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189-56-14

ANALYSIS AND STABILIZATION OF JUNCTION TRANSISTOR OSCILLATORS IN THE LOWER AUDIO RANGE

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SC 1004-A (3-55)

Technical Memorandum

189-56-14

ANALYSIS AND STABILIZATION OF JUNCTION TRANSISTOR OSCILLATORS IN THE LOWER AUDIO RANGE

J. W. Ellis - 1423

ABSTRACT

Transistor oscillators have been analyzed using the low-frequency equivalent circuit for the transistor. Conditions for oscillation and expressions for the frequency of oscillation and for stability of frequency with variations in the parameters of the transistor have been derived. In addition, conditions for reactance stabilization were determined.

Oscillators of very low audio frequency have been designed. Tests were made which indicate the excellent frequency stability with changes in temperature.

Case No. 614.00

September 12, 1956

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ANALYSIS AND STABILIZATION OF JUNCTION TRANSISTOR OSCILLATORS IN THE LOWER AUDIO RANGE

INTRODUCTION

This paper is the outgrowth of a project to study audio-frequency oscillators in which transistors are used as the active element. The chief requirement of the oscillators is a very high degree of frequency stability over a wide temperature range at very low frequency.

In any oscillator there are certain general conditions which must be satisfied. The voltage fed back to the input must be both of the correct phase and amplitude in order to sustain oscillation. There must also be a nonlinear element to limit the amplitude of the oscillations; waveform is usually produced or controlled by resonant elements.

The two most common types of oscillators are those with negative resistance and those with external feedback. Only these oscillators with external feedback were considered in this study of junction transistors.

GENERALIZED FOUR-TERMINAL TRANSISTOR OSCILLATOR

This paper describes a method devised to reduce the dependency of the frequency on the parameters of the transistor. The generalized low-frequency circuit for transistor oscillators is presented below. The frequencies are low enough so that the collector capacity of the transistor can be neglected, and the parameters of the transistors are purely resistive. Z_1, Z_2, Z_3, and Z_m are reactances of the frequency-determining network, and Z_b, Z_c, and Z_e are reactances added externally to increase the stability of the circuit.

Loop equations for the circuit of Figure 1 are as follows:

$$i_1\left(Z_b' + Z_e' + Z_1\right) + i_2\left(Z_e' - Z_m\right) - i_3\left(Z_1 + Z_m\right) = 0 .$$

$$i_1\left(Z_e' - Z_m - R_m\right) + i_2\left(Z_e' + Z_c' + Z_2 - R_m\right) + i_3\left(Z_2 + Z_m\right) = 0$$

$$-i_1\left(Z_1 + Z_m\right) + i_2\left(Z_2 + Z_m\right) + i_3\left(Z_o\right) = 0.$$

For oscillations to start, the determinant of the coefficients of the loop equations above must be equal to zero. If the determinant is less than zero, the oscillations will build up in magnitude until limited by some nonlinearity of the circuit. If the determinant is greater than

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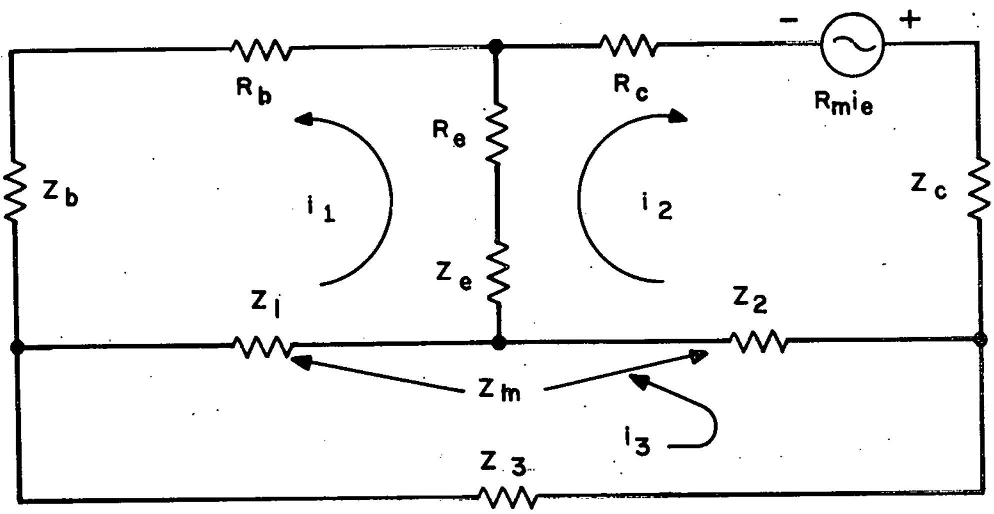

Figure 1

$$i_e = i_1 + i_2, \quad Z_b' = R_b + Z_b = R_b + jX_b$$

$$Z_m = jX_m, \quad Z_c' = R_c + Z_c = R_c + jX_c$$

$$Z_e' = R_e + Z_e = R_e + jX_e$$

$$Z_o = Z_1 + Z_2 + Z_3 + 2Z_m = jZ_c$$

= series impedance of frequency-determining network

zero, the oscillations decrease in amplitude and die out. Furthermore, the three simultaneous equations will have trivial solutions unless the determinant is equal to zero.[1]

The determinant is

$$\begin{vmatrix} \left(Z_b' + Z_e' + Z_1\right) & \left(Z_e - Z_m\right) & -\left(Z_1 + Z_m\right) \\ \left(Z_e' - Z_m - R_m\right) & \left(Z_e' + Z_c' + Z_2 - R_m\right) & \left(Z_2 + Z_m\right) \\ -\left(Z_1 + Z_m\right) & \left(Z_2 + Z_m\right) & \left(Z_o\right) \end{vmatrix} .$$

Expanding the determinant, separating the real and imaginary parts, and setting each equal to zero gives the following:

Real parts equal to zero:

$$\begin{aligned} Z_o\Big[&\left(X_1 + X_b\right)\left(R_c + R_e - R_m\right) + \left(R_b + R_e\right)\left(X_c + X_2\right) + \left(X_e\right)\left(R_b + R_c\right) \\ &+ X_m\left(2R_e - R_m\right)\Big] + \left(2R_e - R_m\right)\left(X_1 + X_m\right)\left(X_2 + X_m\right) \\ &+ \left(X_1 + X_m\right)^2 \left(R_c + R_e - R_m\right) + \left(X_2 + X_m\right)^2 \left(R_b + R_e\right) = 0. \end{aligned} \qquad (1)$$

(1) See "Advanced Engineering Mathematics" by Wylie, pages 63 and 579, 1951 Edition.

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Imaginary parts equal to zero:

$$Z_o\Big[\big(R_bR_e + R_bR_c + R_eR_c - R_bR_m\big) - \big(X_1 + X_b\big)\big(X_e + X_c + X_2\big) - X_e\big(X_c + X_2\big) - X_m\big(2X_e - X_m\big)\Big] + 2\big(X_e - X_m\big)\big(X_1 + X_m\big)\big(X_2 + X_m\big) + \big(X_1 + X_m\big)^2\big(X_c + X_e + X_2\big) + \big(X_2 + X_m\big)^2\big(X_b + X_e + X_1\big) = 0. \qquad (2)$$

COLPITTS OSCILLATOR

The basic Colpitts oscillator with no stabilization was analyzed with Equations 1 and 2.

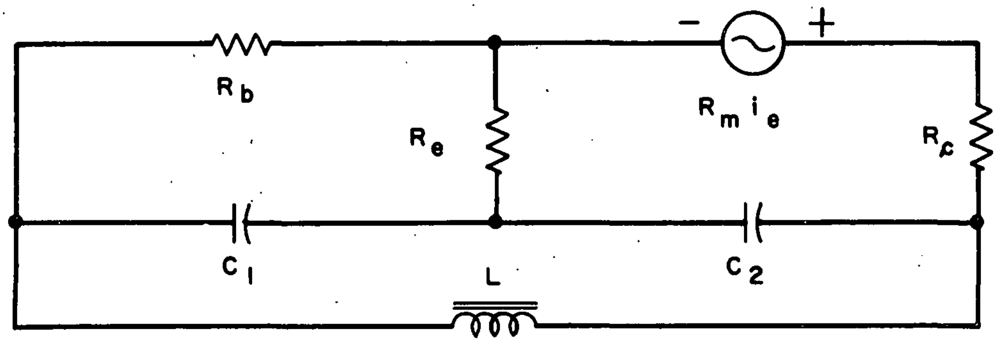

Figure 2

Referring to Figure 2, $Z_b = Z_c = Z_e = X_m = 0$

$X_1 = \frac{-1}{\omega C_1}$, $X_2 = \frac{-1}{\omega C_2}$, $X_3 = \omega L$

$Z_o = j(X_1 + X_2 + X_3)$.

Substituting the above values in Equation 2 gives the following expression for the frequency of oscillation:

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{LC_T} + \frac{1}{KC_1C_2}}$$

where,

$$K = \big(R_bR_e + R_bR_c + R_eR_c - R_bR_m\big)$$

$$C_T = \frac{C_1C_2}{C_1 + C_2}.$$

At resonance $Z_o \approx 0$. Assuming the frequency of oscillation to be at the resonant frequency of the tank and substituting into Equation 1 gives

$$X_1^2\big(R_e + R_c - R_m\big) + X_1X_2\big(2R_e - R_m\big) + X_2^2\big(R_b + R_e\big) = 0.$$

Dividing by $X_2^{\,2}$ and solving for $\frac{X_1}{X_2}$ gives

$$\frac{C_2}{C_1} = \frac{X_1}{X_2} = \frac{\left(R_m - 2R_e\right) \pm \sqrt{\left(2R_e - R_m\right)^2 - 4\left(R_b + R_e\right)\left(R_c + R_e - R_m\right)}}{2(R_c + R_e - R_m)} .$$

Assuming $R_e << R_m$, $R_e << \left(R_c - R_m\right)$

$$4\left(R_b + R_e\right)\left(R_c + R_e - R_m\right) \ll \left(2R_e - R_m\right)^2$$

gives

$$\frac{C_2}{C_1} = \frac{X_1}{X_2} \approx \frac{R_m}{R_c - R_m} \approx \frac{\alpha R_c}{R_c - \alpha R_c} = \frac{\alpha}{1 - \alpha}$$

since $\Delta \leq 0$ for continuous oscillation

$$\frac{C_2}{C_1} \leq \frac{\alpha}{1 - \alpha} \text{ or } \frac{C_1}{C_2} \geq \frac{1 - \alpha}{\alpha} .$$

Again, using Equation 1 and dividing by $X_1^{\,2}$, gives

$$\frac{C_1}{C_2} = \frac{X_2}{X_1} \approx \frac{\left(R_m - 2R_e\right) \pm \sqrt{\left(2R_e - R_m\right)^2 - 4\left(R_b + R_e\right)\left(R_c + R_e - R_m\right)}}{2(R_b + R_e)} .$$

Making the same assumption as before gives

$$\frac{C_1}{C_2} = \frac{X_2}{X_1} \approx \frac{R_m}{R_b + R_e} \approx \frac{R_{21}}{R_{11}} .$$

Therefore, $\frac{C_1}{C_2} \leq \frac{R_m}{R_b + R_e}$ for oscillation.

The above gives for oscillation: $\frac{R_m}{R_b + R_e} \geq \frac{C_1}{C_2} \geq \frac{1 - \alpha}{\alpha}$.

If $\frac{C_1}{C_2} > \frac{R_m}{R_b + R_e}$, oscillations will die out.

If $\frac{C_1}{C_2} < \frac{R_m}{R_b + R_e}$, oscillations will build up until limited by a nonlinearity of the circuit.

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STABILIZATION OF COLPITTS OSCILLATOR

The external series reactance to be added in the base lead for stabilization can be determined as follows. Eliminating $\frac{1}{KC_1C_2}$ from Equation 3 gives

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{LC_T}} .$$

If we assume that $Z_e = Z_c = Z_m = 0$, that $Z_b \neq 0$, and that oscillations are at the natural frequency of the tank, then $Z_o = 0$. Substituting these assumptions in Equation 2 gives

$$X_1{}^2X_2 + X_2(X_1 + X_b) = 0.$$

Solving for X_b gives:

$$X_b = - X_1\left(1 + \frac{X_1}{X_2}\right).$$

To find the stabilization in the collector, it is assumed that:

$$Z_b = Z_e = Z_m = Z_o = 0, \quad Z_c \neq 0.$$

Equation 2 becomes: $X_1{}^2(X_c + X_2) + X_2{}^2X_1 = 0$

$$X_c = - X_2\left(1 + \frac{X_2}{X_1}\right).$$

To find the stabilization in the emitter circuit, it is assumed that:

$$Z_o = Z_b = Z_c = Z_m = 0, \quad Z_e \neq 0.$$

Equation 2 then becomes:

$$2X_eX_1X_2 + X_1{}^2(X_e + X_2) + X_2{}^2(X_e + X_1) = 0$$

$$X_e = \frac{-\left(X_1{}^2X_2 + X_1X_2{}^2\right)}{\left(X_1 + X_2\right)^2} .$$

Since X_1 and X_2 are capacitive, X_b, X_c, and X_e must be inductive.

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HARTLEY OSCILLATOR

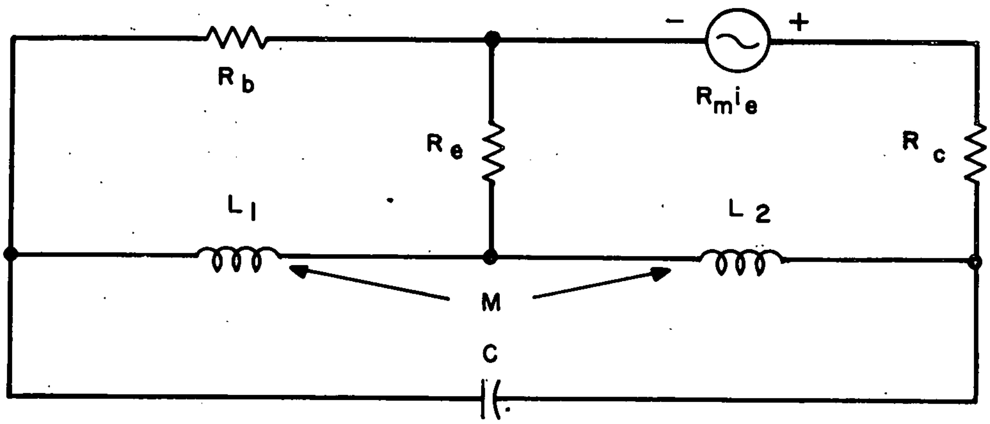

Figure 3

Let $X_1 = \omega L_1$, $X_2 = \omega L_2$, $X_3 = \frac{-1}{\omega C}$, $X_m = \omega M$

$$Z_b = Z_c = Z_e = Z_o = 0.$$

Equation 1 becomes

$$(L_2 + M)^2 (R_b + R_e) + (L_2 + L_m)(L_1 + M)(2R_e - R_m) + (L_1 + M)^2 (R_c + R_e - R_m) = 0$$

$$\frac{L_2 + M}{L_1 + M} = \frac{(R_m - 2R_e) \pm \sqrt{(2R_e - R_m)^2 - 4(R_b + R_e)(R_c + R_e - R_m)}}{2(R_b + R_e)} .$$

Assuming: $R_e \ll R_m$, $R_e \ll (R_c - R_m)$,

$$4(R_b + R_e)(R_c + R_e - R_m) \ll (2R_e - R_m)^2$$

gives: $\frac{L_2 + M}{L_1 + M} \approx \frac{R_m}{R_b + R_e}$;

or, since $\Delta \le 0$ for oscillation,

$$\frac{L_2 + M}{L_1 + M} \le \frac{R_m}{R_b + R_e} .$$

Dividing the above equation by $(L_2 + M)^2$ gives

$$\frac{L_1 + M}{L_2 + M} = \frac{(R_m - 2R_e) \pm \sqrt{(2R_e - R_m)^2 - 4(R_b + R_e)(R_c + R_e - R_m)}}{2(R_c + R_e - R_m)}$$

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Assuming: $R_e \ll R_m$, $R_e \ll \left(R_c - R_m\right)$,

$$4\left(R_b + R_e\right)\left(R_c + R_e - R_m\right) \ll \left(2R_e - R_m\right)^2$$

gives

$$\frac{L_1 + M}{L_2 + M} \approx \frac{R_m}{R_c - R_m} \approx \frac{\alpha}{1 - \alpha} .$$

Since $\Delta \leq 0$,

$$\frac{L_1 + M}{L_2 + M} \leq \frac{\alpha}{1 - \alpha}$$

or

$$\frac{L_2 + M}{L_1 + M} \geq \frac{1 - \alpha}{\alpha} .$$

Therefore, if $\frac{R_m}{R_b + R_e} \geq \frac{L_2 + M}{L_1 + M} \geq \frac{1 - \alpha}{\alpha}$,

oscillation will occur.

STABILIZATION OF HARTLEY OSCILLATOR

Stabilization by adding reactance in base circuit from Equation 2 by assuming

$$Z_e = Z_c = Z_o = 0, \; Z_b \neq 0, \; X_m \neq 0$$

gives

$$X_b = \frac{2X_m\left(X_1 + X_m\right)\left(X_2 + X_m\right) - X_1\left(X_2 + X_m\right)^2 - X_2\left(X_1 + X_m\right)^2}{\left(X_2 + X_m\right)^2} .$$

Stabilization by adding reactance in collector circuit from Equation 2 assuming

$$Z_e = Z_b = Z_o = 0, \; Z_c \neq 0, \; X_m \neq 0$$

gives

$$X_c = \frac{2X_m\left(X_1 + X_m\right)\left(X_2 + X_m\right) - X_1\left(X_2 + X_m\right)^2 - X_2\left(X_1 + X_m\right)^2}{\left(X_1 + X_m\right)^2} .$$

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Stabilization by adding reactance in emitter circuit from Equation 2 assuming

$$Z_b = Z_c = Z_o = 0, \; Z_e \neq 0, \; X_m \neq 0$$

$$X_e = \frac{2X_m(X_1 + X_m)(X_2 + X_m) - X_1(X_2 + X_m)^2 - X_2(X_1 + X_m)^2}{(X_1 + X_m)^2 + (X_2 + X_m)^2} .$$

FREQUENCY OF OSCILLATION OF HARTLEY OSCILLATOR

To determine the expression for the frequency of oscillation without stabilization, assume

$$Z_b = Z_c = Z_e = 0, \; Z_o = j\left(\omega L_1 + \omega L_2 + 2\omega M - \frac{1}{\omega C}\right) .$$

$$Z_1 = \omega L_1, \; Z_2 = \omega L_2, \; Z_3 = \frac{-1}{\omega C}, \; X_m = \omega M.$$

Substituting these values into Equation 2 gives

$$f = \frac{1}{2\pi} \frac{1}{\sqrt{C(L_1 + L_2 + 2M) + (L_1 L_2 - M^2)/K}}$$

where,

$$K = (R_b R_e + R_b R_c + R_e R_c - R_b R_m) .$$

EXPERIMENTAL DATA

Since a constant frequency oscillator was needed, the Hartley oscillator circuit was selected because of the difficulty in obtaining capacitors which would be very stable over the required temperature range. It was thought, also, that the use of only one capacitor with small value of capacitance would help to insure stability. Silvered mica capacitors were used throughout the study. However, a limited amount of work was done on the Colpitts oscillator at room temperatures.

At the outset of the study, the typical Hartley circuit shown below was used.

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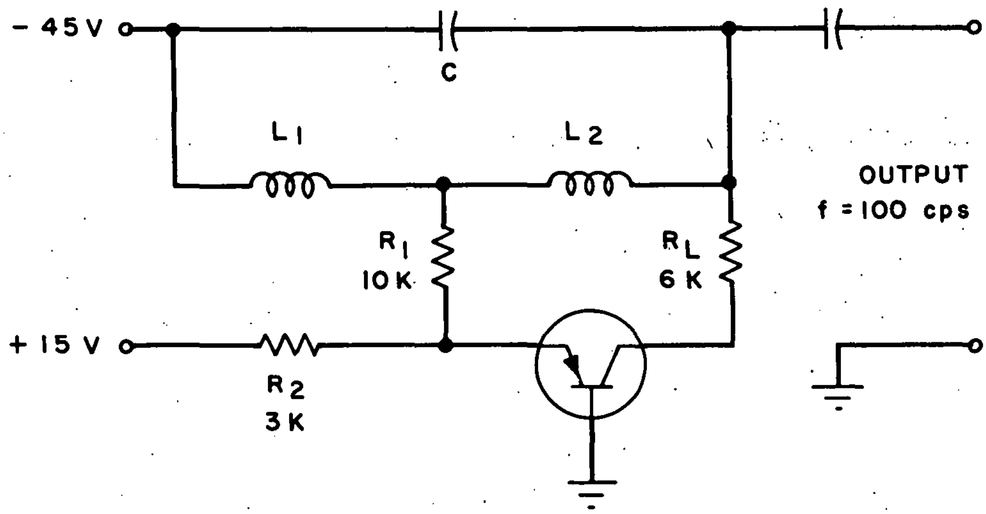

Figure 4

The circuit of Figure 4 operated at a frequency of about 100 cps.* The output waveforms were sinusoidal. The ratio of L_1/L_2 was varied, and it was found that the use of $L_1/L_2 = 1$ gave the best results. For this reason, L_1/L_2 will be equal to one throughout this memo except when indicated.

R_1 of Figure 4 could be replaced by a device which increases its resistance with temperature. This would increase the stability.

Since it was desired to use only one bias supply, the following circuit was used.

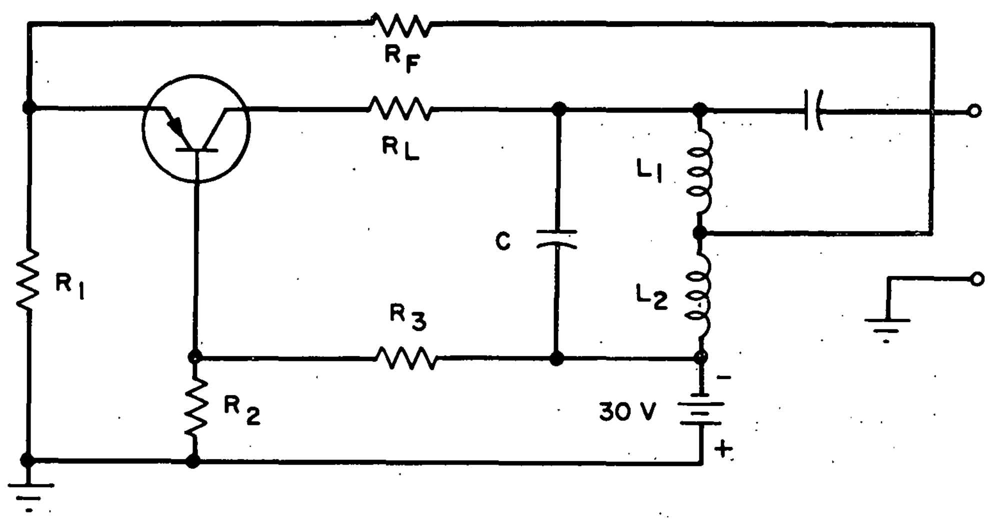

Figure 5

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* This will be the frequency of all circuits discussed hereafter.

The tank circuit can be placed in either the collector circuit or the base circuit. Where only one bias supply was used, the stability seemed to increase with the tank circuit in the collector as shown in Figure 5.

A PNP germanium transistor was used in Figure 5. The temperature was varied over a range from -70° to $+165^{\circ}$F. A counter which gave an accuracy of at least 1 part in 100 was used to measure the frequency. However, the accuracy had to be greater than this and was later improved. At a frequency of 100 cps it was difficult to measure small changes in frequency. The germanium transistor operated very well at room temperature, but the stability became less with a change in temperature. At -45°F the oscillation stopped. The transistor was removed from the circuit, allowed to warm up, and then replaced. It operated well until it became cold again. Several different germanium transistors were used. The stability seemed to depend somewhat on the gain (α) of the transistors. Transistors of higher gain were more stable.

The output and frequency stability increased when a diode or other device which increased in resistance with temperature was placed in the emitter circuit. Thus, the temperature coefficient would depend on the necessary change required, depending on the transistor used.

An NPN silicon transistor was used in the circuit of Figure 5, and the polarity of the bias was reversed. The value of each component was varied without affecting the stability.

The circuit oscillated readily with any of the NPN silicon transistors used. The output wave was almost a perfect sine wave.

Taking the output across R_L changed the waveshape and provided a lower impedance output. This output circuitry change seemed to result in less effect from loading and to increase the stability.

A diode was placed in the base circuit to compensate, as much as possible, for the increase in DC current in the base.

In order to obtain as constant an emitter current as possible, a high-impedance inductor was placed in the emitter circuit.

The DC current in the base changes with temperature. This change in current would tend to change the inductance of the coil if it were placed in the base circuit. The total collector current is equal to $I_{co} + \alpha I_e$ for a grounded base. The α decreases with an increase in temperature, and I_{co} increases. I_{co} for the NPN silicon transistors used is very small even at high temperatures. Therefore, the increase in I_{co} is approximately adjusted by the decrease in αI_e. This adjustment tends to keep the total collector current constant, and stability is therefore increased by putting the tank circuit in the collector circuit.

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The final configuration of the Hartley oscillator is shown below.

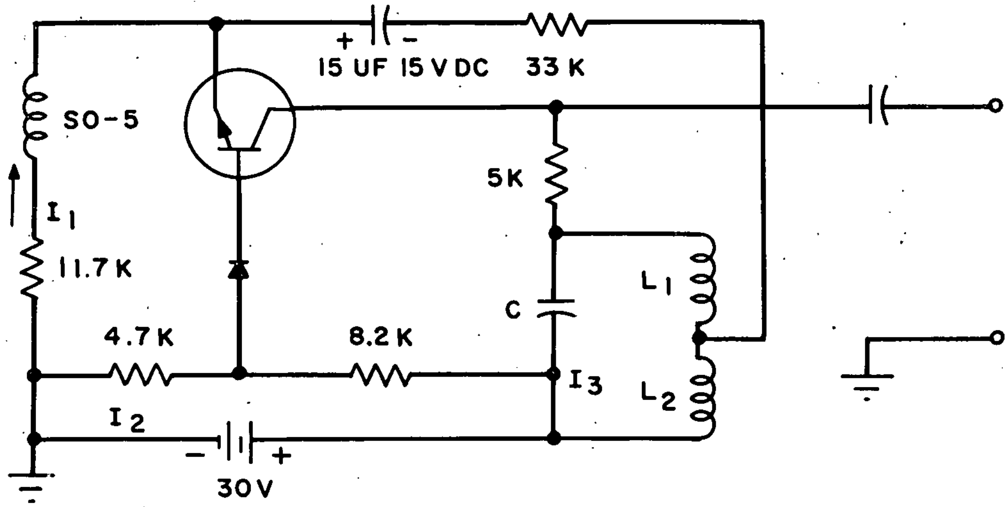

Figure 6

In the circuit of Figure 6 a silicon transistor was used. The frequency stability was very good over the required temperature range. When the temperature was held at 150°F for eight hours, the stability did not change. Output was fairly constant, and the waveshape was an almost perfect sine wave. There was no distortion at either high or low temperature.

Very little data were taken for the Colpitts oscillator. However, enough work was done to indicate that it should operate similar to the Hartley circuit.

The ratio of C_2/C_1 was varied for a Colpitts oscillator, and oscillations were obtained with a ratio of C_2/C_1 as low as 1 and as high as 100. These ratios agree closely with those obtained for Figure 2. However, the circuit used contained external impedance in the collector and emitter circuits. Since the circuit of Figure 2 contained no external impedance or stabilization, it deviated from the practical conditions in which parameters for stabilization were present.

The tank circuit of Figure 6 was kept at room temperature while the rest of the circuit was subjected to changes in temperature. The stability was very good.

When the tank circuit was subjected to temperature changes, the frequency stability was still very good. However, the output was not quite as stable. Curves have been plotted in Figure 7 to indicate the difference.

These tests show that the transistor (a silicon transistor was used) could be stabilized. A large amount of the instability, as indicated by the above test, is the result of changes in the components of the tank circuit when subjected to changes in temperature.

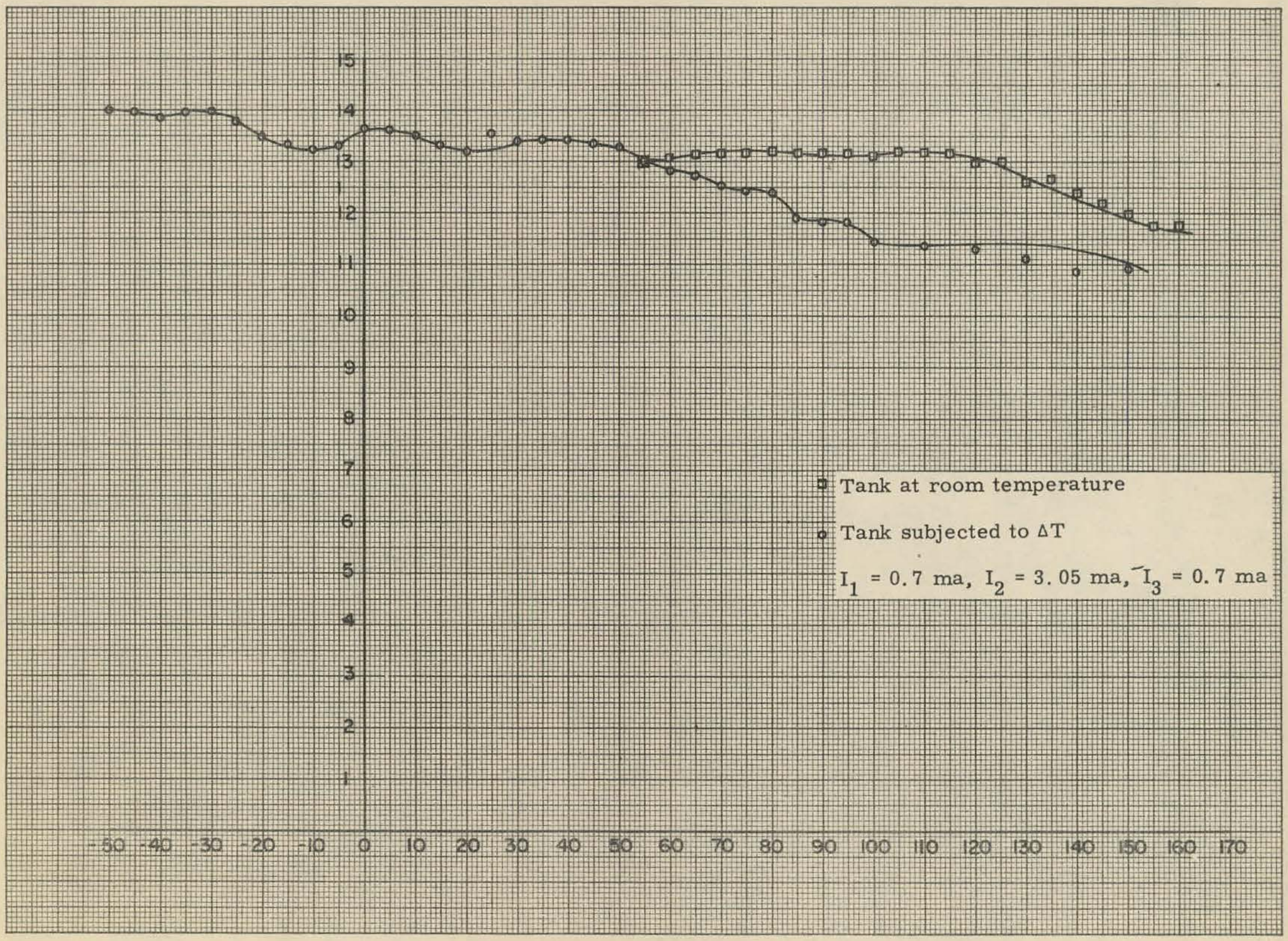

Figure 7

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Thus, if more stable components were available, the over-all stability required could be obtained more easily. In order to take advantage of the miniaturization possible with transistors, these components should be as small as possible.

* The period of the oscillations rather than the frequencies themselves was measured, a more accurate method of determining the drift in frequency. The period was constant out to fourth decimal place.

OTHER OSCILLATOR CIRCUITS

The following circuits were designed. They were operated at room temperature only.

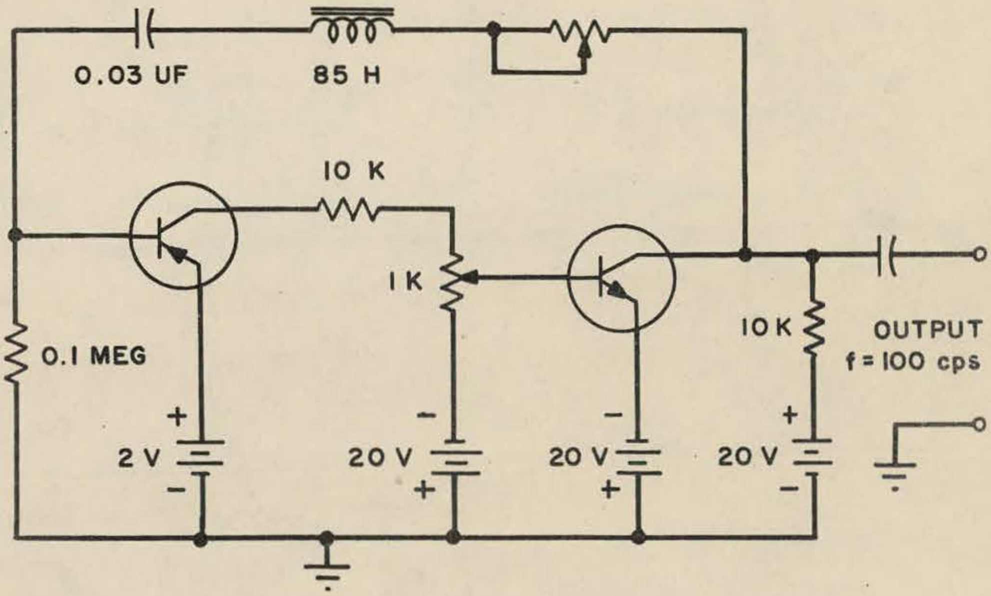

Figure 8 -- DIRECT COUPLED TWO-STAGE OSCILLATOR

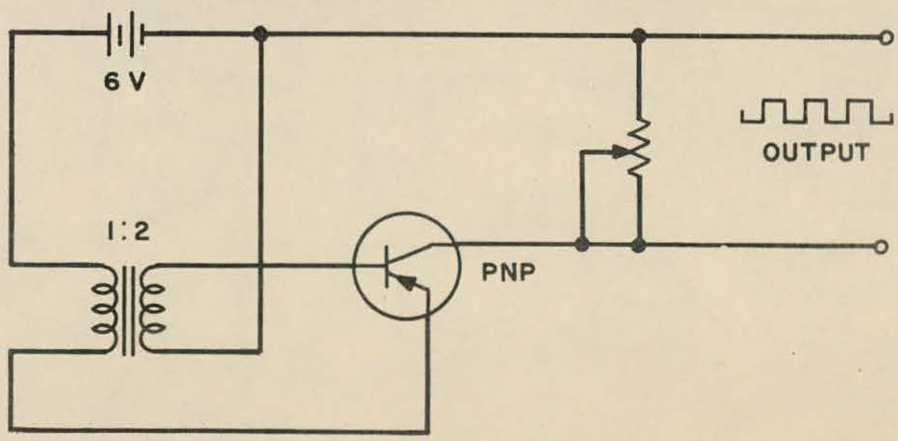

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Figure 9 -- UNSTABILIZED OSCILLATOR

CONCLUSIONS

The work described in this paper was done to determine if oscillator circuits could be transistorized and still be stable. The methods used indicate means of stabilization of a transistor oscillator. A more extensive study would perhaps produce better methods which give a more desirable degree of frequency stability.

It is evident that one of the major difficulties of getting good stability is the stability of the transistor. The silicon transistor is very stable and reduces this difficulty considerably.

Although formulae for stabilization were derived, no experimental verification of the theory was made because of the limited time.

The loading could be improved by using a transformer output and taking the feedback from the output winding. The primary winding of the transformer could be tuned to the desired frequency.

The desired degree of frequency stability was not obtained, but the study showed that a much higher degree of stability could be obtained.

J. W. ELLIS, 1423

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