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Implementation of a Friction Model in an Eulerian Viscoplastic Formulation for Steady Flow

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1 Introduction

The goal of this project was to implement the routines necessary to use the friction model of Wilson and Korzekwa into the finite element analysis program *hickory*, in the case of an Eulerian reference frame. *hickory* is deformation simulation code based on finite element modeling of viscoplastic deformation. When using *hickory*, time-dependent problems are modeled from a Lagrangian reference frame; while steady-state problems are modeled from an Eulerian reference frame. The friction model had been implemented in earlier versions of *hickory*, for use with a Lagrangian reference frame. Additional modifications were required, however, to extend this capability to the case of an Eulerian reference frame. That is the subject of this report.

The necessary modifications were related to the time integration of the friction state variables. In the case of a Lagrangian reference frame, the initial values of the friction state variables are given on all relevant boundary segments. Then, as time increases, the material time derivative is evaluated at all points along the boundary and the state variables are updated independently at each point. In contrast, the application of an Eulerian reference frame to study a steady-state flow requires that each specified boundary segment be a streamline of the flow. As such, an initial value for each state variable must be given at the first point of the streamline, and subsequent values must be determined from previous values by integration along the streamline. Additional routines were added to *hickory* to implement the streamline integration along the boundary.

A plane strain rolling problem was used both to test the implementation and as a source of comparison among friction models. One may model such a rolling problem by a steady flow of material approaching a circular roll, being reduced in thickness along the roll, and leaving the roll with a specified reduction in thickness. The driving force of the rolling process is the frictional force of the roll pulling material through the contact zone. This makes rolling an ideal test problem.

2 The Friction Model

A number of friction models are currently implemented in *hickory*. Each friction model relates the frictional traction to a number of variables, including the relative velocity of the contacting bodies. One simple model is the velocity difference model:

$$\boldsymbol{T}_t = c(\boldsymbol{u}_r - \boldsymbol{u}_w),$$

where $\boldsymbol{T}_t$ is the frictional traction, $\boldsymbol{u}_r$ is the roll velocity, $\boldsymbol{u}_w$ is the workpiece velocity and c is a material constant. In this case, the frictional traction depends continuously on the workpiece velocity. Furthermore, since *hickory* is a velocity based code, the model can be implemented implicitly, i.e. the traction contribution of the workpiece velocity can be incorporated directly into the velocity stiffness matrix.

Another simple model is the Coulomb model:

$$|\boldsymbol{T}_t| = cT_n,$$

where T_n is the normal component of traction and the direction of $\boldsymbol{T}_t$ is opposite that of the relative velocity. Here, the dependence of the frictional traction on the workpiece velocity is not explicitly stated and so must be treated as an external applied force. Numerically, the velocity-implicit friction models are more stable when implemented in a velocity based code such as *hickory*.

The general form of the Korzekwa-Wilson friction model is

$$|\boldsymbol{T}_t| = c_m A;$$

the frictional traction is determined by the fractional contact area, A, and a function of the material state, c_m. As in the Coulomb model, the direction is opposite that of the velocity difference. The first implementation of this model in *hickory* treated c_m as a constant up to the sign. That implementation was later modified so that c_m depended also on the normal component of traction, T_n, making it akin to the Coulomb model. The current implementation treats c_m as a continuous function of the velocity difference so as to make it velocity implicit. Nevertheless, the primary feature of the model, the computation of the fractional contact area, remained the same.

The material derivative of the fractional contact area is computed from the asperity lay vector. The material being modeled is considered to have surface asperities aligned in a certain direction, and as the deformation proceeds, the alignment direction changes with the surface deformation. If the initial lay vector is $\boldsymbol{a}_0$, the lay vector at a later time is $\boldsymbol{a}(t) = \boldsymbol{F}(t)\boldsymbol{a}_0$, where $\boldsymbol{F}(t)$ is the deformation gradient at time t. $\boldsymbol{F}$ satisfies the ordinary differential equation:

$$\dot{\boldsymbol{F}} = \boldsymbol{L}\boldsymbol{F},$$

where $\boldsymbol{L}$ is the velocity gradient. It follows that the equation for the asperity lay is:

$$\dot{\boldsymbol{a}} = \boldsymbol{L}\boldsymbol{a}.$$

The equation is integrated numerically using a midpoint rule:

$$\boldsymbol{a}_{n+1} = \boldsymbol{a}_n + \frac{1}{2}(\boldsymbol{L}_n\boldsymbol{a}_n + \boldsymbol{L}_{n+1}\boldsymbol{a}_{n+1})\Delta\, t.$$

This formula is implicit and so must be applied iteratively. In an Eulerian reference frame, the time step is determined by the spatial subdivision of the streamline and the velocity. In *hickory*, the boundary streamline is first divided into its constituent surface elements. Then each surface element is further subdivided into a specified number of subintervals. Numerical integration then proceeds along this refinement.

Once the asperity lay vector is known, one may form a local coordinate system using the lay vector as the first direction. The second direction is taken to lie on the three-dimensional surface and perpendicular to the lay direction, and the third direction is taken normal to the surface. In this coordinate system, one may keep track of accumulated strains in the first two directions, λ_1 and λ_2. These variables are then used to compute the rate of change in the contact area, A. The exact details of the contact area update are given in the routines provided by Dr. D. A. Korzekwa.

It is also necessary to compute the normal components of traction at each point on the boundary. This is done by first computing the global force vector and then solving a set of variational equations to find an equivalent set of tractions. This set of tractions is given in the standard coordinate system and each component is continuous across surface elements. Then, for each element, the normal and tangential components are computed; note that this introduces discontinuities in the tractions across elements, since the elemental normal vectors are discontinuous. Typically, the tractions computed this way will be noisy and must be smoothed. In our implementation, the tractions are smoothed on each boundary segment independently. First, the normal and tangential components are averaged at the surface element boundaries to produce a continuous field on a given segment; then, the highest frequency component of oscillation is removed by orthogonal projection. The resulting tractions still have some noise, but are consistent enough to use in the friction models.

3 User Input

Using the friction model in the Eulerian case requires the same input as in the Lagrangian but with the addition of one further line specifying the direction of integration along the relevant boundary segments. As in the Lagrangian case, friction models are declared and constants are specified in the same place. Similarly friction surfaces and initial conditions are specified as before. However, one more line must follow the initial values for asperity lay and contact area.

This line contains a list of the boundary segment numbers which require friction state variables, with the sign of the number indicating the direction of integration. A boundary segment given a positive number will be taken in a counterclockwise direction; whereas a boundary segment given a negative number will be taken in clockwise direction. For example, if boundary segments five and six require friction state variables, and if the velocity field follows surface five and flows against surface six, one would input the line

```
5 -6
```

The order does not matter.

If multiple boundary segments are specified, each one is integrated independently, according to the sign of the segment number, as in the input line above, and by the initial values for the friction state variables. A possible enhancement of the implementation would be to allow for consecutive segments with the initial value on a latter segment being taken from the final value of the preceding segment.

4 Examples

A rolling problem provided a good test of the implementation. After transients have died out, the flow of the metal can be assumed to be steady-state, and so can be conveniently modeled in an Eulerian reference frame. In our sample problem, we modeled high temperature rolling of 304 stainless steel using Hart's model to describe the bulk properties. We modeled the friction using the three models described earlier.

The mesh boundary was divided into six segments: the inlet (1); the midroll symmetry plane (2); the outlet (3); the outlet free surface (4); the roll contact surface (5); the inlet free surface (6). The free surfaces and the inlet had traction-free boundary conditions. On the symmetry plane and the roll contact surface we required the condition of no mass flux. Friction boundary conditions were specified on the roll-contact boundary to determine the tangential components of traction. For the Korzekwa-Wilson friction model and the velocity difference model, the friction boundary condition was sufficient to drive the problem, and so the outlet was assumed to be traction free. However, for the Coulomb model, it was necessary to prescribe a velocity condition on the outlet so as to produce a driving normal traction on the roll contact surface; otherwise a zero velocity field would satisfy all the constraints.

The roll surface velocity was given as 1.5 m/s. For the Coulomb model, the horizontal component of the outlet velocity was given as 1.6 m/s, which is consistent with the results of the other models. Typically, the models give a velocity profile slower than the roll speed at the inlet and greater than the roll speed at the outlet, and with a neutral point, the point where the material speed matches the roll velocity, under the roll.

Several data sets were run for each friction model to examine the effects of the friction model parameters on the results. Figure 1 shows a typical frictional traction profile for the velocity difference model, and figure 2 shows a typical profile for the Coulomb model. The difference between the profiles stems from the fact that in the velocity difference model, the frictional traction depends continuously on the velocity; whereas in the Coulomb model, the traction abruptly changes sign at the neutral point. In fact, the current implementation of Coulomb's model uses a smooth scaling function to remove the discontinuity.

As discussed earlier, the current implementation of the Korzekwa-Wilson friction model also uses a scaling function to eliminate abrupt changes in sign, but in this case, the scaling is velocity implicit. In addition, the scaling function takes parameters which make the behavior of the friction model more or less like the velocity difference model. The input parameter. c_1, is

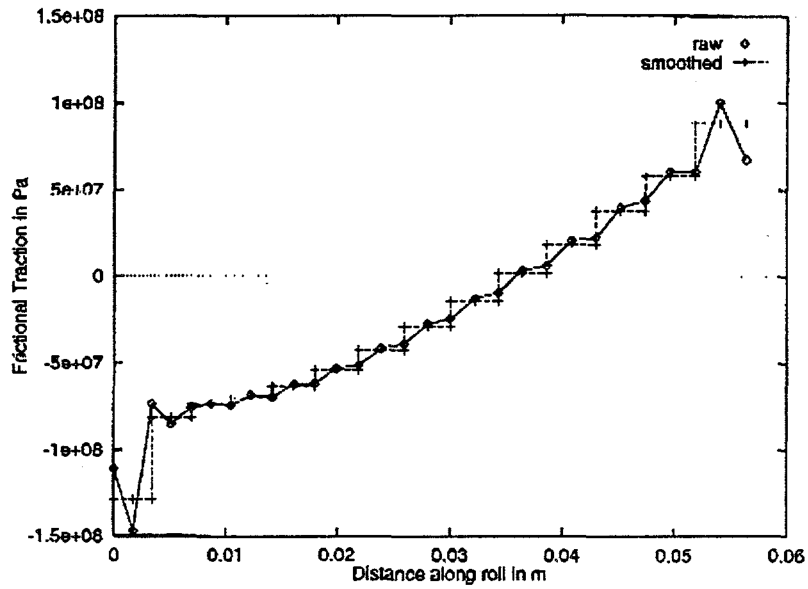

Figure 1: Velocity Difference Model

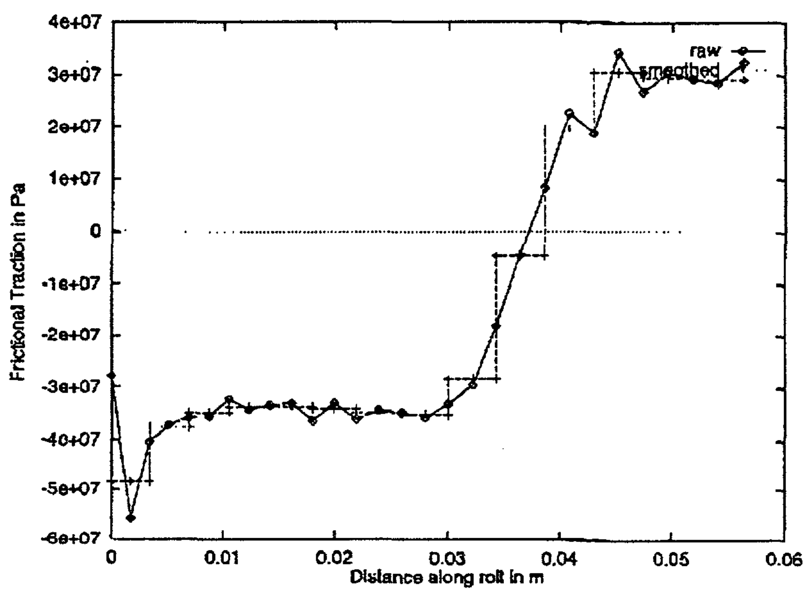

Figure 2: Coulomb Model

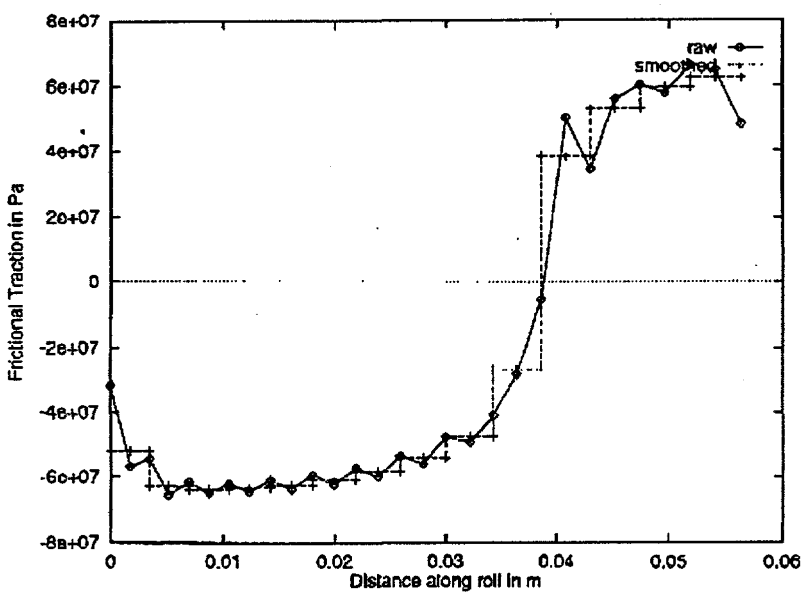

Figure 3: Korzekwa-Wilson friction model $c_1 = 0.1$

a power law type parameter ranging between 0 and 1. When c_1 is close to zero, the Korzekwa-Wilson friction model behaves more like the Coulomb model, and when c_1 is near one, the model behaves more like the velocity difference model. Figures 3 and 4 show the friction traction profiles for the case in which $c_1 = 0.1$ and the case in which $c_1 = 0.8$, respectively.

In the Korzekwa-Wilson friction model, the fractional contact area profile is of particular interest. Figure 5 shows the contact area profiles for three different data sets. The friction coefficient c_3 was given values of $4 \times 10^7, 7 \times 10^7$ and 1×10^8, in all cases the units being n/m^2. The typical behavior is for the contact area to saturate rapidly. In the third case, however, the contact area never saturates because the friction coefficient is too low.

Finally, we should discuss the normal tractions since they are used explicitly in both the Coulomb model and the Korzekwa-Wilson friction model. Figure 6 shows the normal tractions. The normal tractions are somewhat noisy, but generally they peak at contact and diminish until release. Only one value of the normal traction is used per surface element in computing the friction traction. In the current implementation, this value is taken as a weighted average over the surface element nodal points. Figure 6 indicates

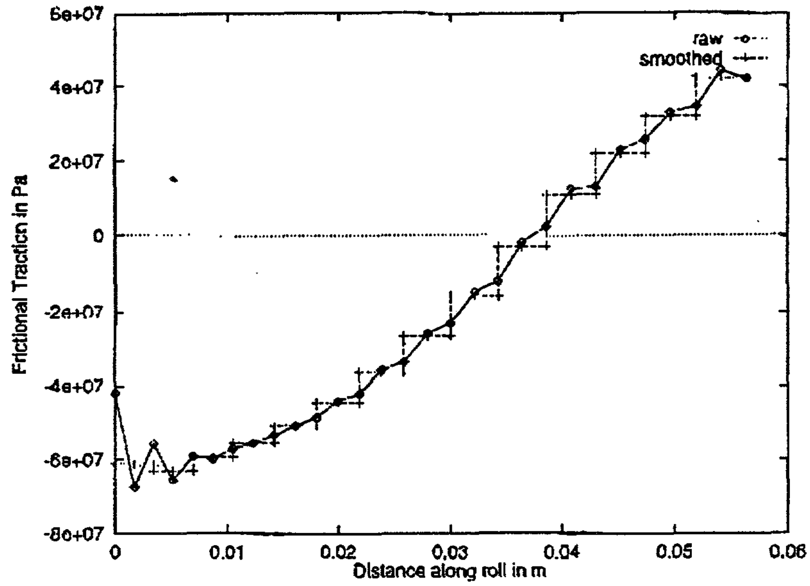

Figure 4: Korzckwa-Wilson friction model $c_1 = 0.8$

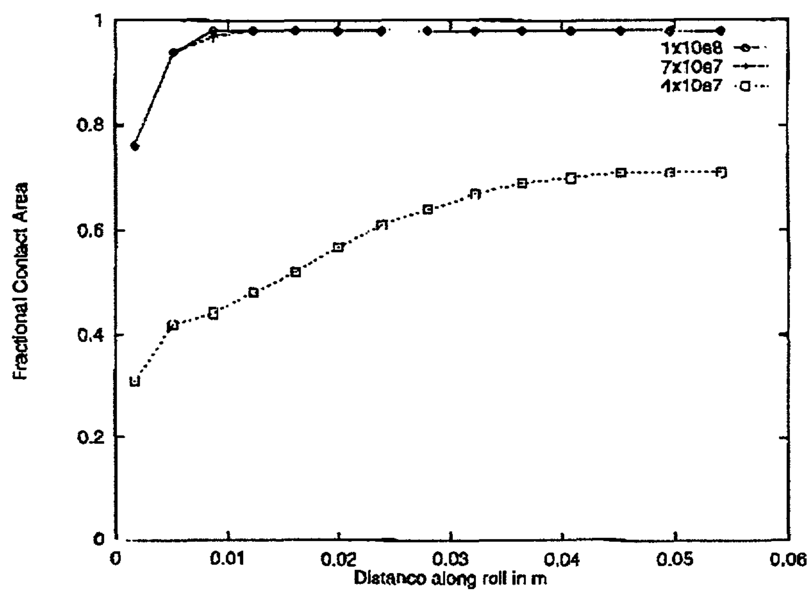

Figure 5: Contact Area Fraction Along the Roll

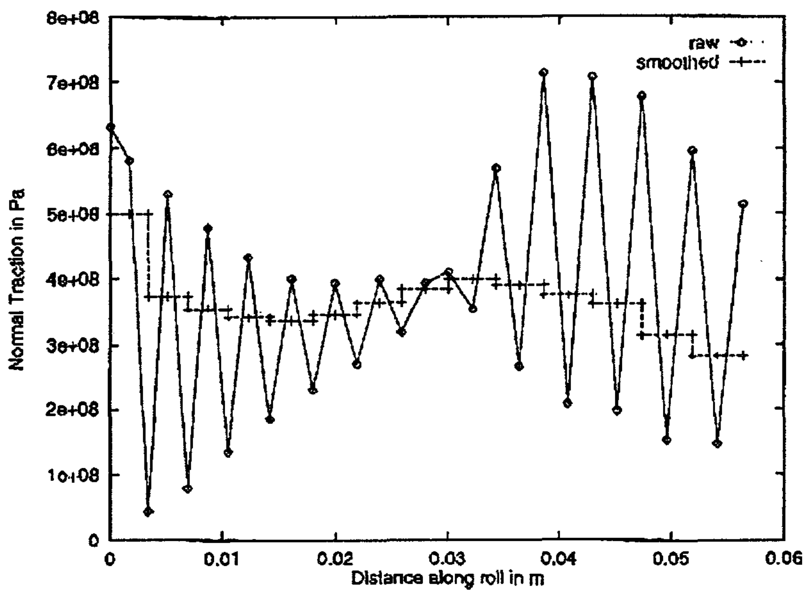

Figure 6: Normal Tractions Along the Roll

the raw normal tractions as given by the traction recovery algorithm and the smoothed (averaged) normal tractions as used in the friction computation. Even though the raw normal tractions are noisy, the smoothed ones are consistently more regular.

5 Summary

The friction modeling capability of hickory was extended to include the model of Korzekwa and Wilson in the case of an Eulerian reference frame. New routines were required which involved integration and update of the friction state variables along segments of the global boundary of the input mesh. The traction evaluation routines were improved to give smoother normal and frictional tractions. A sample rolling problem was run to illustrate features of the current implementation.

6 Acknowledgments

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