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CP-10 SHIELD DESIGN

by

M. Grotenhuis and J. W. Butler

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CP-10 SHIELD DESIGN

by

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CP-10 SHIELD DESIGN

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ABSTRACT

The CP-10 shield and the calculations used in designing it are described.

The shield from the core outward consists of 2 feet of graphite, a sheet of boral, a 6-inch iron thermal shield, and 7.25 feet of ordinary concrete. The radiation escaping is calculated to be 5 mr/hr of gamma photons and 10^{-2} fast neutrons/sq cm-sec.

The top plug, through which all loading and unloading operations are carried out, consists of about 6 feet of laminated iron, lead, and pitch. Immediately above the active core are 15 inches of heavy water and a 12.5-inch aluminum grid filled with a 38.5 per cent pitch. The radiation escaping is calculated to be 4 mr/hr of gamma-rays and 10^{-3} fast neutrons/sq cm-sec.

Included also are the calculations involving heating of the boral and the thermal shield, and the calculation of the shield thickness for the fuel element removal coffin.

I. INTRODUCTION

Calculations are performed to design a shield for the proposed CP-10 reactor. The primary design criterion is low cost; however, more efficient and expensive materials were chosen for the top plug construction in order to save weight and bulk for operational reasons. The particular materials chosen for this study are portland concrete for the radial shield and a laminated iron and pitch construction for the top plug. Permissible radiation levels[1] are the currently accepted values of 0.1r per day for gamma radiation and 200 neutrons/sq cm-sec for fast neutrons.

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[1] "Radiation Hazard Control Manual for Procedure," ANL-4197, March, 1948, p. 3.

II. DESCRIPTION OF REACTOR CORE AND SHIELDING

A. Core

The reactor core is a cylindrical aluminum tank 6 feet high and 8 feet in diameter. The tank contains 40 per cent Bi, 58 per cent D_2O, 2 per cent Al structure, and 8.2 kg of fuel. It was assumed for shielding calculations that the reactor is to be operated at 10 mw.

B. Top Plug

The top plug, shown in Figure 1, consists of laminated layers of iron, lead, and pitch. Boral sheets are placed before the aluminum pitch grid and before each of the two iron layers nearest the core.

C. Radial Shield

The radial shield, shown in Figure 2, consists of 0.25 inch of boral, 6 inches of iron thermal shield, and 7.25 feet of ordinary concrete. The concrete is assumed to have a density of 2.33 gm/cu cm and a composition as listed in Table I.

Table I

COMPOSITION OF PORTLAND CONCRETE[2]

Element	atoms/cu cm x 10^{-24}
Fe	0.000078
H	.0029
O	.0433
Mg	.000121
Ca	.0087
Si	.00942
Al	.000266
S	.000036
C	.00653

[2]H. P. Sleeper, "A Critical Review of ORNL Shield Measurements: Neutron Attenuation," ORNL-436, Dec., 1949, p. 20.

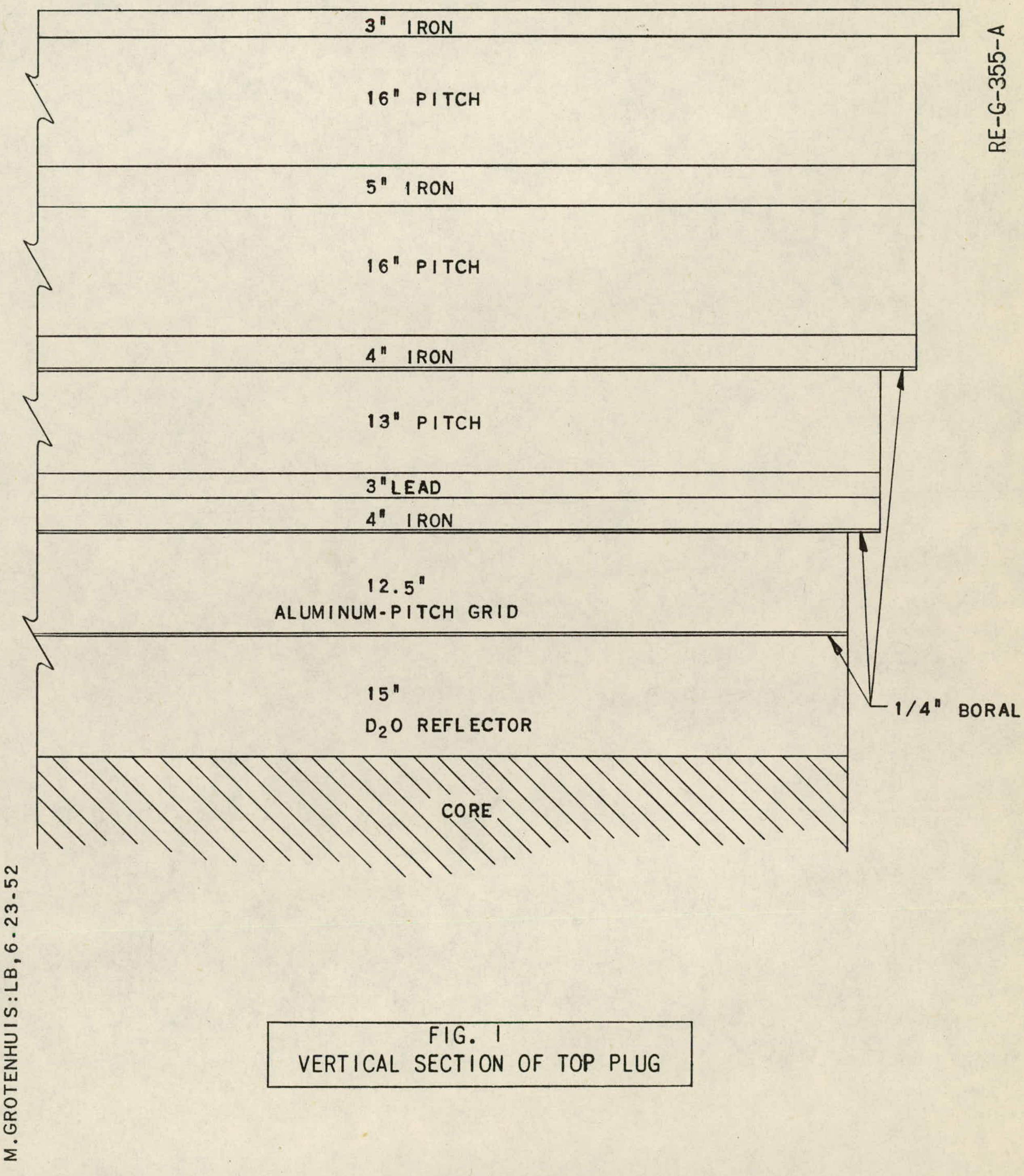

FIG. I
VERTICAL SECTION OF TOP PLUG

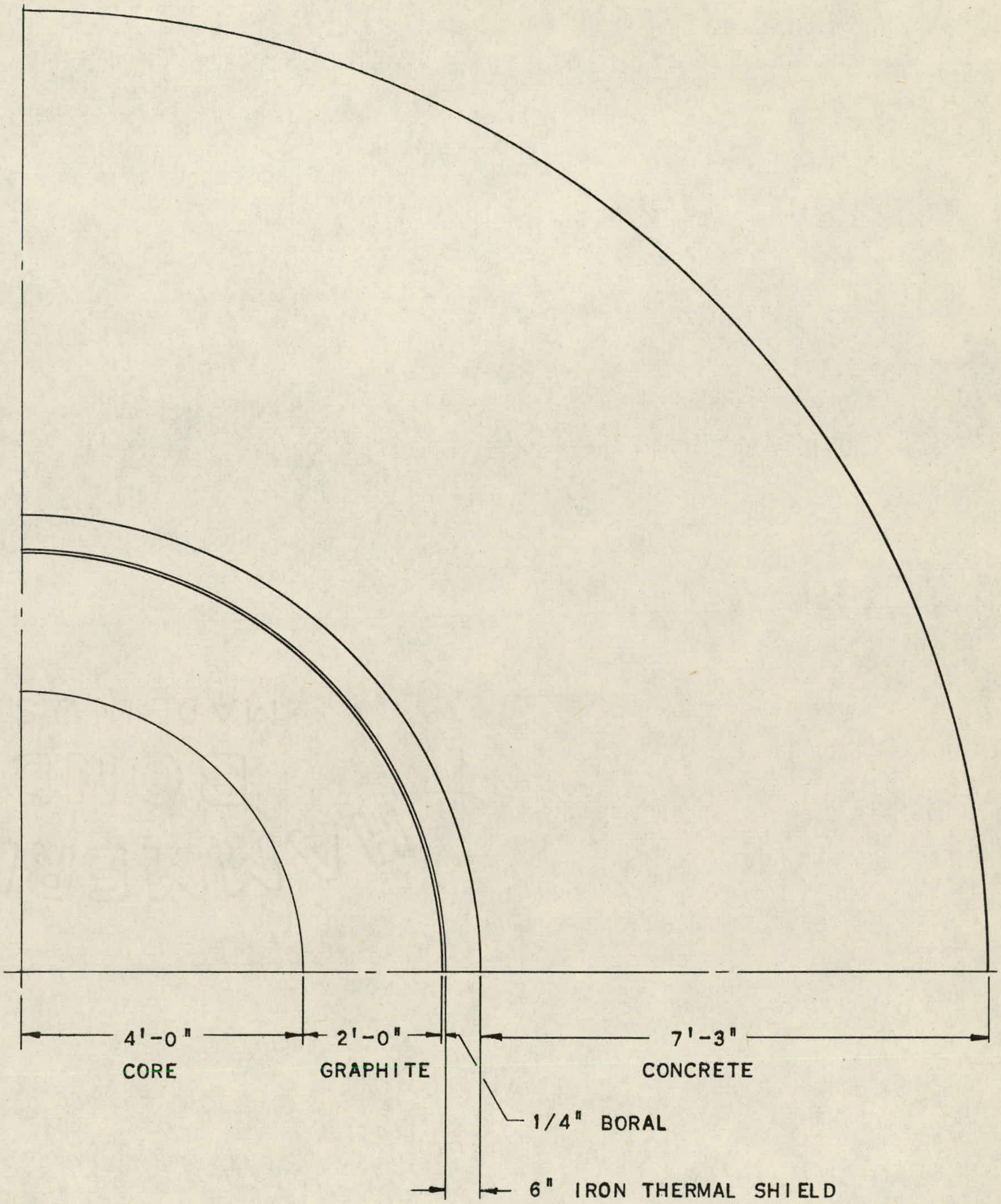

FIG. 2
QUARTER SECTION OF CP-10
SHOWING RADIAL SHIELD

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III. DESIGN CALCULATIONS

A. Top Plug

1. Fast Neutron Flux

To calculate the fast neutron flux through the top plug it is assumed that the pitch can be replaced by an equal volume of water and graphite, and that the neutrons are attenuated according to the single collision theory presented by Welton and Albert.[3] Published[4] removal cross sections, necessary for the application of this theory, are utilized when possible. The removal cross section for graphite is assumed not to differ from that of B_4C, and the removal cross sections of D_2O and Al are estimated as three-fourths of their respective total cross sections in the region of 2 mev.

The expression for the fast neutron flux along the axis of a cylindrical source viewed endwise (Figure 3) is

$$\Phi_f(x) = \int dV \frac{q}{4\pi r^2} N(\rho) e^{-\sigma_c (r-\rho)} \qquad (1)$$

which can be bracketed between the two following expressions.

$$\Phi_f(x) \leqslant \frac{q}{2\sigma_c} \int_0^{\phi_1} d\phi \sin \phi N(\rho) \left\{ 1 - e^{-\sigma_c h \sec \phi} \right\} \qquad (2)$$

$$\geqslant \frac{q}{2\sigma_c} \int_0^{\phi_2} d\phi \sin \phi N(\rho) \left\{ 1 - e^{-\sigma_c h \sec \phi} \right\} \qquad (2A)$$

In ordinary cases these upper and lower estimates are quite close. Throughout this calculation the upper estimate is used.

The definitions and values of the various constants are:

q = source strength = 1.13×10^{11} neutrons/cu cm-sec

σ_c = core removal cross section = 0.107 cm^{-1}

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[3] R. D. Albert and T. A. Welton, "A Simplified Theory of Neutron Attenuation and Its Application to Reactor Shield Design," WAPD-15, November, 1950.

[4] E. P. Blizard and T. A. Welton, "The Shielding of Mobile Reactors (Part I)," Reactor Science and Technology, I, 27, TID-73, January, 1952.

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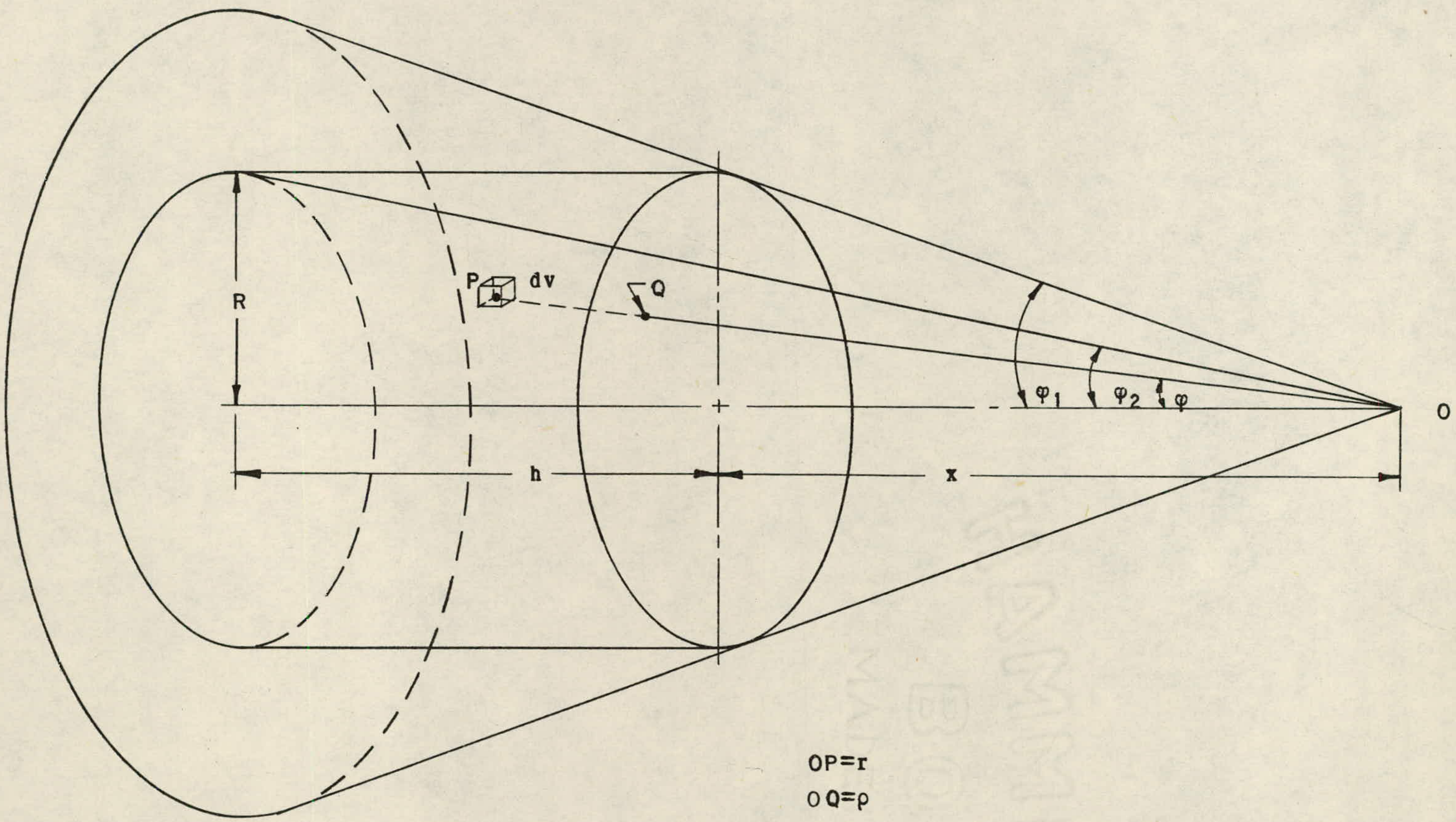

FIG. 3
GEOMETRY FOR CALCULATION OF THE RADIATION FROM A CYLINDRICAL SOURCE VIEWED ENDWISE

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$$N(\rho) = 2.16(\theta\rho)^{0.29} \exp\left[-0.928(\theta\rho)^{0.58} - \bar{\sigma}_r(x)\rho\right]$$

θ = volume fraction of pitch = 0.5385

h = core height = 183 cm

R = core radius = 122 cm

$$\phi_1 = \arctan\left(\frac{R}{x}\right)$$

$$\phi_2 = \arctan\left(\frac{R}{x + h}\right)$$

$$\bar{\sigma}_r(x) = \frac{1}{x}\int_0^x du\ \sigma_r(u)$$

$\sigma_r(x)$ = the removal cross section of the material occurring at point x.

The values of removal cross section used for the plug material are

$\sigma_{D_2O} = 0.1144\ cm^{-1}$

$\sigma_C = 0.0302$

$\sigma_{Fe} = 0.1694$

$\sigma_{Pb} = 0.1122$

$\sigma_{Al} = 0.1131$

A plot of fast neutron flux is obtained by evaluating the integral in Equation (2) for a number of points and drawing a smooth curve as shown in Figure 4. The fast flux at the core surface is a value taken from criticality calculations on the core.

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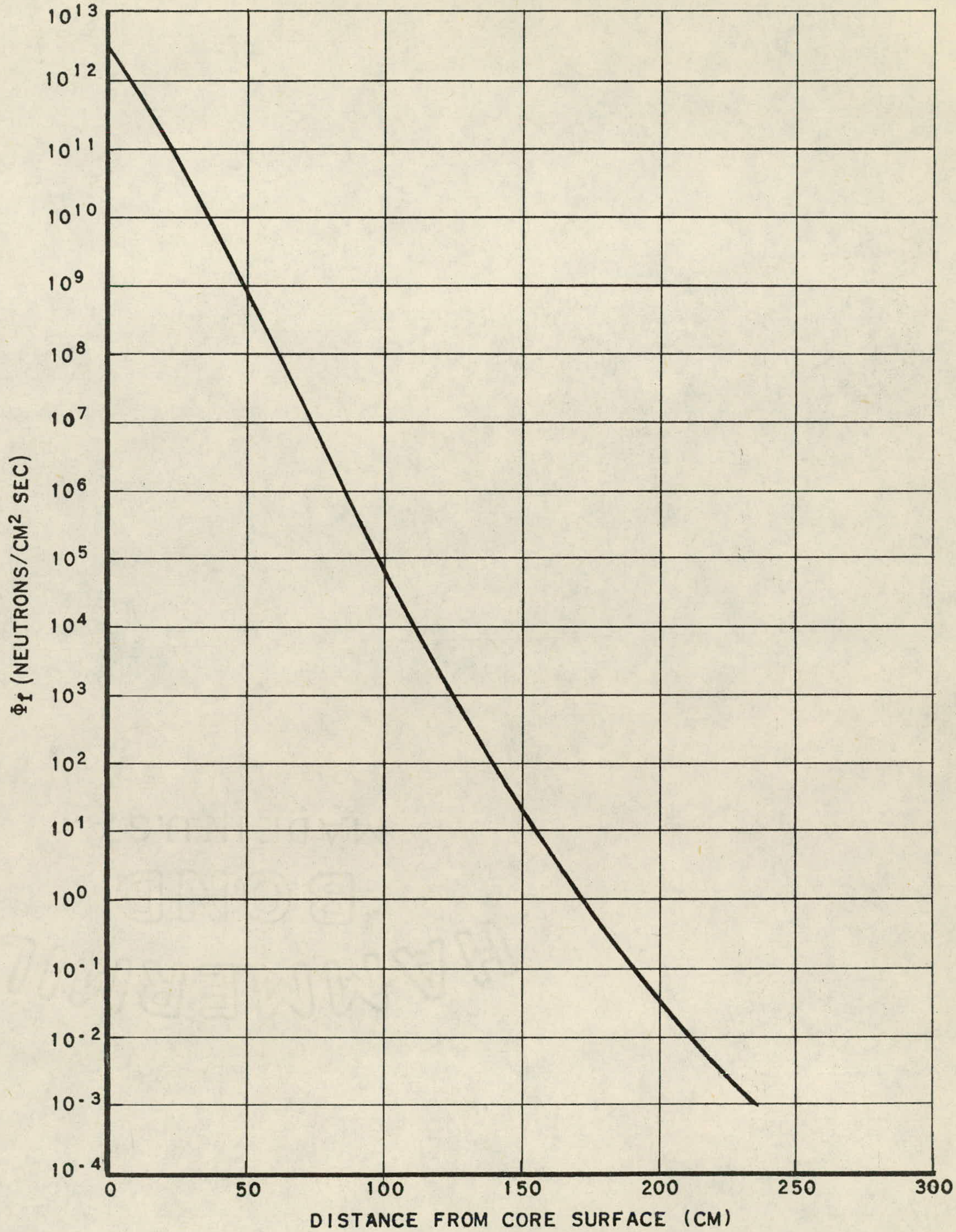

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FIG. 4
FAST NEUTRON FLUX IN THE TOP PLUG

It can be seen from Figure 4 that the fast neutron flux is 10^{-3} neutrons/sq cm-sec at the outer surface of the top plug, a figure well below the tolerance value, although of higher energy neutrons than 2 mev. However, it is desirable, from a weight standpoint, to keep as much of the permissible radiation in gamma photons as possible, as well as having some safety factor.

2. Thermal Neutron Flux

To obtain a plot of thermal neutron flux in the regions where capture gamma-rays are important in designing the shield, the top plug is divided into three sections. The first consists of the reflector and the aluminum grid, the second consists of the iron, lead, and pitch layers, and the third consists of the remainder of the shield. It is found to be advantageous to have a boral sheet on the core side of the aluminum grid as the 7.7-mev capture gammas from aluminum are both numerous and penetrating. A boral sheet is also placed on the core side of the first two iron layers to reduce the capture gamma-rays from those regions.

The thermal flux in the first region is assumed to follow the two-group diffusion equations in slab geometry:

$$\nabla^2\Phi_{fi}(x) - \frac{1}{\tau_i}\Phi_{fi}(x) = 0 \tag{3}$$

$$\nabla^2\Phi_{si}(x) - \frac{1}{L_i^2}\Phi_{si}(x) + \frac{D_{fi}}{\tau_i\, D_{si}}\Phi_{fi}(x) = 0 \tag{4}$$

where _i_ designates the _i_th region (Figure 5). The boundary conditions are

$$\Phi_{f1}(0) = 2.77 \times 10^{12} \qquad \Phi_{s1}(0) = 3.45 \times 10^{13}$$

$$\Phi_{f1}(a) = \Phi_{f2}(0) \qquad \Phi_{s1}(a) = \Phi_{s2}(0)$$

$$J_{f1}(a) = J_{f2}(0) \qquad J_{s1}(a) = J_{s2}(0)$$

$$\Phi_{f2}(b) = 5.37 \times 10^{7} \qquad \Phi_{s2}(0) = 0$$

Applying the boundary conditions with the constants

$$\tau_1 = 126.3\ \text{cm}^2 \qquad \tau_2 = 99.5\ \text{cm}^2$$

$$D_{f_1} = 1.248\ \text{cm} \qquad D_{f_2} = 1.2358\ \text{cm}$$

$$L_1^2 = 4530.7\ \text{cm}^2 \qquad L_2^2 = 34.579\ \text{cm}^2$$

$$D_{s_1} = 0.816\ \text{cm} \qquad D_{s_2} = 0.57135\ \text{cm}$$

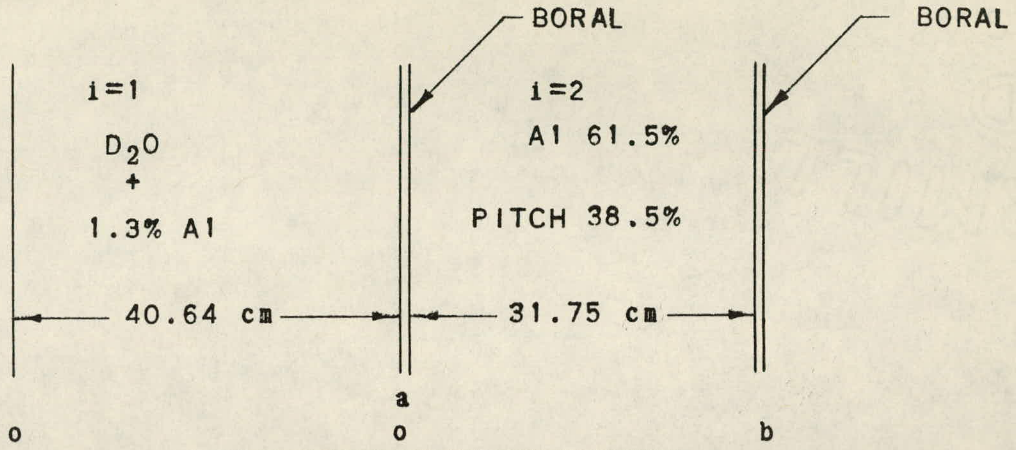

FIG. 5
REFLECTOR AND GRID REGIONS

we obtain as the solutions to (3) and (4) Equations (5) and (6)

$$\Phi_{fi}(x) = A_i \sinh\frac{x}{\sqrt{\tau_i}} + B_i \cosh\frac{x}{\sqrt{\tau_i}} \tag{5}$$

$$\Phi_{si}(x) = C_i \sinh\frac{x}{L_i} + D_i \cosh\frac{x}{L_i} + S_i\, \Phi_{fi}(x) \tag{6}$$

where

$$S_i = \frac{D_{fi}}{D_{si}} \frac{L_i^2}{\tau_i - L_i^2}$$

and the coefficients have the values,

$A_1 = -2.7703546 \times 10^{12}$ $\quad A_2 = -8.8398779 \times 10^{10}$

$B_1 = 2.77 \times 10^{12}$ $\quad B_2 = 8.8099833 \times 10^{10}$

$C_1 = -7.5596989 \times 10^{13}$ $\quad C_2 = 1.0151438 \times 10^{11}$

$D_1 = 3.8857955 \times 10^{13}$ $\quad D_2 = -1.0149891 \times 10^{11}$

The thermal neutron flux in the reflector and grid is plotted in Figure 6.

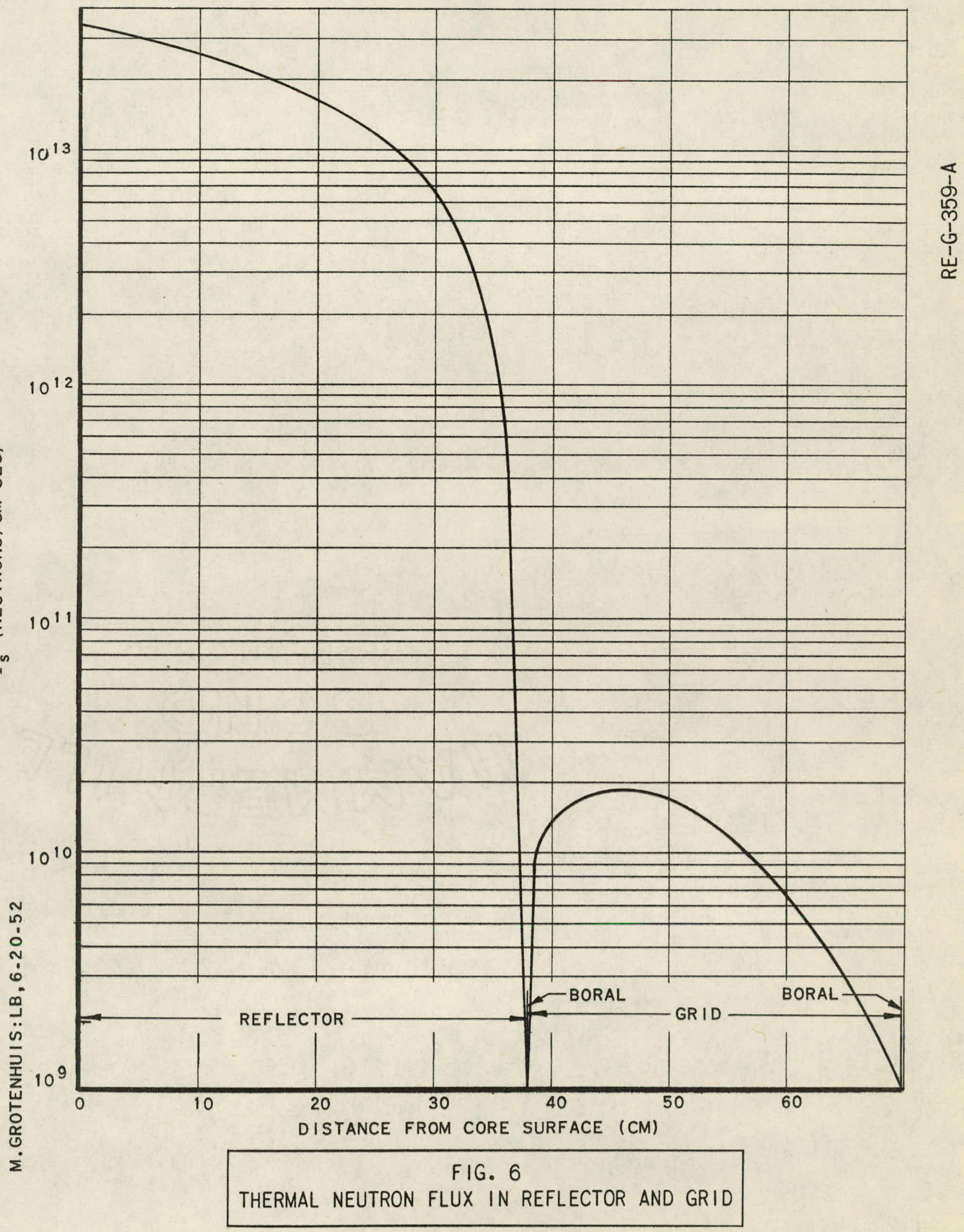

FIG. 6
THERMAL NEUTRON FLUX IN REFLECTOR AND GRID

The thermal neutron flux in the second region is calculated by assuming it follows the one-group diffusion equation in slab geometry (see Figure 7).

$$D_i \nabla^2 \Phi_{si}(x) - \sigma_{ai} \Phi(x) + Q_i(x) = 0 \quad . \tag{7}$$

The boundary conditions are

$\Phi_1(0) = 0$

$\Phi_1(a) = \Phi_2(0)$ $\qquad J_1(a) = J_2(0)$

$\Phi_2(b) = \Phi_3(0)$ $\qquad J_2(b) = J_3(0)$

$\Phi_3(0) = 0$

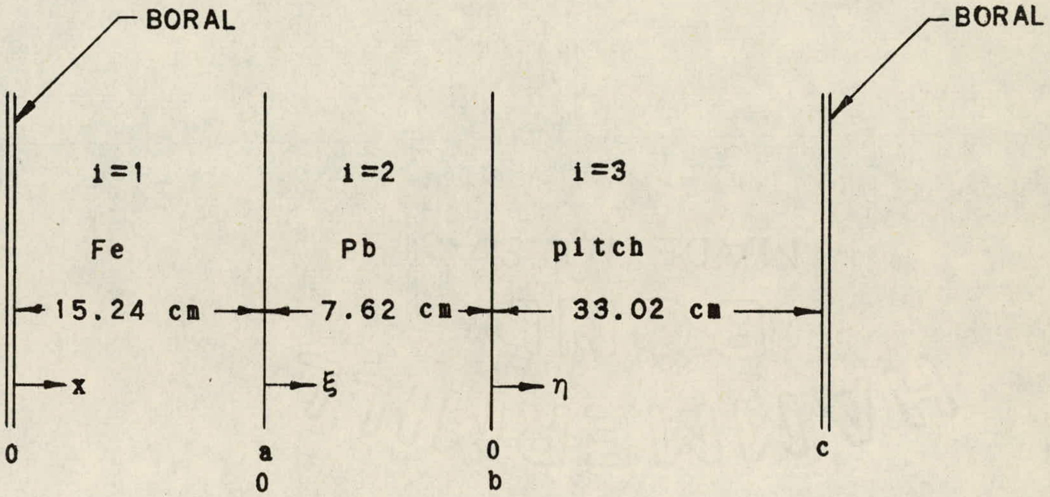

FIG. 7
IRON, LEAD AND PITCH REGIONS

Assuming that $Q_i(x)$ is of the form

$$Q_i(x) = q_i e^{-\sigma_{ri} x} \tag{8}$$

the solution of (7) in each of the three regions is given by

$$\Phi_{s1}(x) = A_1 e^{\kappa_1 x} + A_2 e^{-\kappa_1 x} + \delta_1 e^{-\sigma_{r1} x} \tag{9}$$

$$\Phi_{s2}(\xi) = A_3 e^{\kappa_2 \xi} + A_4 e^{-\kappa_2 \xi} + \delta_2 e^{-\sigma_{r2} \xi} \qquad (10)$$

$$\Phi_{s3}(\eta) = A_5 e^{\kappa_3 \eta} + A_6 e^{-\kappa_3 \eta} + \delta_3 e^{-\sigma_{r3} \eta} + \delta_4 e^{-\sigma_{r4} \eta} \qquad (11)$$

where

$$\delta_i = \frac{q_i}{\sigma_{ai} - D_i \sigma_{ri}^2}$$

and

$\sigma_{r1} = 0.169\ cm^{-1}$ $\qquad$ $\sigma_{a1} = 0.203\ cm^{-1}$

$\sigma_{r2} = 0.112$ $\qquad$ $\sigma_{a2} = 0.007$

$\sigma_{r3} = 0.140$ $\qquad$ $\sigma_{a3} = 0.0215$

$\sigma_{r4} = 0.100$

$D_1 = 0.360\ cm$ $\qquad$ $\kappa_1 = 0.7509\ cm^{-1}$

$D_2 = 1.382$ $\qquad$ $\kappa_2 = 0.07117$

$D_3 = 0.16245$ $\qquad$ $\kappa_3 = 0.3638$

The q's in the various regions are determined as follows: q_1 is the negative divergence of the fast current at the left boundary of region 1; q_2 is determined similarly at the left boundary of region 2 by means of the assumption that the fast current in the second region varies as $\exp(-\sigma_{r1}a - \sigma_{r2}x)$. To calculate q_3 and q_4 it is assumed that the fast current in the third region can be represented by a sum of two exponentials. The ratios of the coefficients and the exponents have the same values as determined by Taylor.[5] The sum of the coefficients is normalized by the condition of continuity of fast current at the boundary between regions 2 and 3. The source term is then given by the negative divergence of this fast current as for q_1 and q_2.

Thus we have for the q_i's

$$q_1 = j_1 \sigma_{r1}$$

$$q_2 = j_1 \sigma_{r2} e^{-\sigma_{r1} a}$$

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[5] J. J. Taylor, "Submarine Thermal Reactor Shield Design," WAPD-23, January, 1951, p. 32.

In region 3, the fast current is represented by

$$J_3(\eta) = j_3 e^{-\sigma_{r3}\eta} + j_4 e^{-\sigma_{r4}\eta}$$

where j_3 and j_4 are determined by

$$j_3 + j_4 = j_1 e^{-(\sigma_{r1}a + \sigma_{r2}b)}$$

$$j_4/j_3 = 2.87 \times 10^{-2}$$

and

$$j_1 = 1.47 \times 10^7 \text{ neutron/sq cm sec} \quad .$$

Then

$$Q_3(\eta) = -\frac{\partial}{\partial \eta} J_3(\eta) = q_3 e^{-\sigma_{r3}\eta} + q_4 e^{-\sigma_{r4}\eta}$$

and

$$q_3 = j_3 \sigma_{r3}$$

$$q_4 = j_4 \sigma_{r4}$$

The values of the δ_i's and q_i's are then found to be

$\delta_1 = 1.289 \times 10^7$ $\qquad q_1 = 2.484 \times 10^6$

$\delta_2 = -2.864 \times 10^7$ $\qquad q_2 = 2.958 \times 10^5$

$\delta_3 = 8.363 \times 10^6$ $\qquad q_3 = 1.532 \times 10^5$

$\delta_4 = 1.579 \times 10^5$ $\qquad q_4 = 3.138 \times 10^3$

Applying the boundary conditions to Equation (7) with the above constants, the following coefficients were determined:

$A_1 = 1.982 \times 10^3$ $\qquad A_4 = 3.469 \times 10^7$

$A_2 = -1.289 \times 10^7$ $\qquad A_5 = -8.804 \times 10^{-1}$

$A_3 = -2.344 \times 10^5$ $\qquad A_6 = -9.517 \times 10^5$

The thermal flux in these regions is plotted in Figure 8.

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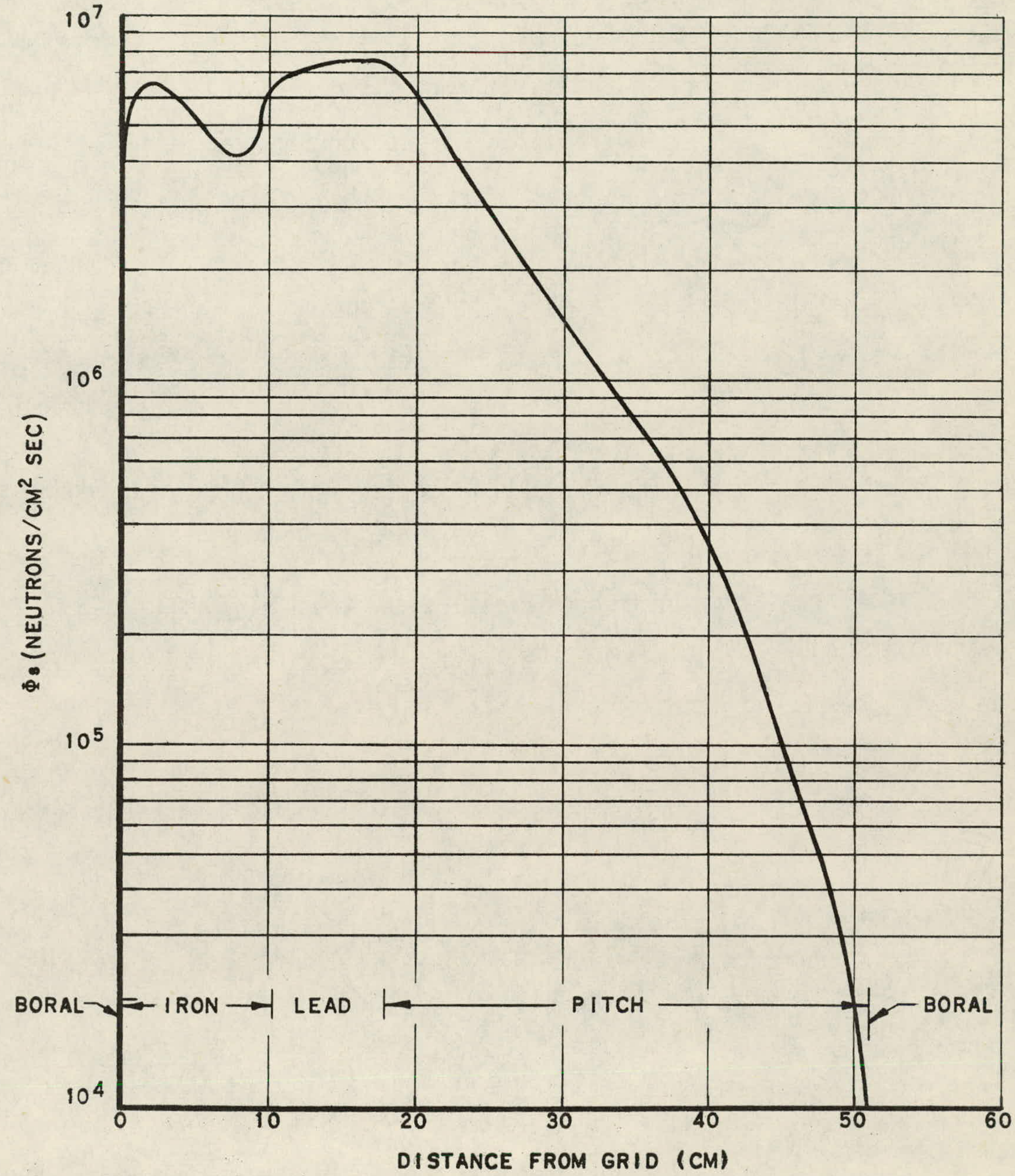

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FIG. 8
THERMAL NEUTRON FLUX IN IRON, LEAD AND PITCH REGION

The thermal flux in the remainder of the top plug was not deemed high enough to produce any appreciable flux of capture gamma-rays at the outer plug surface.

3. Gamma-Ray Flux

The gamma-ray flux through the top plug is calculated assuming exponential attenuation with a build-up term of $1 + \mu r$. The gamma-ray flux is expressed by the following integral

$$\Phi_\gamma(x) = \int dV \frac{q}{4\pi r^2} (1 + \nu(r - \rho) + \bar{\mu}(x)\rho) e^{-\nu(r-\rho) - \bar{\mu}(x)\rho} \quad (12)$$

This is the centerline flux from a cylindrical source viewed endwise (see Figure 3). An upper estimate of this integral is (see discussion under Fast Neutron Flux, p. 9).

$$\Phi_\gamma(x) = \frac{q}{2\nu} \left\{ 2 \left[E_2(\bar{\mu}x) - \frac{E_2(\bar{\mu}x \sec \phi_1)}{\sec \phi_1} \right] + \bar{\mu}x[E_2(\bar{\mu}x) - E_1(\bar{\mu}x \sec \phi_1)] - 2 \left[E_2(\bar{\mu}x + \nu h) - \frac{E_2\{(\bar{\mu}x + \nu h) \sec \phi_1\}}{\sec \phi_1} \right] - (\bar{\mu}x + \nu h)\left[E_1(\bar{\mu}x + \nu h) - E_1\{(\bar{\mu}x + \nu h) \sec \phi_1\}\right] \right\} \quad (13)$$

where

$$E_n(b) = \int_1^\infty dx\, x^{-n} e^{-bx} \quad (14)$$

and

$\mu(x)$ = gamma-ray absorption coefficient of the shield at point x (cm^{-1}) ,

$$\bar{\mu} = \frac{1}{x} \int_0^x du\, \mu(u) ,$$

ν = gamma-ray absorption coefficient of the source (cm^{-1}) ,

$\phi_1 = \arctan (R/a)$,

ϕ_2 = arctan (R/a + h) ,

R = radius of the source = 110 cm ,

h = height of source,

q = source strength (γ/cu cm-sec) = $\overline{\Phi}_s \sigma_a$,

σ_a = thermal neutron absorption cross section of source,

$$\overline{\Phi}_s = \frac{\int_0^h dx \ \Phi_s e^{+\mu x}}{\int_0^h dx \ e^{+\mu x}}$$

, which is a weighted average of the thermal neutron flux in the source.

The exponential integral functions $E_n(b)$ are calculated by using the formula

$$E_n(b) = \frac{e^{-b}}{b + n-1 + f_{n-1}(b)} \ (n \geqslant 1) \quad , \qquad (14\text{-a})$$

where $f_n(b)$ increases steadily from 0 to 1 as b goes from 0 to ∞. A graph of $f_n(b)$ is given in the Project Handbook.[6]

The strengths of the various sources of gamma-rays are listed in Table II. The gamma-ray absorption coefficients are listed in Table III. The calculations indicate that there will be a total escaping gamma-ray flux of 4 mr/hr at the plug surface on the core centerline. This value will be lower toward the edges, perhaps by a factor of two. There are some overestimates in the calculation, such as using the upper estimate (Equation (13)) of the integral in Equation (12).

An estimate of the gamma-ray source strength due to inelastic scattering is included in Table II. The method was similar to that used for calculating capture gamma-ray source strength except that the fast flux and inelastic scattering cross section replaced thermal flux and absorption cross section. The calculations indicate that even with the most pessimistic assumptions, the gamma-rays from inelastic scattering do not contribute significantly to the shield design.

The effects of streaming and penetration of radiation through gaps in the plugs and associated structure are not evaluated in the present report, but will be considered in a later stage of the design.

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[6] P. Morrison, "Radiation Physics," Project Handbook V, 12, Metallurgical Project, CL-697, May, 1945.

Table II

GAMMA-RAY SOURCE STRENGTHS IN CORE AND TOP PLUG

Source	$\sigma_a(cm^{-1})$	E(mev)	$q(\gamma/cm^3\ sec)$	Biological Factor $\left(\frac{mr/hr}{\gamma/cm^2 sec} \times 10^3\right)$
Core Al capture	0.0002838	7.7	1.60×10^{10}	5.08
Core U capture	.000177	7.7	6.64×10^9	5.08
Core Bi capture	.0002940	4	1.10×10^{10}	4.63
Core Bi inelastic scattering	.085	--	$<1.0 \times 10^{11}$	--
Core fission (prompt)	.000983	2.5	7.37×10^{10}	3.62
Core fission (delayed)	--	2.5	2.76×10^{10}	3.62
Core D_2O capture	.0000179	2.5	6.71×10^8	3.62
Core fission (delayed)	--	1	8.27×10^{10}	1.89
Reflector Al capture	.000146	7.7	2.10×10^9	5.08
Reflector D_2O capture	.0000295	2.5	4.25×10^8	3.62
Grid Al capture	.00799	7.7	3.18×10^7	5.08
Grid C capture	.0000840	5	3.34×10^5	5.08
Grid H capture	.00820	2.5	3.26×10^7	3.62
Shield Fe capture	.203	7.7	1.32×10^6	5.08
Shield Fe inelastic scattering	.093	--	$<5.9 \times 10^5$	--
Shield Pb Capture	.007	7.7	5.25×10^4	5.08
Shield Pb inelastic scattering	.083	--	$<5.25 \times 10^4$	--
Shield C inelastic scattering	.000218	5	1.64×10^3	5.08
Shield H inelastic scattering	.0214	2.5	1.61×10^5	3.62

Table III

GAMMA-RAY ABSORPTION COEFFICIENTS (cm^{-1})[7]

E	7.7	5	4	2.5	1
Core	0.230	0.183	0.181	0.191	0.324
D_2O	0.0243	0.030	0.0340	0.0436	0.070
Al	0.0662	0.0628	0.0830	0.102	0.165
Fe	0.236	0.246	0.259	0.290	0.455
Pb	0.514	0.468	0.455	0.466	0.799
Pitch	0.0243	0.030	0.0340	0.0436	0.070
Concrete	0.0548	0.064	0.0705	0.0895	0.144

B. Radial Shield

1. Fast Neutron Flux

The fast neutron flux through the radial shield was calculated by assuming exponential attenuation according to the removal cross sections. The geometry is assumed to be an infinite slab source. From Figure 9 the fast neutron flux can be expressed as the following integral

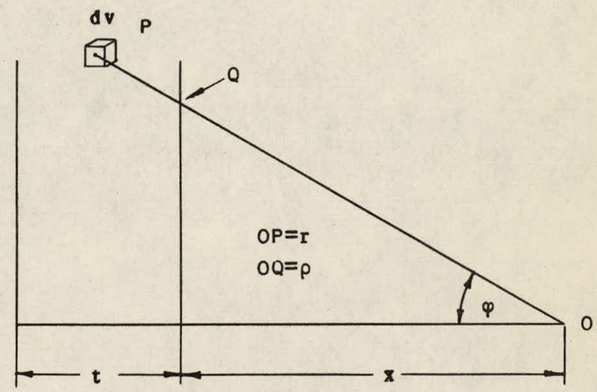

FIG. 9
GEOMETRY FOR INFINITE SLAB CALCULATION

$$\Phi_f(x) = \int dV \, \frac{q}{4\pi r^2} \, e^{-\sigma_c(r-\rho) - \overline{\sigma}_r(x)\rho} \qquad (14)$$

which integrates to

$$\Phi_f(x) = \frac{q}{2\sigma_c} \left\{ E_2(\overline{\sigma}_r x) - E_2(\overline{\sigma}_r x + \sigma_c t) \right\} \qquad (15)$$

[7] J. L. Powell and W. S. Snyder, "Absorption of Gamma-Rays," ORNL-421, March, 1950.

where

q = source strength = 1.13×10^{11} n/cu cm-sec

σ_c = removal cross section for the core = 0.107 cm^{-1}

x = shield thickness

t = source thickness = 120 cm

$\overline{\sigma}_r(x)$ is as defined on p.11

σ_G = removal cross section for graphite = 0.0754 cm^{-1}

σ_{Fe} = removal cross section for Fe = 0.1694 cm^{-1}

σ_{con} = removal cross section for concrete = 0.0909 cm^{-1}

Making use of these constants, Equation (15) yields the value of the fast flux at the outer surface of the shield. This value was calculated to be 10^{-2} neutrons/sq cm-sec. This value is well below permissible levels. However, again it is advantageous to take most of the allowable tolerance in gamma radiation. Although there is uncertainty in the method of calculation, as the amount of hydrogen in the shield is no doubt too low for removal cross sections to accurately describe the flux, the removal cross section used is very close to the actual attenuation cross section observed in experiments with similar concretes.

2. Thermal Neutron Flux

The thermal neutron flux plot through the radial shield is calculated as two separate problems. The first problem is the flux in the graphite reflector; the second is the flux in the iron thermal shield and the concrete.

The thermal neutron flux in the graphite is assumed to follow the two-group diffusion equations.

$$\nabla^2 \Phi_f(x) - \frac{1}{\tau} \Phi_f(x) = 0 \qquad (16)$$

$$\nabla^2 \Phi_s(x) - \frac{1}{L^2} \Phi_s(x) + \frac{D_f}{\tau D_s} \Phi_f(x) = 0 \qquad (17)$$

The boundary conditions are (Figure 10)

$\Phi_f(0) = 7.8 \times 10^{11}$ $\Phi_s(0) = 1.31 \times 10^{13}$

$\Phi_f(a) = 6.3 \times 10^8$ $\Phi_s(a) = 0$

with the constants having the values

$\tau = 372 \text{ cm}^2$ $D_f = 1.113 \text{ cm}$

$L^2 = 2500 \text{ cm}^2$ $D_s = 0.900 \text{ cm}$

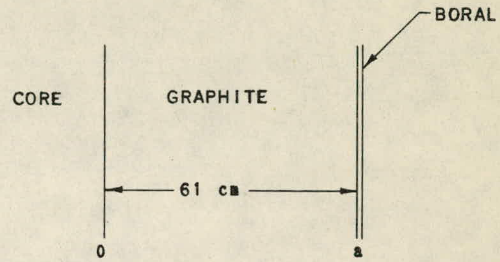

FIG. 10
GRAPHITE REGION

The equations for the fast and slow flux are then

$$\Phi_f(x) = A \sinh \frac{x}{\sqrt{\tau}} + B \cosh \frac{x}{\sqrt{\tau}} \tag{18}$$

$$\Phi_s(x) = C \sinh \frac{x}{L} + D \cosh \frac{x}{L} + S\Phi_f(x) \tag{19}$$

where

$$S = \frac{D_f}{D_s} \cdot \frac{L^2}{\tau - L^2}$$

and the coefficients are

$A = -7.82754 \times 10^{11}$ $C = 1.67729 \times 10^{13}$

$B = 7.8 \times 10^{11}$ $D = 1.40789 \times 10^{13}$

The thermal flux plot in graphite is shown in Figure 11.

The thermal neutron flux in the iron thermal shield and concrete (Figure 12) is assumed to follow the diffusion equation

$$\nabla^2\Phi_i(x) - \frac{1}{L_i^2}\Phi_i(x) + \frac{1}{D_i} Q_i(x) = 0 \quad . \tag{20}$$

The source of thermal neutrons was assumed to be the negative divergence of the fast current where the fast current is represented by

$$J_{fi}(x) = \frac{q}{2\sigma_c} E_3(\sigma_i x + b_i) \tag{21}$$

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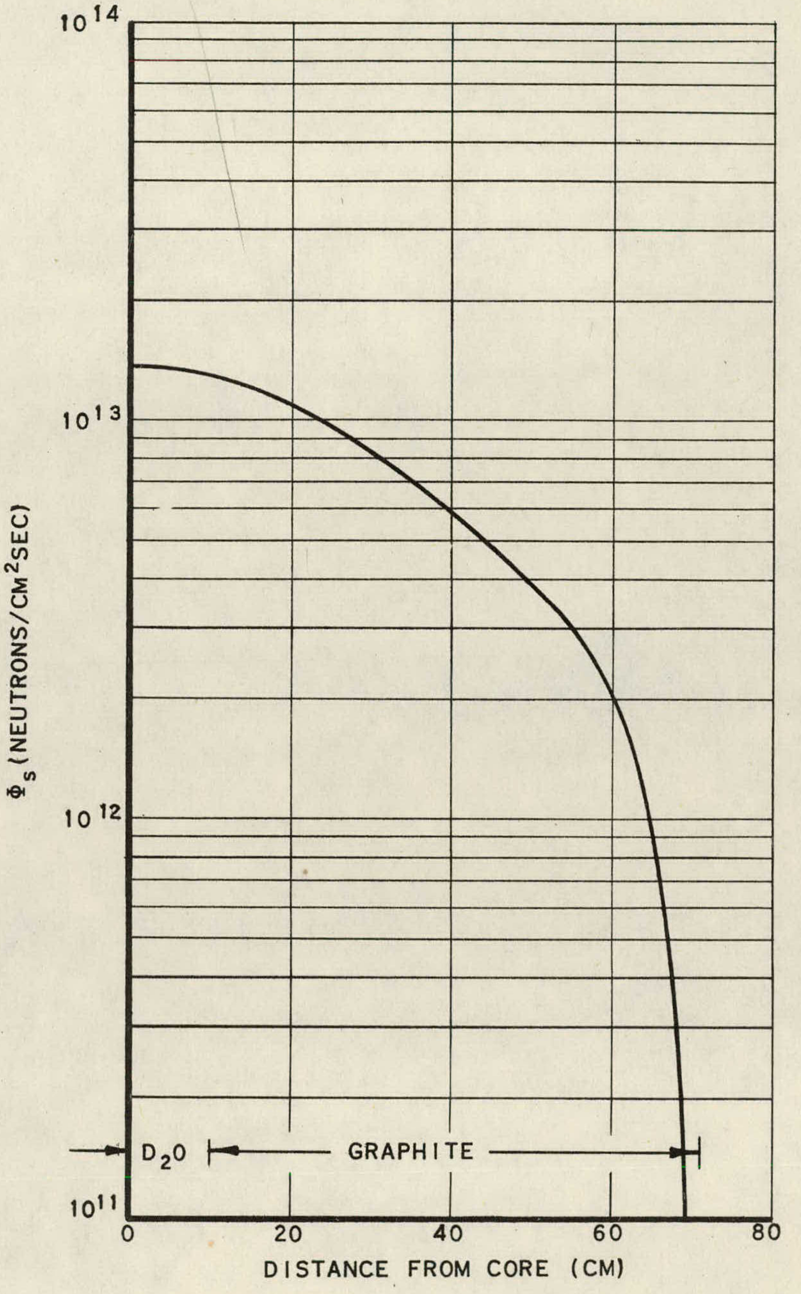

FIG. 11
THERMAL FLUX IN GRAPHITE

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The source of thermal neutrons is then

$$Q_i(x) = \frac{q}{2\,\sigma_c}\,\sigma_i\,E_2(\sigma_i x + b_i) \qquad (22)$$

and was approximated by

$$Q_i(x) \cong \frac{q}{2}\,\frac{\sigma_i}{\sigma_c}\,\frac{e^{-(\sigma_i x + b_i)}}{(\sigma_i x + b_i + 1)} \qquad (23)$$

The boundary conditions are

$\Phi_1(0) = 0$

$\Phi_1(a) = \Phi_2(0)$ $\qquad J_1(a) = J_2(0)$

$\Phi_2(\infty) = 0$

Applying the boundary conditions with the constants having the values

$\sigma_1 = 0.1694\ \text{cm}^{-1}$

$\sigma_2 = 0.0909\ \text{cm}^{-1}$

$\kappa_1 = 0.751\ \text{cm}^{-1}$

$\kappa_2 = 0.0854\ \text{cm}^{-1}$

$D_1 = 0.36$ cm

$D_2 = 1.14$ cm

$q = 1.13 \times 10^{11}$ neutrons/ sq cm-sec

$\sigma_c = 0.107\ \text{cm}^{-1}$

$b_1 = 4.601$

$b_2 = 7.183$

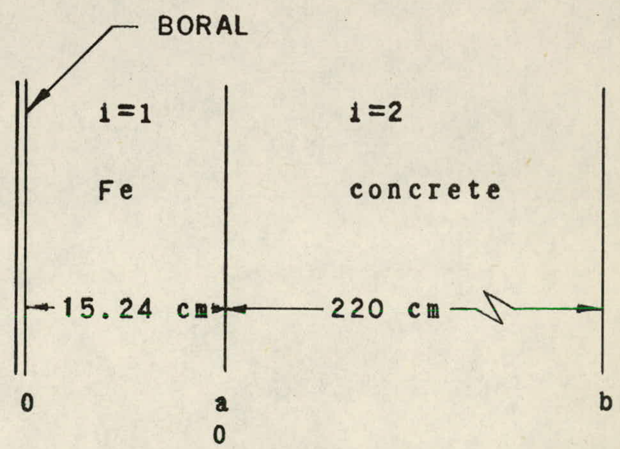

FIG. 12
THERMAL SHIELD AND CONCRETE REGION

the general solution to Equation (20) is

$$U_i(x) = A_i e^{\kappa_i x} + B_i e^{\kappa_i x} \tag{24}$$

A particular solution of (20) is

$$V_i(x) = \frac{K_i}{\sigma_i}\left[e^{-\kappa_i x - \frac{(\kappa_i - \sigma_i)}{\sigma_i}} \left\{ E_i\left[\frac{\kappa_i - \sigma_i}{\sigma_i}(\sigma_i x + b_i + 1)\right] - E_i\left[\frac{\kappa_i - \sigma_i}{\sigma_i}(\sigma_i \alpha_i + b_i + 1)\right]\right\} + e^{\kappa_i x + \frac{\kappa_i + \sigma_i}{\sigma_i}(b_i + 1)} E_1\left[\frac{\kappa_i + \sigma_i}{\sigma_i}(\sigma_i x + b_i + 1)\right]\right] \tag{25}$$

where $\alpha_1 = 0$ and α_2 has the value that makes

$$V_1(a) = V_2(0)$$

and

$$K_i = \frac{q_i \sigma_i e^{-b_i}}{4\sigma_c D_i \kappa_i}$$

The thermal flux is given by

$$\Phi_i(x) = U_i(x) + V_i(x)$$

and imposing the boundary conditions with the given constants, the values

$A_1 = 6.57087 \times 10^2$ $\qquad$ $B_1 = 3.12334 \times 10^8$

$A_2 = 0$ $\qquad$ $B_2 = 6.14024 \times 10^7$

are obtained. The thermal flux is plotted in Figure 13.

3. Gamma-Ray Flux

The gamma-ray flux through the radial shield is calculated assuming exponential attenuation with build-up from an infinite slab source (Figure 9). This is given by

$$\Phi_\gamma(x) = \int dV \frac{q}{4\pi r^2}(1 + \nu(r-\rho) + \mu\rho)e^{-\nu(r-\rho) - \bar{\mu}(x)\rho} \tag{26}$$

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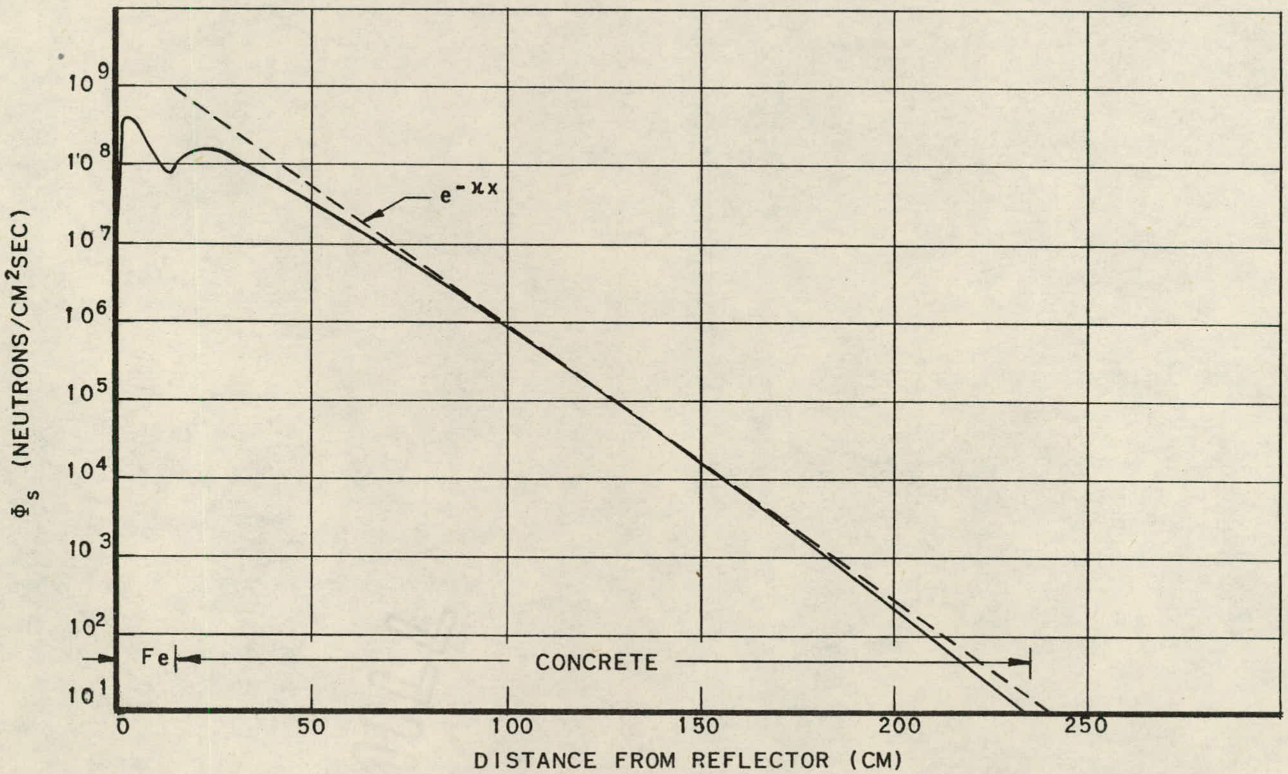

FIG. 13
THERMAL NEUTRON FLUX IN IRON AND CONCRETE

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which integrates to

$$\Phi_\gamma(x) = \frac{q}{2\nu}\Big\{2E_2(\bar{\mu}x) + \bar{\mu}xE_1(\bar{\mu}x) - 2E_2(\bar{\mu}x + \nu h) - (\bar{\mu}x + \nu h)\ E_1(\bar{\mu}x + \nu h)\Big\} \quad (27)$$

All symbols have the same meanings as in Equation (13). To correct approximately for cylindrical geometry one can multiply by $\frac{t}{t + a}$ where t is the distance from the core center to the source boundary and a is the shield thickness.

The core source strengths are as listed in Table II. The remainder of the sources are as listed in Table IV. The gamma-ray absorption coefficients are listed in Table III.

An estimate of the gamma-ray source due to inelastic scattering in the thermal shield is included in Table IV. Again, as in the case of the gamma-rays from inelastic scattering in the top plug (p. 21), a most pessimistic estimate indicates that these gamma-rays do not contribute significantly to the shield design.

Table IV

GAMMA-RAY SOURCE STRENGTHS IN RADIAL SHIELD

Source	E(Mev)	$\sigma_a(cm^{-1})$	$q(\gamma/cm^3 sec)$	Biological Damage Factor $\left(\frac{mr/hr}{\gamma/cm^2 sec} \times 10^3\right)$
Al tank capture	7.7	0.013	1.70×10^{11}	5.08
Reflector capture	5	0.00004	1.44×10^{8}	5.08
Thermal shield capture	7.7	0.203	2.54×10^{7}	5.08
Thermal shield inelastic scattering	--	0.093	$<2 \times 10^{7}$	--

The capture gamma-rays escaping from concrete are calculated by a somewhat different method. The gamma-ray flux from a plane source is given by Figure 14.

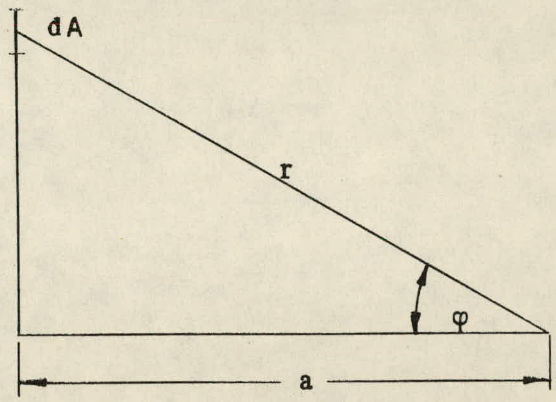

FIG. 14
GEOMETRY FOR GAMMA-RAY FLUX FROM A PLANE SOURCE

$$\Phi_\gamma(a) = \int dA \frac{q}{4\pi r^2} e^{-\mu r}(1 + \mu r) \tag{28}$$

which integrates to

$$\Phi_\gamma(a) = \frac{q}{2}\left\{E_1(\mu a) + e^{-\mu a}\right\} \tag{29}$$

The gamma-ray flux due to an exponentially distributed laminar source $q_0 \exp[-\kappa(a - x)]$ is given by

$$\Phi_\gamma(a) = \int_0^a dx \frac{q_0}{2}\left\{E_1(\mu x) + e^{-\mu x}\right\} e^{-\kappa(a-x)} \tag{30}$$

Integrating,

$$\Phi_\gamma(a) = \frac{q_0}{2}\left\{e^{-\kappa a} \int_0^a dx\, E_1(\mu x)\, e^{\kappa x} + \frac{\left(e^{-\kappa a} - e^{-\mu a}\right)}{(\mu - \kappa)}\right\} \tag{31}$$

A more convenient form for numerical evaluation is obtained by an integration by parts. This yields

$$\Phi_\gamma(a) = \frac{q_0}{2}\left\{\frac{e^{-\kappa a}}{\mu} - \frac{E_2(\mu a)}{\mu} + \frac{\kappa}{\mu}e^{-\kappa a}\int_0^a dx\, E_2(\mu x)e^{\kappa x} + \frac{e^{-\kappa a}}{(\mu-\kappa)} - \frac{e^{-\mu a}}{(\mu-\kappa)}\right\} \tag{32}$$

in which the integral is evaluated numerically.

The capture gamma-ray source in the concrete is the absorption cross section times the thermal neutron flux. The thermal flux for this calculation is taken as

$$\Phi_o\, e^{-\kappa(a-x)}$$

where Φ_o and κ are adjusted so that the value of this expression never falls below that of the calculated thermal flux $\Phi_2(x)$ (Figure 13).

Using the value of κ so obtained and the corresponding values of q_0 for the gamma-rays of each energy given in Table V, total dosage contribution from the concrete capture gamma-rays is calculated from formula (32).

Table V

GAMMA-RAY SOURCE STRENGTHS IN CONCRETE

E(Mev)	$q_o(\gamma/cm^3\ sec)$	Biological Damage Factor $\left(\frac{mr/hr}{\gamma/cm^2\ sec} \times 10^3\right)$
7.7	1.92×10^5	5.08
5	2.22×10^4	5.08
4	5.24×10^6	4.63
2.5	6.88×10^5	3.62

The total escaping gamma-ray dose from the radial shield was calculated to be 5 mr/hr. A plot of gamma-ray flux in the graphite and iron regions is given in Figure 15.

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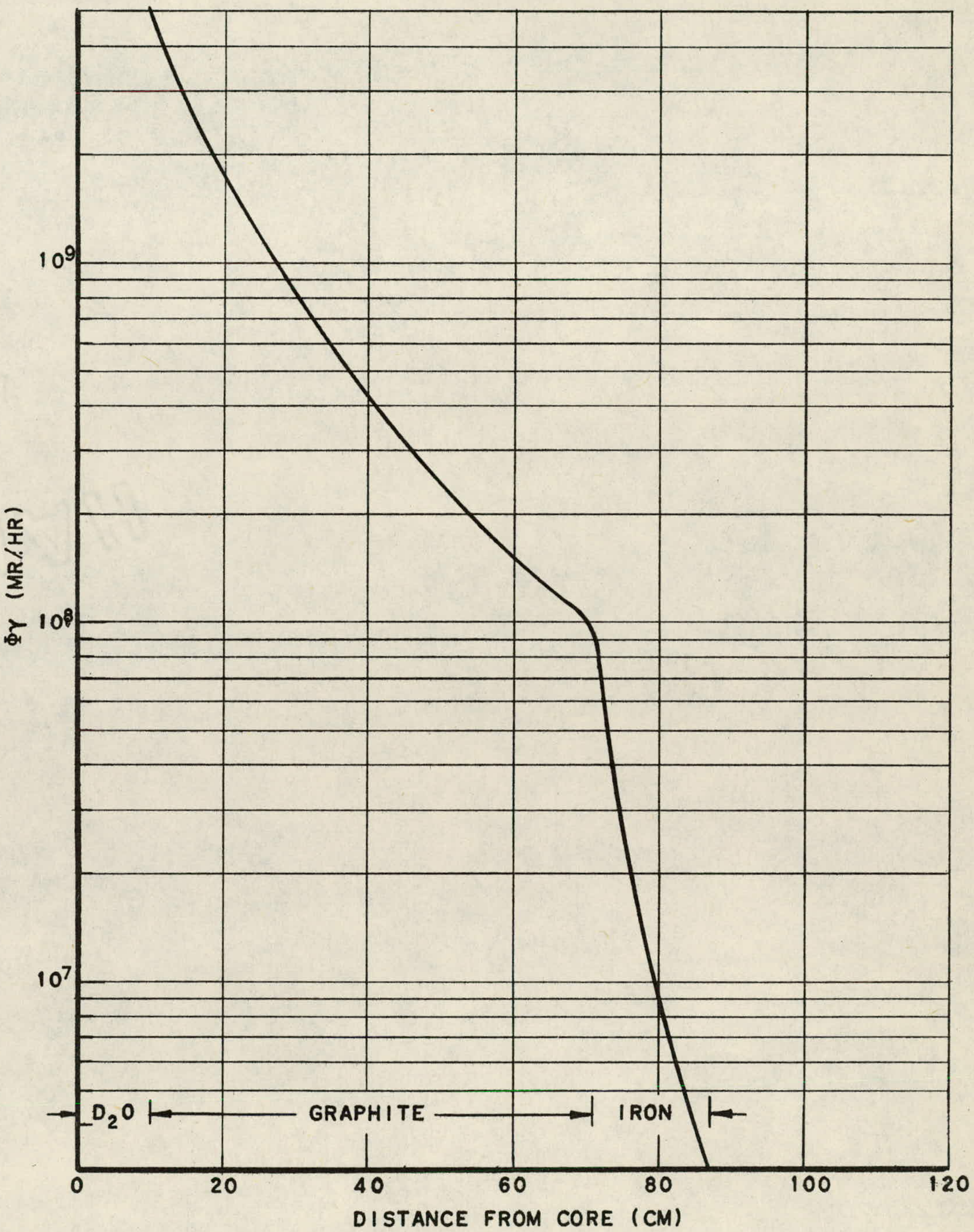

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FIG. 15
GAMMA-RAY FLUX IN GRAPHITE AND IRON

IV. HEATING OF SHIELD COMPONENTS

A. Boral Layer before Aluminum Grid

The heating of the first boral layer in the top plug is calculated by computing the current of thermal neutrons into the boral and assuming them to be totally absorbed. The energy release per neutron absorbed is taken to be 2.8 mev. Since this energy is due chiefly to the kinetic energy of the emitted alpha particle, it is strongly localized at the point of absorption.

The flux in the regions bounding the boral is given by Equations (5) and (6). The current is taken to be proportional to the derivative of the flux

$$J_{si}(x) = - D_{si}\left\{\frac{C_i}{L_i}\cosh\frac{x}{L_i} + \frac{D_i}{L_i}\sinh\frac{x}{L_i} + S_i\Phi_{fi}(x)\right\} \quad (33)$$

Substituting the constants previously tabulated (p. 13), this gives a thermal neutron current of 7.75 x 10^{11} neutrons/sq cm-sec. This amount of thermal neutron current corresponds to a power production of 0.43 watt/sq cm. This is the maximum energy and will occur at the horizontal centerline of the core tank. It will be somewhat lower toward the outer edges of the plug, perhaps by a factor of 5.

B. Boral Layer in Radial Shield

The heating of the boral sheet in the radial shield is calculated in the same manner as that for the top plug, using Equation (19). The thermal neutron current is then

$$J_s = - D_s\left\{\frac{C}{L}\cosh\frac{x}{L} + \frac{D}{L}\sinh\frac{x}{L} + S\Phi_f(x)\right\} \quad (34)$$

This corresponds to a power production of 0.088 watt/sq cm. This is again the maximum value, and it occurs at the half height of the reactor. It will be lower near the top and bottom of the reactor, perhaps by a factor of 2.

C. Thermal Shield

If the gamma-ray flux in the iron thermal shield is denoted by $\Phi_\gamma(E,x)$, then the rate of energy release due to gamma-ray absorption, in mev/cu cm-sec, is given by

$$P(x) = \mu(E)\,\Phi_\gamma(E,x)\,E \quad (35)$$

where

$\mu(E)$ = the gamma-ray absorption coefficient (cm^{-1})

E = energy of the gamma-ray

Taking Φ_γ as given by Equation (27) for each of the sources of gamma-rays listed in Tables II and IV, and the absorption coefficients listed in Table III, the energy production rate per unit volume of the thermal shield can be plotted vs the thickness of thermal shield traversed (Figure 16).

V. FUEL ELEMENT REMOVAL COFFIN

For loading and unloading the reactor it is necessary to have a coffin into which the elements can be withdrawn. The wall thickness of this coffin is designed to shield the most radioactive element without overexposing personnel around the coffin.

For this calculation it is assumed that a fuel element is 6 feet long and has a cross-sectional area of 9.86 sq cm. The element consists of 25.5 per cent Al, 74.4 per cent D_2O, and 0.1 per cent U^{235}. The weight of uranium is 0.3 kg per fuel element. The element will have been exposed for a maximum of 6 months at a power level of 5 mw, followed by one hour of cooling time.

The rate of gamma-ray energy release from fission products is given by the expression[8]

$$\Gamma(t) = 1.6t^{-1.21} \text{ mev/sec fission } (t > 10 \text{ sec}) \quad (36)$$

t = time after fission in seconds

The rate of gamma-ray energy release from a fuel element exposed in a thermal neutron flux Φ_s for T_0 sec and cooled for T_s sec is given by

$$P(T_s) = V\sigma_f^{25}\Phi_s \int_0^{T_0} dt\Gamma(T_s+T_0-t) \text{ mev/sec } (T_s > 10 \text{ sec}) \quad (37)$$

where

$V\sigma_f^{25}$ = the total fission cross section per fuel element.

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[8] J. J. Taylor, "Core Removal Shielding," WAPD-RM-59, June, 1951, p. 9.

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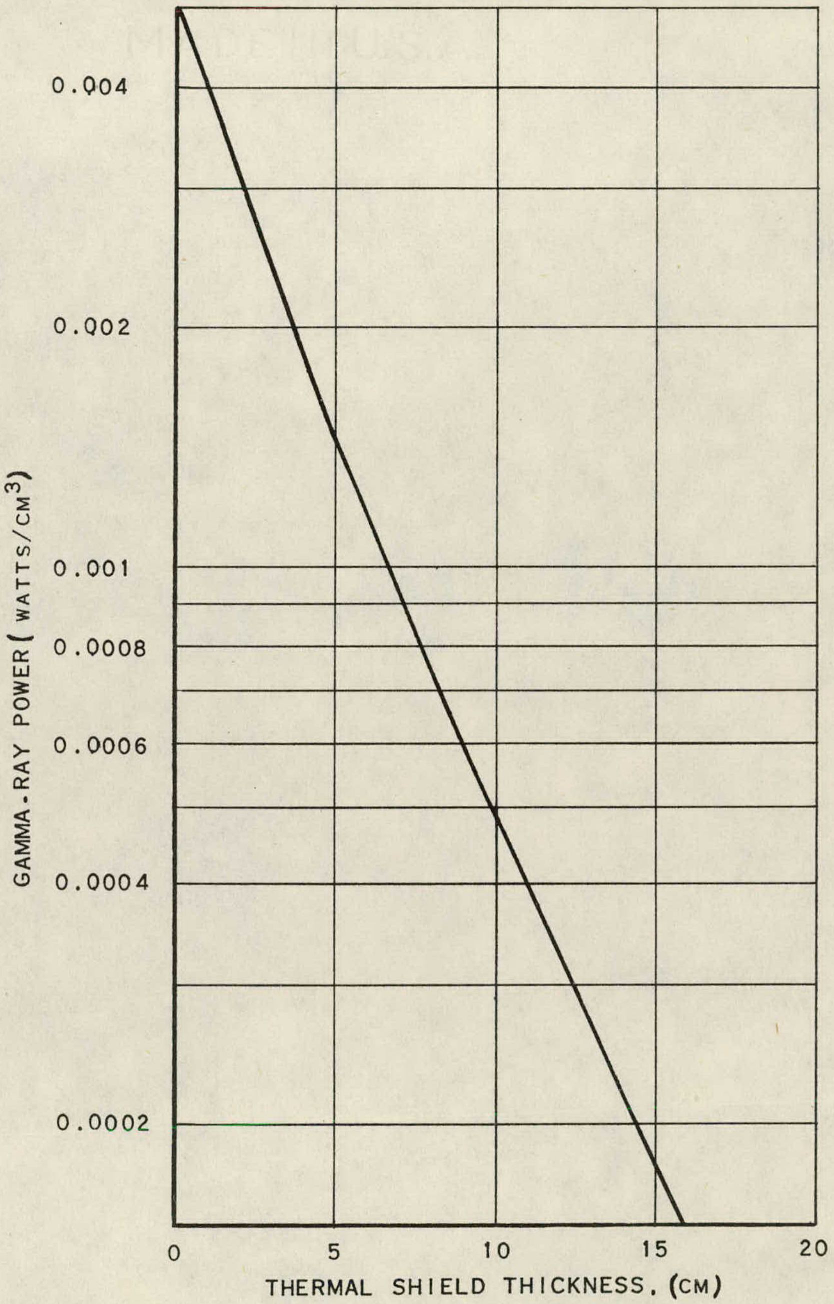

FIG. 16
GAMMA-RAY HEAT IN THERMAL SHIELD

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Using the value

$$V\sigma_f\Phi_s = 1.43 \times 10^{16} \text{ fiss/sec},$$

Equation (37) gives for the rate of gamma-ray energy release:

$$P = 1.09 \times 10^{17} [T_s^{-0.21} - (T_s + T_0)^{-0.21}] \text{ mev/sec.} \quad (38)$$

For an operating time (T_0) of 60 days, Equation (38) gives, for a cooling time (T_s) of one hour:

$$P(1 \text{ hr}) = 1.53 \times 10^{16} \text{ mev/sec.}$$

Estimating that 12 per cent of the energy is in 2.5-mev gamma-rays and the remainder in 1 mev, the source strengths for shielding calculations are then:

$$q_{2.5 \text{ mev}} = 4.01 \times 10^{12} \ \gamma/\text{cm-sec}$$

$$q_{1 \text{ mev}} = 7.38 \times 10^{12} \ \gamma/\text{cm-sec}$$

For all practical purposes the 1-mev source can be neglected as the absorption of 1-mev gamma-rays in lead is much greater than that of 2.5 mev.

The fuel element has a relatively small cross-sectional area and a small gamma-ray absorption coefficient so that for calculation purposes it may be assumed to be a line source of gamma-rays. This will result in a slight overestimate.

The expression for the gamma-ray flux at the edge of a shield surrounding an infinite line source (Figure 17) is:

$$\Phi_\gamma = 2 \int_0^\infty dx \frac{q}{4\pi r^2}(1 + \mu r)e^{-\mu r} \quad (39)$$

$$= \frac{q}{2\pi}\left\{\frac{1}{a} Ki_1(\mu a) + \mu[K_0(\mu a)]\right\} \quad (40)$$

where

$$q = \text{source term } (\gamma/\text{cm-sec})$$

$$Ki_1(\mu a) = \int_0^{\pi/2} d\phi e^{-\mu a \sec \phi} = \int_{\mu a}^\infty du K_0(u)$$

$K_0(\mu a)$ = zero-order bessel function of the second kind.

μ = gamma ray absorption coefficient

a = shield thickness

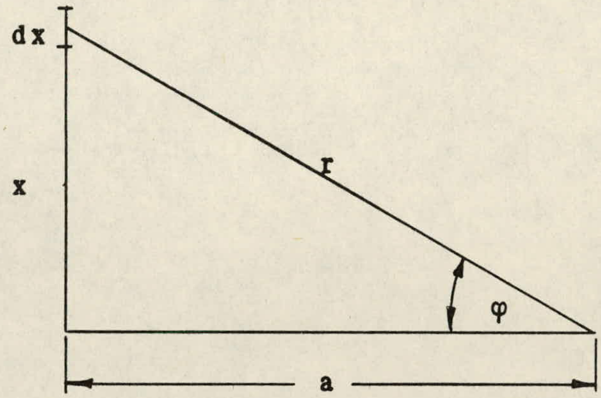

FIG. 17
GEOMETRY FOR INFINITE
LINE SOURCE CALCULATION

The shield thickness (a) necessary for the 2.5-mev gamma-rays to be reduced to 12.5 mr/hr at the shield surface is calculated to be 14.5 inches of lead.

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