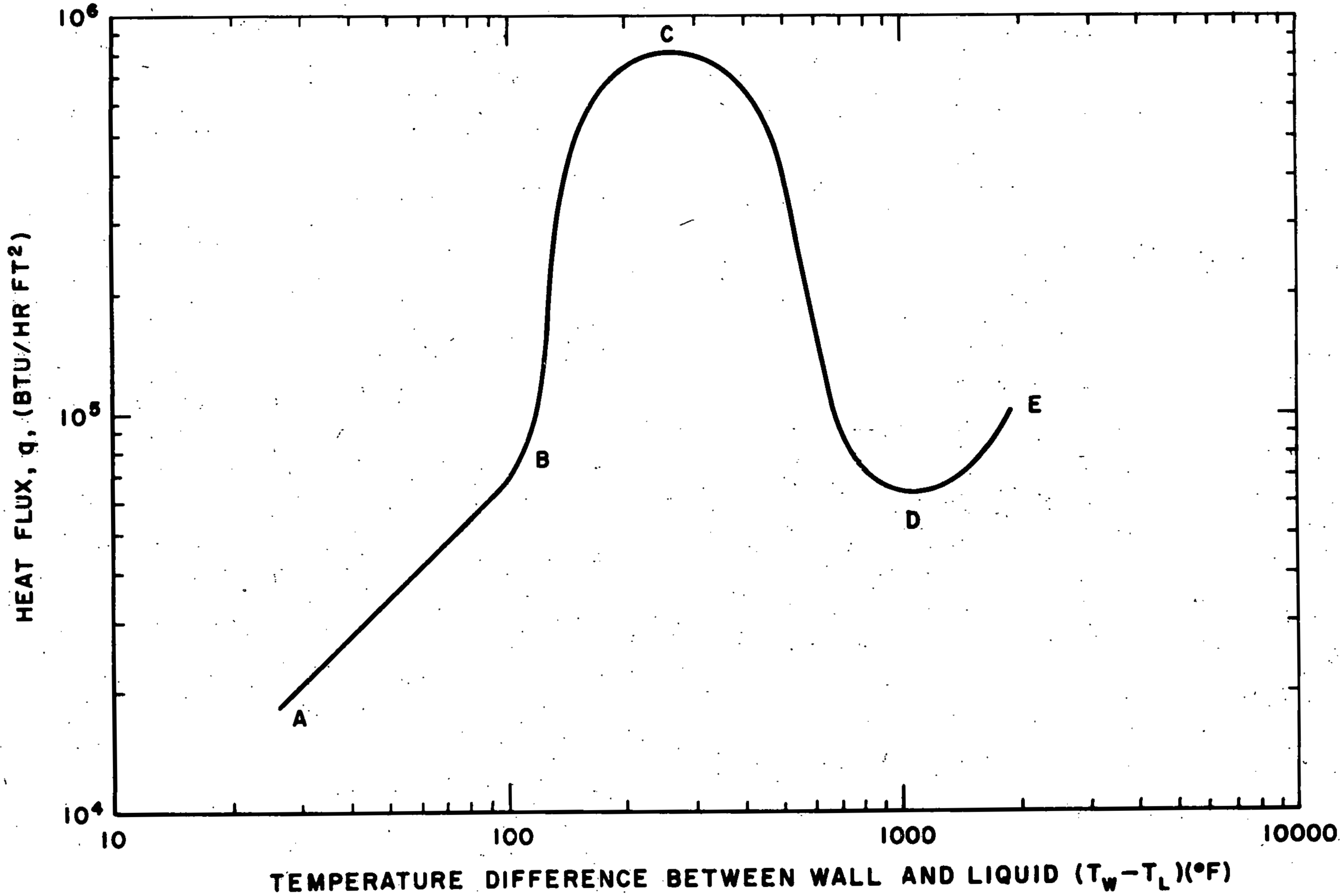

Figure 1. A Typical Curve of Heat Transfer versus Surface Temperature in Boiling.

violent motion. In transition boiling an increase of temperature is followed by a decrease of heat flux until a minimum value is reached at which film boiling starts. This new regime is characterized by an orderly discharge of large bubbles with a regular frequency and at regular intervals. In the film boiling region the heat flux increases with an increase of temperature but at a much slower rate than in nucleate boiling. Consequently, at high heat-transfer rates the temperature of the heating surface can exceed the melting temperature and "burnout" occurs. It is of great practical interest to operate in the nucleate region because of the favorable heat transfer. The problem is to avoid the "burnout" phenomenon.

The temperatures of the wall in nucleate boiling are quite low, for example, with boiling water the temperature of the surface at the point C exceeds the fluid saturation temperature by about 50° F at 14.6 psia and by only 10°F at 2000 psia. Therefore, in many designs the exact surface temperatures are of secondary importance. This is especially true for constant heat-input systems such as a nuclear reactor. The essential information needed by a designer is the limit to the heat transfer rates given by the peak nucleate heat flux, i.e., by the flux corresponding to point C. The temperature at point C is relatively unimportant. An investigation of the conditions leading to this maximum heat flux is therefore of practical and theoretical interest.

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A REVIEW OF NUCLEATE BOILING

Although nucleate boiling does not form the focus of this report, it is desirable to review briefly the features of the nucleate boiling regime. In this section the more successful attacks upon nucleate boiling will be discussed and the inherent limitations of these methods will be considered. An understanding of the problems of predicting the behavior of the nucleate regime will make the advantages of an analysis based upon transitional boiling more easily appreciated.

It was noted previously that designers desire to operate equipment in the nucleate boiling regime. Consequently, with very few exceptions, the research efforts have been directed toward an investigation of the nucleate region only. An important characteristic of nucleate boiling is the sensitivity of the heat flux to the conditions of the solid surface which governs the bubble population. The reason for this becomes evident when the details of the phenomenon are analyzed.

The high heat transfer rates in nucleate boiling are attributed to bubbles which induce a strong agitation of the liquid near the heating surface. This mechanism which was first proposed by Jakob[2] was further investigated and confirmed by Gunther and Kreith[3] and by Rohsenow and Clark.[4] The important factors to consider, therefore, are the bubble population, the rate of bubble formation and the dynamics. The last two factors are a function of the liquid superheat temperature ($T_w - T_s$). However, for a given heat flux this temperature difference is a function of the agitation which in turn depends upon the bubble population. This aspect of the problem was excellently discussed in a paper by Courty and Foust.[5] They point out that if it is assumed that each bubble makes its own "quantized" contribution to the agitation of the liquid and, therefore, to the heat flux, then for any given system the boiling curve in the nucleate region must be an expression for the total number of bubbles on the surface at a given superheat. Consequently, the slope of the boiling curve in the nucleate region represents the change in the liquid superheat temperature

$(T_w - T_s)$ necessary to alter the number of bubbles in such a way as to accommodate a new heat flux. In other words, it appears that for a given heat flux the liquid superheat temperature is not an independent variable but is a sensitive function of surface nucleating condition which governs the bubble population.

The variations of the liquid superheat temperature caused by a change of surface conditions may be small compared to the temperature of the heating surface. Therefore, in many design problems these variations can be neglected when the surface temperature is estimated. However, as all proposed correlations of heat transfer rates in nucleate boiling are of the form

$$q = \text{const.}\ (T_w - T_s)^n \tag{1}$$

these variations of the liquid superheat temperature $(T_w - T_s)$ become very important when the heat flux is to be computed. This result is a consequence of the fact that the exponent n is very large; values between 3 and 24 have been reported in the literature.[5,6] It should be noted also, that in the above equation, both the constant and the exponent n depend on the condition of the surface. Indeed, it is possible to change their values by just changing the surface polishing procedure.[5,6] *
To complicate the problem further, the value of the exponent changes with time because surface characteristics change with prolonged boiling, i.e., the surface exhibits the effect of "aging".

The difficulty which arises when nucleate boiling is considered and an equation of the above form, Equation (1), is used, becomes even more evident when it is realized that experimental data are not yet available which would relate, quantitatively, the variations of the liquid

* This fact is shown on Figure 1A which is reproduced from the paper by Corty and Foust.[5]

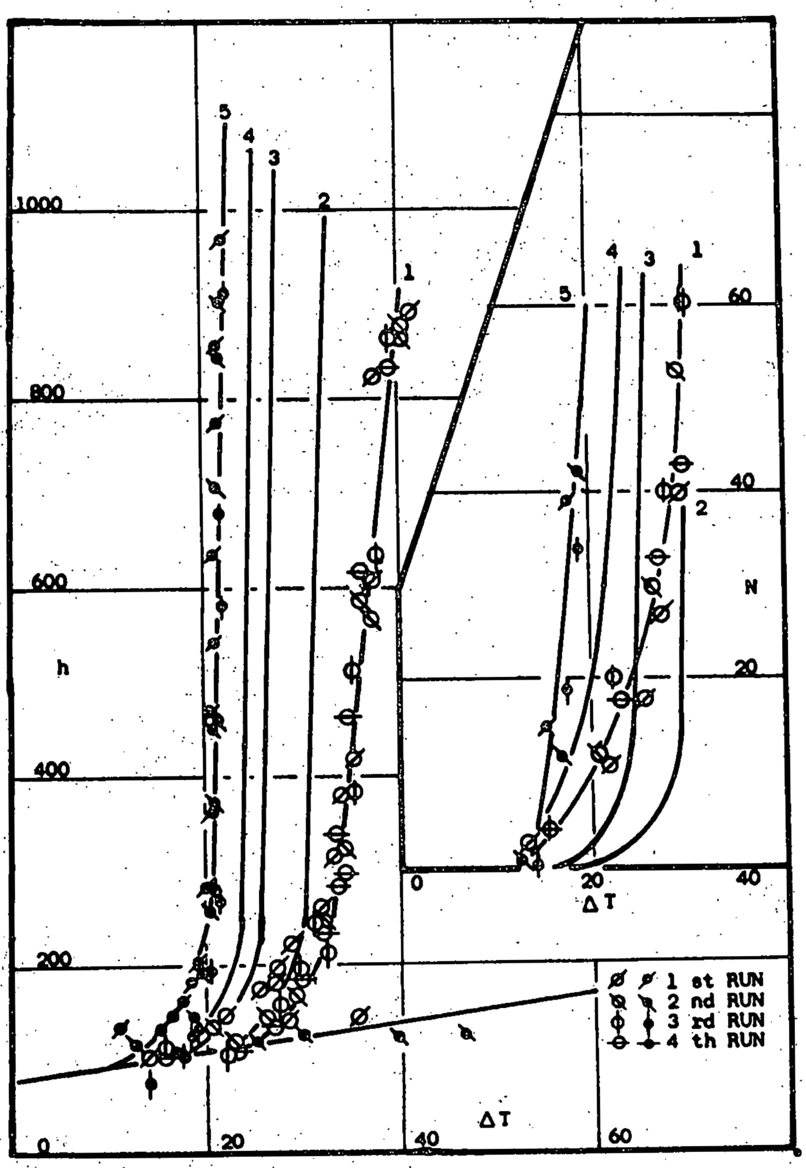


Figure 1A. The Effect of Different Amounts of Roughness on the Heat Transfer in Nucleate Boiling ($h = q/\Delta T$, Btu/hr ft^2 °F; N = bubbles/in^2). This figure is reproduced from the paper by Claude Corty and Allan S. Foust.[5]

superheat temperature, $(T_w - T_s)$, to the surface conditions. The best that can be hoped for from this line of attack is that it will be found that after prolonged boiling various surfaces exhibit similar surface characteristics. If this should happen to be the case it would permit a classification of surfaces and consequently greatly simplify the problem. Data bearing on this point have not yet been gathered.

The accompanying Figure 2 indicates the interaction among bubble dynamics, agitation, superheat and surface conditions. For a given surface (or class of surfaces) some investigators have attempted to find empirical relations among dimensionless groups which arise when the behavior of a single bubble is considered.

This approach to the problem was used by many investigators, among them Gunther and Kreith,[3] Ellion,[7] Kruzhilin,[8] Rohsenow,[9] and Forster and Zuber.[10] Some limited success has been had and we shall now briefly discuss this approach in order to point out why when all things are considered this line of attack cannot be expected to provide any information concerning the peak nucleate heat flux, i. e., the information in which designers of apparatus are most interested.

In nucleate boiling heat transfer rates appear to be independent of the geometry of the system (provided the system is not too small) because the bubbles induce strong localized agitation in the superheated liquid film adjacent to the heating surface. Consequently, all proposed correlations are formulated by considering dimensionless ratios which utilize bubble dimensions rather than the dimensions of the apparatus. In this way there have been proposed a Nusselt modulus, a bubble Reynolds modulus and a Prandtl modulus for the liquid. In pool boiling Ellion[7] has shown that the bubble radius and radial velocity (which he determined from experiments) furnish a suitable characteristic length and velocity for the Reynolds number of the flow adjacent to the heating surface. Ellion, for example, substituted these measured values of bubble radius and velocity in the usual equations for "pipe flow" with the results shown in Figure 3.

Inasmuch as the problem of the growth of a bubble in a superheated

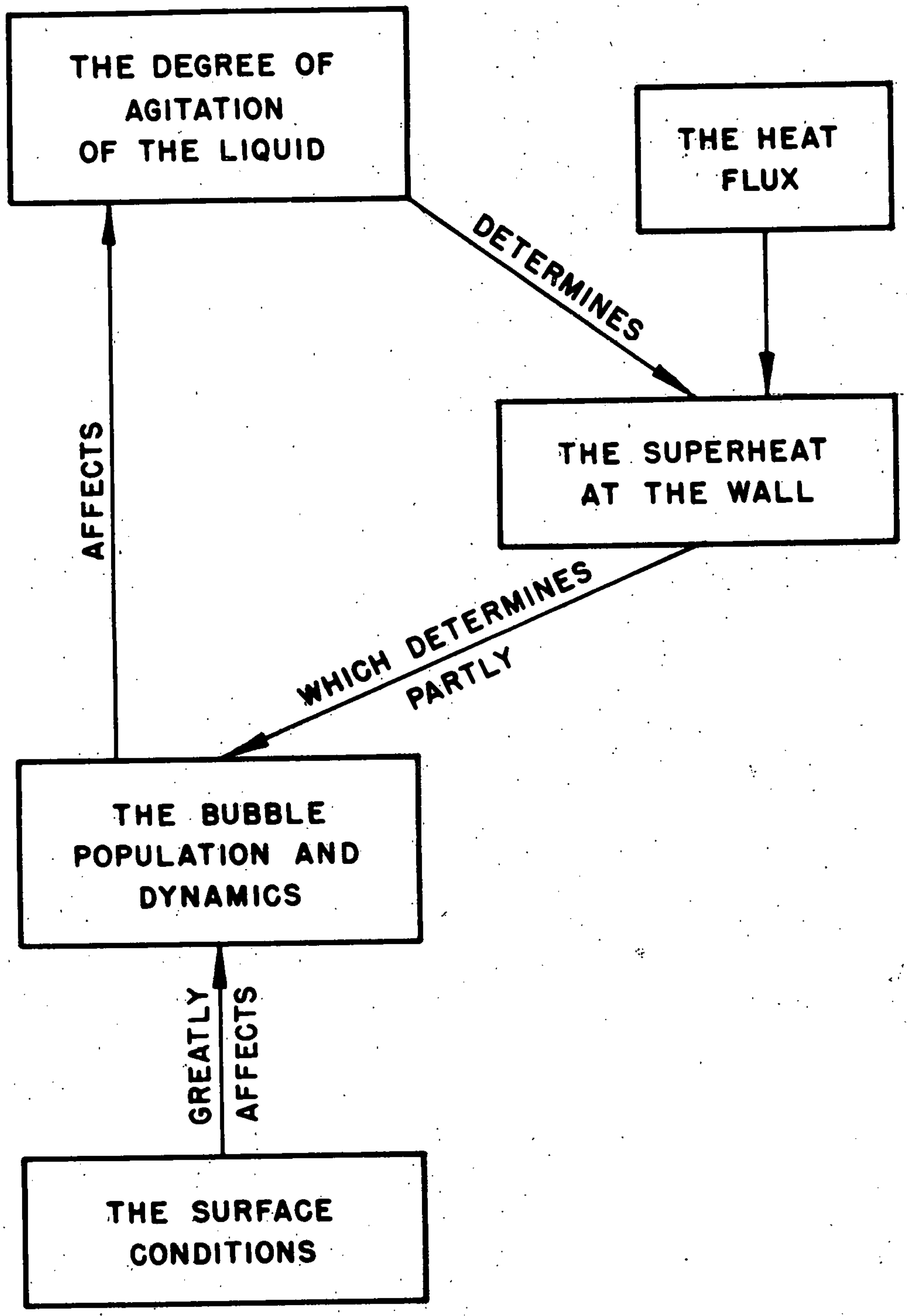


Figure 2. The Interaction of Some of the Parameters Relating to Nucleate Boiling.

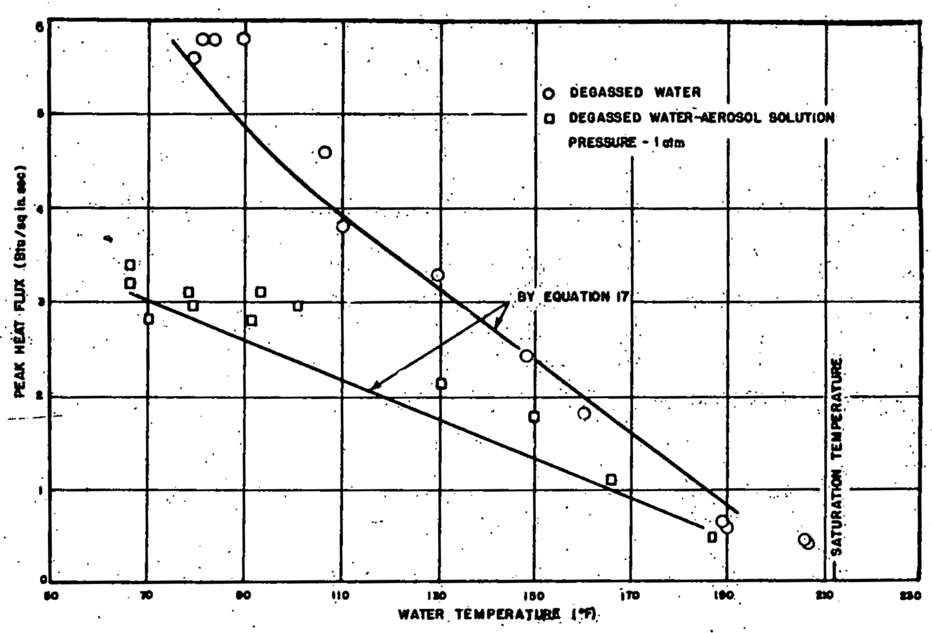


Figure 3. This figure is reproduced from the report by Ellion.[7] It shows the comparison of predicted peak heat flux with experimental data. Equation 17 in Ellion's report is given by: $Nu = C(R\mathring{R}\ \rho_L/\mu)^a\ Pr^b$ where R and $\mathring{R}$ were measured, and

$$C = 0.053, \quad a = 0.8, \quad b = 1.0$$

liquid has been solved,[11, 12, 13] it was proposed[10] that the derived analytical expressions be used for this length and velocity, thus replacing these two variables used by Ellion with one, the superheat.

The bubble growth velocity was found to be

$$\overset{\circ}{R} = \frac{(T_w - T_s)\ c_L\ \rho_L\ \sqrt{\pi a}}{L\ \rho_v\ \cdot 2\sqrt{t}} \tag{2}$$

The product of the bubble radius R and velocity $\overset{\circ}{R}$ is then time independent

$$2 R\overset{\circ}{R} = \left[\frac{(T_w - T_s)\ c_L\ \rho_L \sqrt{\pi a}}{L\ \rho_v\ \sqrt{t}}\right]^2 \tag{3}$$

Consequently, in view of Ellion's results it was reasonable to propose that the above product be used in a pseudo Reynolds modulus as an index of the agitation in the superheated film.[10] It was shown[14] that a useful characteristic length is the product of the Gibbs' critical bubble radius (R_o) and the Weber number

$$\begin{aligned} R' &= \frac{(T_w - T_s)\ c_L\ \rho_L\ \sqrt{\pi a}}{L\ \rho_v} \left[\frac{2\sigma}{\Delta P}\right]^{1/2} \left[\frac{\rho_L}{\Delta P}\right]^{1/4} \\ &= R_o \left[\frac{\rho_L (R\overset{\circ}{R})^2}{2\ \sigma R_o}\right]^{1/4} \end{aligned} \tag{4}$$

These two dimensionless groups include aspects of two of the important phenomena: the bubble dynamics and nucleation.

When Equations (4) and (3) are substituted for the characteristic length and velocity in the bubble Nusselt and Reynolds moduli a relation between the heat flux and the liquid superheat temperature (T_w - T_s) is obtained in terms of the thermodynamic properties of the vapor and the liquid:

$$\frac{q/A\ R'}{(T_w - T_s)\ k} = .0015 \left(\frac{R\overset{\circ}{R}\ \rho_L}{\mu}\right)^{.62} (Pr)^{.33} \tag{5}$$

The exponents and the constant were determined from experimental data at the peak nucleate heat flux reported by Cichelli and Bonilla[15] and by Kazakova.[16] The comparison with experimental results is shown in Figure 4. Recently Perkins and Westwater[17] have reported that the heat transfer rates predicted by Equation (5) are in agreement with their experimental data for boiling methanol in the whole nucleate region.

On the other hand, it is known that Equation (5) cannot be used to predict the behavior of all nucleate boiling systems because the effect of surface roughness is not included. Rohsenow[18] has shown how related dimensionless groups may be used to correlate data from different surfaces. A typical graph prepared by Rohsenow is shown in Figure 5.

These two Figures, 4 and 5, demonstrate clearly that the dimensionless ratios contain pertinent variables in a meaningful way. The diagram (Figure 2) may be interpreted as showing that the bubble dynamics, agitation, and superheat form a "closed loop system" with the surface conditions playing the role of independent "forcing functions." No equation which does not include an allowance for the surface conditions can provide the solution to the problem.

Two important facts should now be noted. Equation (5) is rather complex, i.e., the heat flux is a complicated function of many thermodynamic properties. This complexity is the characteristic not only of Equation (5) but of all proposed correlations for the nucleate region. Another important characteristic is that Equation (5) solves the following problem: Given the liquid superheat what is the corresponding heat flux (or vice versa)? Equation (5), or one like it, will predict the nucleate heat flux for different liquids and at different pressures only if the superheat is known and the surface conditions are similar. However, the equation cannot provide any information concerning the change from nucleate to transition boiling, i.e., the peak flux. This restriction on the utility of the equation is to be expected since in the formulation of the problem, no account is taken of bubble interaction, but rather the dimensionless groups are derived from consideration of a single bubble. Such a restriction applies not only to Equation (5) but also to all correlations which are derived by considering the effect of a single bubble. It seems reasonable that if a quantitative understanding of the change from nucleate to transition

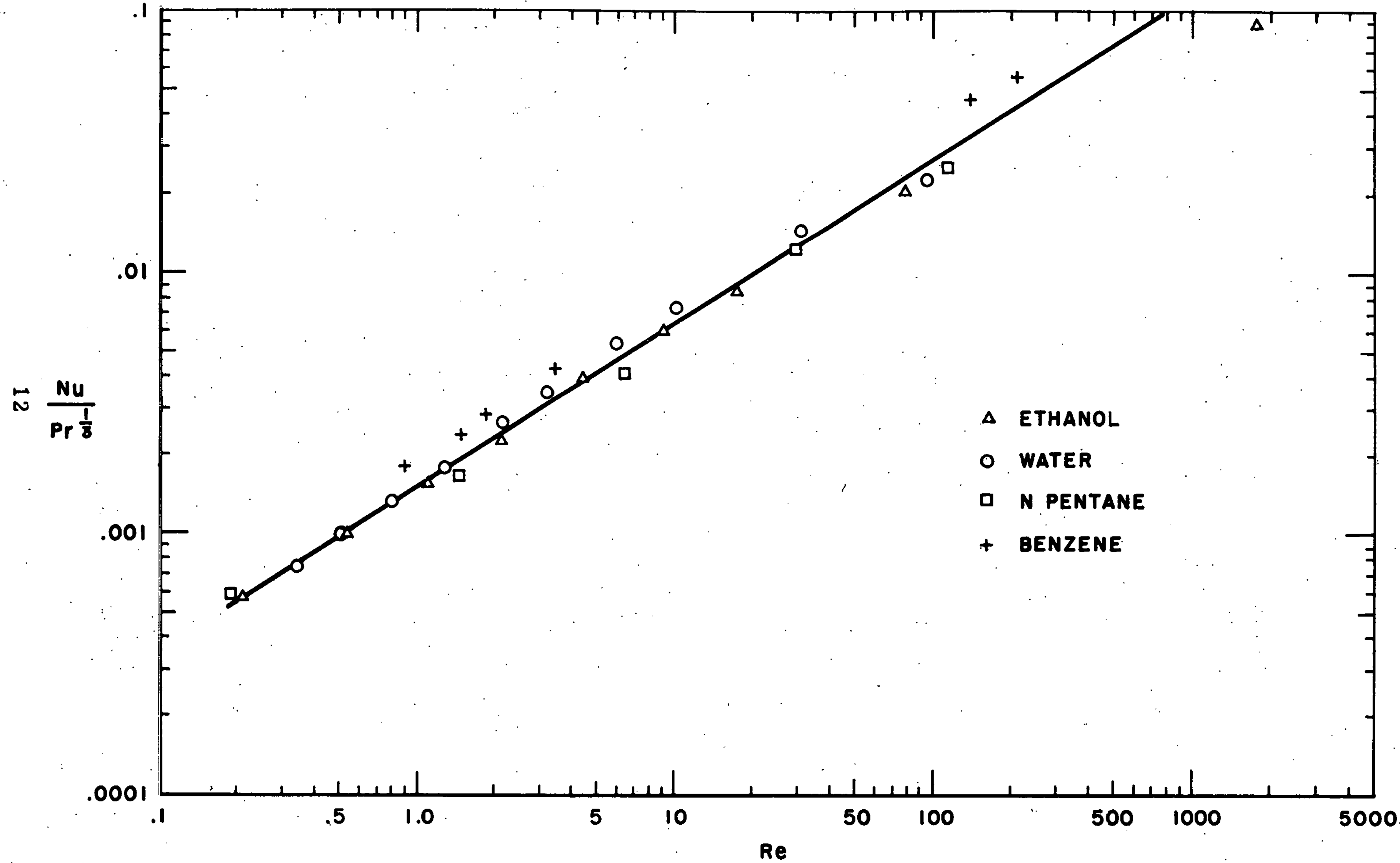


Figure 4. Correlation of Data for Various Liquids at Maximum Heat Flux and Temperature in Pool Boiling. (Reference 10)

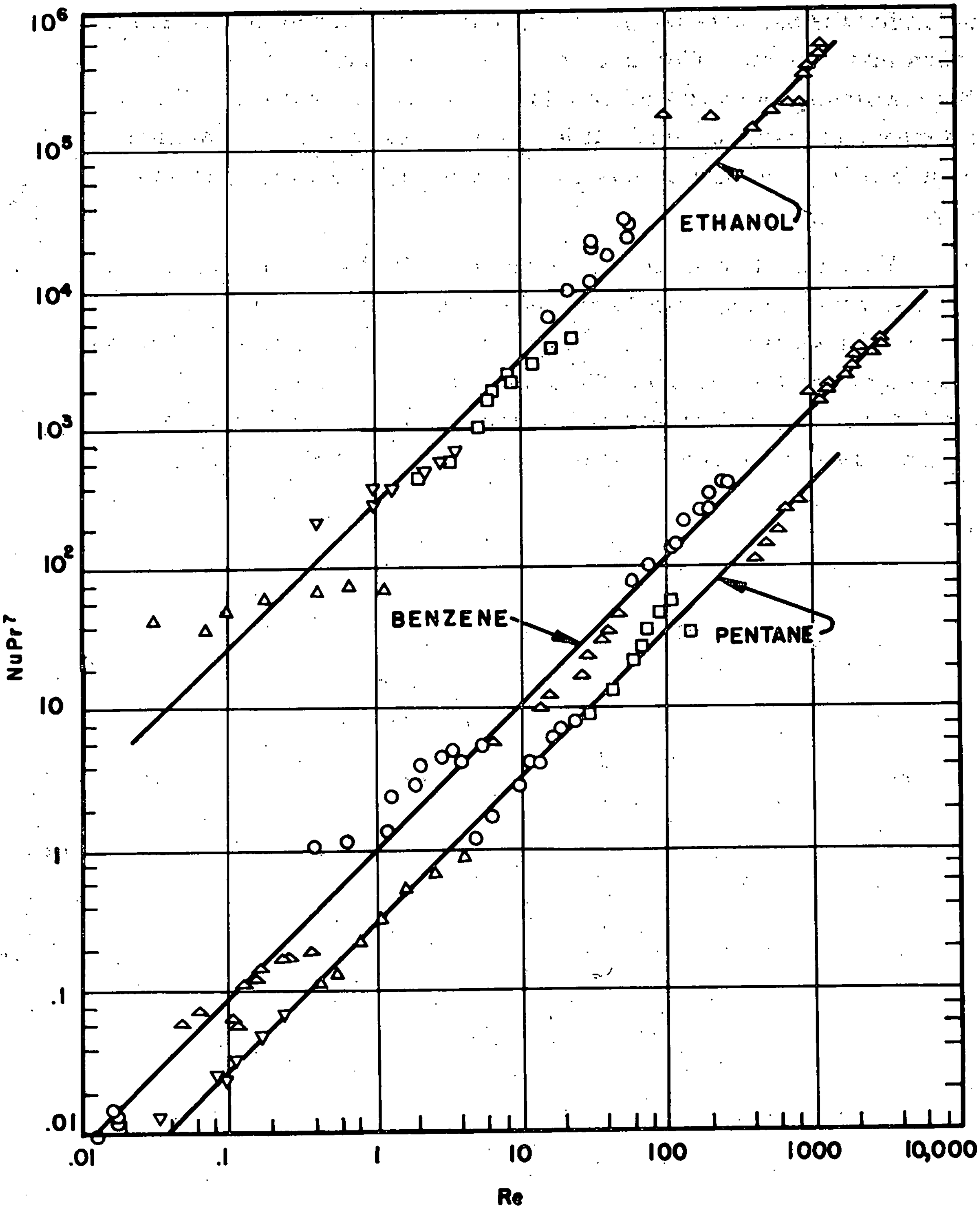


Figure 5. An Empirical Fit of Data in the Nucleate Regime as Obtained by Rohsenow, Griffith and Berenson.[17]

boiling is desired, and if the critical heat flux is to be predicted from an equation for the nucleate portion of the curve, the bubble population has to be taken into account. This is a consequence of the fact that limitation on the maximum nucleate heat flux is brought about because of the mutual interference of many bubbles. It has been noted that no information concerning bubble population is now available either from experiments or theory.

We shall show in the following sections that even if the bubble population were known it would not permit one to solve the "burnout" problem. As we shall see, the "burnout" is caused by interactions which occur away from the surface. This point of view originated with Kutateladze and has been adopted by many (but not all) Russian workers. In the next section the matter will be taken up in detail.

PREVIOUS EMPIRICAL CORRELATIONS OF PEAK NUCLEATE HEAT FLUX

Many investigators have adopted the view that the peak nucleate heat flux marks a transition from one regime to another (just as the critical Reynolds modulus marks the transition from laminar to turbulent flow) and have proposed equations which do not derive from the behavior of the nucleate region alone.

Such contributions are to be found in the work by Kutateladze[19], Sterman[20], Borishanskii[21,22] and by Rohsenow and Griffith.[23] The analysis of Kutateladze will be discussed in greater detail because he was apparently the first to propose that the peak nucleate heat flux was caused by the hydrodynamic instability of two phase flow. It will be seen, also, why Kutateladze's and Borishanskii's method, which consists of analyzing the maximum heat flux by considering the nucleate region, necessarily leads to dimensional analysis and at least one empirical constant. The advantage of an analysis of transition boiling as opposed to nucleate boiling will be seen in the next section, for such an analysis yields an analytical expression for this heat flux without recourse to experimentally determined constants.

Kutateladze noted in 1951 that "the essential feature of the theory of the phenomenon can be derived if one assumes that the crisis in the boiling process is purely a hydrodynamic phenomenon: the destruction of stability of two-phase flow existing close to the heating surface." According to this view the change from nucleate to transition boiling occurs when the velocity in the vapor phase reaches a critical value. Starting from the nonlinear Euler equation of motion and the energy equation he derived, by dimensional analysis, the following equation for the peak nucleate heat flux in pool boiling of liquids at saturation,

$$\frac{q}{L(\rho_v)^{1/2}\left[\sigma g(\rho_L - \rho_v)\right]^{1/4}} = K = \text{const} \qquad (6)$$

The constant K was determined from experiments and its value was found to be

$$K = 0.16 \tag{7}$$

The same equation, using dimensional analysis but a different thought model, was rederived by Sterman.[20]

In two recent papers Borishanskii[21, 22] extended Kutateladze's dimensional analysis to include the effects of viscosity. In the discussion he points out several interesting aspects of nucleate boiling close to the maximum heat flux. These comments are worth repeating here. According to Borishanskii the continuous existence of steady heat transfer rates in the neighborhood of the peak nucleate heat flux leads to the conclusion that there exists a direct steady movement of liquid toward the heating surface and of vapor away from it. Because of the density difference between the two phases, Borishanskii reasoned that a larger part of a cross sectional area close to the heating surface is occupied by vapor than by liquid. Therefore, near the heating surface one can consider a two-phase boundary region whose thickness is of the order of a disengaged bubble. This two-phase boundary region may be visualized as consisting of liquid streams flowing toward the surface and surrounded by vapor. The shapes of the filaments of liquid as they flow towards the wall are not well defined because of the inherent randomness in the bubble dynamics and coalescence in the nucleate regime. (This limitation disappears in transitional boiling as will be shown.)

At low nucleate boiling heat transfer rates the discrete phase appears as a vapor bubble surrounded by a mass of liquid; whereas close to the peak heat flux it is rather a liquid stream surrounded by vapor. Consequently, an analysis of nucleate boiling below the peak flux should consist of an analysis of bubble formation, while close to the maximum flux the analysis should be directed toward the dynamics of a liquid stream filament bounded by a group of bubbles. The change from nucleate to transition boiling occurs when the steady flow of the liquid towards the wall is disrupted, i.e., when at a critical velocity in the vapor phase, the liquid streams are destroyed. Borishanskii noted further that: "This problem seems analogous from a theoretical point of view to the disturbance of steady flow of a liquid stream in gas which is moving coaxially with it. The solution of the problem leads to a relationship between the increment of the oscillation and the wave length. Further analysis of the equation for the limiting case of stable flow leads to the conclusion that the critical boiling point corresponds to the establishment of a definite geometrical structure of the two-phase boundary layer." Borishanskii, therefore, considered that the phenomenon was to be explained by analyzing the stability of a liquid jet surrounded by a moving, coaxial, vapor phase. From the equation which determines the amplitude of the wave and from the energy equation he established, by dimensional analysis, two similarity criteria: K given by Equation (6) as previously found by Kutateladze, and N, given by

$$N = \frac{\rho_L \ \sigma^{3/2}}{\mu^2 \left[g(\rho_L - \rho_v)\right]^{1/2}} \tag{8}$$

By plotting K versus N he found the following approximate relation:

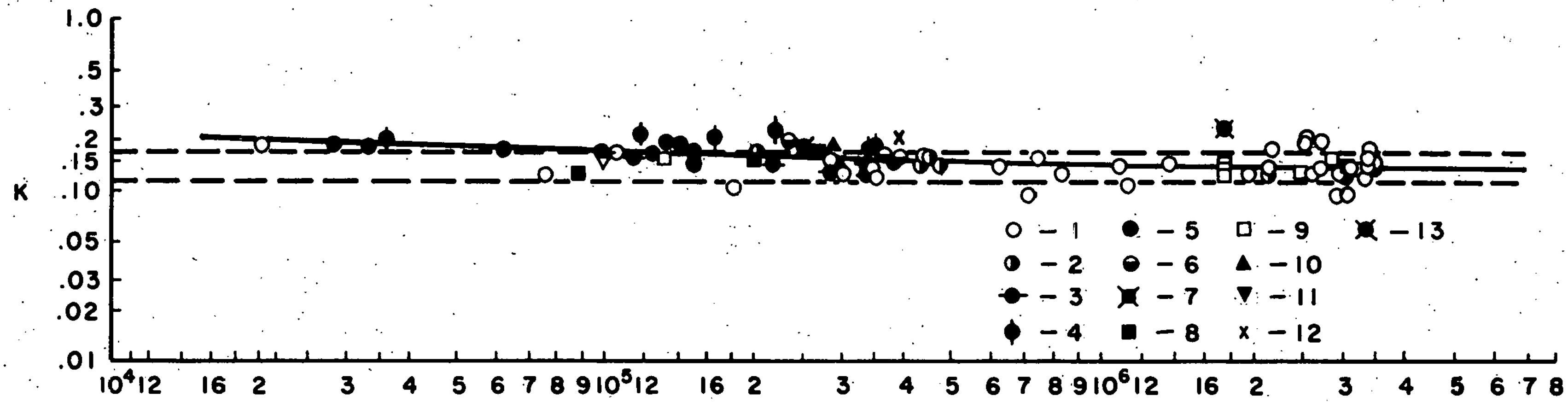
$$K = 0.13 + 4\,N^{-0.4} \tag{9}$$

In order to establish the above correlation 117 experimental data points were used. The experimental data represent the following liquid-solid combination: water boiling on a graphite surface;[24] ethanol, benzene, n-heptane and n-pentane boiling on a chromium plated surface;[15] ethanol[25] and water[26] boiling from a nichrome surface. The correlation and data are shown on Figure 6 which is reproduced from Borishanskii's paper.[21] The viscosity of the liquid appears in Equation (9) only in the additive correction factor N. Inasmuch as the deviation from a horizontal line is small it can be seen from Figure 6 that the effect of the viscosity is also small.

Good agreement with experimental data was achieved, also, by the correlation proposed by Rohsenow and Griffith:[23]

$$\frac{q}{L\rho_v} = c \left(\frac{\rho_L - \rho_v}{\rho_v}\right)^m \tag{10}$$

The constant c = 143 f.p.h. and the exponent m = 0.6 were determined from experimental data by plotting $q/L\,\rho_v$ versus the buoyancy term $\rho_L - \rho_v/\rho_v$.



1	water	6	heptane	11	ethanol
2	benzene	7	propane	12	heptane
3	ethanol	8	propane	13	water
4	pentane	9	water		
5	pentane	10	benzene		

Figure 6. This figure is reproduced from the report by Borishanskii.[21, 22] The solid line (Equation (9)) is his best fit curve to 117 data points. The two dashed lines represent the upper and lower bounds on K (Equation (53)) derived by the considerations in the paper.

Equations (6) and (10) are remarkable because they do not relate heat transfer to a temperature difference. In the work which follows the problem of predicting the heat flux-temperature variations is not considered; it is irrelevant to the design problems which occur in reactors, for example. The authors know of no other circumstances in which the important heat convection characteristics of a system are unrelated to the transport properties of the fluid.

There is, of course, interest in the heat flux-temperature dependence, but it is not as great as interest in the prediction of the peak heat flux, which, as will be seen, is independent of surface temperature.

Equations (6) and (10) provoke an important question: "Why should equations of simple form represent such a complicated phenomenon so well?" When one contrasts Equation (5) with (6) or (10) it is noted that the former contains practically every fluid and vapor thermal property whereas in Equations (6) and (10) not only are these properties absent but even temperature differences play no role. One would expect that the complexity of the heat transfer process would increase as the bubbles become more dense and interact more; the fact that the opposite is true indicates that a new mechanism enters and dominates. The hypothesis that a new mechanism governs the phenomenon gains credence when it is recognized that the critical heat flux not only marks the end of the nucleate region (upon increasing surface temperature) but equally well denotes the beginning of the transitional region. It is an often observed but seldom discussed fact that the decrease in heat transfer that accompanies an increase in surface temperature is found only in transitional boiling and not in any other heat transfer system. It is therefore reasonable to propose that insight into the new mechanism can be obtained by considering transitional boiling.

It will be seen that a well defined geometry is the characteristic of transition boiling. Therefore, an analytical solution of the problem can be attained.[27, 28] In nucleate boiling the flow configuration is not well defined. As noted by Borishanskii, "the form of the liquid streams differs considerably since it is determined by the not well regulated order of the combination of vapor bubbles." Given a field equation and no geometrical data, Kutateladze and Borishanskii found it necessary to use dimensional analysis, which is the best that can be done under the circumstances and which reduces the problem to the determination of an empirical constant.

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TRANSITION BOILING OF LIQUIDS AT SATURATION

A qualitative description, due to Borishanskii, of nucleate boiling close to the peak heat flux was presented in the last section. In this section we shall qualitatively describe transition boiling. The description will be followed by a mathematical formulation of the problem concerning the maximum and minimum heat flux in transition boiling from a horizontal surface. Transition boiling in general will be discussed in a later publication.

To facilitate the presentation, the phenomenon known as Taylor instability is first discussed.

When two liquids of different densities and having a common interface are accelerated in a direction perpendicular to the boundary, any small irregularity of the interface will tend to change in shape. The interface is stable, i.e., the irregularities will tend to smooth out in time, if the acceleration is directed from the denser to the lighter fluid. The interface is unstable, i.e., the irregularities of the interface will grow with time, when the acceleration is directed from the lighter to the heavier medium. A familiar example of this phenomenon is a glass of water turned upside down. The air-water interface instead of remaining plane as it falls will tend to form long liquid spikes which subsequently disintegrate into drops. An analysis of this phenomenon was developed by Taylor[29] using potential theory and neglecting surface tension and viscosity. He found that for an unstable interface the initial growth of the amplitude of the disturbance is exponential in time. The stabilizing effect exerted by the surface tension was recently demonstrated by Bellman and Pennington.[30] Their analysis indicates that disturbances of the interface can be stable or unstable according to whether the wave length is shorter or longer than a critical value. This critical wave length is given by Kelvin's equation (31):

$$\lambda_{01} = 2\pi \left[\frac{\sigma}{g(\rho_L - \rho_v)} \right]^{1/2} \tag{11}$$

They show, also, that the "most dangerous" wave length, i.e., the wave length for which the amplitude of the disturbance grows most rapidly is given by

$$\lambda_{02} = 2 \pi \left[\frac{3 \sigma}{g(\rho_L - \rho_v)} \right]^{1/2} \tag{12}$$

Bellman and Pennington further point out that "This phenomenon explains the hanging of water droplets on the underside of a horizontal surface, such as a ceiling. Such a droplet is undergoing an upward acceleration of 980 cm/sec^2 and will tend to drip because of Taylor instability unless its effective wave length is too small to satisfy Equation (11). For water, the critical wave length is about $\lambda_{01} = 1.73$ cm. Droplets of larger diameter will tend to drip, while smaller ones will tend to hang." Another evidence is cited by Lamb[31] and Milne-Thompson[32] who point to experiments "in which water is retained by atmospheric pressure in an inverted tumbler whose mouth is closed by a gauze of sufficiently fine meshes." The mesh size should not exceed $\lambda_{01}/2$.

The occurrence of capillary waves in stable film boiling has been reported and described by Bromley[33] and by Westwater and Santangelo.[1] In a recent paper Chang[34] observed that in stable film boiling the bubble spacings computed from Equation (11) are in agreement with experimental data results reported by Westwater and Santangelo.[1]

The analytical predictions of Taylor and of Bellman and Pennington were verified by the experiments reported by Lewis[35], and by Allred and Blount.[36] The initial, exponential growth of the disturbance was confirmed. These researchers observed, also, that the initial sinusoidal disturbance of the water-air interface becomes asymmetrical in its final stages. This asymmetrical form consists of spikes of heavy liquid extending into the light fluid, and of rounded regions which may be thought of as bubbles of lighter fluid rising into the heavier fluid. The spacing of these spikes, i. e., the effective "wave length" was found to be in good agreement with the "most dangerous" wave length as given by Equation (12). It is important to note here that this equation was derived from two-dimensional considerations.

Consider now a saturated liquid on a horizontal surface in the nucleate boiling regime. As the heat flux is increased a value is reached when, because of bubble interaction and of the disruption of the liquid streams, an instantaneous vapor film is formed. This value corresponds to the peak nucleate heat flux. The vapor liquid interface of this film is hydrodynamically unstable because the acceleration is directed from the less dense to the more dense medium. It can be expected that, because of agitation, the interface has random initial perturbations, distributed continuously over a spectrum of wave lengths. For a two-dimensional system all perturbations with wave length longer than the critical one, i. e., than λ_{01} are unstable. In view of the initial, exponential growth it can be expected that these unstable perturbations will dominate and that those near λ_{02} will be the first to achieve finite amplitude. Therefore, as a consequence of Taylor's instability a definite geometrical configuration in transition boiling can be expected. For a two-dimensional system this geometry should be characterized by disturbances with wave lengths in the spectrum

$$\lambda_{01} \leq \lambda_0 \leq \lambda_{02} \tag{13}$$

In contrast with the nucleate regime, where the disturbances of the superheated film originate in randomly located surface defects which have nothing to do with one another (except through the complicated interactions described in Figure 2), in the transitional regime the disturbances occur <u>away</u> from the surface and are <u>selected</u> by the properties of the fluid field. The regularity is a field property and this effect dominates over the random properties when the vapor blanket is formed.

The definite geometry can be observed in the excellent high speed motion pictures which have been taken by Westwater and Santangelo. If one focuses attention on the interface quite regularly spaced jets of vapor are seen to be discharged into the liquid. The photographs are taken of boiling from a tube and the observer must not be distracted by the confused picture which presents itself below the tube or far above the interface. Figure 7 is taken from one of the 16 mm. frames of a moving picture provided by Professor Westwater. The arrows indicate the "jets". (The difficulties in enlarging 16 mm frames from a high speed camera should be recognized.)

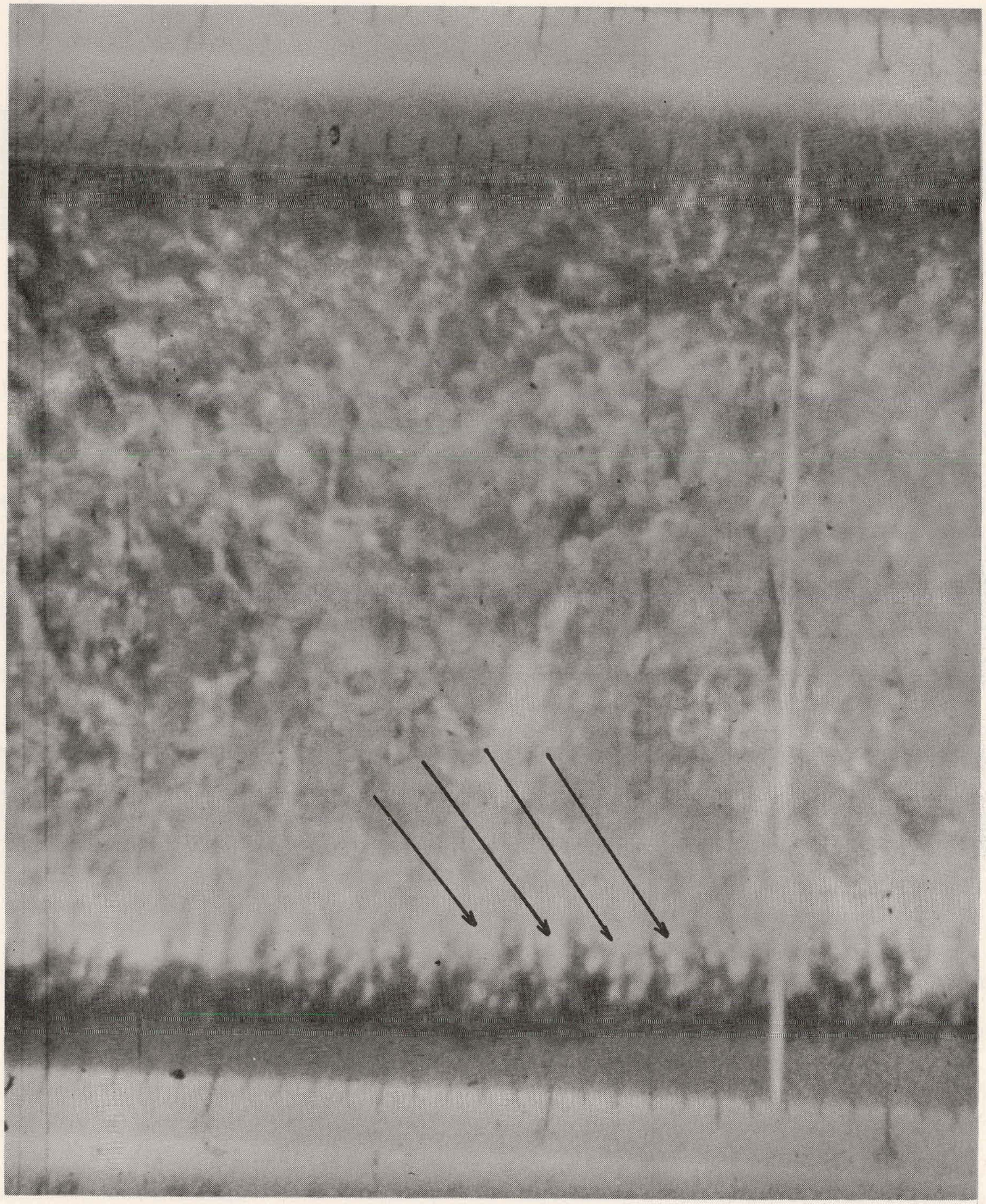

Figure 7. An enlargement from a 16 mm. moving picture film taken by Perkins and Westwater. The arrows indicate the "jets".

Figure 8 shows a high speed still photograph of the nucleate regime.* There is no regularity to the photograph. Figure 9 shows the simultaneous nucleate and transitional boiling. To the left the system is quite irregular; in the center and to the right it will be seen that the vapor shoots vertically upward in three columns, more or less equidistant.

At the conditions represented by the right half of Figure 9, one should analyze the fall of the "spikes" of liquid rather than the "rise of bubbles."

In their downward fall the spikes approach the heated surface and rapid evaporation occurs. The question now arises as to whether there is a solid-liquid contact in transition boiling. The similarity between transition boiling and the "spheroidal state," also known as the "Leidenfrost phenomenon," was first pointed out and discussed by Drew and Mueller.[37] They concluded that the collapse of the spheroidal state corresponds to the change from transmission to nucleate boiling. The criticism advanced by Kruzhilin [8] to their conclusions is not supported by experimental evidence. The temperature of the heated surface in the nucleate region at the instant of formation of the unstable vapor film does not necessarily correspond to the temperature at the instant of collapse of transition boiling. This important fact was clearly demonstrated by Perkins and Westwater.[17] They observed that the change from nucleate to transition boiling occurs over a temperature range instead of at a definite critical temperature. The peak heat flux is constant throughout this range of temperature. Therefore, in the q-T plane instead of a critical point there is a "critical region", i.e., a plateau. The temperature at the two extremities of this plateau are of the same order of magnitude as correctly observed by Drew and Mueller.[37] Because of this region a simultaneous coexistence of nucleate and transition boiling can be expected. It appears, therefore, that in transition boiling no solid liquid contact exists. This fact is confirmed by the moving pictures of Westwater and Santangelo.[1]

*All of the pictures of boiling reproduced herein are provided through the courtesy of Professor Westwater.

Figure 8. Photograph by Westwater and Santangelo[1] showing nucleate boiling of methanol on a horizontal tube. Overall $\Delta T = 67^{\circ}F$, Heat Transfer = 76,900 BTU/hr ft^2.

Figure 9. Photograph of Simultaneous Nucleate (left side) and Transitional (right side) Boiling.[1] Overall $\Delta T = 99.5^{\circ}F$, $q = 172,000$ Btu/hr ft^2.

As the liquid evaporates from the spikes the vapor flows in the region between two spikes. It was noted, already, that this region has a form of a rising bubble. Because of the Taylor instability a release of bubbles at regular intervals can be expected. For large evaporation rates the release of a bubble will appear as a burst. As a row of bubbles is released an unstable interface is again formed. Because of the downward flow of the liquid a spike will be formed now underneath the released bubble and the process is renewed. The process is, therefore, thermally stable because it can continue indefinitely. However it is hydrodynamically unstable, indeed, it is this instability which is the cause of the phenomenon.

The characteristics of transition boiling will be summarized now and qualitatively compared to experimental data.

1) As a consequence of Taylor instability a definite geometrical configuration can be expected in transition boiling. For a two-dimensional system this geometry is characterized by disturbances with wave length in the spectrum

$$\lambda_{01} \leq \lambda_0 \leq \lambda_{02}$$

2) As a consequence of the exponential character of the instability it can be expected that the interface takes the form of spikes of heavy liquid moving downward and of rounded regions of lighter fluid moving upwards.

3) It can be expected that, because of the "spheroidal state," no solid-liquid contact exists in transition boiling. The change from transition to nucleate boiling corresponds to the collapse of the spheroidal state.

4) The temperatures of the solid surface corresponding to the termination of nucleate boiling and of transition boiling do not necessarily coincide. Therefore, at the peak flux nucleate boiling and transition boiling can simultaneously coexist.

5) As a consequence of the definite geometrical configuration

a release of bubbles from the interface at regular intervals can be expected. For large evaporation rates the release of a bubble will appear as a "burst."

6) Because of the release of vapor and of Taylor instability the process exhibits, also, a periodicity in time. The phenomenon is hydrodynamically unstable but thermally stable.

7) Inasmuch as the factors which influence the geometry remain invariant, it can be expected that, in transition boiling, changes in heat transfer rates are associated with changes in frequencies only. The maximum and the minimum heat flux correspond, therefore, to the maximum and the minimum allowable frequencies of the system.

The only quantitative, experimental investigation of transition boiling known to the authors is reported by Westwater and Santangelo.[1] The authors are grateful to Prof. Westwater for the kind permission to reproduce his data and striking photographs in this report. Indeed, they were fortunate to have the result of these outstanding experiments as a guide for the theoretical analysis. Westwater and Santangelo give the following description of transition boiling: "Most prior workers have failed to realize that this boiling is entirely different from both nucleate boiling and film boiling. No active nuclei exist. In fact, no liquid-solid contact exists either. The tube is completely blanketed by a film of vapor, but the film is not smooth nor stable. The film is irregular and is in violent motion."

"Vapor is formed by sudden bursts at random locations along the film. Liquid rushes in toward the hot tube, but before the two can touch, a miniature explosion of vapor occurs and the liquid is thrust back violently. The newly-formed slug of vapor finally ruptures, and the surrounding liquid surges toward the tube. The process is repeated indefinitely." "One observer of these high speed motion pictures has expressed an opinion that occasional liquid-solid contact does occur during transition boiling. If so, these contacts are rate and of exceedingly short duration. The present writers do not believe there is a real contact."

"The frequency of the vapor bursts is surprisingly high. For an overall $T_w - T_s = 133^oF$ (and U of 164 Btu/hr ft^2 ^oF) each inch of the photographed side of the tube exhibited 84 bursts per second. The bursts occur so suddenly and unexpectedly that even in slow motion they resemble explosions."

It is seen that the above observations are in qualitative agreement with the previous statements except for our previously mentioned observation of regularity. A qualitative agreement is obtained also when photographs, published by Westwater and Santangelo[1] and reproduced in this report, are analyzed. We have already remarked on the indeterminate flow configuration in nucleate boiling even at moderate heat transfer rates as seen in Figure 8. This is even more evident at the peak nucleate heat flux discussed in connection with Figure 9. The coexistence of nucleate on the left and of transition boiling on the right hand side of the photograph of the tube permits the comparison. The random spectrum of disturbances is shown on Figure 9 and Figure 10. Spikes of liquid and rounded regions of vapor are seen at the minimum heat flux in transition boiling which is shown on Figure 11. The difference between Figure 11 and Figure 10 will be discussed in the following section when the stabilizing effect of the surface tension and the destabilizing effect of large velocities in the vapor phase, i.e., of Helmholtz instability, are considered. The regularity in space and periodicity in time can be seen in Figure 11a.

In the following section the problem will be formulated mathematically and the results quantitatively compared with experimental data.

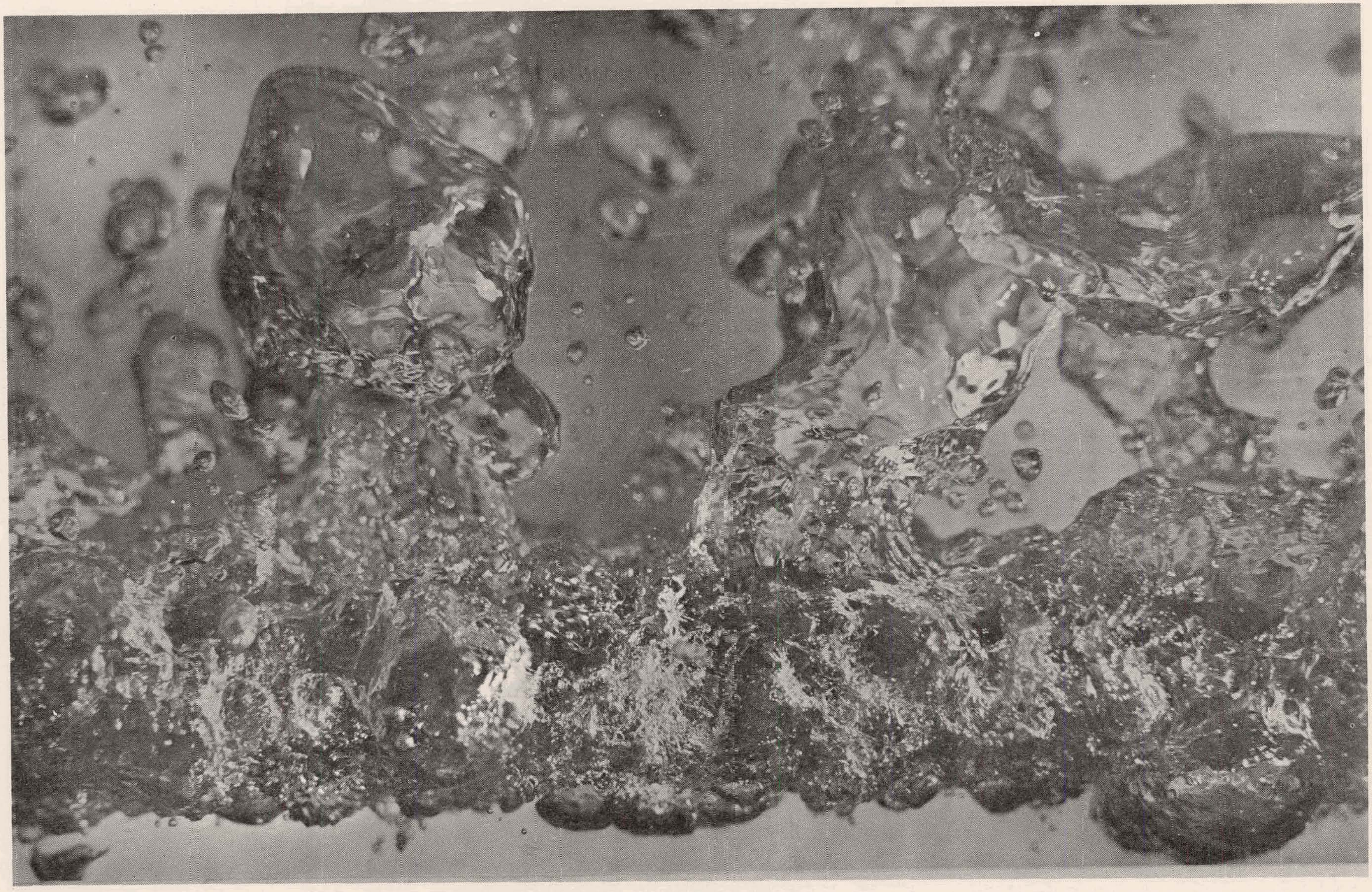

Figure 10. Photograph of Transitional Boiling.[1] $\Delta T = 124\ ^{o}F$, $q = 27,200$ Btu/hr ft^2.

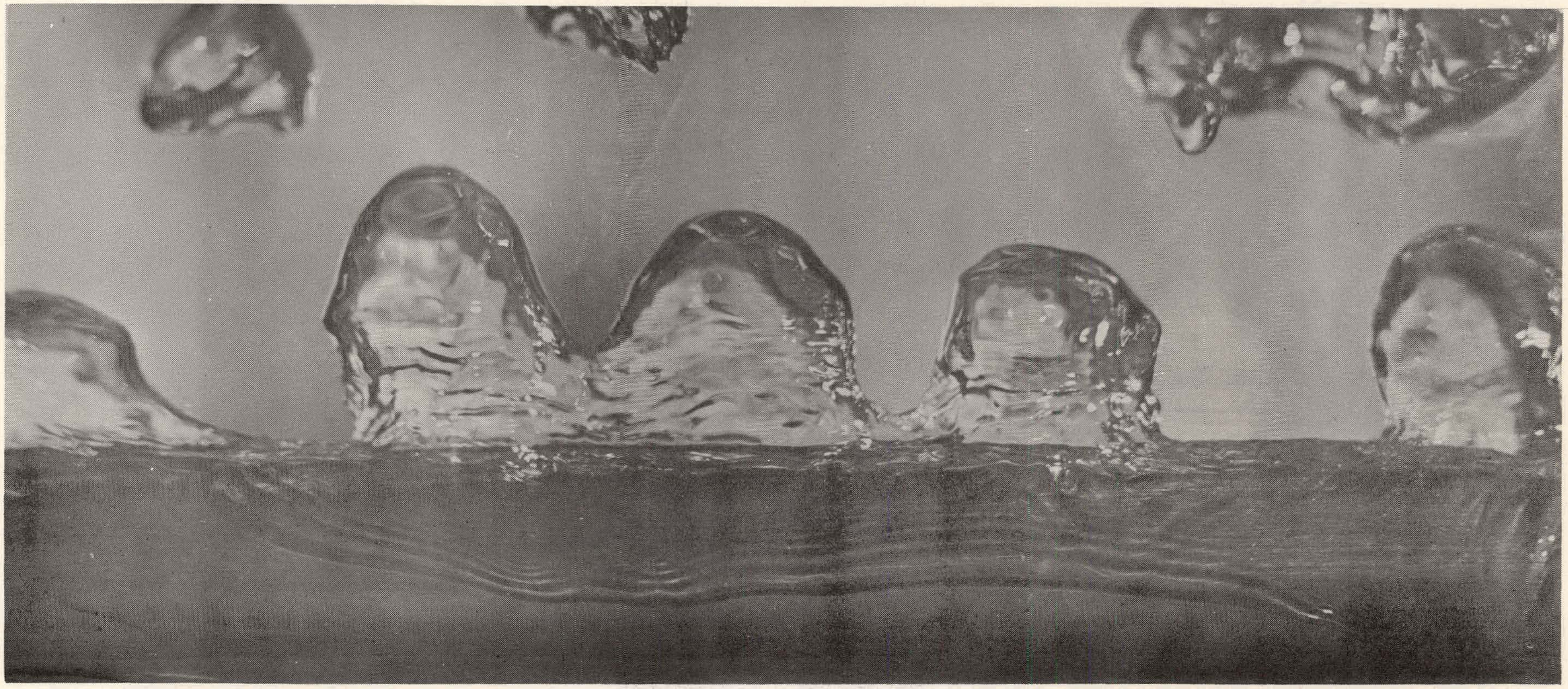

Figure 11. Photograph of Film Boiling,[1] $\Delta T = 148\ ^{o}F$, $q = 12,970\ Btu/hr\ ft^2$

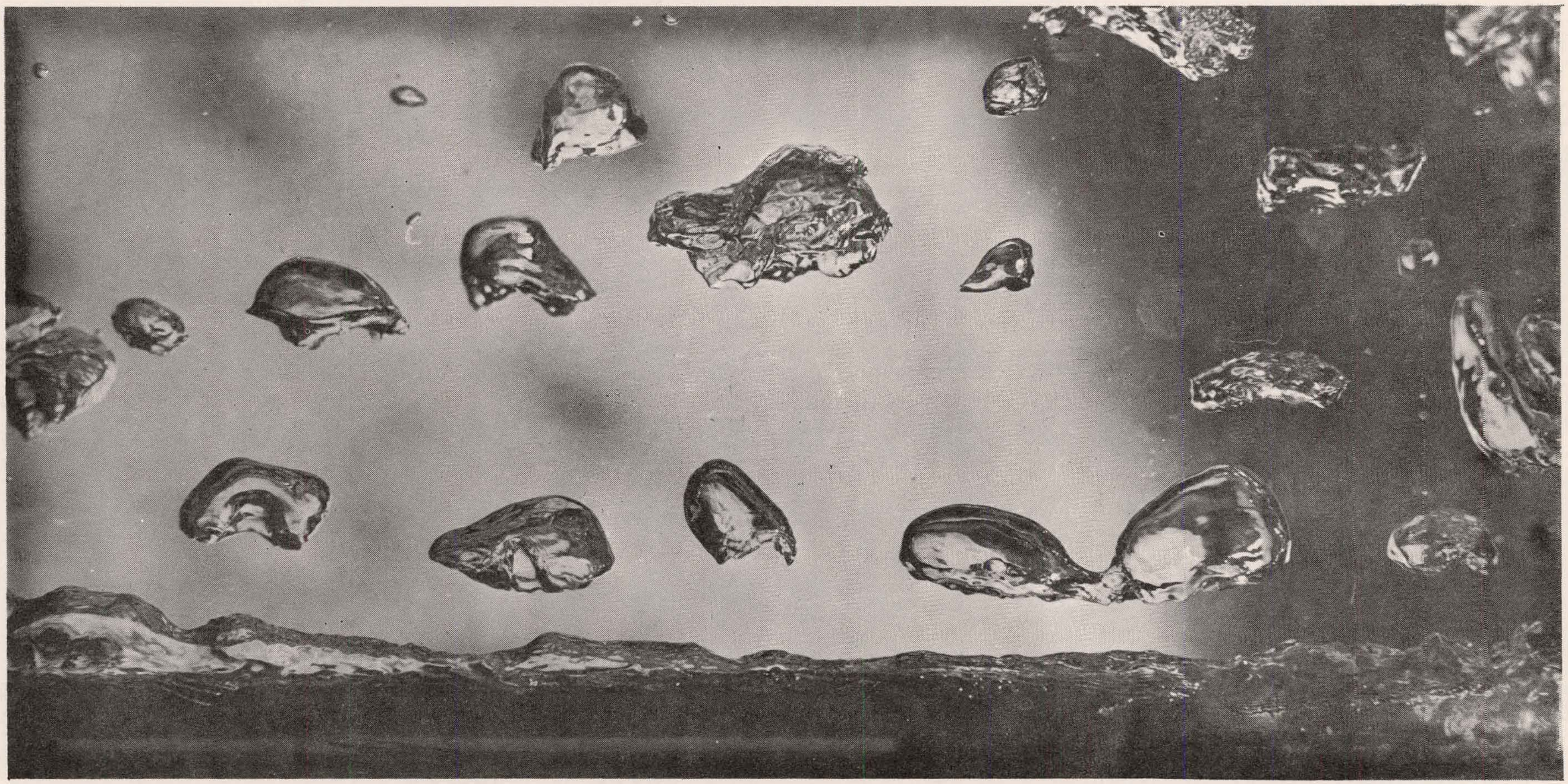

Figure 11A. Photograph of Film Boiling,[1] $\Delta T = 181\ ^{o}F$, $q = 5,470$ Btu/hr ft^2.

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FORMULATION AND SOLUTION OF THE PROBLEM

The geometrical regularity and the periodicity of the process were noted in the previous section. It was noted, also, that inasmuch as the geometry should remain invariant, changes of heat transfer rates should be associated only with changes in the frequency of the system. The phenomenon thus bears a similarity to a release of bubbles from a set of orifices of fixed geometry but with a variable frequency. Although, at first, this similarity appears to be rather tenuous the agreement of the detail and the gross characteristics of the idealized system with experimental data seems to support it. The problem is, therefore, to determine the geometry of the "orifices" and the maximum and minimum frequency of the releases. According to the hypothesis, these frequencies should correspond to the maximum and minimum heat transfer rates in transition boiling from a horizontal surface.

The agreement of the two-dimensional stability analysis[35,36] with experimental results was noted in the preceding section. The analysis which follows is based on the stability of a two-dimensional vortex sheet and the classical results of Helmholtz and Rayleigh[31] are used. In this problem, also, an agreement with experimental results is achieved.[27,28] However, further experimental investigations of transition boiling should be undertaken in order to verify whether this is a general result. An extension of the analysis to three dimensions can, if needed, be made readily.

In order to formulate the problem a simplifying idealization will be made concerning the form of the vapor slugs. It is assumed that these slugs can be approximated by spheres of radius

$$R = \frac{\lambda_0}{4} \tag{14}$$

For a two-dimensional instability it follows from Equations (13), (11) and (12) that the diameter is given within the limits

$$\pi \left[\frac{\sigma}{g(\rho_L - \rho_v)} \right]^{1/2} \leq D \leq \pi \left[\frac{3\sigma}{g(\rho_L - \rho_v)} \right]^{1/2} \tag{15}$$

It was experimentally observed by Faraday and discussed in detail by Rayleigh[38] that an oscillating interface exhibits a double frequency because two trains of waves at right angles traverse the interface. Denoting the frequency by $f = 1/\tau$ the mass of the vapor associated with one bubble is

$$G = \rho_v \frac{4\pi}{3} \left(\frac{\lambda_0}{4} \right)^3 \tag{16}$$

and the number of bubbles per unit area per unit time is

$$n = \frac{2}{\tau} \frac{a}{\lambda_0} \frac{b}{\lambda_0} \tag{17}$$

where a. b is the area of interest. The vapor mass flow rate per unit area is obtained from Equations (16) and (17), thus

$$\overset{o}{m} = \frac{\pi}{24} \rho_v \frac{\lambda_0}{\tau} \tag{18}$$

The heat transfer rate is obtained from an energy balance. In transition boiling of liquids at saturation the only energy requirement of the system is the energy needed to generate the vapor flux given by Equation (18). Therefore, the heat transfer rate from a horizontal surface in transition boiling of liquids at saturation is

$$q = L \frac{\pi}{24} \rho_v \frac{\lambda_0}{\tau} \tag{19}$$

At a constant pressure because the geometry does not change, Equation (19) can be written as

$$q = \frac{\text{constant}}{\tau} \tag{20}$$

The problem now is to determine the minimum and maximum allowable frequencies which, according to the hypothesis, correspond to the minimum and maximum heat flux. These two regions are characterized by a low and a high evaporation rate; consequently a small and a large velocity in the vapor phase can be expected at the minimum and at the maximum heat flux respectively. The stabilizing effect of the surface tension in connection with Taylor instability was discussed by Bellman and Pennington.[30] It was observed, also, in the experiments performed by Allred and Blount.[36] These experiments revealed, also, the effect of the Helmholtz instability. This instability arises at the interface when there is a relative motion of two fluids in a direction parallel to the interface. The mushrooming of the interface observed in these experiments was attributed by Allred and Blount to Helmholtz instability. It is important to note here that the geometry of the interface observed in these experiments, i. e., the form of mushrooms, is identical with the geometry of the interface in transition boiling. This statement can be verified easily by comparing Figures 3. 9 and 5. 6 from Reference 36 (reproduced here as Figures 12 and 13) with photographs published by Westwater and Santangelo.[1] In view of the above considerations it can be expected that the peak flux in transition boiling is characterized by the combined effects of Taylor and Helmholtz instabilities; whereas the minimum flux is characterized by Taylor instability only.

It was noted that at the minimum heat flux the velocities in the vapor and liquid phases are low and that the phenomenon is characterized by the stabilizing effect of the surface tension. Consequently, it was assumed in Reference (27) that the motion of the interface is governed by the surface tension. The angular frequency of such a motion is:[31]

$$n^2 = \frac{\sigma m^3}{\rho_L + \rho_v} \tag{21}$$

where m is the wave number given by

$$m = \frac{2\pi}{\lambda} \tag{22}$$

As the relation between the angular frequency and the period is

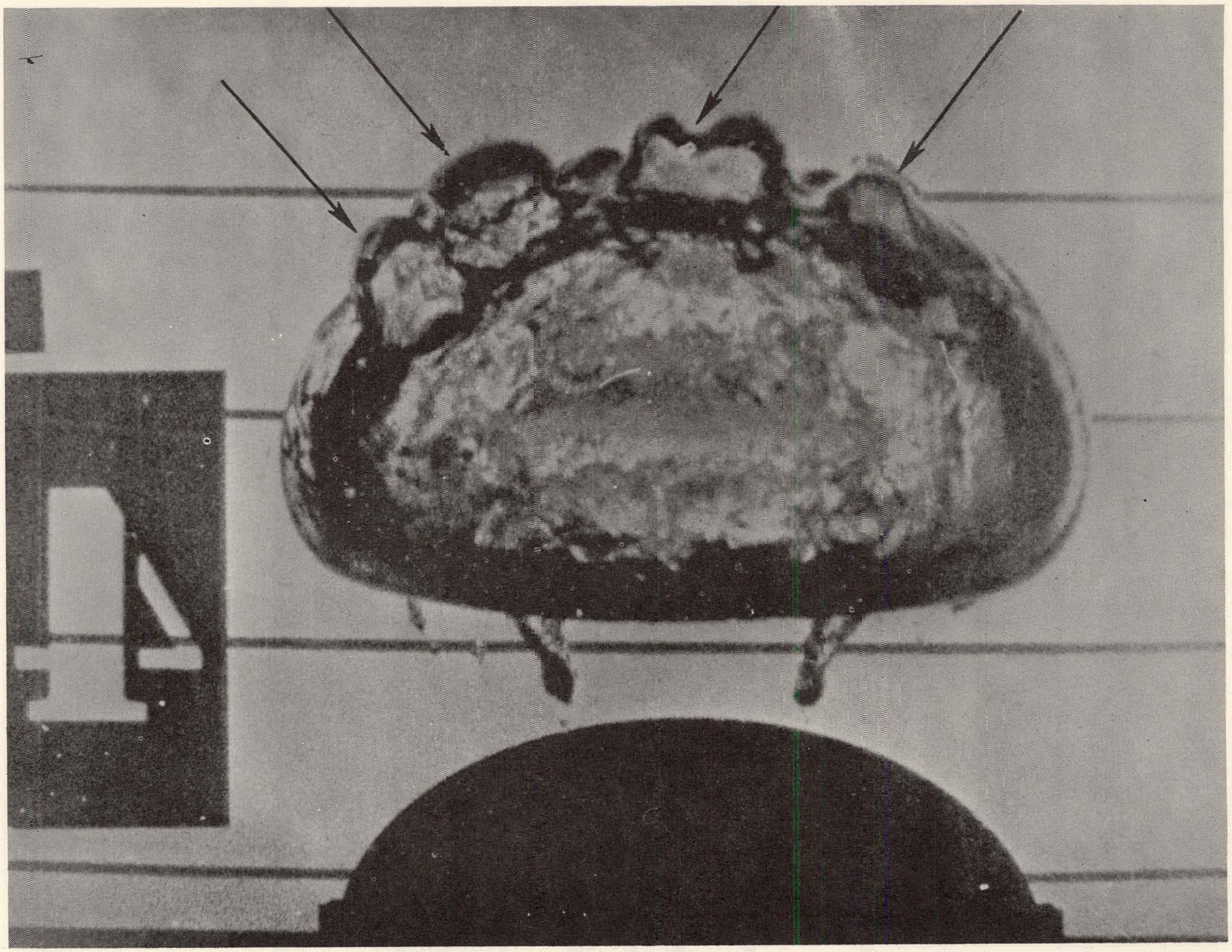

Figure 12. The Growth of "Bubbles" and "Spikes" as a Large Bubble of Air Rises in a Large Volume of Water.[36] (Reprinted by permission of the Los Alamos Scientific Laboratory and the Atomic Energy Commission, Los Alamos, New Mexico.)

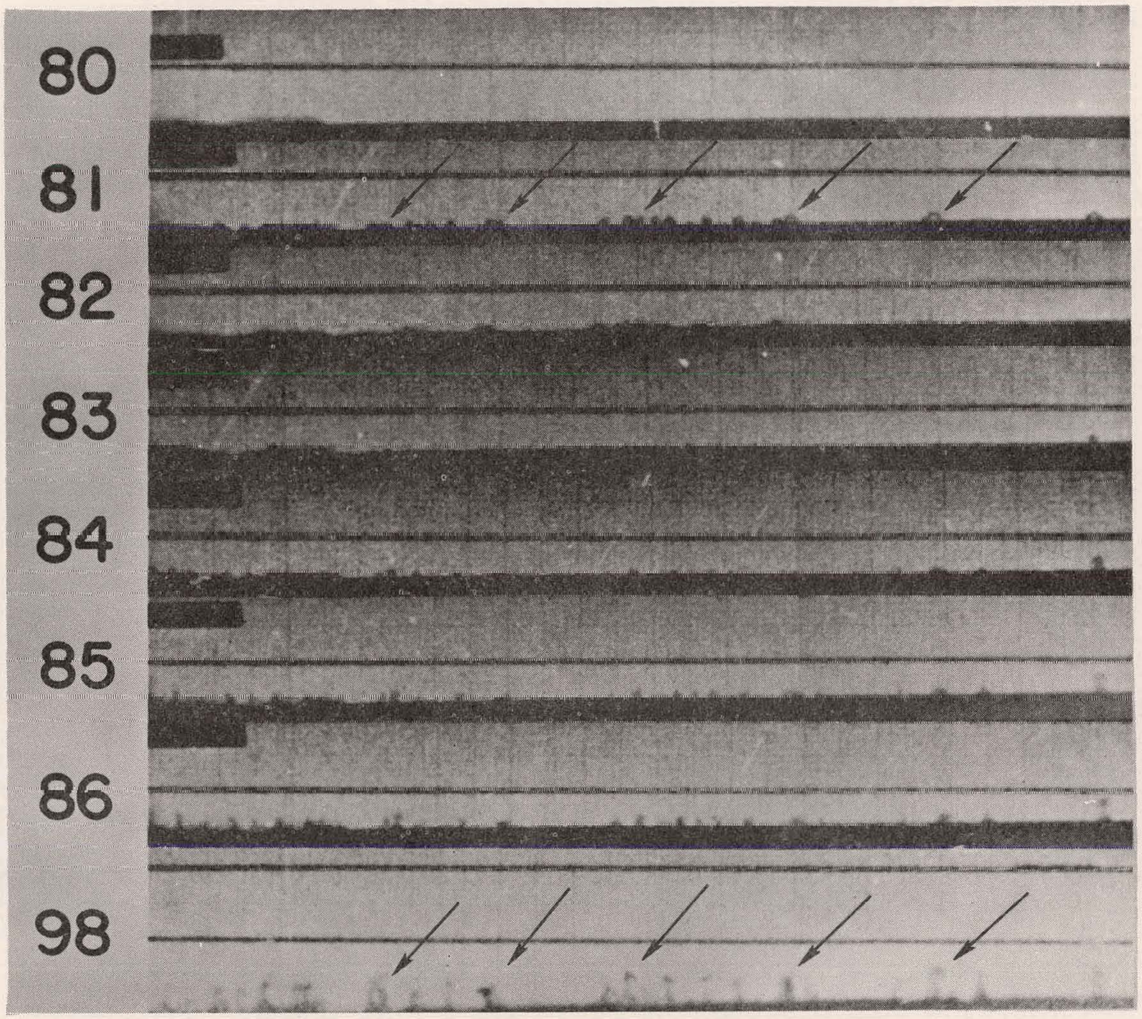

Figure 13. The Growth of Instabilities as a Surface Between Two Liquids is Accelerated Toward the Lighter Fluid. (Reprinted by permission of the Los Alamos Scientific Laboratory and the Atomic Energy Commission, Los Alamos, New Mexico.)

$$\tau = \frac{2\pi}{n} \tag{23}$$

then from Equations (23), (21), (11) and (12) the period corresponding to the wave lengths λ_{01} and λ_{02} is given respectively by

$$\tau_{01} = 2\pi \left[\frac{\sigma}{g(\rho_L - \rho_v)} \right]^{1/2} \left[\frac{(\rho_L + \rho)^2}{\sigma g(\rho_L - \rho_v)} \right]^{1/4} \tag{24}$$

and

$$\tau_{02} = 2\pi \left[\frac{3\sigma}{g(\rho_L - \rho_v)} \right]^{1/2} \left[\frac{3(\rho_L + \rho_v)^2}{\sigma g(\rho_L - \rho_v)} \right]^{1/4} \tag{25}$$

Substituting alternately Equation (11) and (24), and Equation (12) and (25) in Equation (19) it can be expected that, because of the spectrum of unstable disturbances, the minimum heat flux in transition boiling from a plane, horizontal surface can be determined within the limits given by:

$$\frac{\pi}{24} \frac{1}{(3)^{1/4}} L \rho_v \left[\frac{\sigma g(\rho_L - \rho_v)}{(\rho_L + \rho_v)^2} \right]^{1/4} \leq q \leq \frac{\pi}{24} L \rho_v \left[\frac{\sigma g(\rho_L - \rho_v)}{(\rho_L + \rho_v)^2} \right]^{1/4} \tag{26}$$

It should be emphasized here that the above equation holds for a horizontal plane surface only. As discussed in Reference (27), the minimum heat flux in transition boiling from a horizontal tube depends on the stability of two systems of capillary waves. It is, therefore, determined by the motion with higher energy requirement.

An analysis of the minimum heat flux in transition boiling from a plane, horizontal surface can be formulated, also, by considering the asymptotic growth of Taylor instability. It will be seen in the following that this formulation leads to an expression for the heat flux which, at low pressure, exhibits the same dependence on liquid properties as Equation (26).

Allred and Blount[36] have reported that the asymptotic growth of Taylor instability can be predicted from the equation, derived by Davies and

Taylor,[39] for the rise of a large, two-dimensional bubble:

$$V = \frac{2}{3} \left[r\, g \right]^{1/2} \tag{27}$$

where r is the radius of curvature of the bubble vertex. Making the same assumption concerning the radius of the vapor slugs it follows from Equation (14) and Equation (27) that the velocity of rise, i. e., the asymptotic growth of the amplitude is given by

$$V = \frac{2}{3} \left[\frac{\lambda_0}{4}\, g \right]^{1/2} \tag{28}$$

Because of the spectrum of unstable disturbances the wave length λ_0 in the above expression can be determined between the limits given by Equation (13), i. e., by Equation (11) and Equation (12).

If one considers that the bubbles follow one another very closely, then to a good approximation:

$$f = V \frac{2}{\lambda_0} \tag{29}$$

and it follows from Equations (11), (12), (28) and (29) that the period can be determined within the limits:

$$\frac{3}{2} \left[\frac{2\pi}{g} \left(\frac{\sigma}{g(\rho_L - \rho_v)} \right)^{1/2} \right]^{1/2} \leq \tau \leq \frac{3}{2} \left[\frac{2\pi}{g} \left(\frac{3\sigma}{g(\rho_L - \rho_v)} \right)^{1/2} \right]^{1/2} \tag{30}$$

Using Equation (29) for the frequency, the mass flow rate of the vapor associated with one release location is

$$\overset{o}{m} = \rho_v \frac{4\pi}{3} \left(\frac{\lambda_0}{4} \right)^3 V \frac{2}{\lambda_0} \frac{1}{\lambda_0^2} \tag{31}$$

Following the development which led to Equation (19), the analysis, in this case, gives for the range of the minimum heat flux the following expression:

$$\frac{\pi}{48} L \rho_v \frac{2}{3}\left[2\pi g\left(\frac{\sigma}{g(\rho_L - \rho_v)}\right)^{1/2}\right]^{1/2} \leq q \leq \frac{\pi}{48} L \rho_v \frac{2}{3}\left[2\pi g\left(\frac{3\sigma}{g(\rho_L - \rho_v)}\right)^{1/2}\right]^{1/2} \quad (31A)$$

It will be seen that the heat transfer rates predicted by Equation (2 6) and by Equation (31A) are in agreement with experimental data at atmospheric pressure. This could have been expected inasmuch as both equations exhibit at low pressures the same dependence on the density, since $\rho_v << \rho_L$ At high pressure, however, this is not the case. Therefore, experimental data at high pressure are needed to show which equation is a better approximation to the phenomenon.

We shall investigate now the peak flux in transition boiling. It was noted, already, that the peak flux is characterized by the joint effect of Taylor and Helmholtz instabilities. The problem is to determine in what way they manifest themselves and interact. It was noted, also, that phenomenon bears a similarity to a release of bubbles with variable frequency from a set of regularly spaced orifices of fixed geometry. In accordance with the hypothesis, at the peak flux the frequency reaches the maximum. In view of Taylor instability we are lead, therefore, to consider a vapor column i.e., a jet of diameter $\lambda_0/2$. In view of Helmholtz instability we have to investigate the stability of this jet, i.e., to determine what is the maximum velocity in the vapor phase which permits a periodic behaviour. In accordance with the previous analysis the stability of a two-dimensional system is again investigated.

The analysis is most easily made if it is recalled that the stability of these systems is usually investigated by considering the stability of small disturbances.[31, 32] Whether one considers the stability of a plane jet or a round jet will make little difference, provided it is the axial disturbances which are investigated. Since the plane surface is mathematically simpler, it is taken here.

Consider now a coordinate system in which the direction y is parallel to the surface and z is taken perpendicular to the surface. Let the plane y = 0 denote an interface between the vapor leaving the heated surface and the fluid rushing toward it. For a vortex sheet which oscillates under the influence of surface tension, the propagation equation of a small disturbance is:[31,32]

$$c^2 = \frac{\sigma m}{\rho_L + \rho_v} - \frac{\rho_L \, \rho_v}{(\rho_L + \rho_v)^2} (u_v + u_L)^2 \tag{32}$$

The condition of stability is the condition that waves of the prescribed type can propagate, i.e., that c shall be real.

The velocity in the liquid phase is obtained from the equation of continuity

$$\rho_v u_v = \rho_L u_L \; * \tag{33}$$

Substituting u_L from Equation (33) into Equation (32) the critical velocity in the vapor phase is then obtained, thus

$$u_v = \left[\frac{\sigma m}{\rho_v}\right]^{1/2} \left[\frac{\rho_v}{\rho_L + \rho_v}\right]^{1/2} \tag{34}$$

Rayleigh[38] has shown that the critical wave length for a cylindrical jet is**

$$\lambda = 2\pi R \tag{35}$$

where R is the radius of the jet. If the jet is to break into spheres, the wave lengths must be the same axially as circumferentially, hence equation (35) is substituted in Equation (34) to provide a relation between

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* As pointed out by Birkhoff[40] in considering this problem, one may always adopt a coordinate system such that (33) is true.

** Note that λ refers to the wave lengths of disturbances in the interface between ascending and descending streams. λ_{01} and λ_{02} refer to wavelengths in the horizontal interface.

the velocity and wave length.

From the relation between the wave number and the wave length and from the assumption concerning the diameter of the jet, it follows that the critical wave number in Equation (34) is given by

$$m = \frac{4}{\lambda_0} \tag{36}$$

As

$$\lambda_0 = \frac{2\pi}{m_0}$$

the critical velocity in the vapor phase becomes

$$u_v = \left[\frac{\sigma}{\rho_v} \; \frac{4\, m_0}{2\pi}\right]^{1/2} \left[\frac{\rho_L}{\rho_L + \rho_v}\right]^{1/2} \tag{37}$$

The relation between the critical velocity and the frequency is obtained from the equation of continuity, which, because of the double frequency* is given by

$$\rho_v \, \frac{4\pi}{3} \left(\frac{\lambda_0}{4}\right)^3 \frac{2}{\tau} = \rho_v \, \pi \left(\frac{\lambda_0}{4}\right)^2 u_v \tag{38}$$

After cancelling common terms Equation (38) yields:

$$\frac{\lambda_0}{\tau} = \frac{3}{2} \, u_v \tag{39}$$

Substituting u_v from Equation (37) into Equation (39) it follows that

$$\frac{\lambda_0}{\tau} = \frac{3}{\sqrt{2\pi}} \left[\frac{\sigma\, m_0}{\rho_v}\right]^{1/2} \left[\frac{\rho_v}{\rho_L + \rho_v}\right]^{1/2} \tag{40}$$

---

* i.e., allowing for the two trains of waves at right angles.

The peak flux is obtained by substituting Equation (40) into Equation (19), thus

$$q = L \frac{\pi}{24} \rho_v \frac{3}{\sqrt{2\pi}} \left[\frac{\sigma m_0}{\rho_v}\right]^{1/2} \left[\frac{\rho_L}{\rho_L + \rho_v}\right]^{1/2} \tag{41}$$

Because of the spectrum of unstable disturbances the wave number m_0, according to Equation (13), can be determined within the range

$$\sqrt{\frac{g(\rho_L - \rho_v)}{\sigma}} \geqslant m_0 \geqslant \sqrt{\frac{g(\rho_L - \rho_v)}{3\sigma}} \tag{42}$$

The peak flux in transition boiling from a horizontal surface is determined, therefore, within the limits:

$$\frac{\pi}{24} \frac{3}{\sqrt{2\pi}} L \rho_v \left[\frac{\sigma g(\rho_L - \rho_v)}{\rho_v^2}\right]^{1/4} \left[\frac{\rho_L}{\rho_L + \rho_v}\right]^{1/2} \geqslant q$$

$$q \geqslant \frac{\pi}{24} \frac{3}{\sqrt{2\pi}} \frac{1}{(3)^{1/4}} L \rho_v \left[\frac{\sigma g(\rho_L - \rho_v)}{\rho_v^2}\right]^{1/4} \left[\frac{\rho_L}{\rho_L + \rho_v}\right]^{1/2} \tag{43}$$

Equation (43) can be written as

$$K_1 \; L\rho_v \left[\frac{\sigma g(\rho_L - \rho_v)}{\rho_v^2}\right]^{1/4} \left[\frac{\rho_L}{\rho_L + \rho_v}\right]^{1/2} \geqslant q \geqslant$$

$$K_2 \, L \, \rho_v \left[\frac{\sigma g(\rho_L - \rho_v)}{\rho_v^2}\right]^{1/4} \left[\frac{\rho_L}{\rho_L + \rho_v}\right]^{1/2} \tag{44}$$

where K_1 and K_2 are the numerical constants given by

$$K_1 = \frac{\pi}{24} \frac{3}{\sqrt{2\pi}} = 0.157 \tag{45}$$

and

$$K_2 = \frac{\pi}{24} \frac{3}{\sqrt{2\pi}} \frac{1}{(3)^{1/4}} = 0.12 \qquad (46)$$

The algebraic mean is, therefore,

$$K_m = 0.138 \qquad (47)$$

A convenient average value for the peak flux can be obtained by replacing m_0 in Equation (41) by the upper limit obtained from Equation (42); and by approximating the numerical constant $3/\sqrt{2\pi}$ by unity. The resulting equation is then of the form previously derived[27]

$$q = \frac{\pi}{24} L \rho_v \left[\frac{\sigma g (\rho_L - \rho_v)}{\rho_v^2}\right]^{1/4} \left[\frac{\rho_L}{\rho_L + \rho_v}\right]^{1/2} \qquad (48)$$

The value of the numerical constant in this case is

$$K = \frac{\pi}{24} = 0.131 \qquad (49)$$

In order to test the validity of the above analysis the detailed and gross features of the thought model will be compared, in the following section with experimental data. The experimental results reported by Westwater and Santangello[1] in this case, again, are invaluable.

THE COMPARISON OF ANALYTICAL WITH EXPERIMENTAL RESULTS

In the following the theoretical predictions are compared with experimental data for boiling methanol at atmospheric pressure reported by Westwater and Santangelo.[1]

A. The diameter of bubbles at the minimum heat flux.

	Analysis inches	Experiment inches
Eq. (15)	$0.2 \leq D \leq 0.345$	$0.2 \leq D \leq 0.36$

B. The period at the minimum heat flux.

	Analysis seconds	Experiment seconds
Eq. (24)	0.08	0.06
Eq. (25)	0.18	
Eq. (30)	$0.048 \leq \tau \leq 0.063$	

C. The minimum heat flux in transition boiling.

	Analysis Btu/hr ft^2	Experiment Btu/hr ft^2
Eq. (26)	$5000 \leq q \leq 6540$	$q = 5470$
Eq. (31A)	$5500 \leq q \leq 7100$	

D. The relation between the heat flux in transition boiling and the frequency.

It was assumed that in transition boiling changes in heat transfer rates are associated with changes of frequency only. It follows, therefore, from Equation (20) that in transition boiling the following relation should hold

$$\frac{q_1}{q_2} = \frac{\tau_2}{\tau_1} = \frac{f_1}{f_2} \tag{50}$$

where the subscripts one and two refer to two different operating conditions.

Westwater and Santangelo[1] have reported that for an over-all temperature difference $T_w - T_s = 133^{o}F$, and a heat transfer coefficient $U = 164$ Btu/hr ft^2 ^{o}F, each inch of the photographed side of the tube exhibited 84 bursts per second. Whereas at a heat flux of $q = 5470$ Btu/hr ft^2 the frequency was 22 bursts per second per inch length of tube. Substituting these values into Equation (50) it follows that

$$\frac{q_1}{q_2} = \frac{21,800}{5,470} = 3.98$$

$$\frac{f_1}{f_2} = \frac{84}{22} = 3.82$$

As a further check of the hypothesis the ratio of the periods computed from the analysis will be compared with the ratio of the maximum and minimum heat flux determined from experiments.[1] The period at the peak flux is obtained by substituting alternately Equation (11) and Equation (12) into Equation (40), thus

$$\tau_{01} = 2\pi \left[\frac{\sigma}{g(\rho_L - \rho_v)}\right]^{1/2} \frac{\sqrt{2\pi}}{3} \left[\frac{\rho_v}{\sigma} \sqrt{\frac{\sigma}{g(\rho_L - \rho_v)}}\right]^{1/2} \left[\frac{\rho_L + \rho_v}{\rho_v}\right]^{1/2} \tag{51}$$

$$\tau_{02} = 2\pi \left[\frac{3\sigma}{g(\rho_L - \rho_v)}\right]^{1/2} \frac{\sqrt{2\pi}}{3} \left[\frac{\rho_v}{\sigma} \sqrt{\frac{3\sigma}{g(\rho_L - \rho_v)}}\right]^{1/2} \left[\frac{\rho_L + \rho_v}{\rho_v}\right]^{1/2} \tag{52}$$

The corresponding periods at the minimum heat flux are given by Equation (24) and Equation (25) respectively. Substituting the theoretical and experimental values into Equation (50) the following result is obtained:

Analysis		Experiment
$\frac{\text{Eq. (24)}}{\text{Eq. (51)}} = \frac{0.08}{0.0027} = 29.6$		$\frac{q_{max}}{q_{min}} = \frac{172,000}{5,470} = 31.4$
$\frac{\text{Eq. (25)}}{\text{Eq. (52)}} = \frac{0.18}{0.0061} = 29.6$		

E. The peak heat flux in transition boiling.

Inasmuch as the square root term in Equations (43), (44), or (48) is close to unity except in the neighborhood of the thermodynamic critical state it is seen that both Equations (6) and (9) are of the same form as Equation (43). The agreement of the heat transfer rates predicted by Equation (9) with experimental data was discussed already; this agreement is shown also on Figure 6. Therefore, for a comparison of the present analysis with experiments it suffices to compare the value of the coefficients K determined analytically with the values determined by Kutateladze and by Borishanskii from experiments.

	Analysis	Experiments
Eq. (44)	$0.12 \leq K \leq 0.157$	Kutateladze: $K = 0.16$
Eq. (48)	$K = \frac{\pi}{24} = 0.131$	Borishanskii: $K = 0.13$

The values of the constant K given by the theoretical limits

$$\frac{\pi}{24}\frac{3}{\sqrt{2\pi}} = 0.157 \geq K \geq 0.12 = \frac{\pi}{24}\frac{3}{\sqrt{2\pi}}\frac{1}{(3)^{1/4}} \qquad (53)$$

are indicated on Figure 6.

In Figure 14 the heat transfer rates predicted by Equation (48) are compared with experimental data for water reported by Kazakova.[26] Another

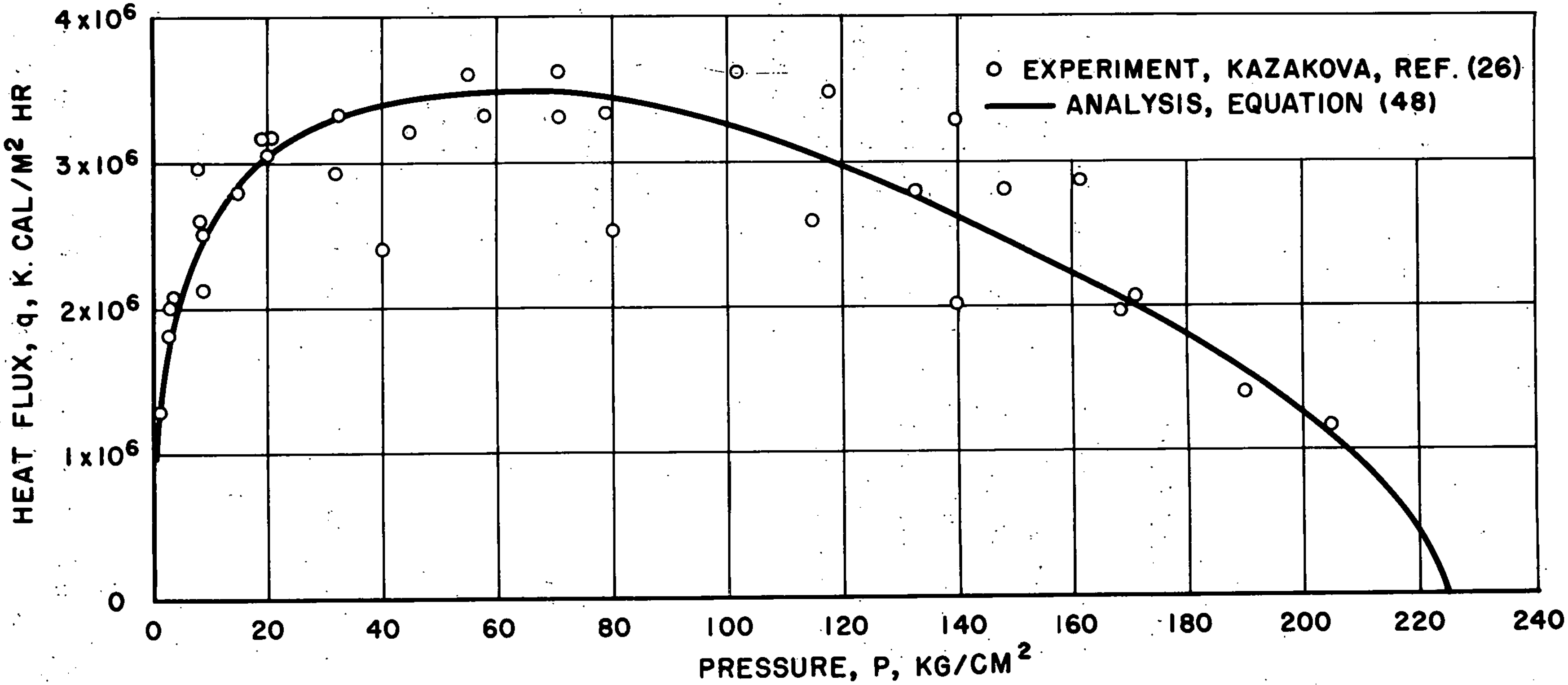

Figure 14. Comparison of Predicted with Experimental Peak Heat Transfer Rates in Pool Boiling for Water at Saturation Temperature.

comparison with experimental data for ethanol reported by Cichelli and Bonilla[15] is shown on Figure 15.

It appears from the above comparisons that this simple idealized system exhibits the detailed and general features of the observed phenomenon. It is seen that the description and statements concerning transition boiling which in a previous section were shown to be in qualitative agreement are, also, in quantitative agreement with experimental results. It should be noted that this agreement was achieved by two dimensional considerations; further experiments should indicate whether a three dimensional modification is necessary.

It is interesting to note that the analysis predicts an inherent uncertainty in determining the exact heat flux. The width of this uncertainty band is approximately $\pm$ 14%.* It follows from the theory that a certain irreproducibility of the experimental results can be expected. The scatter of experimental data is often reported in the literature.

In the following section the analysis will be extended to transition boiling of subcooled liquids.

* This uncertainty should in no wise be considered as a defect in the theory but it is to be expected. For an analogous case see the discussion by Rayleigh (reference 37, pages 364-5) where an uncertainty of $\pm$ 18% represents the best that can be done.

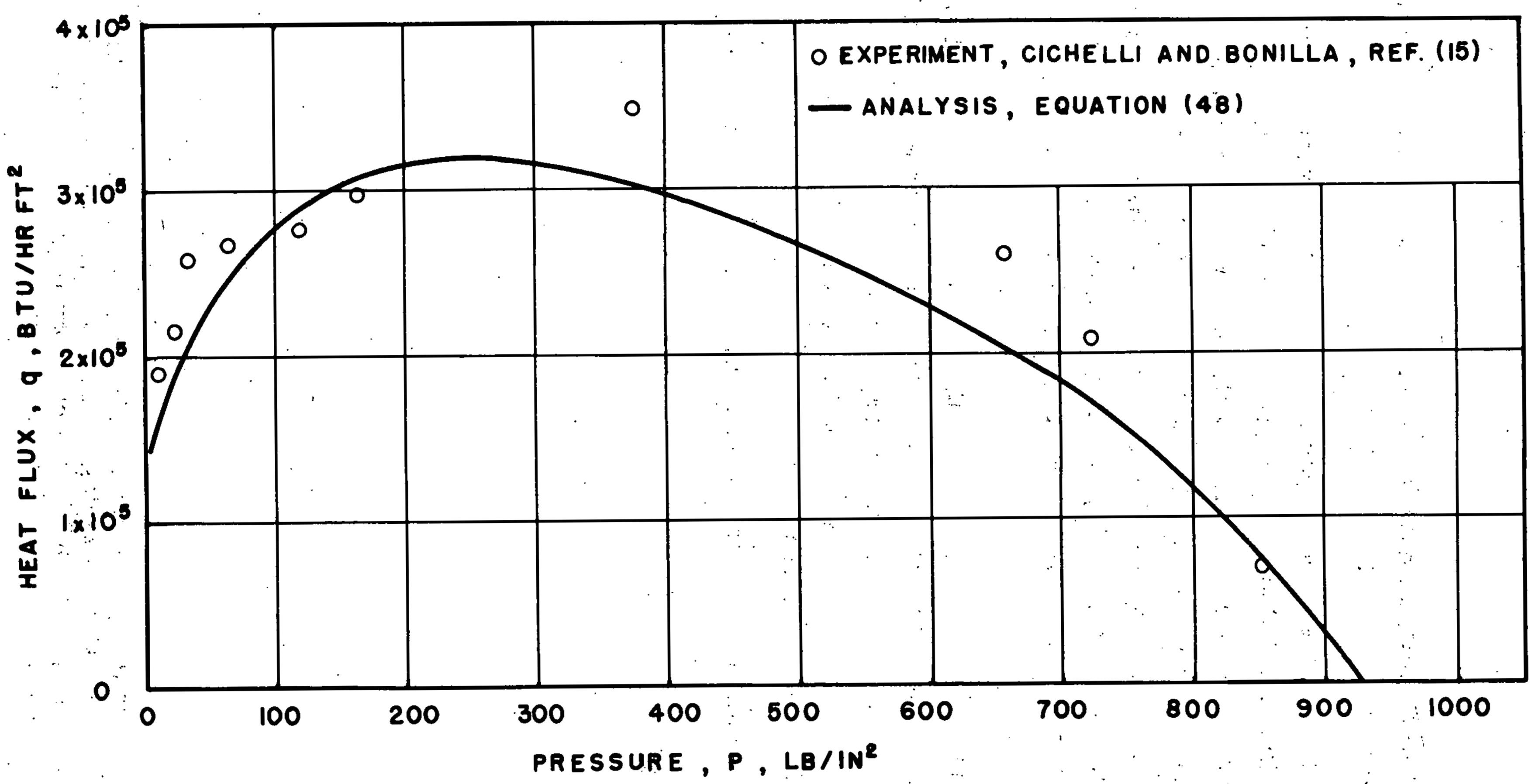

Figure 15. Comparison of Predicted with Experimental Peak Heat Transfer Rates in Pool Boiling for Ethyl Alcohol at Saturation Temperature.

TRANSITION BOILING OF SUBCOOLED LIQUIDS

Equations (43) and (48) which predict the peak flux in transition boiling were determined from stability considerations and an energy balance for liquids at saturation temperature. To extend the analysis to subcooled liquids a basic assumption will be made: The change from nucleate to transition boiling is determined only by the hydrodynamic stability in pool boiling, i. e., in the absence of a forced flow velocity, the change occurs when the vapor flux attains a given value. Therefore, if the liquid is at saturation and the viscosity is neglected, the heat transferred across the solid surface is equal to the energy required for the generation of that particular vapor mass flow. Since the stability is the mechanical aspect of the problem it will be unaffected by whether the liquid is subcooled or not, but depend only on the mass transport. However, as the heat flux across the solid surface is determined by an energy balance it will depend on the subcooling. Therefore, all energy requirements will appear as additive terms to the energy needed for the generation of the critical vapor flow. The problem is now reduced to the determination of energy requirements associated with a boiling, subcooled liquid.

It was already noted by Kutateladze[19] that when the degree of subcooling is low, bubbles will depart from the liquid-vapor interface. The mass of vapor is replaced by an equivalent mass of liquid which, as the bulk temperature is subcooled, has to be heated first up to the saturation temperature before evaporation can occur. It was shown that Equation (48) is a mean value of Equation (43). The analysis which follows will be based, therefore, on Equation (48). It is seen from Equation (48) that the critical vapor mass flow rate away from the surface is given by

$$\overset{o}{m} = \frac{\pi}{24} \rho_v \frac{\lambda_0}{\tau} = \frac{\pi}{24} \rho_v \left[\frac{\sigma g(\rho_L - \rho_v)}{\rho_v^2} \right]^{1/4} \left[\frac{\rho_L}{\rho_L + \rho_v} \right]^{1/2} \qquad (54)$$

Consequently, the heat flux density associated with this enthalpy change of the liquid is

$$q_2 = \left[\frac{\pi}{24} \quad \rho \quad \frac{\lambda_0}{\tau}\right] \quad c_L (T_s - T_L) \tag{55}$$

The energy (q_3) that is transferred from the film interface (which is at saturation temperature) to the subcooled liquid will now be estimated. The problem of the transfer of energy between a vapor-liquid interface and the body of the liquid and the analogous mass transfer problem, i.e., the absorption of a gas into an agitated liquid, were recently analyzed.[41,42,43] The important simplification introduced in these papers is the concept of a "contact time"[41] or "exposure time"[42,43] during which a portion of the liquid is exposed to a constant temperature or gas concentration at the interface. If the assumption is made that this exposure time is short relative to the distances traversed by the liquid the energy will be transferred mainly by conduction. This is equivalent, therefore, to the assumption that the "depth of penetration" is small compared to the "scale of turbulence". In the present problem the interface and, therefore, the temperature distribution is constantly renewed by stirring and agitation. Because the phenomenon exhibits a double frequency (see footnote page 44) it can be expected that the "exposure time" is equal to half a period. Using the plane approximation the average rate of heat transfer from the interface to the subcooled liquid is then:

$$q_3 = k \frac{2}{\tau} \int_0^{\tau/2} \left(\frac{\partial T}{\partial n}\right) dt \tag{56}$$

where n is the normal to the interface.

It is shown in the literature[44,45,13,11] that in transient problems of evaporation and condensation, i.e., of a moving interface the temperature gradient at the interface can be approximated by the temperature difference between the interface and the liquid divided by the thermal diffusion thickness. For a plane interface this gradient is:[11]

$$\frac{\partial T}{\partial n} = \sqrt{\frac{\pi}{4}} \quad \frac{T_s - T_L}{\sqrt{a\,t}} \tag{57}$$

Substituting the above gradient in Equation (56) the average heat transfer rate from the interface to the subcooled liquid becomes upon integration

$$q_s = \sqrt{2\pi}\,\frac{k}{\sqrt{a\tau}}\,(T_s - T_L) \tag{58}$$

In accordance with the approximations leading to Equation (48) the period τ is given by

$$\tau = 2\pi \left[\frac{\sigma}{g(\rho_L - \rho_v)}\right]^{1/2} \left[\frac{\rho_v^2}{\sigma g(\rho_L - \rho_v)}\right]^{1/4} \left[\frac{\rho_L + \rho_v}{\rho_L}\right]^{1/2} \tag{59}$$

and the ratio of the wave length to the period by

$$\frac{\lambda_0}{\tau} = \left[\frac{\sigma g(\rho_L - \rho_v)}{\rho_v^2}\right]^{1/4} \left[\frac{\rho_L}{\rho_L + \rho_v}\right]^{1/2} \tag{60}$$

The heat transferred across the solid surface is obtained again from an energy balance, i.e., by adding (Equations (48), (55) and (58). Consequently, the critical heat flux in transition boiling of a subcooled liquid is expected to be given by

$$q = L\,\frac{\pi}{24}\,\rho\,\frac{\lambda_0}{\tau} + \frac{\pi}{24}\,\rho\,\frac{\lambda_0}{\tau}\,c_L\,(T_s - T_L) + \sqrt{2\pi}\,\frac{k}{\sqrt{a\tau}}\,(T_s - T_L) \tag{61}$$

where τ and λ_0/τ are given by Equations (59) and (60) respectively.

In Figure 16 the heat transfer rates predicted by Equation (61) are compared to the experimental data reported by Gunther and Kreith[3] for water boiling at atmospheric pressure from a horizontal surface. A comparison is shown also with experimental results for ammonia and carbontetrachloride reported by Bartz[45] and by Ellion.[7] Inasmuch as these data were obtained from vertical surfaces, it appears that the heat transfer rates predicted by

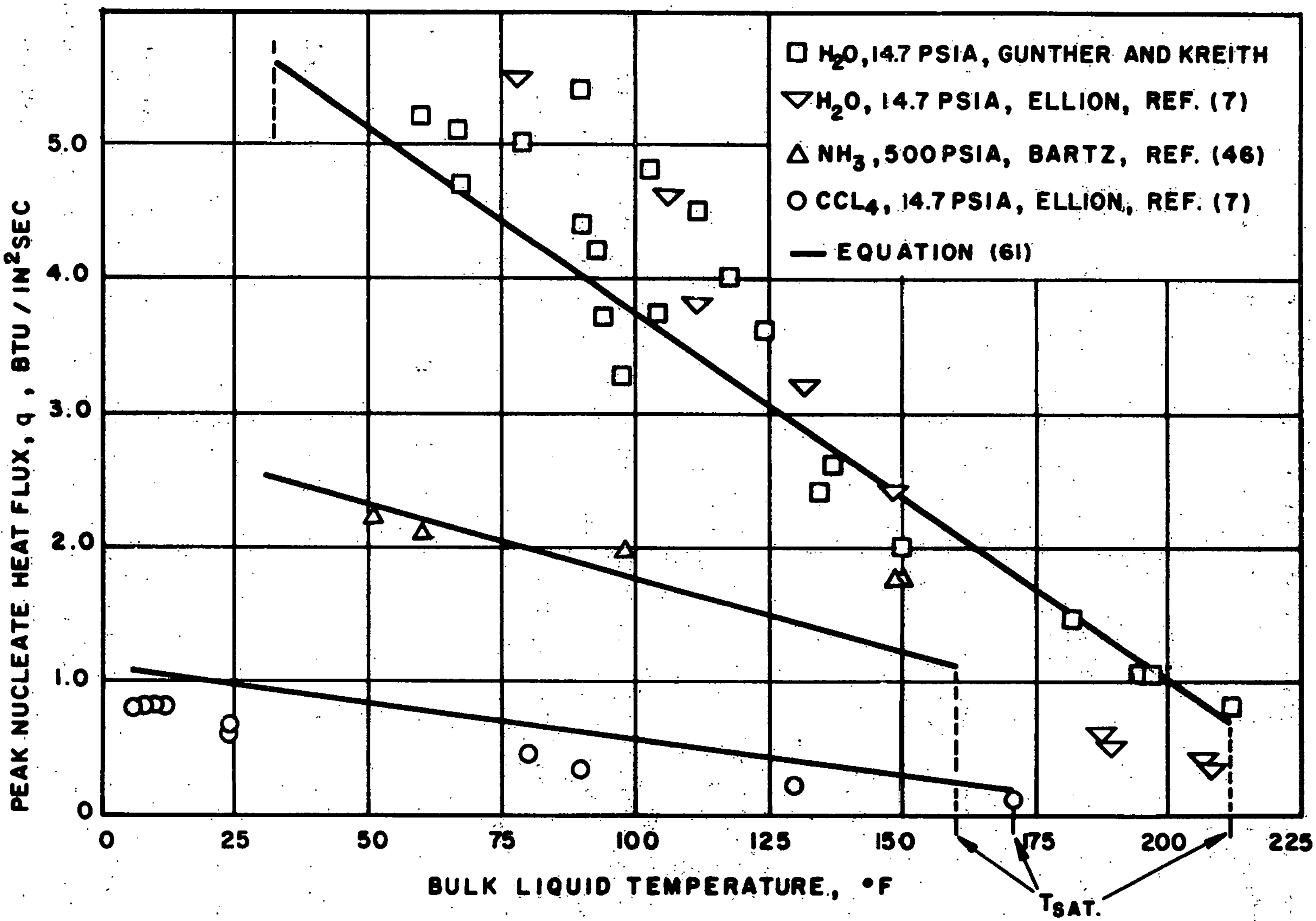

Figure 16. Comparison of Theoretical with Experimental Peak Nucleate Heat Transfer Rates in Pool Boiling of Subcooled Liquids.

Equation (61) can be used as a first approximation for these surfaces also.

One further comparison with experimental data may be made. In reference 47, Leppert, Costello and Hoglund give data on heat transfer to subcooled water containing small amounts of additive. The authors have supplied us with data not given in the original paper, from which the following table was constructed:

Figure No. In Reference	$T_S - T_L$ provided by Authors	Peak Heat Flux Experimental	Peak Heat Flux Analytical
6	47.6°F	9.10 - 11.74x10^5 Btu/hr ft^2	9.89 x 10^5
7	13.4	4.55 - 5.12 x 10^5	5.28 x 10^5

Before concluding this section we shall explore the possibility of applying to the present problem the concept of "contact time" in a forced flow system.[41, 48] Denoting the forced flow velocity by V, the contact time in the present problem is then

$$t = \frac{\lambda_0}{V} \tag{62}$$

The energy associated with this forced flow system becomes, therefore

$$q = \sqrt{2\pi}\; k \left[\frac{V}{\lambda_0 a}\right]^{1/2} (T_s - T_\infty) \tag{63}$$

Thus the total heat flux from the solid surface is

$$q = L \frac{\pi}{24} \rho_v \frac{\lambda_0}{\sqrt{a\tau}} + \frac{\pi}{24} \rho_v \frac{\lambda_0}{\tau} c_L (T_s - T_L) + \sqrt{2\pi} \frac{k}{\sqrt{a\tau}} (T_s - T_L) + \sqrt{2\pi}\, k \sqrt{\frac{V}{a \lambda_{01}}} (T_s - T_L) \tag{64}$$

In Figure 17 the heat transfer rates predicted by the above equation are compared with experimental data reported by Gunther [49] for water flowing through a horizontal, square channel. It should be emphasized, however, that Equation (63) does not take into account the effect of the forced flow velocity on the stability. This effect, as well as the effect of a restricted geometry of the conduit can be expected to have an influence on the peak flux. This aspect of the problem still remains to be solved. The agreement shown on Figure 13 should be taken, therefore, only as an encouraging indication concerning the possibility of extending the analysis to forced flow systems in closed conduits.

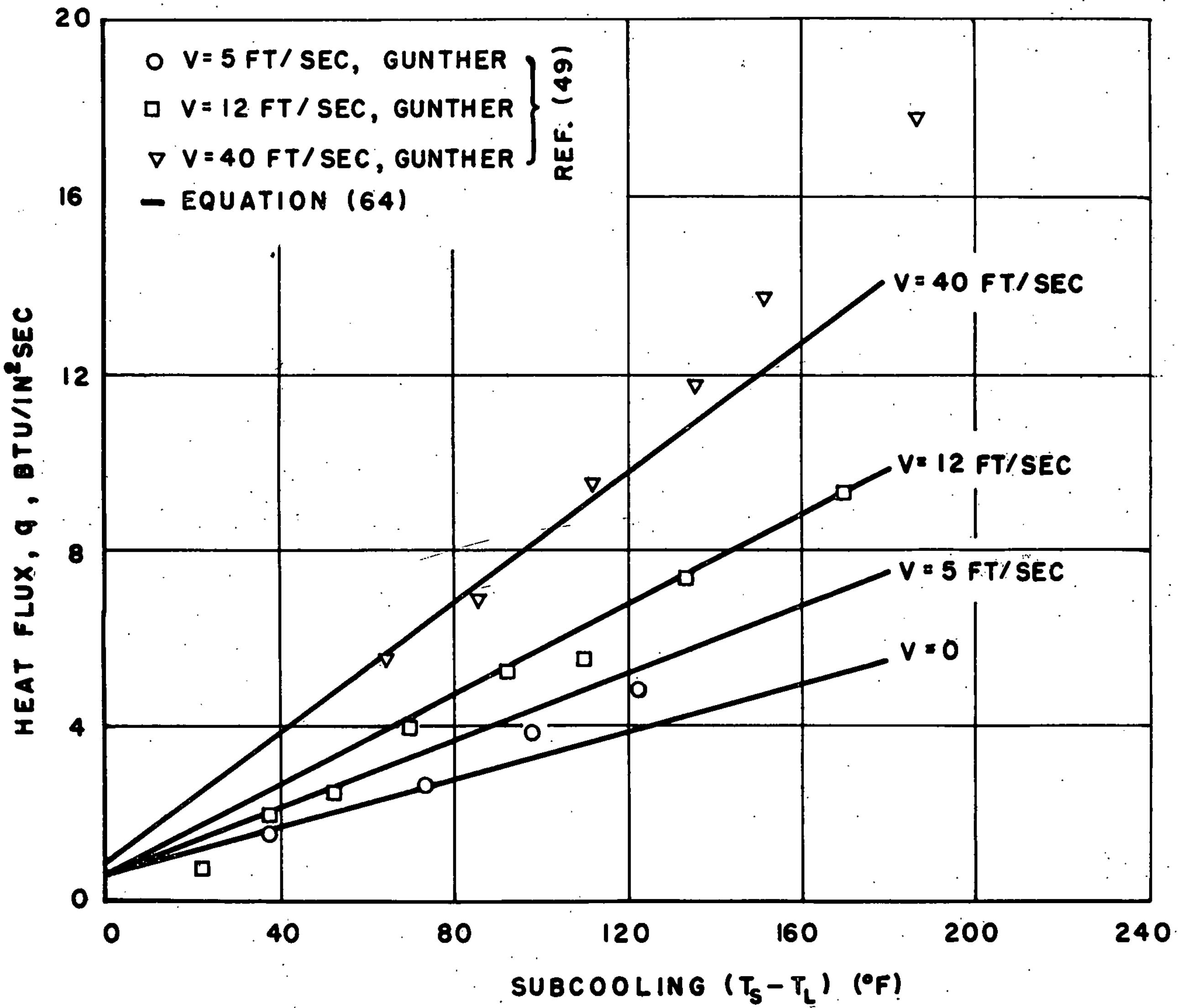

Figure 17. Comparison of Theoretical with Experimental Peak Nucleate Heat Transfer Rates for Water.

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DISCUSSION OF THE THEORY

The agreement of the analytical predictions, based on this simple idealized system, with experimental data was shown in preceding sections. However, more experimental data in transition boiling are needed in order to establish the limitations of the two-dimensional approach. It was noted already that an extension to three dimensions can be made easily. The authors were unable to find experimental data for pool boiling of subcooled liquids at high pressures and test Equation (60) in the high pressure range. It can be expected, however, that for a constant temperature difference (T_s - T_L) the effect of subcooling will decrease with an increase of pressure because the term λ_0 / τ decreases while τ increases with an increase of pressure.

It should be noted that the interface in subcooled boiling is not plane and the treatment of the interface as though it were plane was introduced as the simplest idealization. However, it will follow that no matter what geometry is postulated, the form of the resulting solution is unchanged and only the constant (i. e., $\sqrt{2\pi}$) will be affected in Equation (61). This change will alter each of the slopes of Figure 16 in the same way. The group $k\sqrt{a\tau}$ will still determine the relative slopes for the different liquids and the ratios of slopes will be unaffected by the constant. The agreement in Figure 16 concerning absolute as well as relative slopes serves to indicate that the conception of the process is proper.

Similar remarks apply to the use of a "contact time."

A discussion of the basic hypothesis concerning the hydrodynamic instability, geometrical configuration and frequency of transition boiling was presented in previous sections. No further elaboration will be made here. For the same reason a discussion will not be made here of the interesting theoretical conclusions concerning the inherent uncertainty in predicting the exact heat flux from which a certain irreproducibility of experimental results can be inferred. However, another interesting aspect of the theory concerning the location of the burnout point along a heated conduit for uniform and nonuniform heat transfer rates, will be discussed here in some detail.

Equation (64) is of the following form:

$$q \leq q_1 + q_2 + q_3 + q_4 \tag{65}$$

The left hand side represents the energy transferred from the surface, whereas the right hand side is the energy required to bring about the flow instability. It should be kept in mind that the "burnout" is a local phenomenon. It will occur when the equality sign in Equation (65) is reached. For a uniformly heated conduit of constant cross sectional characteristics the lowest value of subcooling occurs at the downstream end which is, therefore, the location of the burnout point. For a non-uniform, say cosine heat flux distribution as occurs in a reactor the maximum value of q occurs at $x = \ell/2$ where ℓ is the length of the heating element. At that point, however, the subcooling is still high, therefore the inequality sign in Equation (65) may still hold. The subcooling is minimum at the downstream end, i.e., at $x = \ell$, but at this point the heat flux q is low so that again the inequality may hold. The burnout point if it occurs will occur at a point where the equality first occurs, $\ell/2 < x < \ell$. A quantitative investigation of this aspect of the problem will be made and reported at a future date.

We shall conclude this section by deriving from the above theory the analytical expression of the empirical function which appears in the correlation of the peak nucleate heat flux recently proposed by Griffith.[50]

The correlation of Griffith has the following form:

$$\frac{q}{\rho_v (h_v - h_L) \left[\frac{\rho_L - \rho_v}{\mu} \, g \left(\frac{k_L}{c_L \, \rho_L} \right)^2 \right]^{1/3} F} = f(P/P_c) \tag{66}$$

The function $f(P/P_c)$ was determined from experiments and is given in graphical form in a plot versus the reduced pressure (P/P_c). The factor F is given by Griffith as:

$$F = 1 + A_1 \left(\frac{V D \rho_L}{\mu}\right) + A_2 \left[\frac{c_L \rho_L (T_s - T_L)}{L \rho_v}\right]$$
$$+ A_3 \left[\left(\frac{V D \rho_L}{\mu}\right) \frac{c_L \rho_L (T_s - T_L)}{L \rho_v}\right]^m \tag{67}$$

where V is the bulk velocity of the liquid (forced flow). The coefficients, A_1, A_2, A_3 were also found from experiments. In pool boiling the bulk velocity is zero (V = 0) and the coefficient A_2 was determined by choosing the best value from the experiments of Gunther and Kreith.

It will be shown in the following that the analytical expression for the function $f(P/P_c)$ and the value of the constant A_2 can be obtained from the preceding analysis.

In pool boiling (V = 0) with liquids at saturation ($T_s - T_L = 0$; $L = h_v - h_L$) the heat transfer rates given by Equations (61) and (66) must be the same, therefore:

$$f\left(\frac{P}{P_c}\right) = \frac{\pi}{24} \frac{\lambda_0}{\tau} \left[\left(\frac{\rho_L - \rho_v}{\mu}\right) g \left(\frac{k_L}{c_L \rho_L}\right)\right]^{1/3} \tag{68}$$

In pool boiling of subcooled liquids the enthalpy change is $h_v - h_L = L + c_L(T_s - T_L)$; it follows from Equations (60), (66) and (68) that

$$q = \frac{\pi}{24} \frac{\lambda_0}{\tau} \rho_v \left[L + c_L (T_s - T_L)\right] \left[1 + A_2 \frac{c_L \rho_L (T - T_L)}{L \rho_v}\right] \tag{69}$$

rearranging

$$q = \frac{\pi}{24} L \rho_v \frac{\lambda_0}{\tau} + \frac{\pi}{24} \rho_v \frac{\lambda_0}{\tau} c_L (T_s - T_L) +$$
$$\frac{\pi}{24} \frac{\lambda_0}{\tau} \left[1 + \frac{c_L (T_s - T_L)}{L}\right] c_L \rho_L (T_s - T_L) A_2 \tag{70}$$

Comparing Equation (70) with Equation (61) it follows that A_2 is given by:

$$\sqrt{2\pi} = \frac{\lambda_0}{\sqrt{a\tau}} \left[1 + \frac{c_L(T_s - T_L)}{L} \right] A_2 \frac{\pi}{24} \tag{71}$$

As previously mentioned Griffith[50] determined the value of the constant A_2 from experiments with water at atmospheric pressure The value he found was $A_2 = 0.014$. Equation (70) gives for the same conditions the value of $A_2 = 0.021$.

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