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ABSOLUTE REACTIVITY MEASUREMENT FROM TRANSIENT BEHAVIOUR OF A SUBCRITICAL NUCLEAR REACTOR

by

P.Schmid

ABSTRACT

It is shown that absolute reactivities can be measured with good accuracy from the transient behaviour of a subcritical nuclear reactor. The method is characterized by the application of step functions (either in neutron source strength or in reactivity) and by the observation of a time lag of the integrated reactor power.

Simple formulae for the least square fit of experimental data are deduced, and some experimental checks at the Swiss swimming pool reactor SAPHIR are briefly summarized.

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1. Introduction

One of the most familiar methods for absolute reactivity measurements is the observation of stable reactor periods and computation of the corresponding reactivities from the inhour equation (1):

$$\varrho = \frac{k-1}{\beta k} = \frac{1}{\beta k}\frac{\ell}{T} + \sum_i \frac{\beta_i}{\beta}\frac{1}{1+\lambda_i T} \quad (S=0) \quad (1)$$

where

ϱ = reactivity in dollar units

k = effective multiplication factor of chain reaction

β_i = fraction of fission neutrons emitted by the i'th delayed neutron emitter

$\beta = \sum_i \beta_i$ = fraction of fission neutrons which are delayed

λ_i^{-1} = mean life of delayed neutron emitters of type i

ℓ = mean neutron life in finite reactor system

T = stable reactor period

S = fictitious reactor power due to those fissions only which are produced directly by absorption of source neutrons. This is a convenient specification of source strength.

With many reactors this is a convenient way to calibrate the control elements in a period range above -100 sec. The maximum reactivity which can be measured in the supercritical state is mainly determined by safety considerations. In the subcritical state however there is an inherent lower limit of the stable period due to the delayed neutron emitter with 80 sec mean life. Therefore negative reactivities below - 0.3 $ cannot be measured accurately by that method.

A simple way to determine negative reactivities even much below that limit is the following: the reactor power is kept constant at a high level $p_0 \gg S/\beta$ so that $|\varrho_0| \ll 1$. The negative reactivity which shall be measured is then suddenly applied, the power immediately jumping to p_1 * ("prompt jump"). Now ϱ_1 is given by

$$\varrho_1 = 1 - \frac{p_0}{p_1^*}(1-\varrho_0) \cong 1 - p_0/p_1^* . \quad (2)$$

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(1) S. Glasstone and M.C. Edlund: "The Elements of Nuclear Reactor Theory". New York 1952.

If S is constant other negative reactivities ϱ_n can be measured by comparison of corresponding stationary subcritical power levels p_n (2)

$$\varrho_n = \varrho_1 \cdot P_1/P_n \cdot \qquad (3)$$

This method looks promising, but in practice leads to considerable difficulties since most neutron detectors don't give a signal which is representative for the actual power level. Any substantial movement of control elements changes the ratio between detector signal and power level, partly because of direct shadowing effects, partly due to shifting of the neutron flux pattern. In addition very fast recorders are required to allow an effective discrimination of the response from short lived delayed neutron emitters against the "prompt jump".

Therefore it is most desirable to use dynamic properties of the subcritical nuclear reactor which are essentially amplitude independent, e.g. the phase of the subcritical reactor transfer function. Continuous reactivity measurements can be performed if the reactivity undergoes a small harmonic perturbation which generates a corresponding sine function superimposed on the power level change. The phase shift between reactivity perturbation and relative response of the power level is a unique indication of the mean reactivity. This method - though not very accurate - is useful in particular for routine measurements since it provides continuous information, e.g. during start - up. However it requires a considerable equipment specially designed for the purpose.

Axtmann et al. (3) mention the possibility of obtaining the reactivity of a subcritical nuclear reactor from a so called lifetime of the transient (i.e. the time necessary for the level change to reach 99 % of its ultimate value). They assume that all neutrons are promptly emitted, having a common life expectancy of 0.08 - 0.12 sec. Considering this very rough model their results are rather qualitative, but nevertheless they point out an attractive type of experiment.

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(2) J. Bernot et V. Rajewski: "Mesure des réactivités dans une pile". CEA report No 310 (1954).

(3) R.C. Axtmann, G. Dessauer and T.F. Parkinson: "Reactivity Measurements in a Subcritical Pile". DP-48 (Savannah River Labora-

The method proposed in this report implicitly makes use of the phaseshift dependence on reactivity. Instead of small periodic perturbations of ϱ however a step function either in neutron source strength or in reactivity or in both of them is to be performed. The information needed for computation of the reactivity after the step can be obtained by means of a neutron counter with scaler.

2. Experimental Concept.

The initial subcritical state assumed for $t < 0$ shall be characterized by the initial stable power p_o, the initial "non reproduction source power" S_o and the corresponding negative reactivity ϱ_o

$$\varrho_o \equiv \frac{k_o - 1}{k_o \beta} \tag{4}$$

At $t = 0$ reactivity and source power shall suddenly be changed from ϱ_o to ϱ_a and from S_o to S_a respectively, which causes the power level to approach the asymptotic value p_a. Of course either ϱ or S may be constant at $t = 0$. There shall be a neutron counter whose counting rate is given by

$$c(t) = \gamma_o p_o \quad (t<0) \quad \text{and} \quad c(t) = \gamma_a p(t) \quad (t>0) \tag{5}$$

Shadowing effects and shifts of the neutron flux pattern are thus included, but direct counting of source neutrons shall be neglected. The integrated counting rate indicated by the scaler is normalized to zero at $t = 0$ and designated as

$$C(t) \equiv \int_o^t c(t)\,dt. \tag{6}$$

In fig.1 and fig.2 $c(t)$ and $C(t)$ are represented for two typical experiments.

The time lag τ is expected to be characteristic for the reactivity ϱ_a of the final state, essentially independent from the ratio γ_a / γ_o. The equation relating ϱ_a to τ and p_o/p_a shall be derived in the next section.

3. Theory of the Experiment

Let us start the theoretical considerations from the generalization of an equation by M. Ash (4):

$$\ell^* \dot{p}(t) = \varrho(t) p(t) - \int_0^t p'(\tau) W(t-\tau) d\tau + S/\beta \qquad (7)$$

where $\ell^* \equiv \ell/\beta k$ $\qquad \dot{p} \equiv dp/dt$

and

$$W(t) = \sum_i \frac{\beta_i}{\beta} e^{-\lambda_i t} \quad . \qquad p' \equiv dp/d\tau \quad .$$

For

$$S = S_0 \ , \quad \varrho(t) = \varrho_0 \qquad (t<0)$$

$$S = S_a \ , \quad \varrho(t) = \varrho_a \qquad (t>0)$$

the Laplace transform of (7) for $t>0$ is given by

$$\ell^* (s\bar{p}(s) - p_0) = \varrho_a \bar{p}(s) - (s\bar{p}(s) - p_0)\bar{W}(s) + \frac{1}{\beta s} S_a \quad . \qquad (8)$$

If S_a is replaced by $-\beta \varrho_a p_a$ and equation (8) solved for $\bar{p}(s)$ we get

$$\frac{\bar{p}(s)}{p_0} = \frac{1}{s} \cdot \frac{s\{\ell^* + \bar{W}(s)\} - \varrho_a \cdot p_a/p_0}{s\{\ell^* + \bar{W}(s)\} - \varrho_a} \quad . \qquad (9)$$

If p_0 and $p(t)$ $(t>0)$ are partly replaced by c_0/γ_0 and $c(t)/\gamma_a$, respectively, equation (9) yields

$$\bar{c}(s) = c_0 \frac{\gamma_a}{\gamma_0} \cdot \frac{1}{s} \cdot \frac{s\{\ell^* + \bar{W}(s)\} - \varrho_a p_a/p_0}{s\{\ell^* + \bar{W}(s)\} - \varrho_a} \quad . \qquad (10)$$

Since integration of $c(t)$ means division by s of $\bar{c}(s)$, according to the theory of Laplace transforms, we get

$$\bar{C}(s) = c_0 \frac{\gamma_a}{\gamma_0} \cdot \frac{1}{s^2} \frac{s\{\ell^* + \bar{W}(s)\} - \varrho_a p_a/p_0}{s\{\ell^* + \bar{W}(s)\} - \varrho_a} \quad . \qquad (11)$$

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(4) Milton Ash: "Solutions of the Reactor Kinetics Equation for Time Varying Reactivities". J. Appl. Phys. 27, 1030 - 31 (1956)

The only partial fractions of equation (11) which do not correspond to terms decaying as $t \to \infty$ are

$$\gamma_a p_a \frac{1}{s^2} \quad \text{and} \quad R/s\,, \tag{12}$$

where R means the residuum of equation (11) at the pole s = 0.

We get the following expression for R:

$$R = \frac{d}{ds}\left(s^2 \bar{C}(s)\right)_{s=0} = \frac{\gamma_a}{\varrho_a}(p_a - p_o)(\ell^* + \bar{W}(0))\,. \tag{13}$$

With the abbreviation

$$\Theta \equiv \ell^* + \bar{W}(0) = \ell^* + \sum_i \frac{\beta_i}{\beta}\lambda_i^{-1} \quad +) \tag{14}$$

we therefore obtain

$$\lim_{t\to\infty} C(t) = \gamma_a p_a \left[t + \frac{\Theta}{\varrho_a}\left(1 - \frac{p_o}{p_a}\right)\right]. \tag{15}$$

A comparison of equation (15) with the asymptotic representation of C(t) in fig.2 immediately yields the following simple result:

$$\underline{\varrho_a = \frac{\Theta}{\tau}\left(\frac{p_o}{p_a} - 1\right).} \tag{16}$$

If no considerable structural changes are made at t = o, e.g. if a small, weakly absorbing neutron source is moved only, the ratio of counting rates is equal to the power ratio ($\gamma_o = \gamma_a$), so that (16) can immediately be applied.

But even if p_o/p_a is not known accurately from c_o/c_a (unknown γ_o/γ_a) ϱ_a is well determined by τ if $p_o/p_a \ll 1$. If this condition cannot be fulfilled, but still $p_o < p_a$ it is suggested to plot the fictitious values $\varrho_{af}\,(c_o/c_a)$ versus c_o/c_a and to extrapolate that function to $c_o/c_a = o$. If on the other hand $p_o > p_a$ an extrapolation plot is absolutely necessary. But then $\varrho_{af}(c_o/c_a)$ must be extrapolated to $c_o/c_a = 1$. Then again $\gamma_o = \gamma_a$ because in the limit $c_o = c_a$ no control elements are moved, and so any shadowing effect is excluded.

+ For U^{235} as fissionable material and $\ell = 0$ $\Theta = 12{,}4$ sec.

4. Practical Performance of the Experiment and Evaluation of Statistical Errors.

It is recommended to measure the subcritical reactor power by means of a pulse detector in connection with a scaler and to record the arrival of every m^{th} output pulse from the scaler on a chart. The time marks are designated as t_ν, the suffix ν counting these pulse groups with $t_0 = 0$ as reference time. From $\nu = -N$ to $\nu = 0$ the marks are regularly spaced. At zero time there is nearly a discontinuity in the counting rate, so that the distance between successive time marks changes rapidly at first but soon reaches a new equilibrium value at say t_n. From t_n to t_{n+N} the spacing is essentially regular again.

Obviously equation (16) may be transformed to

$$\varrho_a = \frac{\theta}{\tau}\left(\frac{t_{n+N}-t_n}{-t_{-N}} - 1\right). \qquad (17)$$

If the detector counts exclusively neutron induced pulses and if the resolution of detector, amplifier and scaler is sufficiently high to avoid noticeable counting losses the relative error $\Delta\varrho_a/\varrho_a$ of the negative reactivity is mainly given by the statistical error of τ. Therefore this quantity and its statistical error shall be determined from the set $(t_{n+1}; t_{n+2}, \ldots\ldots t_{n+N})$ according to the least square fit method.

As shown in fig.2 t_ν and $C(t_\nu) \equiv C_\nu = \nu \cdot m$ are related by the asymptotic equation

$$\lim_{\nu\to\infty} C_\nu = \gamma_a p_a (t_\nu - \tau). \qquad (18)$$

For $\nu > n$ we may therefore accept the following equation:

$$t_\nu = \tau + \frac{m}{\gamma_a p_a}\nu \equiv \tau + A\nu \qquad (\nu > n). \qquad (19)$$

ν is exact due to the working principle of a scaler, whereas t_ν is subject to a statistical error, given by

$$\Delta t_\nu = \frac{\sqrt{m\nu}}{\gamma_a p_a} = A\sqrt{\nu/m}. \qquad (20)$$

The statistical weights w_ν of t_ν are therefore proportional to

$$w_\nu \equiv \nu^{-1} . \tag{21}$$

The most probable values of τ and A are determined from the least square condition

$$\sum_{\nu=n+1}^{n+N} w_\nu [t_\nu - A\nu - \tau]^2 = \text{minimum}, \tag{22}$$

i.e.

$$\sum_{\nu=n+1}^{n+N} [t_\nu - A\nu - \tau] = 0 , \tag{23}$$

$$\sum_{\nu=n+1}^{n+N} \nu^{-1} [t_\nu - A\nu - \tau] = 0 , \tag{24}$$

or

$$N\tau + \frac{N}{2}(N+2n+1)A = \sum_{\nu=n+1}^{n+N} t_\nu \tag{25}$$

$$\sum_{\nu=n+1}^{n+N} \nu^{-1}\tau + NA = \sum_{\nu=n+1}^{n+N} \frac{t_\nu}{\nu} . \tag{26}$$

The solution for τ is

$$\tau = \frac{\sum_{\nu=n+1}^{n+N} \left(\frac{1}{\nu} - \frac{2}{N+2n+1}\right) t_\nu}{\sum_{\nu=n+1}^{n+N} \frac{1}{\nu} - \frac{2N}{N+2n+1}} . \tag{27}$$

The partial sum of the harmonic series can be approximated by the following series (5):

$$\sum_{\nu=n+1}^{n+N} \frac{1}{\nu} = \ln \frac{n+N}{n+1} + \frac{1}{2}\left(\frac{1}{n+1} + \frac{1}{n+N}\right) + \frac{1}{12}\left[\frac{1}{(n+1)^2} - \frac{1}{(n+N)^2}\right] + \cdots . \tag{28}$$

The denominator of equation (27),

$$D(n,N) \equiv \sum_{\nu=n+1}^{n+N} \frac{1}{\nu} - \frac{2N}{N+2n+1} \quad \text{(cf. appendix!)}, \tag{29}$$

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(5) From the Euler-Maclaurin sum formula for finite series, cf.e.g. E. Madelung: "Die mathematischen Hilfsmittel des Physikers". New York 1943.

is then approximated by

$$D(n,N) \cong \ln \frac{n+N}{n+1} + \frac{N+2n+1}{2(n+1)(n+N)} \left(1 + \frac{N-1}{6(n+1)(n+N)}\right) - \frac{2N}{N+2n+1} \quad . \qquad (30)$$

Finally we obtain

$$\tau = \frac{1}{D(n,N)} \sum_{\nu=n+1}^{n+N} \left(\frac{1}{\nu} - \frac{2}{N+2n+1}\right) t_\nu \quad . \qquad (31)$$

The mean square statistical error $(\Delta\tau)^2$ of τ can be deduced from equations (20), (27) and (29), the result being

$$(\Delta\tau)^2 = \frac{m}{D(n,N)} \cdot \frac{1}{c_a^2} \qquad (32)$$

It is seen that $\Delta\tau$ is proportional to $\sqrt{m}$ and to the reciprocal counting rate c_a^{-1}. As can be verified by inspection of the D(n,N) values given in the appendix $D(n,N) \cong D(\mu n, \mu N)$, so that the introduction of a smaller scaling factor m, with invariant t_n, t_{n+N} and c_a, really reduces $\Delta\tau$.

Using equations (32) and (16) we obtain the relative error of τ which for practical purposes can be identified with the relative error of φ:

$$\left(\frac{\Delta\varphi}{\varphi}\right)^2 \cong \left(\frac{\Delta\tau}{\tau}\right)^2 = \varphi_a^2 \frac{m}{D(n,N)} \cdot \frac{1}{\theta^2 (c_a - c_o)^2} \qquad (33)$$

Thus for given times of observation the relative random error of φ_a is proportional to φ_a and inversely proportional to the difference of the counting rates. This means that φ can be calculated with the same statistical accuracy from both experiments shown in fig.1 and fig.2. As exact step functions and ideal coincidence of the step with the time mark at t_o are impossible a systematic error of τ must be taken into account. If the magnitude of this systematic error is comparable in both cases the experiment with bigger τ, corresponding to the condition $c_o > c_a$, is preferable.

5. Test Experiments at SAPHIR.

In order to test the method proposed in this report various

experiments have been performed at the Swiss swimming pool reactor SAPHIR.

In a first series of experiments the single control rod was dropped from different heights, so that the negative reactivity with two safety rods out and fully inserted control rod could be determined for different fuel configurations. γ_o was considerably different from γ_a, so that the reactivity was to be extrapolated. The results were in good agreement with predictions from reactivity measurements in supercritical and slightly subcritical states which had been performed according to the Nordheim method (inhour formula, cf. (1)).

Later instead of reactivity steps the Po-Be neutron source was moved inside a guiding tube placed at an edge of the reactor core far from the neutron detector (Westinghouse fission counter). γ was certainly unchanged when the source was moved out and in. One typical set of two experiments for a certain fuel configuration, with both safety rods in upper limit and the control rod partly withdrawn from its lower limit, are illustrated in fig. 1 and fig.2.

Time marks were written on a recorder chart with a speed of 2 mm/sec. First τ was roughly determined graphically according to the following formula:

$$\tau \cong \tau_r \equiv t_r - (t_{2r} - t_r) \quad (r = 1, 2, 3, \ldots) \tag{34}$$

The sequence τ_r converged towards τ with growing r, and this showed the lowest reasonable value of r which was then chosen as n in a more rigorous calculation of τ the results of which are given here:

a) Experiment with decreasing counting rate:

$c'_o = 478.5 \pm 3 \text{ sec}^{-1}$; $m' = 1024$
$c'_a = 97.3 \pm 0{,}4 \text{ sec}^{-1}$:, $n' = 20$; $N' = 25$;
$\tau' = 46.8$ sec.

$$\left(\frac{\Delta\tau'}{\tau'}\right)^2 = (0.036)^2$$

Thus $\varrho'_a = - 1.038 \pm 0.038$ dollars

b) Experiment with increasing counting rate.

$c_o = c'_a = 97.3 \pm 0.4 \text{ sec}^{-1}$; $m = 1024$
$c_a = c'_o = 478.5 \pm 3 \text{ sec}^{-1}$; $n = 70$; $N = 115$
$\tau = 9.85$ sec.

$$\left(\frac{\Delta\tau}{\tau}\right)^2 = (0.026)^2$$

Thus

$$\varrho_a = - 1.003 \pm 0.026 \text{ dollars}$$

These figures do not allow for systematic errors which might be introduced e.g. by

I. application of the S(t) step at a $t \neq 0$
II. distortion of the S(t) step function towards a limited ramp function
III. counting losses.
IV. direct counting of source neutrons.

Items I and II are estimated to contribute less than 0,8% and 3 % to the total error in experiment a) and b), respectively. The counting losses are below 1% in each experiment, but it has not yet been investigated how much the time lag τ is influenced by a constant dead time of the counting device. Direct counting of source neutrons must be very unimportant in view of the mutual position of source and detector with respect to the reactor core. As a matter of fact, the good agreement of ϱ'_a and ϱ_a allows to exclude some systematic error sources within the limit given by statistics.

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References.

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(5) From the Euler-Maclaurin sum formula for finite series, cf.e.g. E. Madelung: "Die mathematischen Hilfsmittel des Physikers". New York 1943.

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Appendix: Table of Selected D (n,N) Values.

D(n,N)	n = 5	n = 10	n = 15
N = 5	0,020635	0,004645	0,001733
N = 10	0,082515	0,023610	0,009924
N = 15	0,160560	0,053657	0,024584
N = 20	0,242302	0,090409	0,044239
N = 25	0,322765	0,130857	0,067457
N = 30	0,400034	0,173104	0,093113
N = 35	0,473471	0,215980	0,120370
N = 40	0,542987	0,258762	0,148623

FIG. 1: COUNTING RATES AS FUNCTION OF TIME FOR $\rho = -1.04$ \$

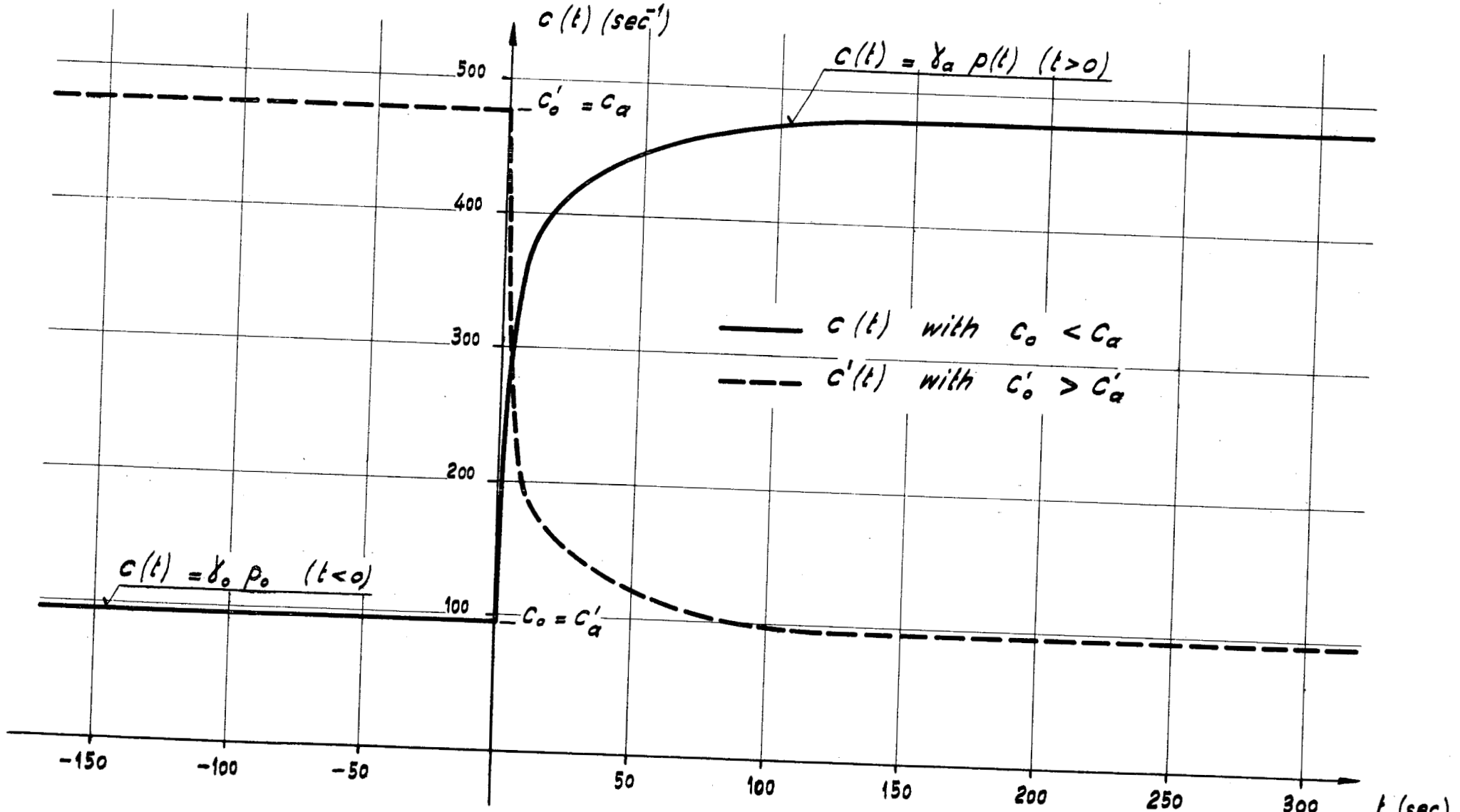

FIG. 2: TOTAL NUMBER OF COUNTS AS FUNCTION OF TIME FOR $\rho_a = -1.04$ $