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MATERIAL REPLACEMENT EXPERIMENTS: THEORY AND MEASUREMENTS FOR THE LADY GODIVA ASSEMBLY

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ABSTRACT

The perturbation theory for material replacement experiments is given through second order thus permitting corrections for sample size. Computed flux and adjoint distribution functions are tabulated for the Lady Godiva assembly enabling the observed danger coefficients for U^{238} and U^{235} to be compared with corresponding predicted values. Consistency of this data is checked by its use in three independent combinations each yielding for the effective fraction of delayed neutrons from fast fission the value $[\beta_f] = 0.0068 \pm 0.0002$. Estimation of the reactivity contributions associated with inelastic scattering are given. give the following connection between central danger coefficient ratios and $[(\nu-1)\sigma_f - \sigma_c]$ ratios for the Topsy assembly (oralloy core + 8½" tuballoy reflector),

$$\frac{\Delta K_o(\text{U-233})}{\Delta K_o(\text{U-235})} = 1.71 \rightarrow \frac{[(\nu-1)\sigma_f - \sigma_c](\text{U-233})}{[(\nu-1)\sigma_f - \sigma_c](\text{U-235})} = 1.78$$

$$\frac{\Delta K_o(\text{Pu-239})}{\Delta K_o(\text{U-235})} = 1.93 \rightarrow \frac{[(\nu-1)\sigma_f - \sigma_c](\text{Pu-239})}{[(\nu-1)\sigma_f - \sigma_c](\text{U-235})} = 1.97$$

Evaluation of transport cross sections by means of replacement measurements in Godiva is illustrated for the several elements carbon, copper, and gold, the values relative to $\sigma_{tr}(\text{Oy}) = 1$ being $\sigma_{tr}(\text{C}) = 0.43 \pm 0.02$, $\sigma_{tr}(\text{Cu}) = 0.55 \pm 0.02$, $\sigma_{tr}(\text{Au}) = 0.91 \pm 0.02$.

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I. Introduction

Many measurements have been made of the reactivity changes caused by the addition of foreign material to small voids located at different radial positions in various spherically symmetric critical assemblies at Pajarito. Some of the results have been given in LA-1159 (Oy Hydride Critical Assemblies) and LA-1251 (Critical Masses of Oy at Reduced Concentrations and Densities).

This report is concerned primarily with the theory of such material replacement experiments, specifically with respect to: 1) the usually minor contributions to reactivity change associated with inelastic scattering and with anisotropic flux components, and 2) perturbations associated with the finite size of the material replacement samples. Detailed application of theory is restricted to the Lady Godiva assembly (the basic untamped critical Oy sphere).

Since nontechnical and nonessential prefatory material has been deleted, the first page of the report is page 5.

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II. Perturbation Theory for Material Replacement Experiments

A. Preliminary Development

A general outline of the perturbation theory has been given by Wigner in CP-3048 and subsequently worked out and applied in detail at Knolls (see, for example, KAPL-71 and 98). In the above reports, neutron transport was considered to be governed by the differential diffusion equation. However, our restriction to spherical assemblies, sufficiently simplifies the perturbation theory without recourse to transport approximations and the following development will utilize the Boltzmann equation in the form,

$$\frac{\partial n}{\partial t}(\vec{r},\vec{v},t) + \vec{v}\cdot\nabla n(\vec{r},\vec{v},t) = \int d\vec{v}'\, n(\vec{r},\vec{v}',t-\tau)\,|v'|\,\sigma(\vec{v}'\to\vec{v},\tau)\,d\tau - \sigma_t n|v| \qquad \text{(II-1)}$$

where $n(\vec{r},\vec{v},t)$ is the neutron density at position $\vec{r}$, velocity $\vec{v}$, and time t; $\sigma_t = \sigma_t(\vec{r},|v|)$ is the total cross section expressed in units of reciprocal length; $\sigma(\vec{v}'\to\vec{v},\tau)\,d\vec{v}\,d\tau$ is the differential cross section (also in units of reciprocal length) for transferring neutrons from velocity $\vec{v}'$ to velocity $\vec{v}$, the process requiring a time τ. The only transfer process considered to be non-instantaneous is that involving delayed neutrons from fission.

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Thus,

$$\sigma(\vec{v}' \to \vec{v}, \tau) = \nu \sigma_f(v') \left[f_o \chi_o(\vec{v}) \delta(\tau) + \sum f_i \chi_i(\vec{v}) \frac{e^{-\tau/\tau_i}}{\tau_i} \right] + \sigma_s(\vec{v}' \to \vec{v}) \delta(\tau) \qquad \text{(II-2)}$$

where $\sigma_f(|v'|)$ is the fission cross section, ν is the average neutrons per fission, f_o is the prompt fraction of fission neutrons and χ_o the associated spectrum ($\int d\vec{v}\, \chi_o(\vec{v}) = 1$); f_i, χ_i , and τ_i are the fraction, spectrum, and decay time of the i^{th} type delayed neutron; $\sigma_s(\vec{v}' \to \vec{v})$ is the scattering cross section from $\vec{v}'$ to $\vec{v}$, and $\delta(\tau)$ is the delta function.

The unperturbed system is taken to be the critical assembly. The perturbed system differs from this by some small material replacement (i.e., the replacement of part of the material composing the original assembly by "foreign material") and, in general, is either supercritical or subcritical. One effect of the material replacement is thus a change in the time behavior of the neutron density in the assembly and the purpose of the perturbation theory is to relate this changed time behavior (or better, the change in reproduction number K) to the neutron cross sections of the foreign material. The neutron transport equation for the unperturbed assembly is then,

$$\vec{v} \cdot \nabla n = \int d\vec{v}'\, n(\vec{r}, \vec{v}')\, |\vec{v}'|\, \sigma(\vec{v}' \to \vec{v}) - \sigma_t n(\vec{r}, \vec{v})\, |\vec{v}| \qquad \text{(II-3)}$$

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and the corresponding adjoint equation,

$$-\vec{v}\cdot\nabla n^*(\vec{r},\vec{v}) = \int d\vec{v}'\, n^*(\vec{r},\vec{v}')\,|\vec{v}|\,\sigma(\vec{v}\to\vec{v}') - \sigma_t n^*(\vec{r},\vec{v})\,|v| \qquad \text{(II-4)}$$

Designating the neutron density and cross sections of the perturbed assembly by n^p and σ^p one has,

$$\frac{\partial n^p}{\partial t} + \vec{v}\cdot\nabla n^p = \int d\vec{v}'\,d\tau\, n^p(\vec{v};\vec{r},t-\tau)\,\sigma^p(\vec{v}'\to\vec{v},\tau) - \sigma_t^p n^p\,|\vec{v}|$$

$$\frac{\partial n^p}{\partial t} = \alpha n^p \qquad \text{(II-5)}$$

By subtracting equation (II-4), multiplied by $n^p(\vec{r},\vec{v},t)$, from equation (II-5) multiplied by $n^*(\vec{r},\vec{v})$, then integrating over $\vec{r}$ and $\vec{v}$, there results:

$$\alpha\int n^p n^*\,d\vec{v}\,d\vec{r} + \int \nabla\cdot(\vec{v}\,n^p n^*)\,d\vec{v}\,d\vec{r} =$$

$$\int d\vec{v}'\,d\vec{v}\,d\vec{r}\,\left[\int d\tau\, n^p(\vec{v};\vec{r},t-\tau)\,|\vec{v}'|\,\sigma^p(\vec{v}'\to\vec{v},\tau) - n^p(\vec{v};\vec{r},t)\,|v'|\,\sigma(\vec{v}'\to\vec{v})\right] n^*(\vec{v},\vec{r})$$

$$-\int d\vec{v}\,d\vec{r}\,\left[\sigma_t^p - \sigma_t\right]|\vec{v}|\, n^p(\vec{v},\vec{r},t)\, n^*(\vec{v},\vec{r}) \qquad \text{(II-6)}$$

which, after setting $\int d\vec{r}\,\nabla\cdot(\vec{v}\,n^p n^*)=0$ (by Gauss' Thm.) and making use of equation (II-2), becomes,

$$\alpha\int n^p n^*\,d\vec{v}\,d\vec{r} + \int d\vec{v}'\,d\vec{v}\,d\vec{r}\; n^p(\vec{v};\vec{r})\,|v'|\left[\nu\sigma_f(v')\sum\frac{\alpha\tau_i f_i\chi_i(\vec{v})}{1+\alpha\tau_i}\right]^p n^*(\vec{r},\vec{v}) =$$

$$\int d\vec{v}'\,d\vec{v}\,d\vec{r}\; n^p(\vec{v};\vec{r})\,|v'|\,\Delta\left[\nu\sigma_f(v')\chi(\vec{v}) + \sigma_s(\vec{v}'\to\vec{v})\right] n^*(\vec{r},\vec{v})$$

$$-\int d\vec{v}\,d\vec{r}\; n^p(\vec{v},\vec{r})\,|v|\,\Delta\sigma_t(v)\, n^*(\vec{v},\vec{r}) \qquad \text{(II-6a)}$$

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where $\chi(\vec{v}) = f_o\chi_o(\vec{v}) + \sum f_i\chi_i(\vec{v})$ is the total fission neutron spectrum and $\Delta\sigma_t = [\sigma_t^p - \sigma_t]$. For a perturbation theory, the changes in cross sections are presumed small and consequently also α and $(n^p - n)$. Thus if, in equation (II-6a), n^p is replaced by n, there results a first order (of approximation) equation relating α to the changes in cross sections by means of the unperturbed distribution functions, $n(\vec{v},\vec{r})$ and $n^*(\vec{v},\vec{r})$. That this first order equation does not contain terms in $(n^p - n)$ is, of course, brought about by the use of the adjoint n^* .

The period, $1/\alpha$, of the perturbed assembly is related to the reproduction number K by the Inhour equation, which may be written as:

$$\frac{K-1}{K} =$$

$$\frac{\alpha\int n^p n^* d\vec{v}d\vec{r} + \int d\vec{v}'d\vec{v}d\vec{r}\; n^p(\vec{v};\vec{r})|v'|\left[\nu\sigma_f(v')\sum \frac{\alpha\tau_i f_i\chi_i(\vec{v})}{1+\alpha\tau_i}\right]^p n^*(\vec{r},\vec{v})}{\int d\vec{v}'d\vec{v}d\vec{r}\; n^p(\vec{v};\vec{r})|v'|\left[\nu\sigma_f(v')\chi(\vec{v})\right]^p n^*(\vec{r},\vec{v})} \qquad \text{(II-7)}$$

A more familiar expression for the Inhour equation is (see, for example, LA-1033),

$$\frac{K-1}{K} = \frac{(1-\gamma\Sigma f_i)\alpha\tau_o}{1+\alpha\tau_o} + \gamma\sum_i \frac{\alpha\tau_i f_i}{1+\alpha\tau_i} \qquad \text{(II-8)}$$

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where τ_o is the lifetime of prompt fission neutrons and γ is the mean effectiveness of the delayed relative to the prompt neutrons. To show that (II-7) is indeed a general expression of the Inhour equation requires an operational definition of K. This definition will be taken as: The reproduction number K for a subcritical assembly having a total neutron multiplication, T_n , when excited by a source having a normal mode distribution, is given by $T_n = 1/(1-K)$ (this relation being equivalent to the statement that each neutron on the average produces K daughter neutrons). Thus, consider a subcritical assembly which has a reproduction number K and is excited by a neutron source, $S(\vec{r},\vec{v})$. The neutron distribution $n^p(\vec{r},\vec{v})$ is given by:

$$\vec{v}\cdot\nabla n^p = \int d\vec{v}'\, n^p(\vec{r},\vec{v}')|v'|\left[\nu\sigma_f(v')\chi(\vec{v}) + \sigma_s(\vec{v}'\to\vec{v})\right]^p - \sigma_t^p n^p|v| + S(\vec{r},\vec{v}) \tag{II-9}$$

The adjoint distribution $n^*(\vec{r},\vec{v})$ for some arbitrary critical assembly characterized by cross section σ is given by:

$$-\vec{v}\cdot\nabla n^* = \int d\vec{v}'\, n^*(\vec{r},\vec{v}')|v'|\left[\nu\sigma_f(v')\chi(\vec{v}') + \sigma_s(\vec{v}\to\vec{v}')\right] - \sigma_t n^*|v| \tag{II-10}$$

Subtracting equation (II-10) multiplied by $n^p(\vec{r},\vec{v})$ from equation (II-9) multiplied by $n^*(\vec{r},\vec{v})$ and integrating over

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$\vec{r}$ and $\vec{v}$ gives:

$$\int d\vec{v}' d\vec{v} \, d\vec{r} \; n^{\dagger}(\vec{r},\vec{v}') |v'| \Delta\left[\nu\sigma_f(v')\chi(\vec{v}) + \sigma_s(\vec{v}'\to\vec{v})\right] n^*(\vec{r},\vec{v}) - \int d\vec{v} d\vec{r} \; n^{\dagger} |v| \Delta\sigma_t n^*$$

$$= -\int S(\vec{r},\vec{v}) \, n^*(\vec{r},\vec{v}) \, d\vec{v} \, d\vec{r} \qquad \text{(II-11)}$$

If $S(\vec{r},\vec{v})$ is a normal mode source, then,

$$\int d\vec{v}' \, n^{\dagger}(\vec{r},\vec{v}') |v'| \left[\nu\sigma_f(v')\chi(\vec{v})\right]^{\dagger} + S(\vec{r},\vec{v}) = T_n \, S(\vec{r},\vec{v})$$

and equation (II-11) becomes,

$$-\left(\frac{1}{T_n - 1}\right) = \frac{K-1}{K} =$$

$$\frac{\int d\vec{v}' d\vec{v} \, d\vec{r} \; n^{\dagger}(\vec{v}',\vec{r}) |v'| \Delta\left[\nu\sigma_f(v')\chi(\vec{v}) + \sigma_s(\vec{v}'\to\vec{v})\right] n^*(\vec{v},\vec{r}) - \int d\vec{v} d\vec{r} \; n^{\dagger} |v| \Delta\sigma_t n^*}{\int d\vec{v}' d\vec{v} \, d\vec{r} \; n^{\dagger}(\vec{v}',\vec{r}) |v'| \left[\nu\sigma_f(v')\chi(\vec{v})\right]^{\dagger} n^*(\vec{r},\vec{v})} \qquad \text{(II-12)}$$

Combining equations (II-12 and (II-6a) then gives the Inhour equation (II-7). Equation (II-12) is the basic relation between reactivity change, K - 1, (which may be positive or negative), and the cross section changes $\Delta\left[\nu\sigma_f(v')\chi(\vec{v})\right]$, $\Delta\sigma_s(\vec{v}'\to\vec{v})$, and $\Delta\sigma_t(v)$. The Inhour equation and the equation $T_n = 1/(1-K)$ are the two relations which afford experimental determinations of K.

The expression (II-7) of the Inhour equation has the rather paradoxical feature of involving the adjoint distribution of a hypothetical critical assembly. However, equation

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(II-12) clarifies this by showing a general connection between the distribution functions of different assemblies, viz., between the $n^p(\vec{r},\vec{v})$ of an assembly with reactivity K (arbitrary) and the $n^*(\vec{r},\vec{v})$ of an assembly with reactivity K = 1.

In the case of a monoenergetic, mono-directional point source of neutrons $S(\vec{r},\vec{v}) = S_o\,\delta(\vec{r}-\vec{r}_o)\,\delta(\vec{v}-\vec{v}_o)$, equations (II-11) and (II-12) may be combined to give,

$$\int dv'\,dv\,dr\; n^p(\vec{r},\vec{v}')\left[\nu\sigma_f(v')\chi(v)\right]^p n^*(\vec{r},\vec{v}) = \frac{K}{1-K}\,S_o\,n^*(\vec{r}_o,\vec{v}_o) \qquad \text{(II-13)}$$

If K is very nearly equal to one, then $n^p(\vec{r},\vec{v})$ is nearly a normal mode distribution regardless of the exciting source $S(\vec{r},\vec{v})$. For this case (II-13) becomes,

$$\frac{T-1}{T_n-1} = \frac{n^*(\vec{r}_o,\vec{v}_o)}{\bar{n}^*} \cong \frac{T}{T_n} \qquad \text{(II-14)}$$

where T is the total multiplication of the neutron source $S_o\,\delta(\vec{r}-\vec{r}_o)\,\delta(\vec{v}-\vec{v}_o)$, T_n is the total multiplication of the normal mode neutron source, and $\bar{n}^*$ is the value of the adjoint averaged over the normal mode source. Equation (II-14) provides a method for the measurement of $n^*(\vec{r},\vec{v})$, and also a reason for the designation of $n^*(\vec{r},\vec{v})$ as the "neutron effectiveness" function.

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B. First Order Perturbation Equation for Spherically Symmetric Assemblies

With the restriction of spherical symmetry, the neutron density and "effectiveness" may be expanded in terms of Legendre Polynomials (see, for example, LA-174),

$$n(\vec{r},\vec{v}) = \sum_{i=0} n_i(r,v) P_i(\mu) \quad ; \quad n^*(\vec{r},\vec{v}) = \sum_{i=0} n_i^*(r,v) P_i(\mu)$$

where μ is the cosine of the angle between $\vec{v}$ and $\vec{r}$. Replacing $n^p(\vec{r},\vec{v})$ by $n(\vec{r},\vec{v})$ in equation (II-12) and utilizing the above expansion, one obtains,

$$\left(\frac{K-1}{K}\right)_o \equiv \Delta K_o = \left\{ \int dv' d\vec{v} d\vec{r}\, |v'| \sum n_i(r,v') P_i(\mu') \Delta\left[\nu\sigma_f(v')\chi(v) + \sigma_s(\vec{v}'\to\vec{v})\right] \sum n_j^*(\vec{r},\vec{v}) P_j(\mu) \right.$$

$$\left. - \int d\vec{v} d\vec{r}\, |v| \sum n_i P_i(\mu) \sum n_j^* P_j(\mu) \Delta\sigma_t \right\} \div \left\{ \int d\vec{v}' d\vec{v} d\vec{r}\, n |v'| \nu\sigma_f \chi n^* \right\} \qquad \text{(II-15)}$$

Considering neutrons from fission and inelastic scattering to be emitted isotropically,

$$\chi(\vec{v}) = \frac{1}{4\pi}\chi(v) \quad ; \quad \sigma^{in}(\vec{v}'\to\vec{v}) = \frac{1}{4\pi}\sigma^{in}(v'\to v)$$

and the elastic scattering cross section to be given in the form

$$4\pi\,\sigma^{es}(\vec{v}'\to\vec{v}) = \sigma^{es}(v) + \sum_{i=1} \sigma_i(v) P_i(\cos[\vec{v}',\vec{v}])$$

equation (II-15) becomes after integrating over μ and μ',

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$$\Delta K_o = \left\{ \int d\vec{r}\, \frac{d\vec{v}'\, d\vec{v}}{(4\pi)^2}\; n_o(r,v')\, v'\, \Delta \left[\nu \sigma_f(v') \chi(v) + \sigma^{in}(v' \to v) - (\sigma_t - \sigma^{es})\, \delta(v'-v) \right] n_o^*(r,v) \right.$$

$$\left. - \int d\vec{r}\, \frac{d\vec{v}}{4\pi} \sum_{i=1}^{\infty} \frac{n_i v\, n_i^*}{2i+1}\, \Delta \left[\sigma_t - \frac{\sigma_i}{2i+1} \right] \right\} \div \left\{ \int d\vec{r}\, \frac{d\vec{v}'\, d\vec{v}}{(4\pi)^2}\; n_o v' \nu \sigma_f \chi(v) n_o^* \right\}$$

(II-16)

Finally, converting from velocity to energy spectra $n(v')\, d\vec{v}'/4\pi = n(E')\, dE'$ and $\sigma(v' \to v)\, d\vec{v}/4\pi = \sigma(E' \to E)\, dE$ and making use of the relation $\sigma_t(E) = \sigma^{es}(E) + \sigma_f(E) + \int dE' \sigma^{in}(E \to E') + \sigma_{c=\text{capture}}(E)$, one obtains for material replacements in the small volume element $[\Delta \vec{r}]$ at r

$$\Delta K_o(r, \Delta\vec{r}, \Delta\sigma) = \left\{ \frac{\Delta \vec{r}}{\int d\vec{r}'\, dE\, dE'\; n_o(r', E')\, v'\, \nu \sigma_f(E') \chi(E) n_o^*(r', E)} \right\} \times$$

$$\left\{ \int dE\; n_o(r,E) \overset{①}{v}\, \Delta \left[\nu \sigma_f(E) \int dE' \chi(E') n_o^*(r,E') - \{ \sigma_f(E) + \sigma_c(E) \} n_o^*(r,E) \right] \right.$$

(II-17)

$$+ \int dE\; n_o(r,E) \overset{②}{v}\, \Delta \left[\int dE'\, \sigma^{in}(E \to E') \left\{ n_o^*(r,E') - n_o^*(r,E) \right\} \right]$$

$$\left. - \sum_{i=1} \int dE\; \frac{n_i(r,E) \overset{③}{v} n_i^*(r,E)}{2i+1}\, \Delta \left[\sigma_t(E) - \frac{\sigma_i(E)}{2i+1} \right] \right\}$$

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The terms on the right hand side of equation (II-17) represent qualitatively the following three causes of reactivity increase: 1) net creation of neutrons; 2) inelastic scattering of neutrons to more effective energies; and 3) scattering of neutrons to more favorable directions of travel. When elastic scattering results in neutron energy loss as in the case of scattering by light elements, the $\sigma^{in}(E\to E')$ in term (2) of equation (II-17) should also include $\sigma^{es}(E\to E')$, and term (3) should read:

$$-\sum_{i=1}^{\infty}\int dE\ \frac{n_i(r,E)v}{2i+1}\left\{n_i^*(r,E)\Delta\sigma_t(E)-\int\frac{d\mu}{2}\,n_i^*(Ef[\mu],r)P_i(\mu)\Delta\sigma^{es}(\mu)\right\} \qquad \text{(II-17a)}$$

where $Ef[\mu]$ is the energy retained after the scattering of a neutron of initial energy E through the angle whose cosine is μ. The third term is zero for $r=0$; for $r\neq 0$ the $i=1$ component (the neutron current scattering term) is predominant and is proportional to the change in transport cross section, $\sigma_{tr}=\sigma_t-\frac{1}{3}\sigma_1$.

The function $\Delta K_0(\vec{r},x)$ representing the reactivity change say per mole for the addition of material (x) at the position $\vec{r}$, permits the immediate evaluation of the total reactivity change associated with the addition of small amounts of (x) in a density distribution $\rho(\vec{r},x)$ moles per unit volume as,

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$$\Delta K = \int \Delta K_o(\vec{r}, x)\, \rho(\vec{r}, x)\, d\vec{r} \qquad \text{(II-18)}$$

The feature of additivity as in (II-18) and the possibility of interpretation by use of the standard functions η and η^* as in (II-17) make the evaluation of $\Delta K_o(\vec{r}, x)$ an appropriate aim of material replacement measurements. Generally complicating this evaluation is a required replacement sample size for which the observed reactivity change, $\Delta K_{obs.}$, does not correspond to the "first order" change, ΔK_o. The next section deals with the "corrections" required for the conversion $\Delta K_{obs.} \rightarrow \Delta K_o$.

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C. Second Order Perturbation Theory

In this section, it is desired to determine the second order contributions to reactivity change associated with the perturbation of the neutron density $[n^p(\vec{r},\vec{v}) - n(\vec{r},\vec{v})]$. This perturbation is assumed to arise from the substitution of foreign material in the small volume element $\Delta\vec{r}$ centered at the position $\vec{r}_o$, the dimensions of $\Delta\vec{r}$ being small as compared to the neutron mean free path in either the foreign or original material. It is also assumed that a suitable displacement of a control rod remote from $\vec{r}_o$ retains the system at critical.

The equations for $n(\vec{r},\vec{v})$ and $n^p(\vec{r},\vec{v})$,

$$\vec{v}\cdot\nabla n = \int d\vec{v}'\, n(\vec{r},\vec{v}')|v'|\,\sigma(\vec{v}'\to\vec{v}) - \sigma_t(v)\, n(\vec{r},\vec{v})|v|$$

$$\vec{v}\cdot\nabla n^p = \int d\vec{v}'\, n^p(\vec{r},\vec{v}')|v'|\,\sigma^p(\vec{v}'\to\vec{v}) - \sigma_t^p(v)\, n^p(\vec{r},\vec{v})|v| \qquad \text{(II-19)}$$

give,

$$\vec{v}\cdot\nabla(n^p-n) = \int d\vec{v}'(n^p-n)|v'|\,\sigma(\vec{v}'\to\vec{v}) - \sigma_t(n^p-n)|v| + S(\vec{r},\vec{v}) \qquad \text{(II-20)}$$

where,

$$S(\vec{r},\vec{v}) = \int d\vec{v}'\, n^p|v'|\,\Delta\sigma(\vec{v}'\to\vec{v}) - n^p|v|\,\Delta\sigma_t \cong \int d\vec{v}'\, n|v'|\,\Delta\sigma(\vec{v}'\to\vec{v}) - n|v|\,\Delta\sigma_t \qquad \text{(II-21)}$$

Thus, $S(\vec{r},\vec{v})$ acts as the neutron source for the distribution (n^p-n). By reason of the assumed smallness of the replacement volume $\Delta\vec{r}$, the distribution (n^p-n) in the

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vicinity of $\vec{r}_o$ is, to first approximation, determined by the streaming of neutrons from this source; that is ,

$$\vec{v}\cdot\nabla(n^p - n) \cong S(\vec{r},\vec{v}) \quad \text{in the vicinity of } \vec{r}_o \qquad \text{(II-22)}$$

with $S(\vec{r},\vec{v}) = S(\vec{r}_o,\vec{v})$ for $\vec{r}$ inside $\vec{\Delta r}$

$= 0$ for $\vec{r}$ outside $\vec{\Delta r}$.

Referring to the general equation (II-12) for reactivity change,

$$\Delta K_o =$$

$$\frac{\int d\vec{v}\,d\vec{v}'\,d\vec{r}\; n^p(\vec{r},\vec{v}')|v'|\Delta\left[\nu\sigma_f(v')\chi(\vec{v}) + \sigma_s(\vec{v}'\to\vec{v})\right] n^*(\vec{r},\vec{v}) - \int d\vec{v}\,d\vec{r}\; n^p(\vec{r},\vec{v})|v|\Delta\sigma_t n^*}{\int d\vec{v}'\,d\vec{v}\,d\vec{r}\; n^p(\vec{r},\vec{v}')|v'|\left[\nu\sigma_f(v')\chi(\vec{v})\right] n^*(\vec{r},\vec{v})} \qquad \text{(II-12)}$$

it is seen that, for the evaluation of the right hand side numerator, only values of $(n^p - n)$ within the replacement volume are required.

Evaluation of $(n^p - n)$ inside $\vec{\Delta r}$:

Letting s represent a coordinate on a line passing through $\vec{\Delta r}$ in the direction $\vec{v}$, then (II-22) may be expressed as:

$$|\vec{v}|\frac{\partial}{\partial s}\left[n^p - n\right] = S(\vec{r},\vec{v}) \qquad \text{(II-22a)}$$

and integrated to give,

$$|v|\left[n^p - n\right] = S(\vec{r}_o,\vec{v})\,\ell(\vec{r},\vec{v}) \qquad \text{(II-23)}$$

where $\ell(\vec{r},\vec{v})$ is the distance from the internal point $\vec{r}$ to

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the surface of the replacement volume $\vec{\Delta r}$ in the direction $-\vec{V}$.

Since an average $\ell(\vec{r},\vec{v})$ will be of the order $[\vec{\Delta r}]^{1/3}$, the right hand side numerator of (II-12) will have the form $[\vec{\Delta r}]\left\{a_1 + a_2[\vec{\Delta r}]^{1/3} + \cdots\right\}$, the similar expression for the denominator being $\left\{b_1 + b_2[\vec{\Delta r}]^{1/3} + \cdots\right\}$. If $b_2 = 0$, then the denominator contributes no second order reactivity changes. This is indeed the case. Rewriting the denominator of (II-12) as,

$$\int d\vec{v}'\,d\vec{\rho}\,d\vec{r}\; n(\vec{r},\vec{v}')\,|v'|\;\textcircled{1}\;[\nu\sigma_f(v')\chi(\vec{v})]\,n^*(\vec{r},\vec{v}) + \int d\vec{v}'\,d\vec{\rho}\,d\vec{r}\; n(\vec{r},\vec{v}')\,|v'|\;\textcircled{2}\;\Delta[\nu\sigma_f\chi]\,n^*(\vec{r},\vec{v})$$

$$+ \int d\vec{v}'\,d\vec{\rho}\,d\vec{r}\;[n^p - n]\,|v'|\;\textcircled{3}\;[\nu\sigma_f(v')\chi(\vec{v})]\,n^*(\vec{r},\vec{v})$$

then the first term is b_1 and the second term is seen to be proportional to $[\vec{\Delta r}]$, i.e., of order higher than the second. The third term is roughly proportional to $\int_{core} d\vec{v}'\,d\vec{r}\,[n_o^p - n_o]\,|v'|$ where $[n_o^p - n_o](\vec{r},|v|)$ is the total or isotropic component of $[n^p - n](\vec{r},\vec{v})$. Integrating (II-22) over a sphere centered at $\vec{r}_o$ and containing $\vec{\Delta r}$ gives,

$$\int_{\text{Sphere surface}} [n^p - n]\,\vec{v}\cdot d\vec{A} = S(\vec{r}_o,\vec{v})\,[\vec{\Delta r}] \qquad \text{(II-22b)}$$

and integrating (II-22b) over $\vec{v}$ gives,

$$\int d\vec{v}\,[n_o^p - n_o]\,|v|\,dA = [\vec{\Delta r}]\int d\vec{v}\,S(\vec{r}_o,\vec{v}) \qquad \text{(II-22c)}$$

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Hence,

$$\int d\vec{v}'\,d\vec{r}\,[n_o^p - n_o]\,|v'| \simeq [\Delta\vec{r}]\,R_c \int d\vec{v}\; S(\vec{r}_o,\vec{v}) \qquad \text{(II-24)}$$

where R_c is the core radius, and the denominator term (3) above is also proportional to $[\Delta\vec{r}]$ (*).

To summarize, the substitution of foreign material in the volume element $\Delta\vec{r}$ centered at r_o, gives a reactivity change which, to second order of approximation, may be expressed as,

$$\Delta K(r_o,\Delta\vec{r},\Delta\sigma) =$$

$$\frac{[\Delta\vec{r}]\int d\vec{v}'\,d\vec{v}\left\{n(\vec{r}_o,\vec{v}')\,|v'| + S(\vec{r}_o,\vec{v}')\,\ell(\vec{v}')\right\}\Delta\left[\nu\sigma_f(v')\chi(\vec{v}) + \sigma_s(\vec{v}'\to\vec{v})\right] n_o^*(\vec{r}_o,\vec{v})}{\int d\vec{v}'\,d\vec{v}\,dr\; n(\vec{r},\vec{v}')\,\nu\sigma_f(v')\chi(\vec{v})\,n^*(\vec{r},\vec{v})} \qquad \text{(II-25)}$$

$$- \frac{[\Delta\vec{r}]\int d\vec{v}\,d\vec{r}\left\{n(\vec{r}_o,\vec{v})\,|v| + S(\vec{r}_o,\vec{v})\,\ell(\vec{v})\right\}\Delta\sigma_t(v)\,n^*(\vec{r}_o,\vec{v})}{\int d\vec{v}'\,d\vec{v}\,d\vec{r}\; n(\vec{r},\vec{v}')\,\nu\sigma_f(v')\chi(\vec{v})\,n^*(\vec{r},\vec{v})}$$

(*) Equation (II-22c) implies that the total density $[n_o^p - n_o]$ falls off from the source as $1/|\vec{r}-\vec{r}_o|^2$, this being due to the neglect of scattering in eqn. (II-22). With scattering included, the progeny of the source neutrons rapidly approach a normal mode distribution which is then counted as part of the unperturbed density $n(\vec{r},\vec{v})$, the density $[n_o^p - n_o]$ falling virtually to zero in the order of several mean free paths from the source. It might thus be better if R_c appearing in (II-24) were replaced by $1/\sigma_t$, a revision, however, which would not alter the conclusion that the term (3) is proportional to $[\Delta\vec{r}]$.

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with,

$$S(\vec{r}_o,\vec{v}) = \int d\vec{v}'\, n(\vec{v}';\vec{r}_o)\,|v'|\,\Delta\sigma(v'\to v) - n(\vec{r}_o,\vec{v})\,|v|\,\Delta\sigma_t \qquad \text{(II-21)}$$

$$\ell(\vec{v}) = \int_{[\Delta\vec{r}]} d\vec{r}\; \ell(\vec{r},\vec{v})$$

where $\ell(\vec{r},\vec{v})$ is the distance from the point $\vec{r}$ to the surface of $[\Delta\vec{r}]$ in the direction $-\vec{v}$.

For the case of spherical symmetry where the distribution functions are considered expressed in terms of Legendre Polynomials, then, letting $(2i+1)\int S(\vec{r}_o,\vec{v})\,\ell(\vec{v})\,P_i(\mu)\,d\mu/2 = [S\ell]_i$, equation (II-25) takes the form of (II-17) with n_i replaced by $n_i + [S\ell]_i$.

Examples:

1) One energy group of neutrons with differential diffusion.

Equation (II-25) becomes for this model,

$$\Delta K(\vec{r}_o, \Delta\vec{r}, \Delta\sigma) =$$

$$[\Delta\vec{r}]\left\{ \frac{(n_o+[S\ell]_o)\,n_o^*(r_o)\,\Delta[(\nu-1)\sigma_f-\sigma_c] - (n_1+[S\ell]_1)\,n_1^*\,\Delta[\sigma_t-\tfrac{1}{3}\sigma_1]}{\int d\vec{r}\; n_o n_o^*\,\nu\sigma_f}\right\} \qquad \text{(II-25a)}$$

where the source term has the expression,

$$S(\vec{r}_o,\mu) = n_o(r_o)\,\Delta[(\nu-1)\sigma_f-\sigma_c] - n_1(r_o)\,P_1(\mu)\,\Delta[\sigma_t-\tfrac{1}{3}\sigma_1] \qquad \text{(II-21a)}$$

Since the neutron path length term $\ell(\vec{v})$ has the property $\ell(\vec{v}) = \ell(-\vec{v})$, then,

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$$[S\ell]_0 = n_0 \Delta[(\nu-1)\sigma_f - \sigma_c] \int \ell(\mu)\frac{d\mu}{2} \equiv n_0 \Delta[(\nu-1)\sigma_f - \sigma_c]\ell_0$$

$$[S\ell]_1 = -n_1 \Delta\left[\sigma_t - \tfrac{1}{3}\sigma_1\right] \int 3P_1^2(\mu)\ell(\mu)\frac{d\mu}{2} \equiv -n_1 \Delta\left[\sigma_t - \tfrac{1}{3}\sigma_1\right]\ell_1 \qquad \text{(II-26)}$$

Since $n_0 + [S\ell]_0 = \bar{n}_0^p$, the value of the perturbed total density averaged over the replacement volume, one has,

$$\bar{n}_0^p = n_0\left[1 + \Delta\{(\nu-1)\sigma_f - \sigma_c\}\ell_0\right]$$

$$\bar{n}_1^p = n_1\left[1 - \Delta\{\sigma_t - \tfrac{1}{3}\sigma_1\}\ell_1\right] \qquad \text{(II-27)}$$

If one designates as ΔK_{oa} the first order reactivity changes associated with fission and capture (i.e., absorption processes) and as ΔK_{os} the first order reactivity changes associated with scattering, then equation (II-25a) becomes,

$$\Delta K(\vec{r}_o, \Delta\vec{r}, \Delta\sigma) =$$

$$\Delta K_{oa}(\vec{r}_o, \Delta\vec{r}, \Delta\sigma)\left\{1 + \Delta[(\nu-1)\sigma_f - \sigma_c]\ell_0\right\} + \Delta K_{os}(\vec{r}_o, \Delta\vec{r}, \Delta\sigma)\left\{1 - \Delta\sigma_{tr}\ell_1\right\} \qquad \text{(II-25b)}$$

Letting f_a and f_s represent the fractions of the first order reactivity change associated with absorptive and scattering processes, respectively, and C_a and C_s represent the correction terms $\Delta[(\nu-1)\sigma_f - \sigma_c]\ell_0$ and $\Delta\left[\sigma_t - \tfrac{1}{3}\sigma_1\right]\ell_1$, respectively, one may rewrite (II-25b) as,

$$\Delta K_{obs.} = \Delta K_0\left\{1 + f_a C_a - f_s C_s\right\} \equiv \Delta K_0\left\{1 + C\right\} \qquad \text{(II-25c)}$$

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The cross section changes $\Delta[(\nu-1)\sigma_f - \sigma_c]$ and $\Delta\sigma_{tr}$ are generally known a priori sufficiently well for the purpose of evaluating the correction terms C_a and C_s. Similarly, the shape of the function $\Delta K_{obs}(\vec{r}_o)$ permits fairly accurate estimates of the fractional reactivity contributions f_a and f_s. Hence the factor C given in (II-25c) may be computed and yield the conversion $\Delta K_{obs} \rightarrow \Delta K_o$.

2) Spherical replacement samples:

For this case, $\ell(\vec{v}) = \ell_o = 3/4$ of the sample radius. Expanded in Legendre Polynomials, the source term becomes,

$$S(\vec{r}_o, E, \mu) = \int dE' \, n_o(E) v' \Delta \left\{ \nu\sigma_f(E')\chi(E) + \sigma^{in}(E' \rightarrow E) \right\}$$

$$- n_o(E) v \Delta \left\{ \sigma_f + \sigma_c + \sigma^{in}(E) \right\} - \sum_{i=1}^{\infty} \Delta \left[\sigma_t - \frac{\sigma_i}{2i+1} \right] n_i v P_i(\mu) = \sum_{i=0}^{\infty} S_i P_i \qquad \text{(II-21b)}$$

With a multigroup representation where 1) Greek letters denote energy groups, 2) Roman letters still designate anisotropic density components, and 3) the flux terms $n_{\alpha i} v_\alpha \equiv N_{\alpha i}$, one has,

$$[S\ell]_{\alpha 0} \equiv \ell_o S_{\alpha o} = \ell_o \left\{ \sum_\beta N_{\beta o} \Delta \left[\nu\sigma_{f\beta}\chi_\alpha + \sigma^{in}_{\beta\alpha} \right] - N_{\alpha o} \Delta \left[\sigma_{f\alpha} + \sigma_{c\alpha} + \sigma^{in}_\alpha \right] \right\}$$

$$[S\ell]_{\alpha i} = \ell_o S_{\alpha i} = -\ell_o \Delta \left[\sigma_t - \frac{\sigma_i}{2i+1} \right]_\alpha N_{\alpha i} \quad (*) \qquad \text{(II-28)}$$

(*) As in (II-17), we ignore the possibility that elastic scattering reduces the neutron energy.

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The parts of the reactivity change due to absorptive processes (fission, capture, inelastic scattering) and scattering become, respectively,

$$\Delta K_a \equiv \Delta K_{oa}(1+C_a) = \frac{[\vec{\Delta r}] \sum_\alpha \left\{ N_{\alpha o} + \ell_o S_{\alpha o} \right\} \left\{ \sum_\beta \left[\nu \sigma_{f\alpha} \chi_\beta + \sigma_{\alpha\beta}^{in} \right] \eta^*_{\beta 0} - \left[\sigma_{f\alpha} + \sigma_{c\alpha} + \sigma_\alpha^{in} \right] \eta^*_{\alpha o} \right\}}{\int d\vec{r} \sum_{\alpha'} \sum_{\beta'} N_{\alpha' o} \nu \sigma_{f\alpha'} \chi_{\beta'} \eta^*_{\beta' o}}$$

$$\Delta K_s \equiv \Delta K_{os}(1+C_s) = - \frac{[\vec{\Delta r}] \sum_\alpha \sum_i \left\{ N_{\alpha i} + \ell_o S_{\alpha i} \right\} \Delta \left[\sigma_t - \frac{\sigma_i}{2i+1} \right]_\alpha \eta^*_{\alpha i}}{\int d\vec{r} \sum_{\alpha'} \sum_{\beta'} N_{\alpha' o} \nu \sigma_{f\alpha'} \chi_{\beta'} \eta^*_{\beta' o}}$$

$$\Delta K_{obs} = \Delta K_a + \Delta K_s \equiv \Delta K_o (1+C) \qquad \text{(II-29)}$$

The material replacement measurements to be discussed in the last section have been made with right cylinder samples having unit height to diameter ratio. For these cylinders, the various moments of $\ell(\vec{v})$ are nearly equal to ℓ_o so that the spherical replacement model applies. More specifically, the measurement denoted by the symbol $\Delta K(r, \vec{\Delta r}, x)$ gives the reactivity difference between an initial configuration in which the volume element $\vec{\Delta r}$ at r is empty and a final configuration in which material (x) occupies $\vec{\Delta r}$. Thus, in general, the initial configuration is also a perturbed configuration. If one designates as K_o the reproduction number of the unperturbed assembly, (original

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material (y), such as U-235, occupies $\Delta\vec{r}$), K_i and K_f the reproduction numbers of the initial and final configurations described above, then,

$$\Delta K(r,\Delta\vec{r},x) = K_f - K_i = (K_f - K_o) - (K_i - K_o)$$
$$= \Delta K(r,\Delta\vec{r},y\to x) - \Delta K(r,\Delta\vec{r},y\to \text{void}) \qquad \text{(II-30)}$$

Equations (II-28), (II-29), and (II-30) then permit the reduction of $\Delta K(r,\Delta\vec{r},x)$ to the first order $\Delta K_o(r,\Delta\vec{r},x)$.

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III. Material Replacement Measurements in Lady Godiva

A. Predicted Flux and Adjoint Distributions

The distribution functions of neutron flux, N, and adjoint, N^*, listed in Tables I and II, are based on the P_3 transport approximation (see, for example, LA-174) and the three-group cross sections listed in Table III together with the physical conditions: Oy density = 18.75 gms/cm^3 and U-235 concentration = 93.5%. The resultant predicted critical radius is 8.965 cm. (For the Lady Godiva assembly, the Oy has a mean density of 18.7_5 gms/cm^3 and a U-235 concentration of 93.7%. Under normal operating conditions, the Oy ball is a slightly prolate spheroid with a polar to equatorial diameter ratio of 1.027, the effective critical radius being 8.73_2 cm. An LA report covering the Lady Godiva assembly is being written by R. E. Peterson.) The three energy groups, high, intermediate, and low, are designated by the subscripts α, β, and γ, so that the flux distribution in the low energy group, for example, is represented as,

$$N_{\gamma 0}(r) + N_{\gamma 1}(r)\cos\theta + N_{\gamma 2}(r)\left[\frac{3\cos^2\theta - 1}{2}\right] + N_{\gamma 3}(r)\left[\frac{5\cos^3\theta - 3\cos\theta}{2}\right] = N_{\gamma}(r,\theta)$$

The deficiencies of the P_3 approximation are most marked for the values of the flux and adjoint near the sphere surface

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TABLE I. PREDICTED NEUTRON FLUX DISTRIBUTION FOR GODIVA

R(cm)	$N_{\alpha 0}$	$N_{\alpha 1}$	$N_{\alpha 2}$	$N_{\alpha 3}$	$N_{\beta 0}$	$N_{\beta 1}$	$N_{\beta 2}$	$N_{\beta 3}$	$N_{\gamma 0}$	$N_{\gamma 1}$	$N_{\gamma 2}$	$N_{\gamma 3}$
0	.3280	.0000	.0000	.0000	.3527	.0000	.0000	.0000	.3193	.0000	.0000	.0000
1.04	.3236	.0341	.0011	.0000	.3478	.0324	.0008	.0000	.3147	.0211	.0003	.0000
2.08	.3103	.0664	.0045	.0001	.3334	.0631	.0032	.0001	.3011	.0412	.0014	.0000
3.12	.2889	.0954	.0100	.0002	.3106	.0907	.0071	.0002	.2792	.0594	.0031	.0001
4.17	.2604	.1194	.0173	.0003	.2792	.1137	.0123	.0004	.2500	.0748	.0053	.0001
5.21	.2260	.1374	.0262	.0004	.2418	.1309	.0187	.0005	.2147	.0867	.0081	.0002
6.25	.1871	.1483	.0365	.0003	.1996	.1417	.0266	.0002	.1747	.0947	.0116	-.0002
7.29	.1452	.1517	.0482	-.0006	.1538	.1453	.0365	-.0013	.1314	.0985	.0166	-.0017
8.33	.1018	.1469	.0616	-.0025	.1055	.1416	.0502	-.0059	.0849	.0979	.0259	-.0071
8.54	.0931	.1449	.0645	-.0030	.0955	.1399	.0537	-.0074	.0751	.0972	.0288	-.0092
8.75	.0844	.1426	.0675	-.0036	.0854	.1379	.0575	-.0092	.0651	.0964	.0324	-.0119
8.965	.0755	.1397	.0707	-.0042	.0749	.1356	.0618	-.0114	.0544	.0955	.0368	-.0154
9.17	.0671	.1367	.0738	-.0048	.0648	.1330	.0664	-.0139	.0439	.0944	.0420	-.0196
9.38	.0585	.1331	.0770	-.0054	.0543	.1301	.0717	-.0169	.0326	.0932	.0485	-.0250

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TABLE II. PREDICTED NEUTRON EFFECTIVENESS OR FLUX ADJOINT DISTRIBUTION FOR GODIVA

R(cm)	$n^*_{\alpha 0}$	$n^*_{\alpha 1}$	$n^*_{\alpha 2}$	$n^*_{\alpha 3}$	$n^*_{\beta 0}$	$n^*_{\beta 1}$	$n^*_{\beta 2}$	$n^*_{\beta 3}$	$n^*_{\gamma 0}$	$n^*_{\gamma 1}$	$n^*_{\gamma 2}$	$n^*_{\gamma 3}$
0	.2512	.0000	.0000	.0000	.2656	.0000	.0000	.0000	.3127	.0000	.0000	.0000
1.04	.2478	-.0261	.0009	.0000	.2619	-.0244	.0006	.0000	.3082	-.0207	.0003	.0000
2.08	.2377	-.0508	.0034	.0000	.2511	-.0476	.0024	-.0001	.2950	-.0403	.0014	.0000
3.12	.2213	-.0729	.0076	-.0001	.2335	-.0684	.0053	-.0002	.2735	-.0581	.0030	-.0001
4.17	.1995	-.0913	.0132	-.0003	.2102	-.0857	.0093	-.0003	.2449	-.0732	.0052	-.0002
5.21	.1732	-.1051	.0200	-.0003	.1820	-.0987	.0142	-.0004	.2103	-.0849	.0079	-.0002
6.25	.1434	-.1135	.0279	-.0002	.1501	-.1068	.0201	-.0002	.1712	-.0928	.0113	.0001
7.29	.1114	-.1162	.0368	.0005	.1156	-.1094	.0276	.0010	.1288	-.0965	.0162	.0016
8.33	.0782	-.1126	.0471	.0020	.0792	-.1065	.0379	.0043	.0833	-.0959	.0253	.0070
8.54	.0715	-.1111	.0493	.0024	.0717	-.1052	.0405	.0055	.0737	-.0953	.0282	.0090
8.75	.0648	-.1094	.0516	.0029	.0641	-.1036	.0434	.0068	.0638	-.0945	.0317	.0117
8.965	.0580	-.1072	.0541	.0034	.0562	-.1018	.0466	.0084	.0534	-.0936	.0361	.0151
9.17	.0515	-.1049	.0564	.0039	.0487	-.0998	.0500	.0102	.0431	-.0926	.0411	.0193
9.38	.0449	-.1022	.0590	.0049	.0408	-.0975	.0540	.0123	.0320	-.0914	.0475	.0247

TABLE III. CROSS SECTIONS FOR OY (93.5% U-235) AND U-238 AS GIVEN IN LA-1272

A. Oralloy:

Energy group	Energy (Mev)	Velocity cm/shake	σ_a	σ_{in}	σ_f	σ_{tr}
α	1.6- ∞	24	.003	1.233	1.154	3.9
β	0.4-1.6	12	.063	1.032	1.172	5.0
γ	0.0-0.4	6	.219	--	1.402	8.0

B. Uranium-238:

α		.05	1.7	.50	3.9
β		.10	1.5	.05	5.0
γ		.20	--	.00	8.0

C. Oralloy and Uranium-238:

Average neutrons per fission: $\nu = 2.5$

Fission spectrum: $\chi_\alpha = 0.475$, $\chi_\beta = 0.425$, $\chi_\gamma = 0.100$

Inelastic scattering spectrum:

$(\alpha \rightarrow \beta) = 0.333$

$(\alpha \rightarrow \gamma) = 0.667$

$(\beta \rightarrow \gamma) = 1.000$

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where the predicted values of N_0, N_1, n_0^*, and n_1^* are too large by perhaps 5-10%. However, it is likewise true that, near its surface, the neutron distribution in Godiva does not correspond to that of an hypothetical untamped sphere due to mounting structure and the small surface irregularities required for reactivity adjustments.

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B. Predicted Versus Measured Danger Coefficients for Oy (Oralloy) and Tu (Tuballoy)

Figure 1 gives the predicted first order reactivity change $\Delta K_o(\Lambda, Oy)$ (expressed as change in reproduction number, K, per gram atom Oy) for the addition of Oy at the position Λ. Figure 2 gives a similar plot of $\Delta K_o(\Lambda, Tu)$. In these two figures, the total reactivity change is also shown resolved into the contributions associated with the $N_o n_o^*$ distributions (i.e., fission, capture, and inelastic scattering), the $N_1 n_1^*$ distributions (i.e., transport scattering) and the remainder (essentially due to the $N_2 n_2^*$ distributions).

Table IV gives a summary of Godiva Oy and Tu replacement measurements. The quantity, $\Delta K(\Lambda, \vec{\Delta\Lambda}, X)$, listed in the second column of Table IV corresponds to the reactivity difference between an initial configuration in which there is a ½" x ½" cylindrical void space centered at the position Λ (on the equatorial plane) and a final configuration in which there is a ½" x ½" cylindrical sample of material (X) at the position Λ. The reactivity changes were determined in the "cent" unit by means of positive period measurements and the Inhour equation [using the delayed neutron data given by Hughes, et al, Phys. Rev. 73, 111, (1948)]. The third column of Table IV gives the uncorrected danger

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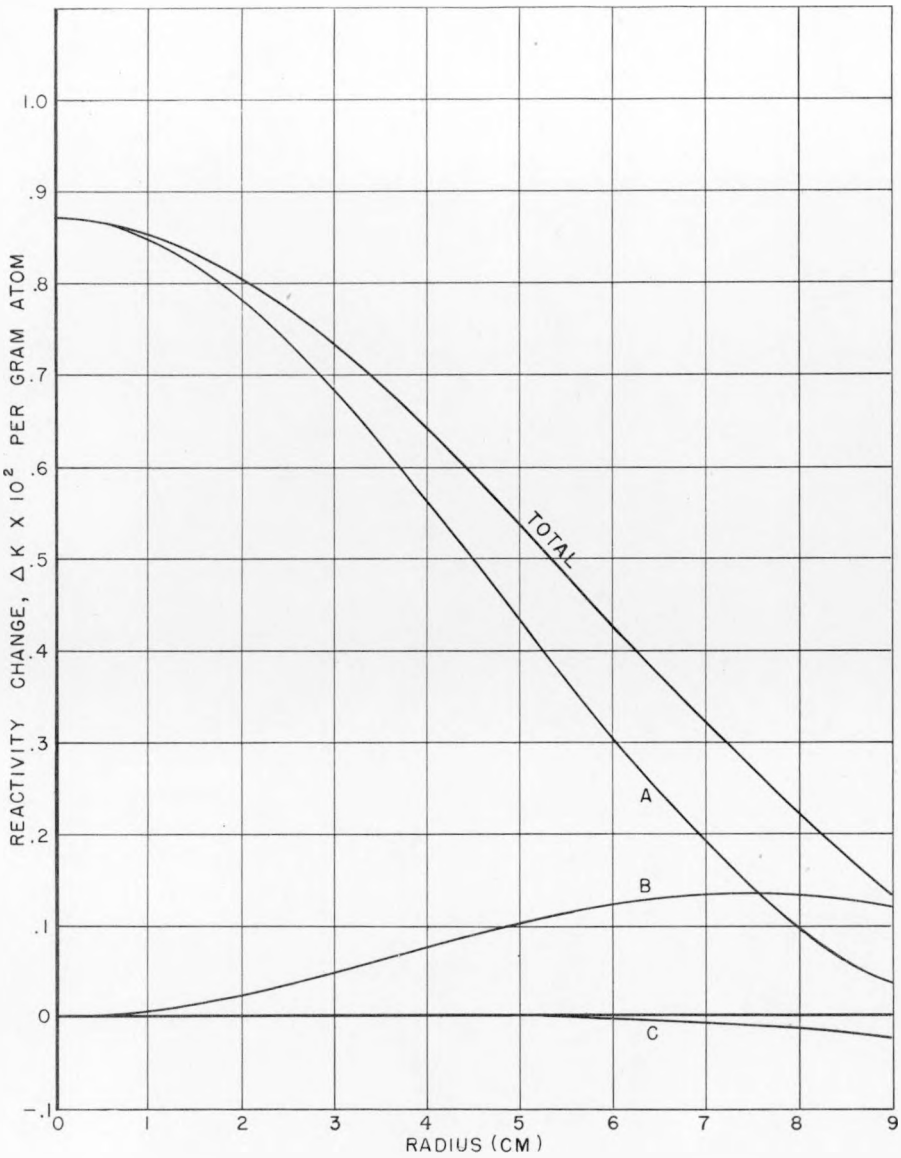

FIG. 1. Predicted reactivity change per gram atom for the addition of Oy (93.5% U-235) at the radial position, R, of a critical untamped Oy sphere. The several curves represent the partial reactivity contributions associated with A) capture, fission, and inelastic scattering; B) transport scattering; and C) the N_2 and N_3 neutron density components.

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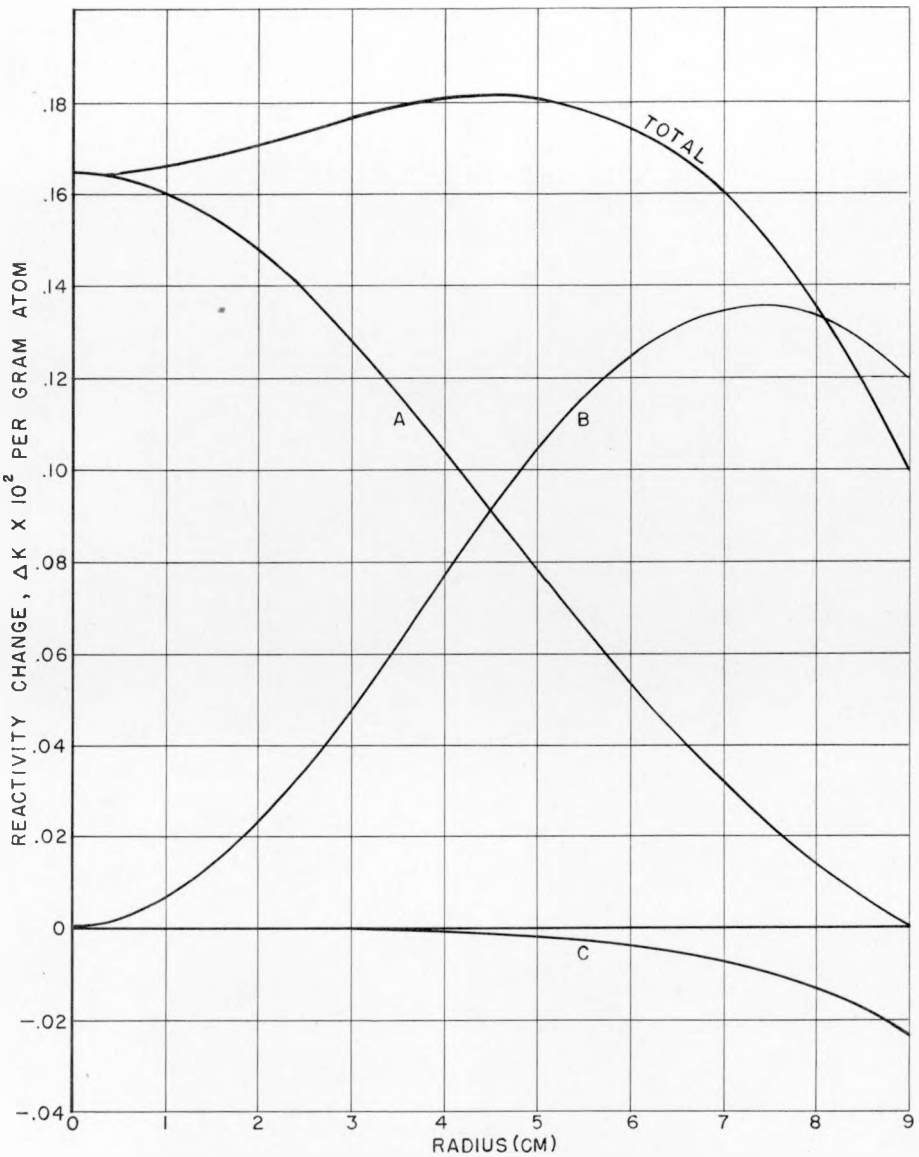

FIG. 2. Predicted reactivity change per gram atom for the addition of Tu (99.3% U-238) at the radial position, R. of a critical untamped Oy sphere. The several curves represent the partial reactivity contributions associated with A) capture, fission, and inelastic scattering, B) transport scattering, and C) the N_2 and N_3 density components.

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TABLE IV.

Radius (cm)	$\Delta K(r, \vec{\Delta r})$ (¢/sample)	$\Delta K(r)$ (¢/gm atom)	$1 \div (1+C)$	$\Delta K_o(r)$ (¢/gm atom)
A. Oy Replacements (30.075 gm sample)				
0.08	17.32	135.5	1.049	142.1
1.03	17.04	133.3	1.047	139.6
3.15	14.02	109.7	1.034	113.4
3.79	12.91	101.0	1.026	103.6
4.42	11.63	91.0	1.017	92.5
4.90	10.76	84.2	1.009	85.0
6.05	8.19	64.1	.986	63.2
6.38	7.49	58.6	.979	57.4
6.71	6.83	53.4	.971	51.9
7.35	5.60	43.8	.955	41.8
7.98	4.24	33.2	.939	31.2
8.14	3.91	30.6	.938	28.7
8.78	2.35	18.4	~ .99	18.2
9.41	1.37	10.7	1.089	11.7
B. Tu Replacements (29.098 gm sample)				
0.08	2.68	21.9	1.13	24.7
2.03	2.82	23.1	1.082	25.0
3.63	3.36	27.5	1.015	27.9
4.42	3.42	28.0	0.984	27.6
4.90	3.55	29.0	.967	28.0
6.05	3.51	28.7	.932	26.7
6.38	3.44	28.1	.923	25.9
7.35	3.27	26.8	.900	24.1
7.98	2.76	22.6	.890	20.1
8.14	2.59	21.2	.888	18.8
8.78	1.57	12.8	~ .99	12.7
9.41	0.95	7.8	1.159	9.0

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coefficient, (reactivity change per sample) ÷ (gram atoms per sample). The fourth column gives the correction factor $1 \div (1 + c)$ by which the observed reactivity change must be multiplied in order to yield the reactivity change per gram atom, $\Delta K_o(\mathcal{R})$ listed in the last column, this corresponding to the infinitesimal replacement. Figure 3 gives a plot of this corrected $\Delta K_o(\mathcal{R}, Oy)$ and $\Delta K_o(\mathcal{R}, Tu)$ data together with a graph of the corresponding predicted functions normalized to agree with the $\Delta K_o(\mathcal{R} = 0, Oy)$ datum.

B.1. Remarks on the Correction Factors

These corrections have been determined from the relations given at the end of the preceding section. As an illustration: for the central Oy slug replacement, the one group relation is,

$$1+C = 1+\ell_o\Delta\{(\nu-1)\sigma_f-\sigma_c\} = 1-\ell_o\{(\nu-1)\sigma_f-\sigma_c\}_{Oy} \qquad \text{(III-1)}$$

Averaging $\{(\nu-1)\sigma_f-\sigma_c\}_{Oy}$ over the computed flux spectrum yields the value 1.766 barns or 0.085 cm^{-1}. The effective neutron path length for the ½" slug is $\ell_o = 0.54$ cm, thus giving $1 + C = 1 - 0.046$, or $[1 \div (1 + C)]_{Oy} = 1.048$.

The difference between the 1.048 and the value 1.049 given in Table IV is due to the difference between one group and three group evaluation of 1 + C. In the former case one learns only that the mean neutron flux in the

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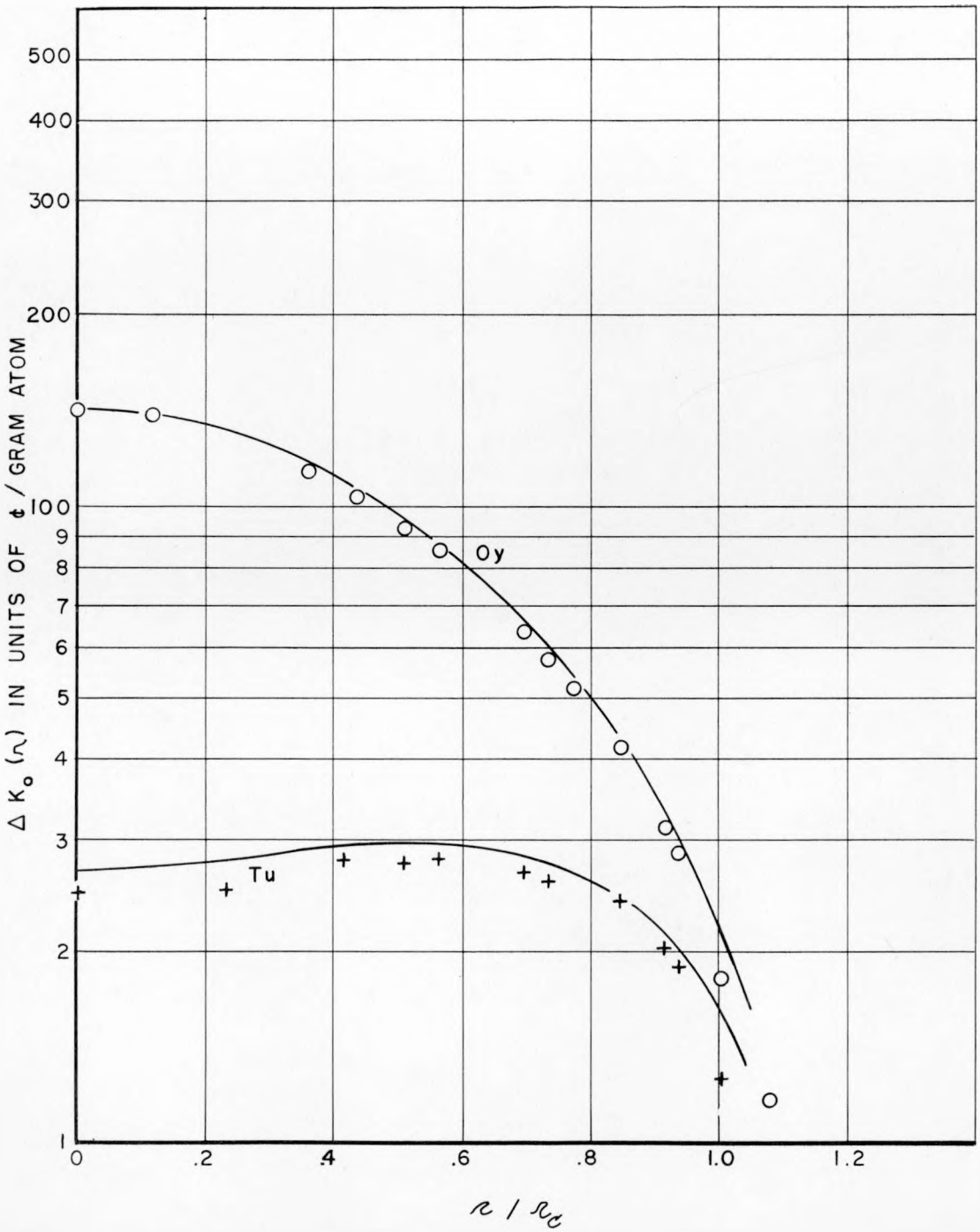

FIG. 3. ΔK_0(Oy) and ΔK_0(Tu) in units of ¢/gm atom. Predicted (solid curves) vs. measured (crosses and circles).

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perturbation region is depressed 4.6% beneath the unperturbed flux. In the latter case, one obtains,

$$N^{p}_{\alpha o} = N_{\alpha o}(1 - 0.054_3)$$
$$N^{p}_{\beta o} = N_{\beta o}(1 - 0.047_9)$$
$$N^{p}_{\gamma o} = N_{\gamma o}(1 - 0.034_6)$$

(mean flux perturbation over a ½" x ½" cyl. central void produced by removal of Oy) (III-2)

$$\rightarrow \quad (N^{p}_{\alpha o} + N^{p}_{\beta o} + N^{p}_{\gamma o}) = (N_{\alpha o} + N_{\beta o} + N_{\gamma o})(1 - 0.046)$$

Thus the central flux is not only depressed but its spectrum is "softened" by the void.

Similarly, for the case of the Tu slug replacements, one obtains,

$$N^{p}_{\alpha o} = N_{\alpha o}(1 - 0.095_6)$$
$$N^{p}_{\beta o} = N_{\beta o}(1 - 0.062_8)$$
$$N^{p}_{\gamma o} = N_{\gamma o}(1 + 0.037_0)$$

(mean flux perturbation over a ½" x ½" central cyl. produced by substitution of Tu for Oy) (III-3)

The relations (III-1) through (III-3) are all consequences of the second order perturbation theory. Experimental determinations of the dependence of $(1 + C)_{Oy}$ on sample size are in excellent agreement with prediction.

"25" and "28" foil activation measurements over the equatorial plane of the Little Eva assembly (Oy core plus ~ 4" Tu

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tamper) with and without Oy or Tu in the central 3/4" diameter sphere, have given the results (LA-1487):

3/4" diameter central sphere Oy → void:

$$\left.\begin{aligned} A_{\text{"25"}}{}^{p} &= A_{\text{"25"}}(1 - 0.05_5) \\ A_{\text{"28"}}{}^{p} &= A_{\text{"28"}}(1 - 0.06_0) \end{aligned}\right\} \text{ measured}$$

3/4" diameter central sphere Oy → Tu: (III-4)

$$\left.\begin{aligned} A_{\text{"25"}}{}^{p} &= A_{\text{"25"}}(1 - 0.04_5) \\ A_{\text{"28"}}{}^{p} &= A_{\text{"28"}}(1 - 0.09_0) \end{aligned}\right\} \text{ measured}$$

From equations (III-2) and (III-3), the central flux values given in Table I and the fission cross sections given in Table III, one obtains, after scaling the perturbation up to a 3/4" diameter sphere:

3/4" diameter central sphere Oy → void:

$$\left.\begin{aligned} A_{\text{"25"}}{}^{p} &= A_{\text{"25"}}(1 - 0.061) \\ A_{\text{"28"}}{}^{p} &= A_{\text{"28"}}(1 - 0.072) \end{aligned}\right\} \text{ predicted}$$

3/4" diameter central sphere Oy → Tu: (III-5)

$$\left.\begin{aligned} A_{\text{"25"}}{}^{p} &= A_{\text{"25"}}(1 - 0.048) \\ A_{\text{"28"}}{}^{p} &= A_{\text{"28"}}(1 - 0.122) \end{aligned}\right\} \text{ predicted}$$

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Since the predicted values apply to the untamped Oy sphere, the agreement between (III-4) and (III-5) regarding total flux depression and spectral change in the perturbed region is considered good.

B.2. Determination of γ_f, the Effective Fraction of Delayed Fission Neutrons

Since the ratio of the "cent" unit to absolute unit of reactivity change is $\gamma_f/100$, a comparison for example of the predicted Oy danger coefficients given in Fig. 1, and the corresponding "cent" values listed in Table IV would yield the value of γ_f, provided, of course, that the predicted danger coefficients were correct. Unfortunately, this proviso is incorrect, quantitatively, due to the limitations not only of the P-3 calculation but also the cross section values. The following three methods for the evaluation of γ_f will rely on Tables I, II, and III, only in a minor way. (Except for detail, these are the same three methods used for a different purpose in LA-1278, "The Inhour Equation for the Tu Tamped Oy Sphere.")

First Method. Use of the Central Oy Replacement Measurement.

The reactivity change for central Oy addition is,

$$\Delta K_o(r,\vec{\Delta r},0_y) = \vec{\Delta r}\,\frac{\left\{\sum N_\alpha \sigma_{f\alpha} \sum \nu\chi_\beta \eta^*_\beta - \sum N_\alpha(\sigma_{f\alpha}+\sigma_{c\alpha})\eta^*_\alpha + \sum\sum N_\alpha \sigma^{in}_{\alpha\beta}(\eta^*_\beta-\eta^*_\alpha)\right\}}{\int d\vec{r}\,\sum\sum N_\alpha \sigma_{f\alpha}\nu\chi_\beta \eta^*_\beta} \qquad \text{(III-6)}$$

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Designating $\Sigma N_{\alpha o}\sigma_{f\alpha}\Sigma\nu\chi_\beta\eta^*_{\beta o}$ by $\nu F(r)$ and adopting the normalization F(r = 0) = 1, the denominator of the right hand side of equation (III-6) is $\nu V\overline{F}$, where V is the core volume and $\overline{F}$ is the value of F averaged over V. A major error in the danger coefficients plotted in Figs. 1 and 2 arises from a predicted critical volume some 8% larger than the Godiva volume. This particular error could have been essentially eliminated by scaling up the cross sections of Table III and would have resulted in an $\sim$ 8% reduction in all danger coefficient values. A second error is due to the P_3 approximation, which gives too high values for neutron flux near the core surface, and thence, too large a value for $\overline{F}$. This error can be avoided by using the experimental value of $\overline{F}$. Thus, the function F(r) is the normalized product of the fission rate $\Sigma N_{\alpha o}\sigma_{f\alpha}$, measurable by "25" foil activation, and fission neutron effectiveness $\Sigma\chi_\beta\eta^*_{\beta o}$, measurable by mock fission source multiplication. Figure 4 gives a plot of experimental values of $\Sigma N_{\alpha o}\sigma_{f\alpha}$ and $\Sigma\chi_\beta\eta^*_{\beta o}$, together with experimental values of "28" foil activation and the corresponding predicted (P_3) functions. (The inclusion of the "28" foil activation serves to indicate general consistency of the data.) One thus obtains,

$$\overline{F}_{experimental} = \overline{F}_{predicted} \times (1 - 0.046 \pm 0.02)$$
$$= 0.2803 \times 0.954 = 0.267 \pm 2\% \qquad \text{(III-7)}$$

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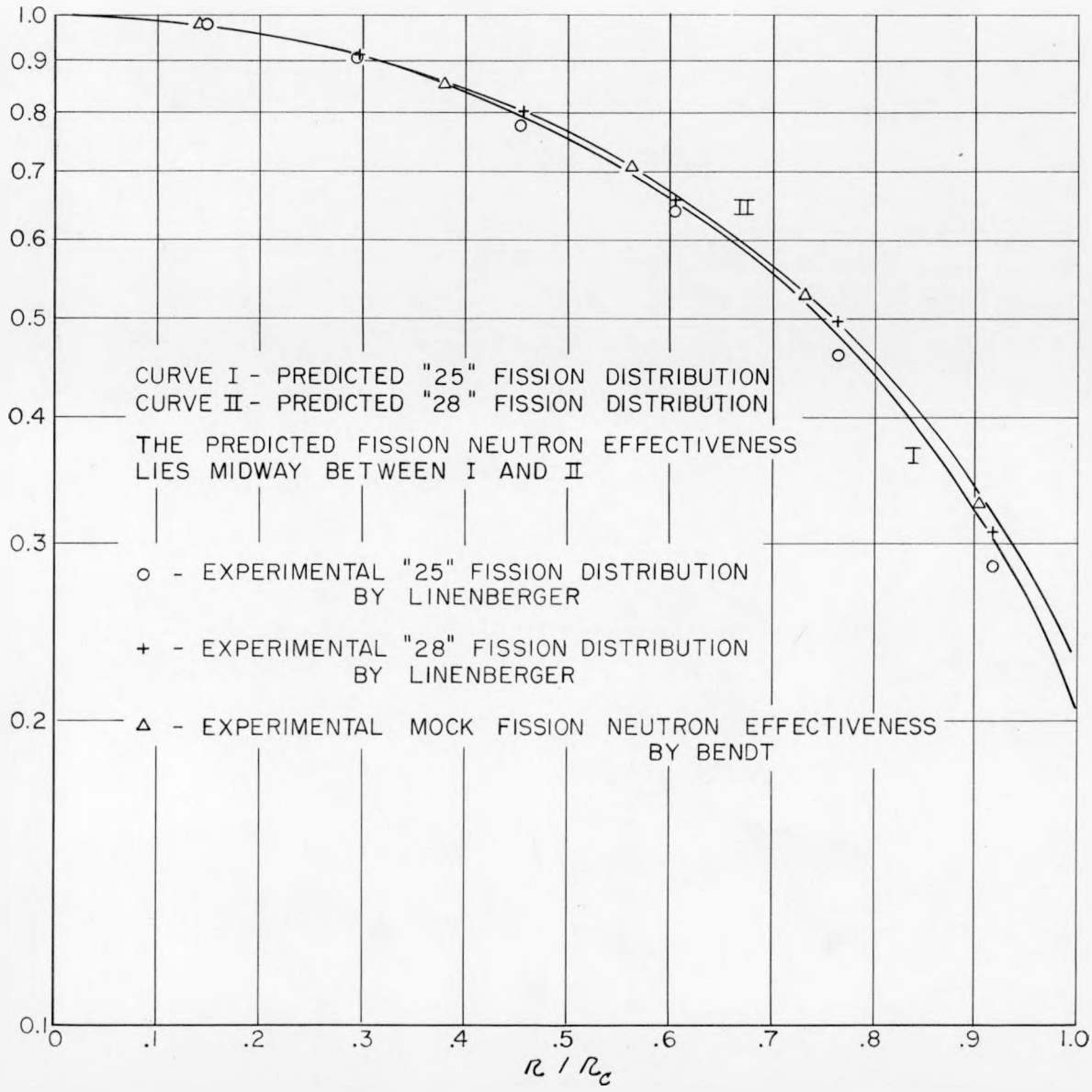

FIG. 4. Flux and adjoint radial distributions in Godiva.

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One may define the quantity "α" by,

$$\Delta K_o(r=0,0y) \equiv \frac{\vec{\Delta r}}{\sqrt{F}} \quad \frac{(\nu - 1 - "\alpha")}{\nu} \qquad \text{(III-8)}$$

which using equation (III-6) gives,

$$"\alpha" = \frac{\sum N_{\alpha o}(\sigma_{f\alpha} + \sigma_{c\alpha})\eta^*_{\alpha o} - \sum\sum N_{\alpha o}\sigma_{f\alpha}\chi_\beta \eta^*_{\beta o} - \sum\sum N_{\alpha o}\sigma^{in}_{\alpha\beta}(\eta^*_{\beta o} - \eta^*_{\alpha o})}{\sum N_{\alpha o}\sigma_{f\alpha} \sum \chi_\beta \eta^*_{\beta o}} \qquad \text{(III-9)}$$

Were it not for the energy dependence of η^*_o, "α" would be equal to the ratio of the capture to fission cross section σ_c / σ_f, a fairly small number. Actually, the increased effectiveness of the lower energy neutrons and the inelastic scattering make "α" $< \sigma_c / \sigma_f$. From Tables I, II, and III, one obtains,

$$"\alpha"_{pred.} = 0.032 \qquad \text{(III-10)}$$

Thus, with the values $(\vec{\Delta r})$ = 12.55 ± 0 cm^3/gram atom, V = 2789 ± 0 cm^3, ν = 2.5, and $\Delta K_o(r=0,0y)$ = 142.1 ± 1.0 ¢/gm atom.

Equation (III-8) becomes,

$$[\gamma f]_I = 0.0069_7 \pm 0.0001_6 \qquad \text{(III-11)}$$

The ± 0.0001_6 includes only experimental uncertainties.

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Second Method. Use of the Difference Between Oy and Tu Replacement Measurements.

This method assumes, as in Table III, that transport cross sections of Oy and Tu are identical and hence that the reactivity change accompanying substitution of Oy for Tu at any position r is due purely to fission, capture, and inelastic scattering. Again, if η_o^* were independent of energy this reactivity change would have the simple form,

$$\left[\Delta K_o(r,Oy) - \Delta K_o(r,Tu)\right] \sim \frac{\left\{\left([\nu-1]\sigma_f - \sigma_c\right)_{Oy} - \left([\nu-1]\sigma_f - \sigma_c\right)_{Tu}\right\}}{(\nu\sigma_f)_{Oy}} \frac{F(r)\vec{\Delta r}}{\bar{F}V} \qquad \text{(III-12)}$$

which, if summed over the core volume, would yield,

$$\sum_{core}\left[\Delta K_o(r,Oy) - \Delta K_o(r,Tu)\right] \sim \left(\frac{\nu-1-\sigma_c/\sigma_f}{\nu}\right)_{Oy}\left[1 - \frac{\{(\nu-1)\sigma_f - \sigma_c\}_{Tu}}{\{(\nu-1)\sigma_f - \sigma_c\}_{Oy}}\right] \qquad \text{(III-13)}$$

In the actual case of energy dependent η_o^*, one may define the parameter β by,

$$\sum_{core}\left[\Delta K_o(r,Oy) - \Delta K_o(r,Tu)\right] \equiv \frac{\nu-1-\beta}{\nu}\left[1 - \frac{\Delta K_o(r=0,Tu)}{\Delta K_o(r=0,Oy)}\right] \qquad \text{(III-14)}$$

and obtain the predicted value, $\beta = 0.018$. That β must be smaller than $(\sigma_c/\sigma_f)_{Oy}$ is due to an over-compensation of inelastic scattering reactivity contribution brought about by use of the term $\Delta K_o(r=0,Tu) \,/\, \Delta K_o(r=0,Oy)$ rather than

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$[\{\nu-1\}\sigma_f-\sigma_c]_{Tu}$ / $[(\nu-1)\sigma_f-\sigma_c]_{Oy}$ as in (III-13). The data listed in Table IV give,

$$\sum_{core}\left[\Delta K_o(r,Oy)-\Delta K_o(r,Tu)\right] = 12350¢ - 5220¢ = 7130 \pm 80¢ \qquad \text{(III-15)}$$

which, with the observed value $\left[\Delta K_o(r=0,Tu)/\Delta K_o(r=0,Oy)\right] = 0.174$ and the predicted value of β yields,

$$[\gamma f]_{II} = 0.0068_7 \pm 0.0001_0 \qquad \text{(III-16)}$$

Again, the cited uncertainty of $\pm\ 0.0001_0$ includes only experimental causes. Although avoiding the need for fission rate and neutron effectiveness distributions, there is the disadvantage in the Second Method of the assumption of equal Oy and Tu transport cross sections.

Third Method. Mass Increment, ΔM_c, Between Delayed and Prompt Critical.

Although the value of $\Delta K_o(r=R_c,Oy)$ may be read from Fig. 4, the surface irregularities of Lady Godiva make it preferable to take advantage of the scaling law between core density and critical dimensions which in terms of reactivity change have the expression (LA-1251),

$$\frac{1}{V}\cdot\int_{core} d\vec{r}\ \Delta K_o(r,Oy) = 3\Delta K_o(r=R_c,Oy) \qquad \text{(III-17)}$$

From the value $\Sigma\,\Delta K_o(r,Oy) = 12350¢$ given in (III-15) and

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$V = 2789\ cm^3$, one obtains,

$$\Delta K_0(r=R_c, Oy) = 18.5\,¢/gm.\,atom \qquad \text{(III-18)}$$

This value for Oy surface mass gives 1.27 kg Oy per 100¢ or

$$\Delta M_c / M_c = 0.0243 \qquad \text{(III-18a)}$$

We shall use, initially, the one-group extrapolated end-point method for determining the dependence of ν_{cr} (number of neutrons per fission required for critical) on sphere size. With the standard notation f = excess neutrons per collision and α = capture to fission ratio, one has,

$$\frac{\delta\nu_{cr}}{\nu_{cr}} = \frac{\nu_{cr}-1-\alpha}{\nu_{cr}}\,\frac{\delta f_{cr}}{f_{cr}} = \frac{\nu_{cr}-1-\alpha}{\nu_{cr}}\left(\frac{M_c}{f_{cr}}\frac{df_{cr}}{dM_c}\right)\frac{\delta M_c}{M_c}$$

and the relation between $[\gamma f]$ and mass increment ΔM_c becomes,

$$[\gamma f] = \frac{\nu-1-\alpha}{\nu}\left(-\frac{1}{3}\,\frac{R_c}{f_{cr}}\frac{df_{cr}}{dM_c}\right)_{f_{cr}=f}\frac{\Delta M_c}{M_c} \qquad \text{(III-19)}$$

where the log derivative term is evaluated by means of the equations of the end-point method; namely,

$$\left.\begin{aligned} 1+f_{cr} &= \frac{k/\sigma}{\tan^{-1} k/\sigma} \\ k\left(R_c + \frac{.71}{[1+f_{cr}]\sigma}\right) &= \pi \end{aligned}\right\} \rightarrow \left(-\frac{1}{3}\,\frac{d\log f_{cr}}{d\log R_c}\right)_{f_{cr}=.34} = 0.481 \qquad \text{(III-20)}$$

The one-group values $f = 0.34$ and $\alpha = 0.074$ follow from Tables I, II, and III, and (III-19) becomes,

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$$[\gamma f] = 0.274 \frac{\Delta M_c}{M_c} \quad \text{(by one-group extrapolated end-point method)} \qquad \text{(III-21)}$$

The log derivative term is a slowly varying function of f_{cr}, a 7% change in the latter resulting in a 1% change in the former. The evaluation (III-21) is thus rather insensitive to the cross-sectional uncertainties. One feature not taken into account is that as surface mass is added, leakage is reduced and the competing process of inelastic scattering is enhanced. This softens the flux spectrum and increases the values of the effective one-group cross sections. Thus, the addition of surface mass ΔM_c should give, by reason of this spectral effect, a higher reactivity change $-\Delta\nu_{cr}/\nu_{cr}$, than predicted by a one-group method. By means of the three-group cross sections given in Table III, one obtains,

$$[\gamma f] = 0.280 \frac{\Delta M_c}{M_c} \quad \text{(by three-group extrapolated end-point method)} \qquad \text{(III-22)}$$

The relations (III-18a) and (III-22) give,

$$[\gamma f]_{III} = 0.0068_0 \qquad \text{(III-23)}$$

Of the three methods for evaluating γf, only the last relies on the calculation of a major quantity, viz., $\left(-\frac{R}{f}\frac{df}{dR}\right)$, a quantity intimately connected with the relative reactivity

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contributions of fission and scattering. Despite the complexity of the scattering process in Oy, the error in the value of $\left(-\frac{R}{f}\frac{df}{dR}\right)$ obtained by the extensively checked (LA-53 and LA-53A) end-point method is presumably small. In the cases of the first two methods, the quantities "α" and β, although small, are poorly determined percentage-wise mainly because of uncertainties in the inelastic scattering cross sections. An indication of error in the tabulated inelastic scattering cross sections of U-238 is given by central Godiva data on a) U-238 to Oy fission ratio, b) U-238 capture to U-238 fission ratio, and c) U-238 to Oy danger coefficient ratio.

	$\sigma_f(28)/\sigma_f(oy)$	$\sigma_c(28)/\sigma_f(28)$	$\Delta K_o(28)/\Delta K_o(oy)$
Predicted	0.146	0.64	0.189
Observed	0.167*	0.49*	0.168

* communicated by G. A. Linenberger (III-24)

By means of (III-24) one finds that the reactivity contribution to $\Delta K_o(28)$ due to inelastic scattering is approximately 50% of the predicted contribution. Since the reactivity change due to inelastic scattering is mainly associated with that part which scatters neutrons into the low energy group, this is in qualitative agreement with results (a larger magnitude of inelastic scattering in the high energy group but smaller neutron transfer to the low energy group than indicated by

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Table III) obtained from measured leakage spectra of small Tu spheres containing central mock fission sources (LA-1497; P. Bendt and J. E. Bendt). If the reactivity contribution to $\Delta K_o(Oy)$ due to inelastic scattering has been similarly overestimated (as is likely), then the value of γf given for each of the three methods is too large, and by as much as $\sim$ 2%. In view of these uncertainties, the summary of γf evaluations becomes,

$$[\gamma f]_{Av.} = 0.0068 \pm 0.0002 \qquad \text{(III-25)}$$

Note: The relative neutron effectivenesses in the three different energy groups α , β , and γ are computed to be,

$$\gamma_\alpha = \frac{\int d\vec{r}\, \Sigma N_{\rho o} \sigma_{f\beta} n^*_{\alpha o}}{\int d\vec{r}\, \Sigma \Sigma N_{\rho o} \sigma_{f\beta} \chi_\gamma n^*_{\gamma o}} = 0.963; \quad \gamma_\beta = 1.007; \quad \gamma_\gamma = 1.148$$

Since the measurements of Hughes et al (Phys. Rev. 73, 111, 1948) place the mean energy of the delayed neutrons in the 400-600 kev region, one would expect the γ for delayed neutrons to be intermediate between γ_β and γ_γ . A value γ = 1.05 $\pm$ 0.05 gives f = 0.0065 $\pm$ 0.0005 which is somewhat lower than the Hughes et al value of $0.0075_5 \pm 0.0005$ associated with thermal fission.

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C. One-Group Neutron Creation Cross Sections, $[(\nu-1)\sigma_f-\sigma_c]$, and Transport Cross Sections, σ_{tr}

C.1. Relative values of $[(\nu-1)\sigma_f-\sigma_c]$ for U-233, U-235, and Pu-239 in the central Topsy spectrum, and for U-235 and Pu-239 in the central Godiva spectrum.

With the fast spectra of the Topsy and Godiva assemblies, the central reactivity change contributions due to inelastic scattering and absorption are of the same order of magnitude for many of the non-fissionable elements. However, for the above fissionable elements, the reactivity contribution from neutron production far outweighs the inelastic scattering effect and central danger coefficient ratios correspond quite closely to $[(\nu-1)\sigma_f-\sigma_c]$ ratios. The present purpose is to illustrate how close this correspondence is by evaluating the effect of inelastic scattering (or more exactly, the net downgrading of the scattered neutron spectrum compared to the spectrum incident on the sample material) on the basis of estimated flux and adjoint distributions.

The one-group cross section $[(\nu-1)\sigma_f-\sigma_c]$ is defined in the customary way as an average over the flux spectrum:

$$[(\nu-1)\sigma_f-\sigma_c] \equiv \int[(\nu-1)\sigma_f(E)-\sigma_c(E)]N(E)dE \Big/ \int N(E)dE \qquad \text{(III-26)}$$

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Since the fission neutron spectra for uranium and plutonium are essentially identical, then, with the normalization $\int \chi(E)\,\eta^*(E)\,dE = 1$, the relation between central danger coefficient (reactivity change per gram atom) and cross sections may be expressed as,

$$\Delta K_o(r=0,x) = \frac{A}{\int N(E)\,\nu\sigma_f(E)\,dE\,d\vec{r}} \left\{ \int [(\nu-1)\sigma_f - \sigma_c]_x N(E)\,dE - \int [\sigma_f+\sigma_c]_x [\eta^*(E)-1]\,dE\,N(E) + \int \sigma_x^{in}(E\to E')\,N(E)\,[\eta^*(E')-\eta^*(E)]\,dE\,dE' \right\}$$

(III-27)

where, if the cross sections of (X) are expressed in barns, the constant A is Avogadros Number divided by 10^{24}. For the ratio of danger coefficients, one then has,

$$\frac{\Delta K_o(r=0,x)}{\Delta K_o(r=0,y)} = \frac{[(\nu-1)\sigma_f - \sigma_c]_x\,[1+C_x]}{[(\nu-1)\sigma_f - \sigma_c]_y\,[1+C_y]} \qquad \text{(III-28)}$$

with,

$$C_x = \frac{\int \sigma_x^{in}(E\to E')\,[\eta^*(E')-\eta^*(E)]\,dE\,dE' - \int [\sigma_f+\sigma_c]_x\,N(E)\,[\eta^*(E)-1]\,dE}{\int [(\nu-1)\sigma_f - \sigma_c]_x\,N(E)\,dE}$$

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Table V lists the measured danger coefficient values, the computed correction values C_x, and the resultant $[(\nu-1)\sigma_f - \sigma_c]$ ratios. For the evaluation of the C_x it has been assumed that the fission cross section of Pu-239 is energy independent, that the fission cross section of U-233 has the same energy dependence as U-235 (i.e., $\sigma_f(23,E) / \sigma_f(25,E)$ = constant) and that the spectrum of inelastically scattered neutrons is the same for U-233, U-235, and Pu-239; namely, that given in Table III. As was mentioned earlier, there is some experimental evidence that the inelastic scattering of neutrons to the low energy group is not as strong as indicated by Table III and that, consequently, the computed C_x values are too large.

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TABLE V.

$[(\nu-1)\sigma_f - \sigma_c]$ RATIOS FROM CENTRAL MATERIAL REPLACEMENT MEASUREMENTS

A. Topsy: Oy Core + 8½" Tu Reflector

Material	$\Delta K_0(\lambda=0, x)$ (¢/gm atom)	$\frac{\Delta K_0(x)}{\Delta K_0(U\text{-}235)}$	$C_x^{(*)}$	$\frac{[(\nu-1)\sigma_f-\sigma_c]_x}{[(\nu-1)\sigma_f-\sigma_c]_{U\text{-}235}}$
Oy(95.4%U-235)	199			
Tu	27			
"U-233"(98.2%U-233)	346			
U-235	(206)	1.00	.009	1.00
U-233	(352)	1.71 ± 0.01	- 0.031	1.78
Pu-239	397	1.93 ± 0.01	- 0.012	1.97

B. Godiva: Bare Oy

Oy(93.7%U-235)	142			
Tu	25			
U-235	(149)	1.00	.029	1.00
Pu-239	287	1.93 ± 0.01	- 0.001	1.98

(*) Assumed that σ^{in}(U-235): σ^{in}(U-233): σ^{in}(Pu-239) = 1 : 2/3 : 2/3

C. 2. Relative values of transport cross sections, σ_{tr}, from material replacement measurements.

Figure 5 gives plots of $\Delta K_o(r,x)$ measured in Lady Godiva for gold (a strong neutron absorber), copper (a typical medium Z element), hydrogen (a strong neutron energy degrader), and carbon (a weak neutron energy degrader). With all these elements, the primary reactivity contribution for replacements near the core surface derives from transport scattering. With the exception of the fissionable and hydrogenous materials, $\Delta K_o(r,x)$ has a maximum in the vicinity of $r \simeq 3/4\, r_{crit.}$. From Figs. 1 or 2, it can be seen that a maximum in this region is expected for a pure scatterer, and that this effect is almost purely associated with transport averaged cross sections.

The one-group transport cross section, $\sigma_{tr}(x)$, is here defined as the average of the energy dependent $\sigma_{tr}(x,E)$ over the $N_1 n_1^*$ spectrum:

$$\sigma_{tr}(x) \equiv \int \sigma_{tr}(x,E)\, N_1(E)\, n_1^*(E)\, dE \Big/ \int N_1 n_1^*\, dE \qquad \text{(III-29)}$$

If one resolves the total reactivity change, $\Delta K_o(r,x)$ into the fission, capture, and inelastic scattering part, $\Delta K_{oa}(r,x)$, (i.e., that part proportional to the $N_o n_o^*$ distribution), and the scattering part, $\Delta K_{os}(r,x)$, then for

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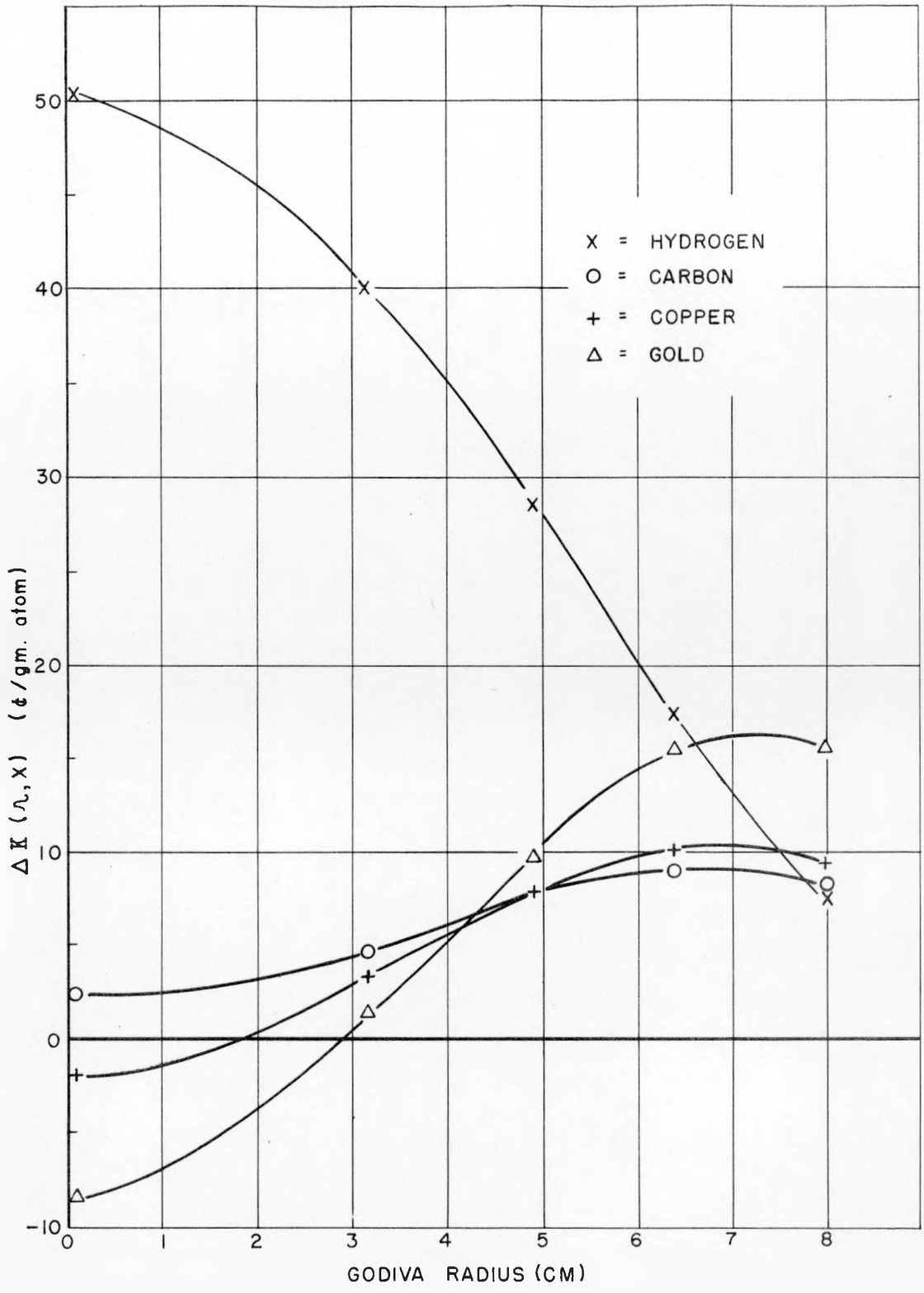

FIG. 5. Material replacement data taken in Godiva for hydrogen, carbon, copper, and gold.

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values of r in the vicinity of $3/4\, r_{crit.}$, one has,

$$\Delta K_o(r,X) = \Delta K_{oa}(r,X) + \Delta K_{os}(r,X)$$
$$= \Delta K_{oa}(r,X) + \sigma_{tr}(X)\, F(r) \qquad \text{(III-30)}$$

where the function $F(r)$ is independent of the material (X). With a one-group treatment, the predicted radial dependence of $\Delta K_{oa}(r,X)$ is the same as those of the two experimentally available functions, 1) the flux squared distribution, $N_o^2(r)$, as measurable by "25" foil activation, and 2) the $\Delta K_o(r,Oy) - \Delta K_o(r,Tu)$ distribution (again assuming $\sigma_{tr}(Oy) = \sigma_{tr}(Tu)$. With the three-group treatment for Godiva, the flux squared distribution (or more accurately the squared "25" fission rate distribution) falls off slightly more rapidly with increasing radius than does the Oy-Tu danger coefficient difference. This feature is also observed experimentally, one having,

$$\frac{\Delta K_o(r \sim r_c, Oy\text{-}Tu)}{\Delta K_o(r=0, Oy\text{-}Tu)} \simeq 1.1\, \frac{N_o^2(r \sim r_c)}{N_o^2(r=0)} \qquad \left(\begin{array}{l}\text{predicted and}\\ \text{observed}\end{array}\right) \qquad \text{(III-31)}$$

For the evaluation of transport cross sections, we shall use in conjunction with (III-30) the recipe,

$$\frac{\Delta K_{oa}(r,X)}{\Delta K_{oa}(r=0,X)} \equiv \frac{\Delta K_o(r,Oy\text{-}Tu)}{\Delta K_o(r=0,Oy\text{-}Tu)} \equiv G(r) \qquad \text{(III-32)}$$

The transport cross section $\sigma_{tr}(X)$ relative to that of Oy is then expressed as,

$$\frac{\sigma_{tr}(X)}{\sigma_{tr}(Oy)} = \frac{\Delta K_o(r,X) - \Delta K_o(r=o,X)\,G(r)}{\Delta K_o(r,Oy) - \Delta K_o(r=o,Oy)\,G(r)} \qquad \text{(III-33)}$$

Referring to Tables I and II, it is seen that although the total flux and adjoint functions have very similar radial dependences, the small quantities, such as $[n^*_{o\beta} - n^*_{o\alpha}]$, which enter in the inelastic scattering or energy degradation part of reactivity change, fall off much more rapidly with increasing radius. Thus, in the case of hydrogen, for example, where the central danger coefficient is primarily determined by the neutron energy degradation, it is known that $\Delta K_{oa}(r,H) < \Delta K_{oa}(r=o,H)\,G(r)$ and that, consequently, any value of $\sigma_{tr}(H)/\sigma_{tr}(Oy)$ obtained from (III-33) will be too low.

Table VI lists values of $\Delta K_o(r, X = H, C, Au, Cu)$ and their reduction to $\sigma_{tr}(X)/\sigma_{tr}(Oy)$ ratios by means of (III-33) for several positions of r (the 6.38 and 7.98 cm positions giving the smallest experimental uncertainty). Since the effective transport cross section of Oy in the Godiva spectrum is $\simeq$ 5.2 barns, it is seen that the predicted hydrogen transport cross section $\sigma_{tr}(H) \simeq 0.2 \times 5.2 \simeq 1$ barn is indeed much too low. Reanalysis of the hydrogen reactivity

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measurements by means of the relations (II-17) and (II-17a) indicates $\sigma_{th}(H) \simeq 0.38 \times 5.2 \simeq 2.0$ barns, which is about the expected value. With the exception of hydrogen and fissionable elements, the term $\Delta K_o(r=o,x)\, G(r)$ is not only a small correction term but quite accurately represents the nonscattering reactivitv contributions.

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TABLE VI

RELATIVE VALUES OF TRANSPORT CROSS SECTIONS
FROM MATERIAL REPLACEMENT MEASUREMENTS
AND EQUATION (III-33)

Radius r (cm)	0.08	3.15	4.90	6.38	7.98
$\Delta K_o(r,H)$	50.3	40.1	28.5	17.3	7.3
$\Delta K_o(r,C)$	2.4	4.6	7.8	8.9	8.2
$\Delta K_o(r,Cu)$	- 1.8	3.3	7.9	10.1	9.4
$\Delta K_o(r,Au)$	- 8.4	1.3	9.6	15.3	15.5
$G(r)$	1.0	0.74_0	0.48_6	0.26_8	0.09_5
$\sigma_{tr}(H)/\sigma_{tr}(Oy)$	--	0.35	0.25	0.20	0.14
$\sigma_{tr}(C)/\sigma_{tr}(Oy)$	--	0.34	0.41	0.43	0.45
$\sigma_{tr}(Cu)/\sigma_{tr}(Oy)$	--	0.56	0.55	0.55	0.54
$\sigma_{tr}(Au)/\sigma_{tr}(Oy)$	--	0.91	0.86	0.91	0.92

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