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UNCLASSIFIED

DEPARTMENT OF PHYSICS

REVIEW OF PARAMETRIC INSTABILITIES

Principal Investigator

George Schmidt

Paper Presented at AGARD-NATO Meeting,

in Edinburgh, Scotland

November 1973

Research Report

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Introductory Survey to Session III No. 4

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Summary

Parametric instabilities produced by electromagnetic waves propagating in a magnetic field free plasma are reviewed. The discussion is based on the use of the ponderomotive force as the basic physical mechanism responsible for these instabilities. Decay of the electromagnetic wave into an electron and ion wave, the oscillating two stream instability, filamentation instability, stimulated Raman and Brillouin scattering, resistive and reactive quasi modes can all be treated in a unified fashion, in a uniform medium.

If the plasma is bounded the threshold power is non-zero even in the absence of damping, and may be the dominant factor in determining threshold and growth rate values. The threshold for stimulated Raman scattering increases in the presence of plasma density gradients, while temperature gradients have a similar effect on simulated Brillouin scattering. The finite cross section of the pump wave determines the undamped threshold for the filamentation instability. In a nonuniform plasma stimulated Raman backscattering becomes very strong in the neighborhood of the point where the pump frequency is twice the local plasma frequency.

Discussion

In this review I will attempt to give a description, in terms of simple physical processes, of a class of parametric instabilities that occur in a plasma under the influence of an electromagnetic wave. To do so the first part of this talk will be devoted to the simplest of cases; that of a plane electromagnetic wave interacting with a uniform, magnetic field free plasma in the presence of small perturbations. The physical insight gained by studying this simple case can then be used to study more complex (and more realistic) situations where the plasma has a finite size, density and temperature gradients, etc. The effects of a background magnetic field will not be considered here.

Since in Maxwell's equations the fields are linear functions of the charges and currents, all nonlinearities in electromagnetic wave propagation come from the charges and currents being nonlinearly related to the fields. In the case of plasmas this nonlinearity can be ultimately reduced to the nonlinearity in the particle motion. To the lowest order one obtains a nonlinear force acting on the particle, the ponderomotive force.[1]

One writes the expanded equation of motion in a wave field as $\ddot{\underline{r}} + \ddot{\tilde{\underline{r}}} = \frac{q}{m}[\underline{E}(\underline{r}) + (\tilde{\underline{r}} \cdot \nabla)\underline{E} + \dot{\tilde{\underline{r}}} \times \underline{B}$ + higher order terms] where $\tilde{\underline{r}} = -\frac{q}{m\omega^2}\underline{E}$ is the oscillating part of the particle coordinate and $\underline{r}$ is the slowly varying part. By averaging this equation over an oscillation period one finds

$$\ddot{\underline{r}} = -\frac{q^2}{m^2\omega^2}[<(\underline{E} \cdot \nabla)\underline{E}> + <\dot{\underline{E}} \times \underline{B}>] = -\frac{q^2}{m^2\omega^2}\nabla\frac{<E^2>}{2}$$

so the average nonlinear force on the particle may be written as

$$<\underline{F}> = m\ddot{\underline{r}} = -q\nabla\psi$$

where $\psi = \frac{q}{2m\omega^2} \langle E^2 \rangle$ is the ponderomotive potential. This formula can easily be generalized for two or more waves. The net effect of this force is to push particles away from regions of high field intensity, toward minima of the field intensity. Note that the acceleration is inversely proportional to the mass square, so the nonlinearity in the ion motion can be ignored. In a plasma, however, electrostatic coupling between electrons and ions transmits the ponderomotive force from electrons to ions.

We will now use the ponderomotive force concept to discuss parametric plasma processes.

Consider for example an electromagnetic wave propagating in a plasma, and let there be a small perturbation on the wave intensity perpendicular to the direction of propagation. This results in a force pushing electrons into the weaker field regions, dragging ions behind them, due to electrostatic coupling. The plasma dielectric function $\varepsilon = 1 - \frac{\omega_p^2}{\omega^2}$ will therefore be increased (ω_p decreased) in regions of high field intensity. According to the wave equation, however, the intensity increases further where the index of refraction is higher, forcing out even more particles from this region until the wave has broken up into filaments[(2)]. These are like wave guides or light pipes generated by the nonlinearity of the medium.

Let us look now at a periodic density perturbation, like an electron or ion wave propagating (say) parallel with the electromagnetic wave. This density perturbation acts as an optical grating and scatters some of the light backwards if the light wavelength is twice the wavelength of the plasma perturbation, or in general if $k = 2k_o \cos\theta$, where θ is the angle between the wave vector of incoming radiation k_o and that of the plasma wave k. Now the interference of the incoming and scattered wave gives rise to a spatially varying intensity (a standing wave) and hence a sinusoidal ponderomotive force with wave number k. The phase of this ponderomotive force is such as to drive the density perturbation in the plasma even higher, producing more scattering and so on. This process is called stimulated Raman scattering if the plasma wave is an electron mode, and stimulated Brillouin scattering for an ion wave[3]. These waves obey the frequency and wave number matching conditions, $\omega_o = \omega + \omega_1$, $k_o = k + k_1$ and the process may be thought of as the decay of a photon into another photon plus a plasmon or phonon.

The plasma wave need not always be an eigenmode[4]. If the electromagnetic wave is sufficiently strong, such that the ponderomotive force dominates over the electrostatic restoring force, the wave is driven off resonance and a "reactive quasi mode" results. Such a plasma wave does not satisfy the linear dispersion relation. Another interesting case arises when the plasma wave interacting with the electromagnetic wave is a strongly damped one. Such a wave has no sharp resonance, but driven by the beat wave of the incoming and scattered e.m. wave, it will absorb energy from them. This absorption of energy (and momentum) is accompanied by scattering leading to instability. Here a photon decays into a scattered photon with lower frequency and wavenumber, while the difference in energy and momentum is absorbed by plasma particles. This "resistive quasi mode" scattering is reminiscent of nonlinear Landau damping. Both the resistive and reactive quasi mode exists in Raman and Brillouin scattering as well.

If the electromagnetic wave frequency ω_o is close to the plasma frequency ω_p a new situation arises that has been extensively investigated[5,6]. The wavelength of the electromagnetic wave becomes very large, so that this case may be treated as an interaction of a uniform oscillating electric field interacting with the plasma. Consider a plasma density fluctuation under the influence of this field E_o, when $\omega_o < \omega_e$ the electron oscillation frequency (Bohm-Gross frequency). The electrons oscillation in this field as driven at the frequency ω_o produce their own oscillating electric field E_k due to space charge as shown. The ponderomotive force produced by the superposition of E_o and E_k drives more plasma into regions of higher density, increasing it further[7]. This is the oscillating two stream instability, a purely growing mode, like the filamentation instability.

When $\omega_o > \omega_e$ the phase of electron oscillations changes by π, and the ponderomotive force changes sign, driving the oscillation of the density perturbation. This is equivalent to the driving of an ion wave, and the process may be thought of as the decay of a photon into a phonon and a plasmon[6]. The frequency and wave number matching conditions are again satisfield. If E_o is sufficiently large a reactive quasi mode again arises[5].

All the modes discussed can be derived as special cases of a general dispersion relation[3,4]. In the presence of damping mechanisms all these processes occur only when the driving wave intensity exceeds a certain threshold. One may characterize the driver by the ratio of oscillatory electron velocity in the wave field to the speed of light $\frac{V_o}{c} = \frac{eE}{m\omega_o c} \approx 1,\ 6.10^4 \frac{\sqrt{P}}{\omega_o}$ where P is the power density in Watts/m^2. The parameter regimes for different instabilities are tabulated at the end of the paper.

The results can be briefly summarized as follows:

For Raman and Brillouin scattering and reactive quasi mode scattering, one obtains the lowest thresholds and highest growth rates, when the scattered light has the same plane of polarization as the incoming light and the directions of propagation are opposite, that is for the case of backscattering. The directional dependence of resistive quasi modes is more complicated, depending on the parameters of the problem. Filamentation, the oscillating two stream instability, and decay into electron and ion modes grow fastest when the plasma waves propagate at right angles to the driving wave. The threshold power necessary to excite Brillouin backscattering exceeds that for Raman backscattering by the factor

$$\left.\frac{P_B}{P_R}\right)_t \approx \frac{\Gamma_i}{\Gamma_e}\sqrt{\frac{M}{m}}\, k\lambda_D$$

where Γ_i and Γ_e are the damping rates of the ion and electron wave respectively and λ_D is the Debye length. This is usually a large number. The growth rates much above threshold are related by

$$\left(\frac{\gamma_R}{\gamma_B}\right)^2 \approx k\lambda_D \sqrt{\frac{M}{m}}$$

Comparing now the filamentation instability with Brillouin backscattering one finds

$$\left.\frac{P_F}{P_B}\right)_t \approx \frac{\omega_{pi}}{2\Gamma_i k_o \lambda_D} \quad \text{and} \quad \left(\frac{\gamma_B}{\gamma_F}\right)^2 \approx \frac{c}{2c_s}$$

where c_s is the ion sound velocity.

It would seem therefore that unless $k_o\lambda_D$ is very small, Raman backscattering dominates over Brillouin scattering and the latter over filamentation in the under dense region. Computer simulation experiments support this contention[8]. Considering more realistic configurations however, where the plasma size is limited and temperature and density gradients are present, one is led to different conclusions.

Consider first the effect of finite plasma size[9] along the direction of propagation extending from $x = o$ to $x = \ell$. At $x = \ell$ there can be no backscattered radiation present, it has to build up within the slab. It escapes at $x = o$ and is no longer accessible to drive plasma waves. The plasma waves on the other hand propagate forward, have zero amplitude at $x = o$ and are absorbed in the boundary layer at $x = \ell$. The loss of backscattered and plasma wave energy represent losses not very different from damping. One may estimate the equivalent damping rate for each wave as V/ℓ, where V is the absolute value of the group velocity of the wave. In an infinite plasma the threshold condition may be written simply as $\gamma_o > \sqrt{\Gamma_1 \Gamma_2}$ where γ_o is the growth rate in the absence of damping and Γ_1, Γ_2 are the damping rates of the decay modes. Replacing $\Gamma \to \frac{V}{\ell}$ one is led to the threshold estimate for a finite plasma slab in the absence of damping as

$$\frac{\gamma_o \ell}{\sqrt{V_1 V_2}} > 1$$

Indeed the detailed threshold calculation, solving the coupled mode equations with the proper boundary conditions, yields the same answer, except that one on the right hand side is to be replaced by $\frac{\pi}{2}$. This yields essentially identical conditions for Brillouin and Raman stimulated backscattering

$$\frac{V_o}{c}\frac{\ell}{\lambda_D} > \pi$$

If the actual damping of a wave becomes comparable or larger than the equivalent loss damping, it should also be included raising the threshold further.

A density or temperature gradient[10] in the plasma has an effect similar to that of finite size. Consider for instance Raman backscattering in a plasma with a density gradient. At some point (say at $x = o$) resonant back-scatter occurs with the local frequency and wavenumber matching condition satisfied. Further upstream the driver and backscattered wave produce a ponderomotive force to drive electron plasma oscillations. However, since the plasma density there is different, the local eigenfrequency or wave number does not match that of the driver and only small amplitude oscillations will be driven. This mismatch leads to an effective interaction length $\ell' \sim \Delta k^{-1}$ where Δk is the wavenumber shift for the plasma wave of given frequency in the density gradient. If the incoming and backscattered wave also suffer wavenumber shifts, this must also be included. In a linear density gradient Δk is a linear function of x , and ℓ' is easily calculated. In a finite plasma with a density gradient the smaller of ℓ or ℓ' will determine the threshold. Clearly a temperature gradient has a similar effect on stimulated Brillouin scattering, while a density gradient has no effect on this instability.

The quasi modes and the filamentation instability do not depend on resonances hence they are not sensitive to density and temperature gradients. An interesting threshold exists for the filamentation instability, due to the finite size of the beam (or plasma) in the direction perpendicular to beam propagation[2]. As the incident power decreases, the filament cross section proportionally increases. Once the filament cross section exceeds that of the beam, filamentation can no longer occur. This results in the threshold for filamentation

$$P_t \approx 10^4 \, (T_e + T_i) \left(\frac{\omega_o}{\omega_p}\right)^2 \qquad \omega_o >> \omega_p$$

where P is given in Watts and the electron and ion temperatures in electron volts.

A special case is presented by Raman scattering in a nonuniform plasma near the point where the local plasma frequency is half the driving wave frequency[11]. It follows from the frequency matching condition that the frequency of the backscattered wave is the local plasma frequency or in other words, the backscattered wave is at its cutoff. At cutoff however, the group velocity of the wave is very small, (its wavelength very large), leading to an accumulation of wave energy. Hence, one may expect a strong enhancement of the instability in this region. This has indeed been seen in computer simulation[12]. When the effects of the plasma waves generated by this process are taken into account it is found that the backscattered wave can be partially or totally trapped in a finite region. In such a case the threshold and growth rates are near the homogeneous plasma values.

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Table I

Dependence of growth rate on pump strength for electron wave scattering.

$\frac{v_0}{c} \simeq \frac{eE_0}{m\omega_0 c}$	growth rate
$\frac{\Gamma_p}{(\omega_p\omega_0)^{1/2}} \gg \frac{v_0}{c} \gg \left(\frac{\Gamma_-\Gamma_p}{\omega_0\omega_p}\right)^{1/2}$	$\left(\frac{v_0}{c}\right)^2 \frac{\omega_0\omega_p}{\Gamma_p}$
$\left(\frac{\omega_p}{\omega_0}\right)^{1/2} > \frac{v_0}{c} > \frac{\Gamma_p}{(\omega_p\omega_0)^{1/2}}$	$\left(\frac{v_0}{c}\right)(\omega_p\omega_0)^{1/2}$
$\left(\frac{v_0}{c}\right) > \left(\frac{\omega_p}{\omega_0}\right)^{1/2}$ Reactive quasi mode	$\left(\frac{v_0}{c}\right)^{2/3} (\omega_p^{\,2}\omega_0)^{1/3}$
$\frac{v_0}{c} > \left(\frac{\Gamma_-}{\omega_0}\right)^{1/2} k_0\lambda_D$ Resistive quasi mode	$\left(\frac{v_0}{c}\right)^2 \omega_0 \,\mathrm{Im}\, \frac{1}{\chi_e}$

(From J. Drake et. al. U.C.L.A. Report PPG-158, 1973)

Table II

Growth rates of the ion wave scattering

$\frac{v_0}{c}$	growth rate
$\frac{\Gamma_a}{\omega_{pi}}\left(\frac{c_s}{c}\right)^{1/2} > \left(\frac{v_0}{c}\right) > \left(\frac{2\Gamma_a\Gamma_-}{\omega_0\omega_{pi}} k_0\lambda_D\right)^{\frac{1}{2}}$	$\frac{\omega_{pi}}{2}\left(\frac{v_0^2}{c_s c}\right)\left(\frac{\omega_{pi}}{\Gamma_a}\right)$
$\sqrt{\frac{2c_s}{c}}\, k\lambda_D > \frac{v_0}{c} > \frac{\Gamma_a}{\omega_{pi}}\left(\frac{c_s}{c}\right)^{1/2}$	$\frac{v_0}{(2cc_s)^{1/2}}\,\omega_{pi}$
$\frac{v_0}{c} > \left(\frac{\omega_{pi}}{\omega_0}\right)^{1/2}(k\lambda_D)^{3/2}$ Reactive quasi mode	$\left(\frac{v_0}{c}\right)^{2/3}(\omega_{pi}^2\omega_0)^{1/3}$
$\frac{v_0}{c} > \frac{v_e}{c}\left(\frac{2\omega_0\Gamma_-}{\omega_p^2}\right)^{1/2}\left[\mathrm{Im}\,\frac{1}{1+k^2\lambda_D^2\chi_i}\right]^{-1/2}$ Resistive quasi mode	$\frac{1}{2}\left(\frac{v_0}{v_e}\right)^2\frac{\omega_p^2}{\omega_0}\,\mathrm{Im}\,\frac{1}{1+k^2\lambda_D^2\chi_i}$

(From J. Drake et. al. U.C.L.A. Report PPG-158, 1973)

Table III

Filamentation instability

Threshold	Growth rate for $\frac{V_o}{c} \gg \frac{V_{oT}}{c}$
$\frac{V_{oT}^2}{c^2} = \frac{\Gamma_-}{\omega_o}\left(1 + \frac{T_i}{T_e}\right) k_o^2 \lambda_D^2$	$\gamma = \frac{V_o}{c}\,\omega_{pi}$

Table IV

Inhomogeneous thresholds

Stimulated Raman backscatter	Stimulated Brillouin backscatter
$\left(\frac{V_o}{c}\right)^2 \gg \frac{1}{k_o L_n}$	$\left(\frac{V_o}{c}\right)^2 \gg \frac{\lambda_D^2 k_o}{L_T}$
L_n is the density scale length	L_T is the temperature scale length

References

1., H. Boot, R. Harvie 1957. Charged Particles in a Non Uniform Radio Frequency Field. Nature 180, 1187 (1957).

A. V. Gaponov, M. A. Miller 1958. Potential Wells for Charged Particles in a High Frequency Electromagnetic Field. J. Exptl. Theoret. Phys. U.S.S.R. 34, 242 (1958) [Soviet Physics JETP 7, 168 (1958)].

G. Schmidt 1966. Physics of High Temperature Plasmas, Academic Press 1966, p. 47-49.

2., P. Kaw, G. Schmidt and T. Wilcox 1973. Filamentation and Trapping of Electromagnetic Radiation in Plasmas. Phys. Fluids 17, 1522 (1973).

3., L. M. Gorbunov 1968. Perturbation of a Medium by a Field of a Strong Electromagnetic Wave. J. Exptl. Theoret. Phys. U. S. S. R. 55, 2298 (1968). [Soviet Phys. JETP 28, 1220 (1969)].

4., J. Drake, P. K. Kaw, Y. C. Lee, G. Schmidt, C. S. Liu and M. N. Rosenbluth, 1973. Parametric Instabilities of Electromagnetic Waves in Plasmas. U.C.L.A. Report PPG-158.

5., K. Nishikawa 1968. Parametric Excitation of Coupled Waves I and II. J. of the Phys. Soc. of Japan, 24, 916 and 1152 (1968).

6., V. P. Silin 1965. Parametric Resonance in a Plasma. J. Exptl. Theoret. Phys. U.S.S.R. 48, 1679 (1965). [Soviet Physics JETP 21, 1127 (1965)]

D. F. DuBois, M. V. Goldman 1965. Radiation Induced Instability of Electron Plasma Oscillations. Phys. Rev. Letters, 14, 544 (1965).

7., A. Y. Wong, G. Schmidt 1973. Physical Interpretation of the Oscillatory Two Stream Instability. U.C.L.A. Report PPG-151.

8., D. W. Forslund, J. M. Kindel and E. L. Lindman 1973. Nonlinear Behavior of Stimulated Brillouin and Raman Scattering in Laser Irradiated Plasmas. Phys. Rev. Letters. 30, 739 (1973).

9., D. Pesme, G. Laval and R. Pellat 1973. Parametric Instabilities in Bounded Plasmas. Phys. Rev. Letters, 31, 203 (1973).

10., M. N. Rosenbluth 1972. Parametric Instabilities in Inhomogeneous Media. Phys. Rev. Letters, 29, 565 (1972).

M. N. Rosenbluth, R. Z. Sagdeev 1972. Comments on Plasma Physics and Controlled Fusion 1, 129 (1972).

11., J. F. Drake, Y. C. Lee 1973. Absolute Raman Scattering Instabilities in an Inhomogeneous Plasma. U.C.L.A. Report PPG-156.

12., J. Kindel 1973. Private Communication.