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February 15, 1958

REACTOR CONTROLS ANALOG FACILITY (RCAF)

OPERATIONS MANUAL

F. P. Green

OAK RIDGE NATIONAL LABORATORY
operated by
UNION CARBIDE CORPORATION
for the
U.S. ATOMIC ENERGY COMMISSION

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CONTENTS

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INTRODUCTION

An electronic analog computer is an engineering tool developed during the last decade to assist the engineer and scientist in the solution of problems which are expressible as a group of algebraic and differential equations in a single independent variable, usually time. There are two basic types of computers: the digital, which uses numerical binary arithmetic to perform its operations, and the analog, which uses electrical and electromechanical components displaying transfer functions similar to those of the system being studied to perform its operations. Commonplace examples representing digital and analog devices are the desk calculator and the slide rule, respectively.

In the electronic analog computer, the magnitudes of currents represent the magnitudes of dependent variables, regardless of their dimensional description. The currents are produced by applying known or derived input signal voltages across the properly selected input impedances. The mathematical operations upon these variables produce the desired unknown variables as output voltages, whose magnitudes vary with time and which may be recorded on strip-chart recorders, oscilloscopes, or digital read-out devices.

The independent-variable scale can be selected arbitrarily as "real" time or as compressed or expanded time to suit the range of the variables and time constants involved. If real time is selected, a "simulation" is performed whereby the computer can act as an artificial component of an actual system to be controlled by an operator or an automatic controller. Since the analog computer may serve as a model of a specific kinetic process, it can be connected by suitable transducers to the very system it simulates, for the purpose of controlling that system. By specifying the desired demand parameters, the controlling functions are defined by the difference between the measured variables and their demands.

In all cases the computer can be made to perform with an accuracy of the order of 1%, and can reduce the solution time of a complex problem by an order of magnitude. This, in effect, reduces the effort in mathematical manipulation so that the engineer can concentrate on applying his judgment, experience, and analytical ability to the description and interpretation of the problem and its solution. It is also possible to use "branch operation," a computer procedure in which the course of the problem solution is determined by the values generated during its solution.

APPLICATION OF THE RCAF

The principal uses of the analog installation include simulation for reactor component and reactor control system design, simulation of typical and new reactor designs for training of students and reactor operators, and accelerated- or retarded-time-base computation of dynamics problems in the sciences and engineering. The large number of problems now being handled attests to the versatility of the facility by virtue of several removable problem – or patch – boards. This feature has permitted personnel in other Laboratory divisions to investigate systems whose kinetic equations are impractical to solve by conventional means.

When the equations have been derived for the closed-loop dynamics of a system, responses to step or sinusoidal inputs can be readily calculated by standard mathematical analyses; therefore the advantages of the analog computer in servomechanism analysis may not be immediately obvious. The advantages of the analog computer are that (1) complex multiloop system analysis can be handled with no more difficulty than a single-loop analysis, (2) complex input functions can be introduced with no complication, (3) changes in parts of the system can be made rapidly, and (4) nonlinear conditions, which are often used to improve system operation and which are usually formidable to manipulate mathematically, can be handled easily.

An attractive feature of analog machines is that parts of the actual dynamic system being studied may be included as part of the analog simulation. This is particularly useful when the system contains some nonlinear elements whose behavior cannot be described in simple terms.

The analog computer simultaneously presents many intermediate variables, besides the primary variables of solution, which usually are of considerable interest in the detailed elucidation, selection, or design of components of the system.

The validity of computer studies depends upon the interrelation of input data accuracy and the analog used. When the problem is one with which there has been little or no experience, the only safe procedure is to vary the coefficients of the analog and observe the sensitivities of the resulting responses. A high system sensitivity to such changes indicates that one of the following conditions exists:

1. The analog setup has been poorly chosen. The equations describing the system may have been arranged in such a manner that an attempt is being made to divide by a quantity which is very nearly zero or, perhaps, to subtract two large, very nearly equal numbers in an open-loop simulation. This common situation is usually rectified readily by rearrangement of the block diagram.

2. The constants and variables in a critical area have been measured inaccurately, resulting in the analog system being designed on the basis of entirely erroneous concepts.

3. The system for which the simulation was made has been designed in such a manner that, even with a proper analog setup and accurate measurements, difficulties are being encountered with analog analysis. When this condition exists, it is recommended that the designs be reviewed for easement of the problem, rather than using a system which would possibly involve high manufacturing cost, difficult maintenance, and high failure probability.

Mathematical Approach

The first method of computer application consists in the mathematical, or differential analysis, approach. When the computer is used as a differential analyzer, or, perhaps more literally, as an "integrating synthesizer," only *particular* solutions to differential equations are represented. Driving functions and initial conditions must be specified explicitly. The operating sequence listed below should be followed:

1. Express the known or expected relations existing at each point of interest in a system by writing the local equations and by indicating other special relations in graphic form. Indicate the parameters to be varied, their magnitudes and dimensions, and their expected ranges. The greatest return obtainable from analog computation occurs when n equations are explicitly indicated in $(n + 1)$ variables, and the machine is set up to find solutions to them all.

2. Express the relations in an operational block-diagram form.

3. Reduce the numerical quantities in the problem by scaling or normalizing to set them in a suitable form for computation; that is, transform the system equations to machine equations.

4. Patch onto a board all the operational blocks, apply all the driving functions and initial conditions, set the parameters, and patch into recorders all the variables which are to be observed.

5. Verify the analog behavior by checking and calibrating against some known or expected performance, such as steady-state values.

6. Explore the adjustment of parameters in order to determine the areas of stability, and perhaps change the configuration in control sections to arrive at the optimum response and stability condition, that is, the smallest steady-state error, the smallest rms error, the smallest dynamic lag, or the largest region of stability.

7. Check and calibrate the records for plausibility, accuracy, and proper dimensions.

Pragmatic Approach

The second method of application consists in the simulator, model, or pragmatic approach. For the investigator not familiar with loop or nodal equation representation of a system, "direct simulation" can simplify the analysis through bypassing most of the higher mathematics. The computer circuit is built from functional blocks which are direct analogs of components of the system to be simulated. Table 1 presents some electrical, mechanical, and thermodynamic analogies with their equivalent operational analogs. The operating sequence for this method is similar to that for the mathematical method beginning with step 2.

The following elucidation of step 3 applies to both methods and consists in the suggestion that a list be prepared of expected maximum and minimum values of the output variables. An important fact regarding the time analogy selection, that is, simulation vs fast- or slow-time computation, is that the time scale factor is independent

Table 1. Simple Illustrative Analogs

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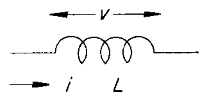
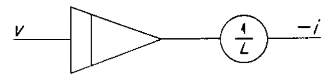
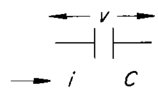
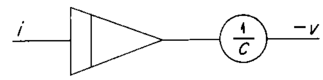
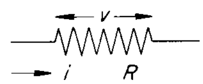
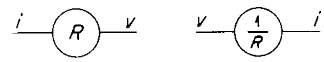
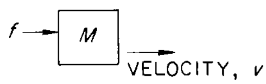
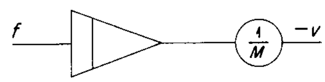
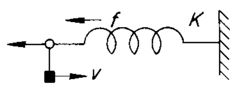
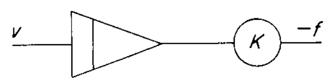
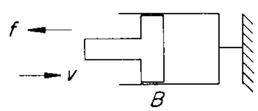
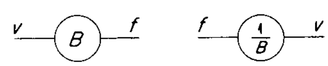
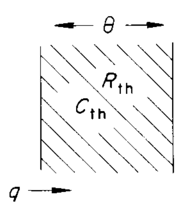
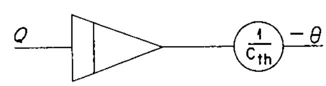
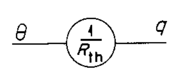
ORNL—LR—DWG 30330

VARIABLE	SYMBOL	EQUATION	ANALOG
ELECTRICAL			
INDUCTANCE		$i = \frac{1}{L}\int v\,dt$	
CAPACITANCE		$v = \frac{1}{C}\int i\,dt$	
RESISTANCE		$v = iR$ $i = v/R$	
MECHANICAL			
MASS		$v = \frac{1}{M}\int f\,dt$	
SPRING		$f = K\int v\,dt$	
DAMPER		$f = Bv$ $v = f/B$	
THERMAL			
		$\theta = \frac{1}{C_{th}}\int Q\,dt$ $q = \theta/R_{th}$	 

ELECTRICAL
i = CURRENT, amp
v = VOLTAGE, v
L = INDUCTANCE, henrys
C = CAPACITANCE, farads
R = RESISTANCE, ohms
t = TIME, sec

MECHANICAL
v = VELOCITY, fps
f = FORCE, poundals
M = MASS, lb
K = SPRING CONSTANT, poundals / ft
B = DAMPING COEFFICIENT, poundal - sec / ft
t = TIME, sec

THERMAL
q = HEAT FLOW, Btu / sec
θ = TEMPERATURE DIFFERENCE, °F
C_{th} = THERMAL HEAT CAPACITY, Btu / °F
Q = HEAT STORED, Btu

of the dependent-variable scale factors. Thus the first concern should be with the determination of the dependent-variable scale factors, a determination which becomes a completely algebraic process. Once the true-time analog has been considered and scale factors have been determined, it is comparatively easy to account for slow- or fast-time operation.

In most problems the system parameters will be known numerically in consistent units. Determine the scale factors and analog coefficient settings so that the expected voltage ranges will be handy for recording and so that the coefficient potentiometer settings can be made greater than 010. Usually the scale factor, k_v, is the maximum expected variable value divided by the maximum permissible voltage. Round off this quotient to the next convenient larger number to present an easily interpolated recorded scale. Set all coefficient potentiometers as close to unity as possible. This may seem to be a rather arbitrary requirement; however, when the action of most real systems in nature is formulated mathematically with an accuracy and significance consistent with the accuracy of measurement and determination of parameters, the so-called "loop gains" in a system will have similar orders of magnitude. The gain can usually be adjusted to a value near unity when time units are defined appropriately. In other words, a unit lag having a time constant one-hundredth that of another in a system will have little effect on the system's dynamic performance, or, alternatively, a part of a system having a natural frequency 100 times that of another part will have a small effect on studies made near the lower frequency. To summarize, step 3 can be stated as follows:

Physical System Term		**Analog System Term**
System coefficient (a) × system variable (v)	→	System coefficient × scale factor (k_v) × analog variable (e_v); $ak_v e_v$ = analog coefficient $(K_v) \times e_v$

A similar procedure for selection of a time scale factor may now be in order. Needless to say, systems whose time constants of interest cover a range of much greater than 100 to 1 should be run twice, each observed with a different time scale factor.

PROBLEM PRESENTATION

Listed below is the information that should be supplied and determined when a problem is submitted to the RCAF. Additional copies of this guide are available.

Initial Problem Presentation Guide

Information Relating to the Problem

- Physical Description
  - Sketch or block diagram
  - Mathematical equations of lowest convenient order
  - Glossary of symbols
- Purpose of Analysis
  - Number of runs with parameters to be varied, including dimensions
  - Initial conditions and boundary conditions
  - Numerical values of constants
  - Estimated ranges of variables
  - Estimation of time constants
  - Accuracy of results

Information Relating to the RCAF

- Problem suitability to analog computation
- Manpower time
- Machine time and schedule
- Patch-board space

CONTROL SYSTEMS AS AN EXAMPLE (*1*)

Control implies the safe regulation of a system under all conditions, involving all the aspects of a system from the basic design to the operating technique. Proper control includes the accident as well as the normal routine, for the operating personnel must be safeguarded from injury and the system from excessive damage.

There are many control problems which may be anticipated or detected that can then be solved by experimenting with test samples of the materials or components to be employed and by devising an operating technique to prevent serious consequences. The kinetic response, however, depends upon the entire system. Coupling exists between all components, and a perturbation initiated in one unit will affect each of the others. Simulation of the entire system offers a means of detecting and solving the control problems which are likely to be encountered in the kinetics, a means of mapping areas of controllability and synthesizing the necessary auxiliary equipment required for kinetic stability. Proper consideration of the kinetic behavior early in the design may prevent the inclusion of elaborate or questionable auxiliary control equipment. At this stage of, for example, a power system design, a simulator of flexible design can be made to follow frequent design changes and to indicate the import of each change upon the controllability of the system before the engineer is committed to the modification.

At a later stage in the power system development, the auxiliary equipment, which was synthesized by means of the simulator, may be constructed and tested by being coupled to the simulator and used for the simulator control.

The use of a simulator for mapping, or surveying, an area of controllability does not impose stringent requirements upon the accuracy of each unit. Where doubt exists as to the validity of a result or where the margin of safety is small, more accurate methods of computation, such as the use of a digital computer, should be employed. The accuracy obtained is a function of the tolerances of the impedances used and of the amplifiers. Amplifier errors are discussed in the sections "The Operational Amplifier" and "Linear Operations."

SYMBOLS

A discussion of analog computer circuits involves an individual designation of a large number of different quantities. In order to standardize the conventions used in this manual, the following procedures are utilized.

Lower-case italic letters denote the instantaneous value of a time-variant function; thus, e represents voltage; i, current; t, time; etc. When specific problem variables are referred to, subscripts are used to distinguish one quantity from another. These are defined as the discussion proceeds.

Upper-case italic letters designate fixed values of voltage, current, circuit parameters, constants, etc. Table 2 presents the graphic symbolism used throughout this manual. No industry-wide conventions on symbolism have yet been determined; therefore a careful observance of definitions should be made whenever reference material is studied.

Panel markings on the RCAF are in capital, roman letters. No effort has been made in this manual to follow the manufacturer's style when using certain panel markings in equations or in drawings. Rather, the designations defined above have been adhered to.

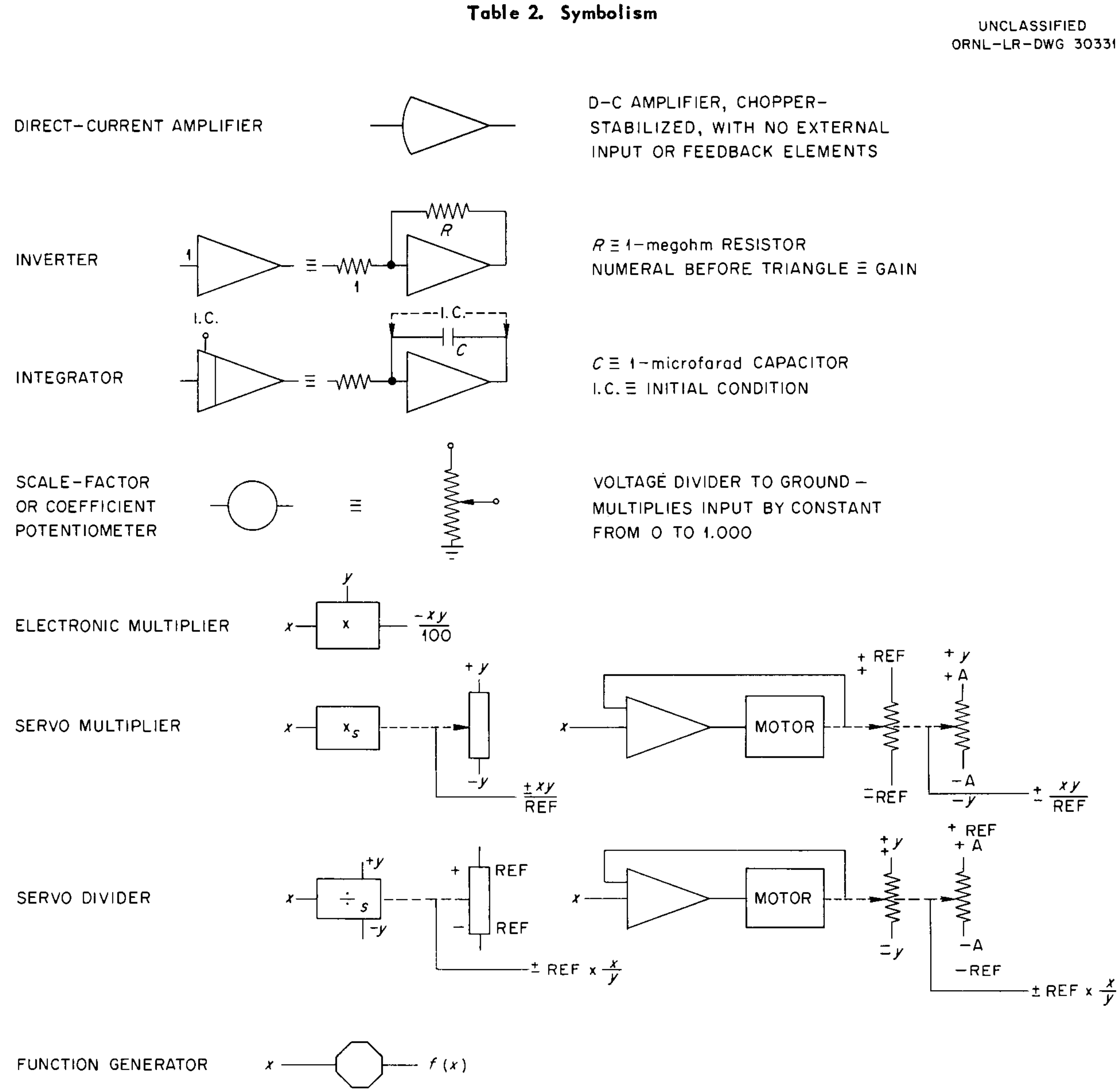
The wide range of subject matter requires repetition of some letter symbols in different chapters to represent different physical quantities. Each repetition is defined within the chapter in which it appears.

KINETIC EQUATIONS: THE BASIS OF A NUCLEAR REACTOR SYSTEM SIMULATION (*1*)

Production of power in a reactor will raise the temperature of both the fuel and the moderator. Some means in the form of a fluid coolant is provided to remove this heat from the critical lattice. Since the rate of change of power production in the reactor is usually a function of either the fuel or moderator temperature or of both, then the kinetic equations of the complete power plant system, including both the reactor as a heat source and the heat exchangers as a sink, must be considered if it is desired to determine the behavior of a part of the system. A simplified set of these equations usually can be written without making assumptions which are difficult to justify. The various parameters are constants or known functions of time determined by a specific reactor design. If the design is altered, these parameters may likewise be expected to change.

These equations are differential equations, most of which are linear with constant coefficients. However, the differential equation involving the mean reactor flux or reactor power as a function of time is nonlinear in that one of the coefficients usually involves the power, temperature, and maybe some other time-variant parameter. The type of nonlinearity is not singularly defined for

Table 2. Symbolism



reactors in general but depends upon the specific design.

Circulation of the fluid coolant in the system introduces another type of nonlinearity in some of the equations, because an exact representation of the differential equations for a system exhibiting a pure transport lag in some of its parameters would require an infinite set of linear equations. It is customary to represent such transport lags approximately by finite sets of first-order linear equations, with constant coefficients, in which the driving or forcing function of one such equation is the dependent variable of the preceding one in the set.

A shift in flux or power distribution in the critical lattice during power or temperature transients would have to be represented analytically by additional equations to describe these variables as a function of position within the reactor, as well as of time. It is inconvenient to utilize the analog computer for solving the reactor equations involving functions of space and time when both these quantities are considered as independent variables.

The complete set of equations representing the entire reactor power plant can be linearized, that is, considered linear for a small perturbation in either a driving function or one of the parameters. The response of the system to such a perturbation can then be determined. Stability of the linearized representation of the system can be determined by Nyquist's criterion (*2*). This is the general procedure for analyzing a reactor power plant for stability.

A power plant such as that described above would be considered inherently stable if the analysis of the equations by Nyquist's criterion indicates stability. Naturally, a reactor design engineer would prefer that his design be inherently stable. Stable designs usually simplify the control equipment, because the fail-safe features can be more easily designed. If the equations indicate that the power plant is unstable and if it is impractical to alter the design parameters sufficiently to eliminate the instability, an external control mechanism in most practical designs can be provided to prevent these oscillations. The equations of this external control system, when added to the previous set of equations, provide a new set which can again be analyzed for stability. The control system usually involves the motion of a mass accelerated by a force and accordingly can be represented as a second-order linear differential equation with constant coefficients. The driving function of this equation is proportional to an error signal made up of perturbations in one of the dependent variables or in a linear combination of these variables taken from the previously determined unstable system. The polarity of such a driving function is chosen to make the system degenerative rather than regenerative in response to a small perturbation.

COMPONENT IDENTIFICATION OF THE RCAF

While a flexible analog computer may appear to be a complex system, it will in general consist of a multiplicity of similar units. Addition, subtraction, multiplication by constants, differentiation, and integration are the only operations necessary for the solution of linear differential equations with constant coefficients. These operations can be carried out by means of d-c "operational amplifiers" with appropriate feedback circuits. As many as 58 of these amplifiers are available in the RCAF, all derived from the "chopper-stabilized direct-current amplifier" design developed by RCA but acquired from three sources of manufacture. The nonlinear operation of multiplication in the analog computer is more complicated than the previously mentioned operations. Seven channels of multiplication, including two of the many methods of multiplication available, are used in this facility, each with its inherent accuracy and response time limitations. Other special devices are also incorporated in the computer to handle many nonlinear situations. The entire equipment is capable of displaying the kinetics of systems whose variables of interest do not exceed a range of 10^4.

Computer Group 16-31R, Electronic Associates, Inc.

The basic facility is shown in Fig. 1. The double-width relay rack B is the Electronic Associates console, the control point of the computer, which includes the patch-bay at left center, the control panel at right center, the attenuator assembly at the top, and 20 operational amplifiers, available as summers or integrators. All critical passive elements are temperature-controlled in an oven and are held at 0.01% of nominal value. Just above right center are two triple-potentiometer servo multipliers; a servo voltmeter has recently been inserted where one of the blank panels appears on the photograph. The Computer Group 16-31R can be operated independently of or in conjunction with the other linear and nonlinear component assemblies.

Prepatch Panel. – The computer makes use of a patch-bay assembly into which a prepatch panel may be inserted. The panel provides terminals to bring out all units and networks to fit the requirements of the problem. Construction of the equipment confines all leakages to ground paths, thereby minimizing terminal-to-terminal leakage. To facilitate problem setup, multicolor coding and lettering are used. Although not necessary, it is usually desirable to remove the prepatch panel to insert or remove patch cords and plugs. The panel is equipped with an interlock which removes all plate power from the computer when the panel is disengaged. The prepatch panel locking lever, when in the locked position, may be depressed to remove reference voltage from the panel to allow cords to be inserted or removed without danger of shorting the reference voltage.

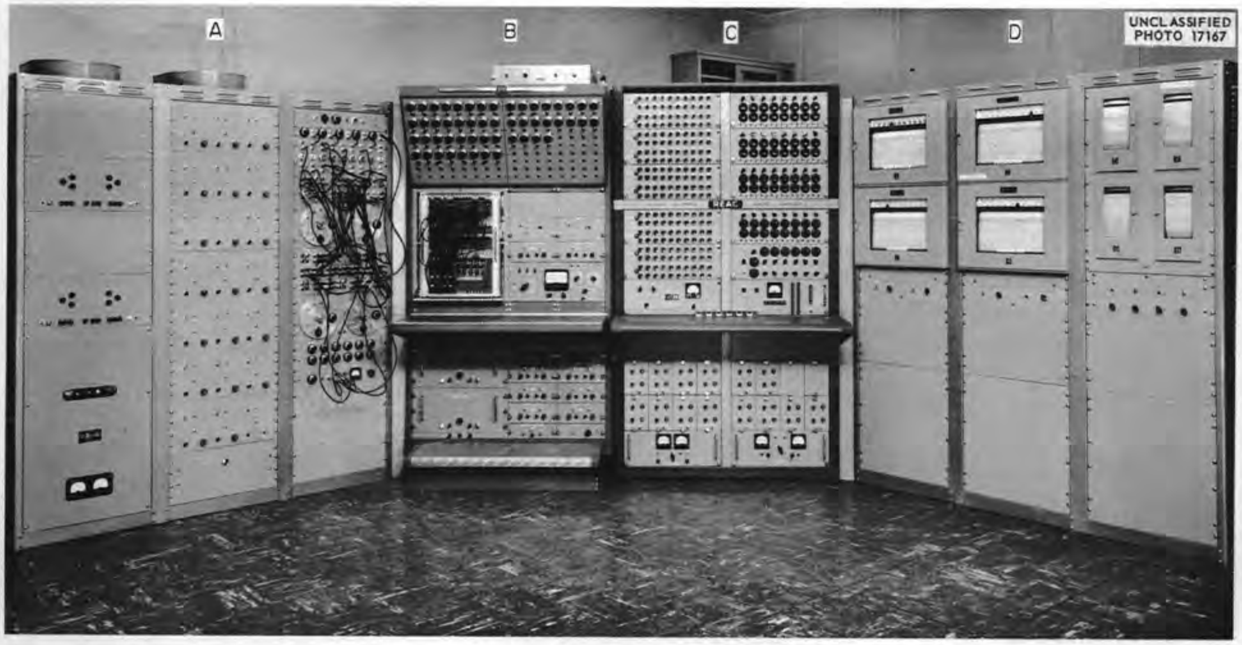


Fig. 1. Reactor Controls Analog Facility.

Control Panel. – The control panel contains the control, monitoring, and chart drive switches, three dpdt function switches, and a VTVM. Provisions are made in the circuitry to check all rack a-c and d-c potentials, attenuator voltages, and amplifier output voltages.

Attenuator Panels. – The 80 manual 10-turn attenuators located on the inclined panel are individually fused and provided with a transfer switch to enable selection of either +100-v or −100-v reference or to make the high end of the potentiometer available at the patch panel.

Dual D-C Amplifiers. – Two dual d-c amplifiers are situated above the control panel and eight dual amplifiers below; each amplifier has its own input grid, zero-Balance-test push button, balance indicator neon light, and zero-adjust potentiometer.

Regulated Power Supplies. – On the console top in a separate chassis are located the +100-v and −100-v reference supplies, the source of the basic computing voltages. Below the console desk are located the +300-v, −300-v, 110-v, and 200-v supplies and the 19-v vibrator drive unit, which supplies a frequency of about 90 cps to the d-c amplifier choppers.

Servo Components. – The servo multipliers and servo voltmeter are electromechanical devices which position a rotating shaft to correspond to an input voltage between ±100 v, thereby permitting multiplication of two variables (using auxiliary potentiometers on the same shaft) and external indication of the servo input variable.

REAC Nonlinear Unit, Reeves Instrument Corp.

Double rack C is the Reeves Instrument Corp. nonlinear unit, 400 series, which contains five electronic multipliers at the upper left, the multiplier calibration panel at the left center, four diode function generators at the upper right, the function generators' calibration and control panel at the right center, the 14 dual d-c amplifiers just below the desk, and power supplies along the bottom.

Function Generators. – The function generators in the nonlinear system provide for more complex transforms than are readily set up on the patchboard. These generators utilize variable bias on a set of diodes to approximate any continuous function by a series of line segments, whose break points and slopes may be varied at will. As the magnitude of the independent variable increases, the string of diodes is made to come into conduction one by one, driving the summing junction of the output amplifier to a higher or lower potential, depending upon the polarities involved. A selector switch on each unit, representing one of the total of 40 segments, permits the polarity of the input voltage and the orientation of the diode to be reversed. Each break-point potentiometer controls the cutoff voltage at the input to the diode, and each slope potentiometer controls the output amplifier gain for each segment.

Electronic Multipliers. – The electronic multipliers work on the "quarter-square" principle and are built around two diode function generators of the type described above in "Function Generators." The product, z, is obtained by a variation of the method whereby $-[(x + y)/2]^2$ and $[(y - x)/2]^2$ are added to yield $-xy/100$ at the output. However, in this case, the vertices of the parabolas approximated in the function generators would lie at the origin, where a given error in voltage is, percentagewise, at its worst. The functions actually chosen place the point $z = 0$, $f(z) = 0$ in a region of slow monotonic increase in slope. Division of the product by 100 ensures that even should both x and y go as high as 100 v, the product will never exceed the 100-v operating range.

Dual D-C Amplifiers. – Each Reeves d-c amplifier has its own front-panel Balance indicator and zero-adjust potentiometer, but the Balance-test selection on all these amplifiers is made on the amplifier calibration panel where the input grid zero can be read on a built-in VTVM.

Regulated Power Supplies. – The left chassis holds the +200-v, –200-v, and +270-v supplies, and the right chassis holds the +400-v and –400-v supplies, both with front-panel metering.

Strip-Chart Recorders, Minneapolis-Honeywell Regulator Co.

Triple rack (recently expanded to quadruple-rack) D contains eight rectilinear strip-chart recorders as the primary output equipment for the computer. These recorders contain pen movements capable of $4\frac{1}{2}$ sec (Nos. 1–4), $2\frac{1}{2}$ sec (Nos. 5–7), and $\frac{1}{3}$ sec (No. 8) minimum full-scale traverse, with variable chart-drive speeds up to a maximum of 8 in./min (Nos. 1–7), or 2.5 sec/division, and 1.0 sec/division (No. 8). Each recorder is supplied with a control panel containing a polarity reversing switch and full-scale range switch, 1, 5, 20, and 100 v. In addition, recorders 5–8 are fed by recorder preamplifiers.

ORNL Designed Equipment

Triple rack A, surveying from left to right, contains power supplies for the ORNL designed and constructed equipment in the first rack; the 20 operational amplifiers from the original Reactor Controls Computer and the combination 24-v relay, 100-v regulated, and 200-v booster supply in the second rack; and the special-components and interconnection panels in the third rack.

D-C Amplifiers. – Each panel accommodates three d-c amplifiers with controls and neon indicator similar to the Electronic Associates amplifiers.

Special Components. – The small panel at the top of the rack houses a calibration circuit for monitoring the output of the four synchronous, delay networks located below. Five auxiliary coefficient potentiometers occupy the narrow panel just below. The large panel, third from the top, comprises a group of precision resistors and capacitors for patching-in special transfer functions. The two large dials on the next panel down control two synchronous capacitor-storage delay lines for the simulation of multisecond transport lags (*3*). Below the time lags is the connector panel tying the ORNL components into the Electronic Associates patch-bay. The next panel down contains two more lag lines. The lowest unit in the rack houses a group of special-purpose feedback circuits used for simulating the delayed-neutron contributions in nuclear reactor problems.

Basic Reactor Control Console. – Dissociated from the computer proper is a special-purpose console attachable to the special-components rack for reactor control problems which mounts three SB-1 grip-handle switches arranged to control three motor-operated potentiometers through variable speed-reducer drives. These controls can be wired into a simulation in such a way as to permit reactor operator and student training on the computer in a realistic manner.

ORNL Regulated Power Supplies. – The ORNL equipment power supplies in the left rack are Kay-Lab +300-v and –300-v units in the upper half, and a Sorensen 6-v heater supply in the lower half of the rack, all provided for the ORNL d-c amplifier operation.

THE OPERATIONAL AMPLIFIER (*1*)

The operational amplifier is a basic constructional unit of the simulator. Figure 2 shows a simplified schematic diagram of a typical, single-ended, operational amplifier. It is a high-gain, d-c amplifier with an odd number of stages of gain. The odd number of stages, usually three, produces a polarity reversal through the circuit so that feedback networks connecting the output to the input will be degenerative. The over-all gain is the product of the individual gains of each of the three stages and is usually near 40,000. A triangle, such as that shown in Fig. 3, is frequently used as a schematic symbol of an operational amplifier and will be so employed in this report. The n denotes the number assigned to a particular amplifier in a circuit employing more than one such unit. The triangle represents the amplifier only; the feedback and input networks are shown separately.

Ideally, the output voltage of such a circuit would equal the input times the gain of the amplifier, provided that the zero adjustment had been made properly. However, numerous effects may alter this condition. Heating of components, with a resultant change in their ratings, voltage drifts in the power supply, and aging of the vacuum tubes are but a few of the causes for deviation of the output from the ideal. Drifts may occur at any point in the circuit, but for convenience in discussing the relative stability of various amplifiers and for purposes of analysis, the combined effect of all drifts is considered as a single drift of the voltage applied to the input grid.

In the circuit of Fig. 4 a feedback impedance, Z_2, degeneratively couples the output, e_2, to the input, while an input impedance, Z_{11}, connects the input to a driving source, e_{11}; e_g represents

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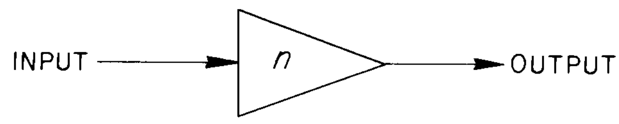

Fig. 3. Block-Diagram Representation of Operational Amplifier.

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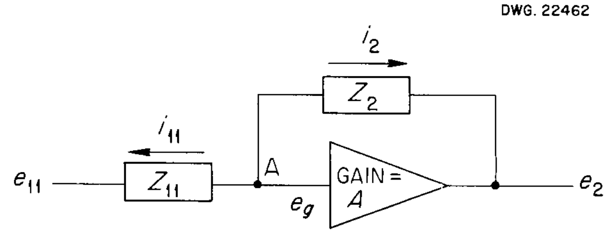

Fig. 4. Operational Amplifier with Feedback and Input Impedances Connected.

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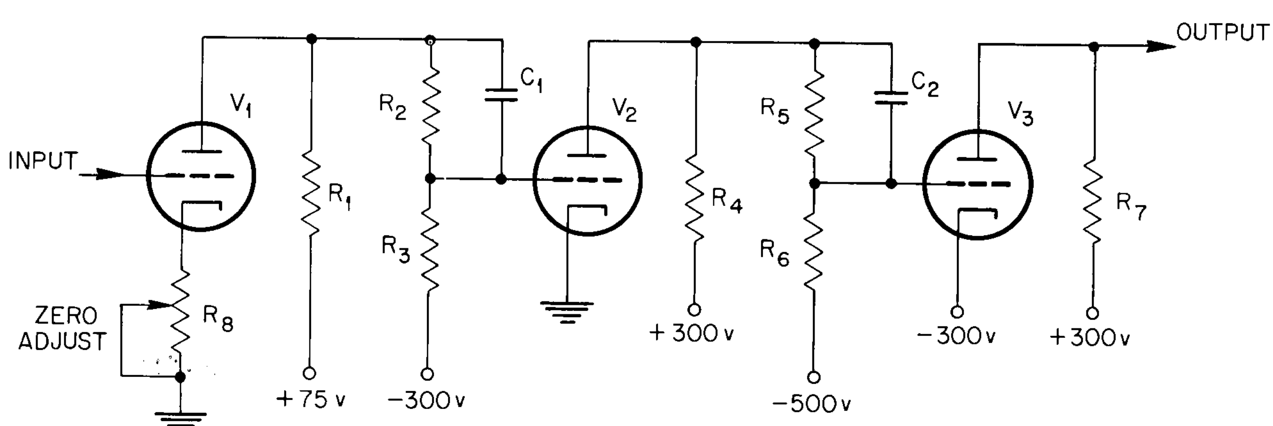

Fig. 2. Conventional Single-Ended Operational Amplifier.

the normal input signal e developed by e_{11} plus a second input e_d representing the effective input drift signal, as defined above.

An operating condition of the amplifier shown schematically in Fig. 2 is that there is no input grid current in the vacuum tube V_1. Accordingly, the current i_{11} through Z_{11} is equal and opposite to the current i_2 through Z_2, and the following relations exist between i_{11}, i_2, e_{11}, e_2, Z_{11}, Z_2, e_g, and e_d:

(1) $$i_{11} = -i_2 \ ,$$

(2) $$i_{11} = \frac{e_{11} - e_g}{Z_{11}} \ ,$$

(3) $$i_2 = \frac{e_2 - e_g}{Z_2} \ .$$

By using Eqs. 2 and 3 in Eq. 1,

(4) $$\frac{e_{11} - e_g}{Z_{11}} = -\frac{e_2 - e_g}{Z_2} \ ,$$

and solving for e_2,

(5) $$e_2 = -\frac{Z_2}{Z_{11}} e_{11} + e_g \left(1 + \frac{Z_2}{Z_{11}}\right) \ ;$$

also

(6) $$e_g = e + e_d \ .$$

If Z_2 is made equal to zero and Z_{11} is infinite, that is, open circuit, then the normal input signal resulting from e_{11}, which is e, will be zero and Eq. 5 becomes

(7) $$e_2 = e_d \ .$$

Thus the input drift signal may be measured in an operational amplifier by connecting the output to the input directly. The output then equals the effective input drift signal. A typical value of this input drift signal for conventional single-ended operational amplifiers is 50 mv. This may be reduced by a push-pull connection in which some of the drifts are balanced out. Such a "differential" amplifier may have drifts as low as 5 mv.

The second term on the right of Eq. 5 is neglected in all applications of the operational amplifier since this term represents the output error and should be small when compared with the first term on the right of this equation for a well-designed amplifier:

(8) $$\text{Error} = e_g \left(1 + \frac{Z_2}{Z_{11}}\right) \ .$$

When e_d is extremely small over the useful range of the amplifier output,

(9) $$e_g = \frac{e_2}{A} \ .$$

Since the gain, A, is quite large, the error will normally be negligible except when the ratio of impedances is very large. An example for which the latter condition is true is the condition when Z_2 is capacitance only and the operational amplifier with pure reactance feedback Z_2 is operating as an integrator.

When e_d is large, the error term will be negligible only for large values of e_2 and the accurate operating range of e_2 is greatly diminished. A dynamic operating range of three decades is normally desirable. With a maximum value in e_2 of 100 v, the minimum limit of 0.1 v will be too low for accuracy in the conventional single-ended d-c amplifier because of the input drift signal.

To extend the operating range to three decades, the RCAF employs a stabilized, single-ended operational amplifier which was developed by the RCA Laboratories (4). Figure 5 shows a simplified

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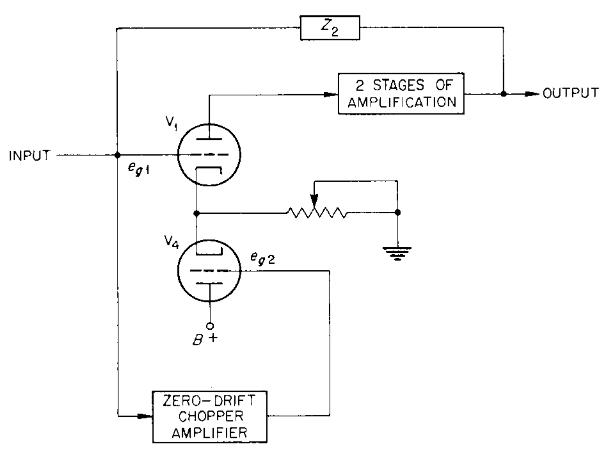

Fig. 5. Simplified Block Diagram of Stabilized Operational Amplifier.

schematic diagram of this amplifier, while a complete schematic diagram is shown in Fig. 6. The amplifier (Fig. 5) is composed of a conventional, single-ended amplifier with a no-drift chopper amplifier added to the input circuit. The chopper amplifier has a very low frequency response; so high-frequency components in the input signal are amplified exactly as they would be in the conventional amplifier. Very low frequencies, however, will be amplified by the chopper circuit, and the behavior of the amplifier will be modified by the feedback from the chopper amplifier.

A drift in any portion of the conventional circuit appears as a d-c offset at the input, shown as e_{g1}, Fig. 5; e_{g1} is amplified by the chopper amplifier by a factor of A_1 and applied to V_4 as e_{g2}. The cathode connection of V_4 to V_1 couples e_{g2} to V_1 so that the effective input to the first tube equals the arithmetic sum of e_{g1} and e_{g2}. In Eq. 5, e_g represents the total input required for the conventional circuit; so it follows that

$$e_g = e_{g1} + e_{g2} = e_{g1}(1 + A_1) \tag{10}$$

or

$$e_{g1} = \frac{e_g}{1 + A_1} \ . \tag{11}$$

The actual low-frequency input is represented by e_{g1}, and for zero feedback impedance it represents the value of the effective input drift signal. Therefore, the chopper amplifier reduces the low-frequency-input drift signal by the factor $1/(1 + A_1)$. The zero-frequency gain of the chopper amplifier used is approximately 2000, so that the drift signal of the conventional portion of the circuit is reduced by a factor of approximately 5×10^{-4}. Thus, if the drift of the conventional amplifier portion were 100 mv, it would be reduced to 0.05 mv by the chopper amplifier. Such a signal is negligible when compared with the normal input signal, e, and will be neglected in later analyses involving the operational amplifier.

The chopper-stabilized amplifier provides another distinct advantage over the unstabilized amplifier in that the error given by Eq. 8 will be reduced by the factor $1/(1 + A_1)$ for low-frequency-input signals. In the case in which the operational amplifier is used as an integrator, Z_2 in Eq. 8 is of the form $-e_2/C\dot{e}_2$, where $\dot{e}_2$ is the time rate of change of e_2 and C is capacitance. For a slowly varying or steady input signal, $\dot{e}_2$ is quite small or zero and hence Z_2 is large. It has already been stated that for large ratios of Z_2/Z_{11} in Eq. 8 the error is not necessarily negligible. The attenuation factor $1/(1 + A_1)$ is most effective for these conditions and accordingly diminishes the error most when it needs most to be diminished. This characteristic of the chopper-stabilized amplifier will be referred to again in connection with explanations of operational amplifier applications.

Figure 7 shows the physical appearance of the amplifier used in the simulator. The two tubes in the front and the chopper in the center comprise the stabilizing amplifier. The chopper is driven at 60 or 90 cps. The three tubes at the rear of the chassis form the conventional, single-ended, d-c amplifier. Characteristics of this circuit are the following:

Chopper amplifier gain	2000
d-c amplifier gain	40,000
Zero-frequency gain	80,000,000
d-c amplifier frequency response	to 500 kc
Chopper amplifier frequency response	0.5 cps
Drift	
High-frequency components	<0.2 mv
Low-frequency components	<0.05 mv

The functional behavior of this amplifier as a basic unit of the simulator is determined by the input and feedback networks employed. These networks vary considerably from stage to stage. To provide flexibility, neither the input nor the feedback networks are included within the amplifier but can be coupled to the amplifier by means of coaxial connectors on an adjoining chassis.

The normal signal on the input grid of a stabilized, d-c amplifier exceeds the drift signal by a sufficient margin to allow the drift signal to be ignored in all but a few critical applications. In the following discussions, e_g, unless specified, refers only to the signal on the input grid which results from normal operation of the amplifier and not from drifts of its components. This e_g results from the finite gain of the amplifier. The gain depends on the frequency, being very high at very low frequencies and lower at the higher frequencies. For

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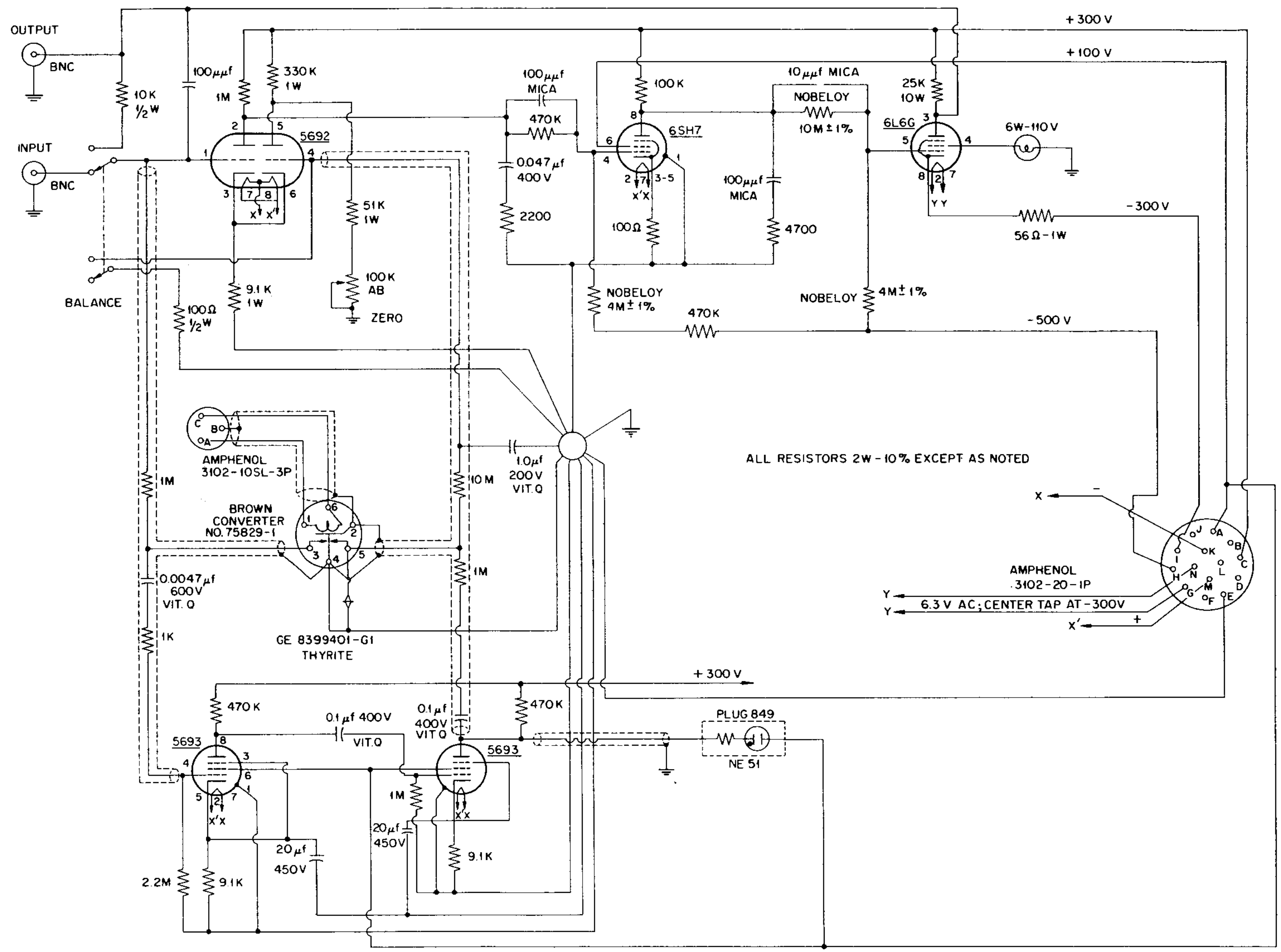

Fig. 6. Circuit Diagram of Chopper-Stabilized D-C Amplifier.

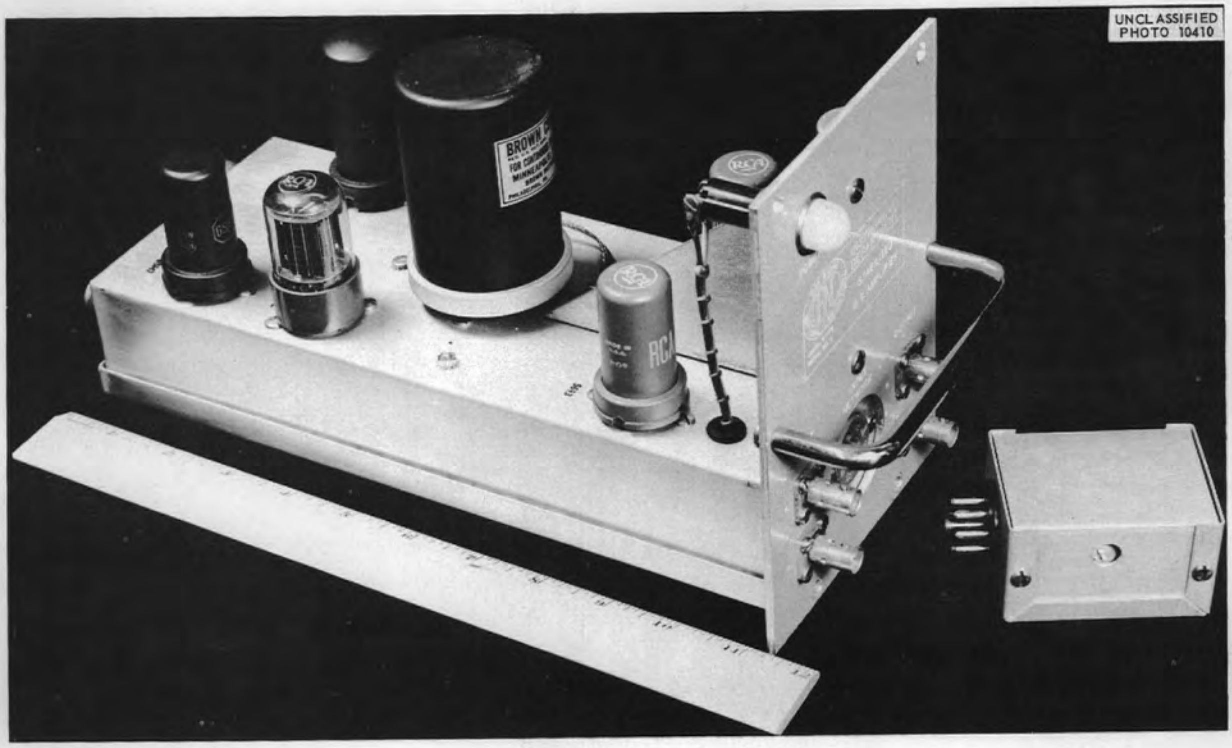

Fig. 7. Chopper-Stabilized D-C Amplifier with Plug-In Network.

this reason the value of e_g depends on the frequency. Considering a single frequency component, it follows that

(12) $$e_g = -\frac{e_2}{A} ,$$

where A is the gain (a complex quantity) of the amplifier at the frequency being considered. Consider the circuit of Fig. 8. Let μ be defined such that

(13) $$e_g = \mu i_{11} .$$

The μ thus defined is the input impedance of the operational amplifier from its grid to ground as presented to i_{11}. Combining Eqs. 12 and 13 yields

(14) $$\mu i_{11} = -\frac{e_2}{A} ,$$

but

(15) $$i_{11} = -i_2 ;$$

therefore

(16) $$\mu i_2 = \frac{e_2}{A}$$

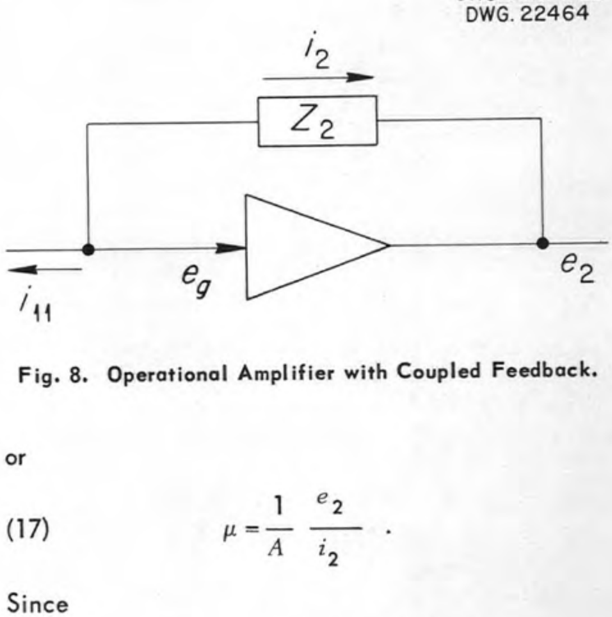

Fig. 8. Operational Amplifier with Coupled Feedback.

or

(17) $$\mu = \frac{1}{A} \frac{e_2}{i_2} .$$

Since

(18) $$e_2 \gg e_g$$

for large values of A, then

(19) $$\frac{e_2}{i_2} = Z_2$$

and

(20) $$\mu = \frac{Z_2}{A} \ .$$

Both Z_2 and A are complex, and it follows that, in general, μ will be a complex impedance.

As an example, consider that both Z_2 and A are real and that

$$Z_2 = 10^6 \text{ ohms} \ ,$$

$$A = 4 \times 10^4 \ .$$

Then

(21) $$\mu = \frac{10^6}{4 \times 10^4} = 25 \text{ ohms} \ .$$

From Fig. 6 it may be seen that the chopper amplifier's input circuit is connected between the input grid and ground and is therefore in shunt with μ. The impedance of this circuit is 10^6 ohms and has a negligible effect on the operational amplifier's normal functions. In later applications additional shunting resistances will be considered from the input grid to ground, and in each case the shunting effect upon the input grid must be negligible for accurate results. The amount of shunting which may be tolerated is determined by the accuracy required in the circuit and may differ from problem to problem. In general, the shunting impedance should be

(22) $$Z_{shunt} > 1000\,\mu$$

or

(23) $$Z_{shunt} > 1000 \frac{Z_2}{A} \ .$$

Since A is complex, the phase of e_2 will differ from that of e_g. It is conceivable, therefore, that for a particular Z_2 and A the feedback would be "in phase" instead of 180 deg out of phase as implied in the preceding analysis. At such a frequency, where regenerative feedback occurs, the circuit would oscillate were the loop gain greater than unity. The phase lag in the amplifier increases with frequency. The frequency at which regenerative feedback occurs is raised by phase-correcting networks in the amplifier. The magnitude of A decreases with frequency. The relation of A to the phase shift is such that for all commonly encountered feedback networks the system is stable and will not oscillate. A more detailed discussion is available in the literature (*2, 4, 5, 6*).

LINEAR OPERATIONS

A d-c amplifier maintains, throughout its operating range, an input voltage which is negligible in comparison with the output. Consequently, the potential at point A (Fig. 4) may be considered zero. Furthermore, since input grid current of the amplifier is vanishingly small, current i_{11} is equal in magnitude and opposite in sign to i_2. The error introduced by these assumptions for the stabilized d-c amplifier is less than the probable error of the measuring equipment except for extremely small values of e_2 and for cases in which a time derivative or integration of the input grid voltage is involved. For the latter two cases further discussion of the probable error will be made. For all other applications this error will be ignored in the following explanations. Thus, in general,

(24) $$i_{11} = \frac{e_{11}}{Z_{11}}$$

and

(25) $$i_2 = \frac{e_2}{Z_2} \ .$$

It follows that

(26) $$\frac{e_{11}}{Z_{11}} = -\frac{e_2}{Z_2}$$

or

(27) $$e_2 = -\frac{Z_2}{Z_{11}} e_{11} \ .$$

The output, e_2, equals the negative of the input, e_{11}, modified by the ratio Z_2/Z_{11}. In general, these impedances are complex and therefore frequency-sensitive. When e_{11} is specified, e_2 may be computed. A sufficient description of the system's kinetic response for many purposes may be obtained by specifying either a step change or a periodic function of time for e_{11} and by describing the behavior of e_2. For arbitrary variations of e_{11}, the impedance ratio, Z_2/Z_{11}, denotes an operation performed on e_{11} to obtain e_2. Operations include scaling, differentiating, integrating, and combinations of them. Operational calculus is well suited for analog circuit analysis, but only a brief description of this method is included at the end of this section, because classical calculus is more widely used and understood. Classical analysis will be used principally to describe the operations of such circuits. Usually this form of analysis will deviate least from the form in which a problem is presented.

If, in Eq. 27, Z_{11} and Z_2 are pure resistances, then

$$e_2 = -\frac{R_2}{R_{11}} e_{11} = -A_{11} e_{11} \quad . \tag{28}$$

Here the output equals the input multiplied by a scalar A_{11} which is equal to the ratio of the two resistances. Such a circuit may be used for amplifying or attenuating a function for use at another point in the circuit. A reversal of polarity accompanies the magnitude change. The change in sign may be the sole purpose of such a circuit in a computer. Since the complexity of the problems which may be handled depends on the number of amplifiers available, the number of sign reversals should be kept low to leave as many amplifiers as possible available for more useful applications.

If, in the circuit shown in Fig. 4, Z_2 is a capacitance while Z_{11} remains a resistance,

$$i_{11} = \frac{e_{11}}{R_{11}} \tag{29}$$

and

$$i_2 = C_2 \frac{de_2}{dt} \quad ; \tag{30}$$

therefore

$$\frac{e_{11}}{R_{11}} = -C_2 \frac{de_2}{dt} \tag{31}$$

and, by integrating each side,

$$e_2 = -\frac{1}{R_{11}C_2} \int e_{11}\, dt + C_1 \quad . \tag{32}$$

The output is equal to the time integral of the input plus a constant of integration, C_1. The RC product is a scaling factor upon the output and is chosen in such a manner that the output does not exceed the voltage limit of the amplifier. The factor RC must also conform with the level of output required by the problem being solved. This produces an integrator circuit. High-frequency noise and signals contribute little to the output e_2, thus permitting these circuits to be used in cascade without decreasing the signal-to-noise ratio.

There is an error in the output of an integrator which may cause difficulty in some applications. To show where this arises, a more exact derivation of the circuit operation follows.

The current i_{11} is

$$i_{11} = \frac{e_{11} - e_g}{R_{11}} \quad , \tag{33}$$

and

$$i_2 = C_2 \frac{d(e_2 - e_g)}{dt} \quad ; \tag{34}$$

therefore

$$\frac{e_{11} - e_g}{R_{11}} = -C_2 \frac{d(e_2 - e_g)}{dt} \quad . \tag{35}$$

Upon integrating each side of Eq. 35,

$$e_2 = \int -\frac{1}{R_{11}C_2} e_{11}\, dt + C_1 + \int \frac{1}{R_{11}C_2} e_g\, dt + e_g \quad . \tag{36}$$

The first two terms on the right of Eq. 36 constitute the desired integral, while the last two terms on the right are errors. The last term equals the output divided by the gain of the amplifier. For most purposes this will be negligible in comparison with e_2 because of the large gain (of the order of 40,000) of the amplifier. Although the magnitude of e_g may be ignored, the time integral of this term involved in the third term on the right may become appreciable. For short-time integrations this may be ignored, and, in many other applications involving longer periods, the accumulation of error is resisted by feedback in the computer.

To determine the magnitude of error, let the input grid signal be represented as a single frequency such that

$$e_g \equiv E_g \cos \omega t \quad . \tag{37}$$

Then

(38) $$\frac{1}{R_{11}C_2}\int_0^t e_g\,dt = \frac{1}{R_{11}C_2}\int_0^t E_g \cos \omega t\,dt = \frac{E_g \sin \omega t}{R_{11}C_2\omega}\ .$$

The maximum error, E_{max}, is

(39) $$E_{max} = \frac{E_g}{R_{11}C_2\omega}\ .$$

Since

(40) $$e_2 = -Ae_g\ ,$$

where

(41) $$e_2 = -E_2 \cos \omega t\ ,$$

it follows that

(42) $$E_g = \frac{E_2}{A}\ ;$$

then Eq. 39 becomes

(43) $$E_{max} = \frac{E_2}{A\omega R_{11}C_2}\ .$$

The maximum error will be less than 0.1% of the output whenever

(44) $$R_{11}C_2A\omega > 1000\ .$$

This error term becomes significant only at very low frequencies, because the magnitude of A is increased by the chopper amplifier. At zero frequency this gain is approximately 80,000,000.

For an integrator using a 1-μf, polystyrene, Western Electric capacitor and a 1-megohm input resistance, the decay of the output occurs with a time constant of 3 hr with the use of the stabilized d-c amplifier.

The error arises from the current through the input resistances which is required to sustain a voltage of e_g on the input grid. The feedback capacitor, C_2, furnishes this current, and the charge is decreased by its flow. Where paralleled networks are employed in the input circuit, the value of R_{11}, as used in the preceding paragraphs in describing the error, should be the effective resistance of the entire circuit.

If, in the circuit of Fig. 4, Z_{11} is a capacitance and Z_2 a resistance, then a detailed analysis gives

(45) $$i_{11} = C_{11}\frac{d(e_{11} - e_g)}{dt}$$

and

(46) $$i_2 = \frac{e_2 - e_g}{R_2}\ .$$

Therefore

(47) $$C_{11}\frac{de_{11}}{dt} - C_{11}\frac{de_g}{dt} = -\frac{e_2}{R_2} + \frac{e_g}{R_2}$$

or

(48) $$e_2 = -R_2C_{11}\frac{de_{11}}{dt} + R_2C_{11}\frac{de_g}{dt} + e_g\ .$$

The first term of Eq. 48 is the derivative of the input voltage times a scalar, R_2C_{11}. The second two terms are errors. The third term may be neglected because it will always be negligible in comparison to the output, for reasons which were given in connection with the integrator. The second term, however, requires further examination.

A single-frequency input and the resultant output are represented by Eqs. 37 and 42. This error term then becomes

(49) $$R_2C_{11}\frac{de_g}{dt} = R_2C_{11}\frac{d[(E_2/A)\cos \omega t]}{dt} = -\frac{R_2C_{11}\omega E_2}{A}\sin \omega t\ .$$

The maximum error is, accordingly,

(50) $$E_{max} = \frac{R_2C_{11}\omega E_2}{A}\ .$$

For this error to be less than 0.1% of the output of the amplifier, it is required that

(51) $$\frac{A}{R_2C_{11}\omega} > 1000\ .$$

The error increases with frequency so that the high gain of the amplifier at low frequencies is not effective in reducing the error. For a gain of 40,000 and a unity RC product, if the error is not to exceed 0.1% of the output, then

$$\frac{40{,}000}{\omega} > 1000 \tag{52}$$

and

$$\frac{1}{\omega} > \frac{1}{40} \; ; \tag{53}$$

thus

$$\omega < 40 \; . \tag{54}$$

Since

$$\omega = 2\pi f \; , \tag{55}$$

then

$$f < 6.3 \text{ cps} \; . \tag{56}$$

From this it may be seen that a differentiator's use is restricted when accuracy is required and that, when used, care should be taken in determining the degree of accuracy obtained. Furthermore, noise components appearing at the input are amplified by the differentiator by a factor equal to ω. Consequently, cascading differentiators can easily degrade the output signal to the point where it is unreliable. For a limited use, therefore,

$$e_2 = -R_2 C_{11} \frac{de_{11}}{dt} \; . \tag{57}$$

Figure 9 shows a more generalized circuit employing the operational amplifier. For this circuit, input networks connected to n sources provide current common to the amplifier input which must

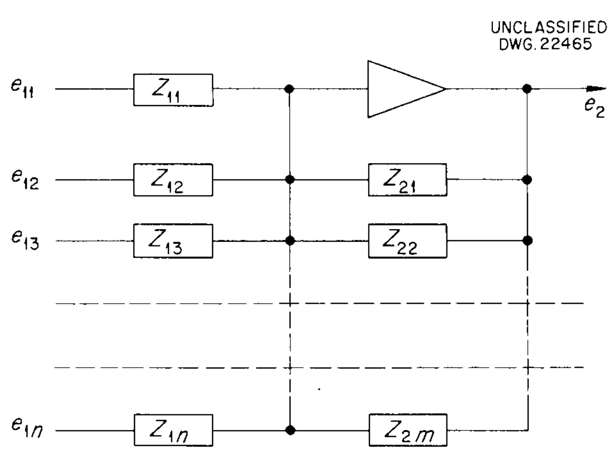

Fig. 9. Generalized Circuit Employing Operational Amplifier.

equal the current from the output through M parallel paths. Thus, for the ith input,

$$i_{1i} = \frac{e_{1i}}{Z_{1i}} \; , \tag{58}$$

and for the kth feedback

$$i_{2k} = \frac{e_2}{Z_{2k}} \; . \tag{59}$$

Since the sum of all input currents must equal the sum of all feedback currents, then

$$\sum_{i=1}^{i=n} \frac{e_{1i}}{Z_{1i}} = - \sum_{k=1}^{k=M} \frac{e_2}{Z_{2k}} \; . \tag{60}$$

The operations denoted by Eq. 60 may include any linear combination of those described in the preceding paragraphs. For example, suppose that n equals 2 and M equals 1 and each is resistive. From Eq. 60,

$$\frac{e_{11}}{R_{11}} + \frac{e_{12}}{R_{12}} = - \frac{e_2}{R_{22}} \; , \tag{61}$$

or

$$e_2 = - \frac{R_{22}}{R_{11}} e_{11} - \frac{R_{22}}{R_{12}} e_{12} \; . \tag{62}$$

Thus a linear combination of two inputs is obtained at the output in what may be called an "adder."

If, in the above examples, Z_{12} were capacitive, the result, since it is a summation of currents as denoted by Eq. 60, is

$$e_2 = - \frac{R_{22}}{R_{11}} e_{11} - R_{22} C_{12} \frac{de_{12}}{dt} \; . \tag{63}$$

If

$$e_{12} = e_2 \; , \tag{64}$$

then

$$e_2 = - \frac{R_{22}}{R_{11}} e_{11} - R_{22} C_{12} \frac{de_2}{dt} \; , \tag{65}$$

which is a linear, first-order, differential equation with e_{11} as a "source" and $R_{22}C_{12}$ as the time constant. In other words, e_2 behaves as X in the following equation:

(66) $$T\frac{dX}{dt} + X = Bf(t) \ ,$$

where

(67) $$T = R_{22}C_{12} \ ,$$

(68) $$B = -\frac{R_{22}}{R_{11}} \ .$$

The same result may also be obtained by treating the circuit as one with a single input and a parallel combination of two feedback impedances as shown in Fig. 10.

The behavior of the output as obtained in the preceding paragraph may also be obtained with a single input network and a single feedback resistance such as those shown in Fig. 11. For this circuit

(69) $$i_{11} = i_a - i_c \ ,$$

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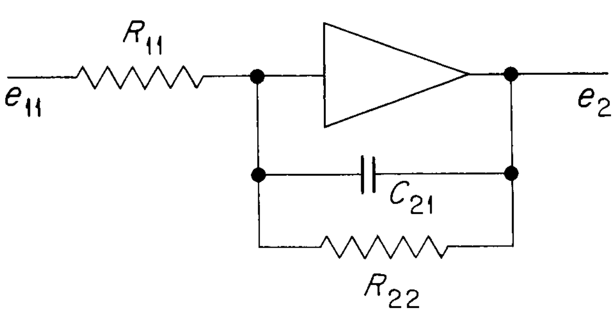

Fig. 10. First-Order Linear System.

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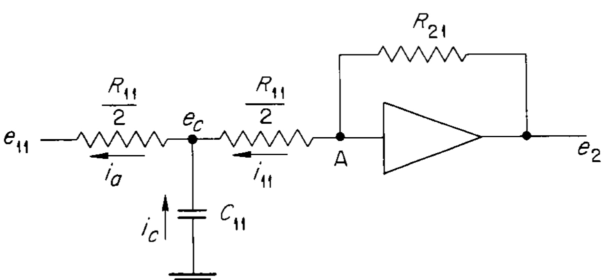

Fig. 11. Alternate Form of First-Order Linear System.

(70) $$i_{11} = \frac{2e_c}{R_{11}} \ ,$$

(71) $$i_c = C_{11}\frac{de_c}{dt} = \frac{C_{11}R_{11}}{2}\frac{di_{11}}{dt} \ ,$$

(72) $$i_a = \frac{2(e_{11} - e_c)}{R_{11}} = \frac{2e_{11}}{R_{11}} - i_{11} \ .$$

Upon substituting Eqs. 71 and 72 in Eq. 69,

(73) $$i_{11} = \frac{2e_{11}}{R_{11}} - i_{11} - \frac{C_{11}R_{11}}{2}\frac{di_{11}}{dt} \ ,$$

or

(74) $$i_{11} + \frac{R_{11}C_{11}}{4}\frac{di_{11}}{dt} = \frac{e_{11}}{R_{11}} \ .$$

Since the input current must equal the feedback current,

(75) $$i_{11} = -\frac{e_2}{R_{21}} \ ;$$

and if this is used in Eq. 74 to eliminate i_{11}, then

(76) $$-\frac{e_2}{R_{21}} - \frac{R_{11}C_{11}}{4R_{21}}\frac{de_2}{dt} = \frac{e_{11}}{R_{11}} \ ,$$

or

(77) $$e_2 + \frac{R_{11}C_{11}}{4}\frac{de_2}{dt} = -\frac{R_{21}}{R_{11}}e_{11} \ .$$

This is the same form as Eq. 66 if

(78) $$T = \frac{R_{11}C_{11}}{4}$$

and

(79) $$B = -\frac{R_{21}}{R_{11}} \ .$$

Two or more parallel input networks of the type shown in Fig. 11 may be employed in a single amplifier. Such a circuit with two parallel inputs is shown in Fig. 12. It can be shown that the solution for the output of this circuit is of the following form:

(80) $$e_2 = -e_{21} - e_{22} \ ,$$

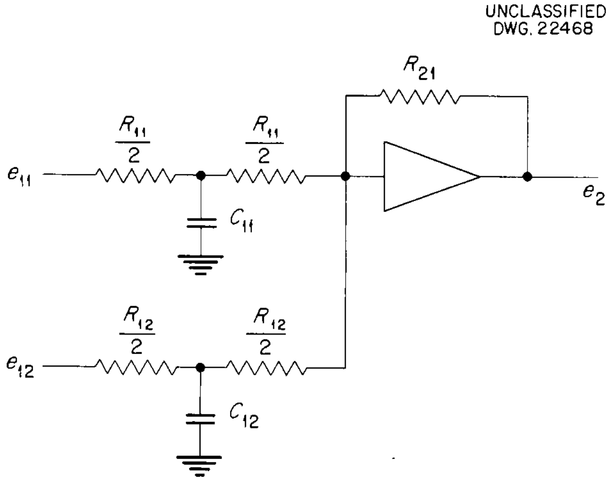

Fig. 12. Multiple First-Order Linear System.

$$T_1 \frac{de_{21}}{dt} + e_{21} = B_1 e_{11} \quad , \tag{81}$$

$$T_2 \frac{de_{22}}{dt} + e_{22} = B_2 e_{12} \quad . \tag{82}$$

The term e_2 is common to all terms on the right side of Eq. 60 and may be factored:

$$\sum_{k=1}^{k=M} \frac{e_2}{Z_{2k}} = e_2 \sum_{k=1}^{k=M} \frac{1}{Z_{2k}} \quad . \tag{83a}$$

The summation on the right of Eq. 83*a* equals the sum of the reciprocals of parallel impedances which, in turn, equals the reciprocal of the effective impedance of the network. Thus

$$\sum_{k=1}^{k=M} \frac{1}{Z_{2k}} = \frac{1}{Z_2} \quad . \tag{83b}$$

Placing Eq. 83*b* in Eq. 60 and cross multiplying,

$$e_2 = - \sum_{i=1}^{i=n} \frac{Z_2}{Z_{1i}} e_{1i} \quad . \tag{84}$$

The term e_2 is a linear combination of n inputs. The ratio of Z_2 to Z_{1i} determines the functional relation of e_2 and e_{1i}. This ratio may be defined as the transference of the respective input to the output.

ANALYSIS BY OPERATIONAL CALCULUS

Operational calculus allows these transferences to be stated in a simplified form (5, 6). As previously explained, the classical method will be employed in this report, but a brief discussion of the transforms equivalent to the transferences of the circuits discussed in this section will follow. Only those transforms relevant to a single input will be discussed, since multiple inputs involve only linear combinations of them.

Each impedance may be placed in its equivalent operational form. Thus, for R, the operational form is R, while for C, the operational form is $1/Cp$, where p is an operator. Inductance is not used in practical operational amplifiers because of the difficulty of obtaining inductors with sufficiently high q; if it were used, it would be represented by Lp. If the impedances comprising the input and feedback networks are given values corresponding to the operational form and if each network is reduced to a single complex quantity by the conventional rules of combining series and parallel impedances, then a ratio of the feedback, Z_2, to the input, Z_{11}, may be obtained, involving the operator p. By this means the following transfer operators are obtained.

Scalar

$$\begin{aligned} Z_{11} &= R_{11} \\ Z_2 &= R_2 \\ \frac{Z_2}{Z_{11}} &= \frac{R_2}{R_{11}} \end{aligned} \tag{85}$$

Integrator

$$\begin{aligned} Z_{11} &= R_{11} \\ Z_2 &= \frac{1}{C_2 p} \\ \frac{Z_2}{Z_{11}} &= \frac{1}{R_{11} C_2 p} \end{aligned} \tag{86}$$

Differentiator

$$Z_{11} = \frac{1}{C_{11} p}$$

(87a) $$Z_2 = R_2$$

$$\frac{Z_2}{Z_{11}} = R_2 C_{11} p$$

To extend this approach to a more complex system, consider the circuit shown in Fig. 10. For this circuit

$$Z_{11} = R_{11} \quad ,$$

(87b) $$Z_2 = \frac{(R_{22})(1/C_{21}p)}{R_{22} + (1/C_{21}p)} = \frac{R_{22}}{R_{22}C_{21}p + 1} \quad ,$$

$$\frac{Z_2}{Z_{11}} = \frac{R_{22}}{R_{11}} \left(\frac{1}{R_{22}C_{21}p + 1} \right) \quad .$$

For the equation

(88) $$T\frac{dX}{dt} + X = By \quad ,$$

where the operator p is used to represent the operation d/dt,

(89) $$(Tp + 1) X = -By$$

or

(90) $$X = -\frac{B}{Tp + 1} y \quad .$$

From Eq. 90 the transference of y to X equals that denoted by Eq. 87b if

(91) $$\frac{R_{22}}{R_{11}} \left(\frac{1}{R_{22}C_{21}p + 1} \right) = \frac{B}{Tp + 1} \quad ,$$

where

(92) $$B = -\frac{R_{22}}{R_{11}} \quad ,$$

(93) $$T = R_{22}C_{21} \quad .$$

These are the same requirements as those obtained earlier for equivalence between the differential equation and the electrical circuit of Fig. 10.

PASSIVE AND ACTIVE NETWORK TABULATIONS

The passive network transfer relations developed above and several more involved arrays are shown in Table 3 (7). Table 4 shows the linear operations using a single amplifier and combinations of passive element input and feedback arrays. Typical input test functions and circuit output responses are also shown. Note that in checking circuits with differentiating input elements a ramp function is used instead of a step function.

In many cases use of passive element networks as input and feedback circuits of high-gain d-c amplifiers is inadequate for transfer function generation. Examples are found in cases where

1. the time constants, or damping ratios, vary,
2. limiters are required,
3. components of desired passive elements are unavailable,
4. initial conditions are required on quantities which are not available as amplifier outputs,
5. rates or accelerations which are unavailable in the passive element setup must be recorded.

Tables 5 and 6 contain circuits for the simulation of all possible transfer functions which are represented by any combination of three-terminal, 2-, 3-, 4-, or 5-element R-C networks (*8*).

In some cases there are more analog circuit gains to be determined from time-constant relations than there are independent equations. In these cases, assume one of the gains and solve for those remaining from the equations given. Equations for the gains assumed are omitted from Tables 3–6. All gains are shown as potentiometer settings. The data in the tables are based on the time constants $T_1 > T_2 > T_3 > T_4$.

Table 6 contains a number of second-order linear systems, wherein b is the system damping ratio. In general, the procedure for transfer function synthesis using a minimum number of amplifiers requires setting the differential equation in integral form so that all derivatives of the input variable disappear, and assembling the transform by means of operational units in Table 5.

Table 3. Passive Network Transfer Relations

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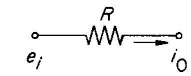
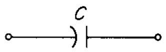
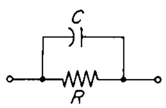
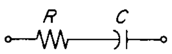
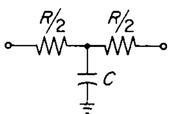
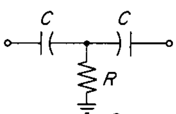
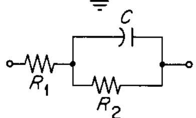
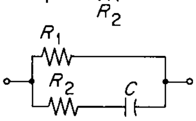
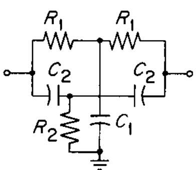
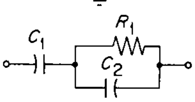
ITEM NO.	CIRCUIT	TRANSFER IMPEDANCE, $Z_T = \frac{e_i}{i_0}$	TIME CONSTANT	GAIN*	INVERSE RELATIONS
1		R		R	
2		$\frac{1}{pC} = \frac{1}{C}\int dt$, WHERE $p = \frac{d}{dt}$		$\frac{1}{C}$	
3		$\frac{R}{1+pT}$	$T = RC$	R	$C = \frac{T}{R}$
4		$\frac{1+pT}{pC}$	$T = RC$	$\frac{1}{C}$	$C = \frac{T}{R}$
5		$R(1+pT)$	$T = \frac{RC}{4}$	R	$C = \frac{4T}{R}$
6		$\frac{2}{pC}\,\frac{(1+pT)}{pT}$	$T = 2RC$	$\frac{2}{C}$	$R = \frac{T}{2C}$
7		$(R_1+R_2)\frac{(1+p\theta T)}{pT}$	$T = R_2 C$ $\theta = \frac{R_1}{R_1+R_2}$	(R_1+R_2)	$R_1 = \theta(R_1+R_2)$
8		$R_1\frac{(1+p\theta T)}{(1+pT)}$	$T = (R_1+R_2)C$ $\theta = \frac{R_2}{R_1+R_2}$	R_1	$R_2 = \frac{\theta T}{C} = \frac{R_1\theta}{(1-\theta)}$
9		$2R_1\frac{(1+pT_1)}{(1+p^2T_1T_2)}$	$T_1 = \frac{R_1C_1}{2}$ $T_2 = R_2C_2$ $R_1C_1 = 4R_2C_2$	$2R_1$	$C_1 = \frac{2T_1}{R_1}$ $C_2 = \frac{T_2}{R_2}$ $T_1 = 2T_2$
10		$\frac{1}{pC_1}\,\frac{(1+pT)}{(1+p\theta T)}$	$T = \frac{R_1C_1}{1-\theta}$ $\theta = \frac{C_2}{C_1+C_2}$	$\frac{1}{C}$	$R_1 = \frac{T(1-\theta)}{C_1}$ $C_2 = \frac{C_1\theta}{(1-\theta)}$

* IMPEDANCE CONSIDERED IN STEADY-STATE GAIN DETERMINATION: $\frac{Z\ \text{FEEDBACK}}{Z\ \text{INPUT}}$ WITH e_i CONSTANT INTO R AND e_i CONSTANT INTO C.

Table 4. Single Amplifier Transfer Relations

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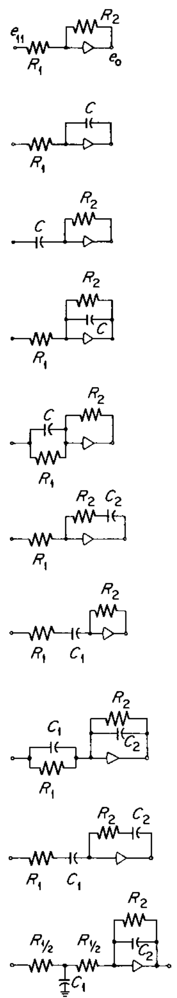
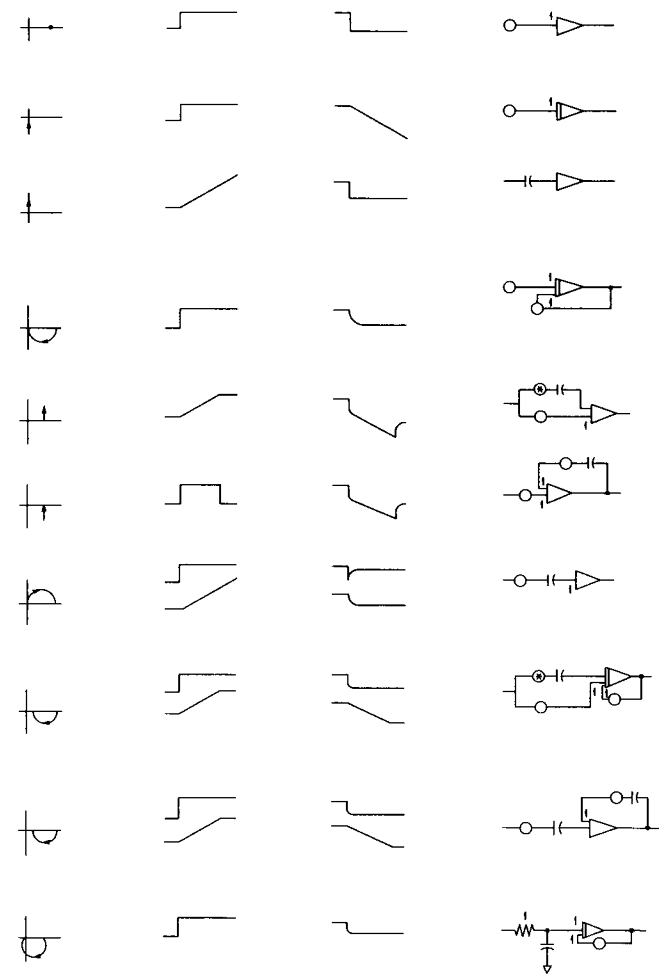
ITEM NO.	CIRCUIT	TRANSFER FUNCTION	TIME CONSTANT	GAIN	POLAR PLOT	INPUT FUNCTION	OUTPUT RESPONSE	BLOCK REPRESENTATION	DESCRIPTION
11.		$e_0=-\frac{R_2}{R_1}e_{11}$		$\frac{R_2}{R_1}$					INVERTER; SCALE CHANGER
12.		$e_0=-\frac{1}{T}\int e_{11}\,dt$	$T=R_1C$	$\frac{1}{T}$					INTEGRATOR
13.		$e_0=-T\frac{de_{11}}{dt}$	$T=R_2C$	T					DIFFERENTIATOR
14.		$e_0=-\frac{R_2}{R_1}\frac{e_{11}}{(1+pT)}$	$T=R_2C$	$\frac{R_2}{R_1}$					FIRST-ORDER LAG
15.		$e_0=-\frac{R_2}{R_1}(1+pT)\,e_{11}$	$T=R_1C$	$\frac{R_2}{R_1}$					AUGMENTER; PROPORTIONAL + DERIVATIVE
16.		$e_0=-\frac{(1+pT)}{pR_1C_2}e_{11}$	$T=R_2C_2$						AUGMENTER; SCALE CHANGER + INTEGRATOR
17.		$e_0=-\frac{pR_2C_1}{1+pT}e_{11}$	$T=R_1C_1$						LEAD; DERIVATIVE X LAG
18.		$e_0=-\frac{R_2}{R_1}\frac{(1+pT_1)}{(1+pT_2)}e_{11}$	$T_1=R_1C_1$, $T_2=R_2C_2$	$\frac{R_2}{R_1}$					
19.		$e_0=-\frac{C_1}{C_2}\frac{(1+pT_2)}{(1+pT_1)}e_{11}$	$T_1=R_1C_1$, $T_2=R_2C_2$	$\frac{C_1}{C_2}$					
20.		$e_0=-\frac{R_2}{R_1}\frac{e_{11}}{(1+pT_1)(1+pT_2)}$	$T_1=\frac{R_1C_1}{4}$, $T_2=R_2C_2$	$\frac{R_2}{R_1}$					SECOND-ORDER LAG





⊛ 30-K HELIPOT IN SERIES WITH C TO DAMP ELECTRONICS ASSOCIATES AMPLIFIER OSCILLATION

Table 5. Transfer Functions* – Items 1–12

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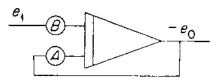
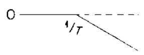
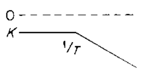
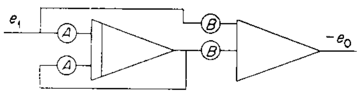
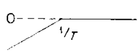
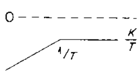
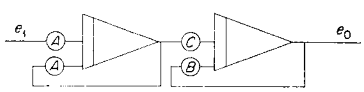
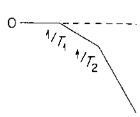
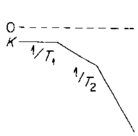


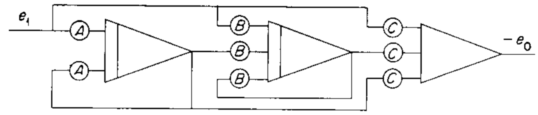
Item No.	Bode Plot	Transfer Function	Time Constants	Gains
1		$\frac{1}{TP+1}$	$T=\frac{1}{A}$	$A=B=\frac{1}{T}$
2		$\frac{K}{TP+1}$	$T=\frac{1}{A}$ $K=\frac{B}{A}$	$A=\frac{1}{T}$ $B=AK$

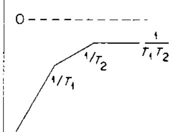
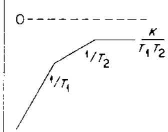


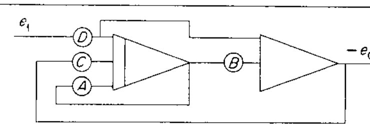
Item No.	Bode Plot	Transfer Function	Time Constants	Gains
3		$\frac{TP}{TP+1}$	$T=\frac{1}{A}$	$A=\frac{1}{T}$ $B=1$
4		$\frac{KP}{TP+1}$	$T=\frac{1}{A}$ $K=BT$	$A=\frac{1}{T}$ $B=AK$

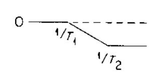
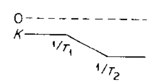
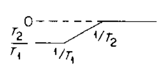
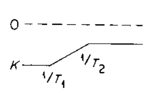


Item No.	Bode Plot	Transfer Function	Time Constants	Gains
5		$\frac{1}{(T_1P+1)(T_2P+1)}$	$T_1=\frac{1}{A}$ $T_2=\frac{1}{B}$	$A=\frac{1}{T_1}$ $B=\frac{1}{T_2}$ $C=B$
6		$\frac{K}{(T_1P+1)(T_2P+1)}$	$T_1=\frac{1}{A}$ $T_2=\frac{1}{B}$	$A=\frac{1}{T_1}$ $B=\frac{1}{T_2}$ $C=\frac{K}{T_2}$



Item No.	Bode Plot	Transfer Function	Time Constants	Gains
7		$\frac{P^2}{(T_1P+1)(T_2P+1)}$	$T_1=\frac{1}{A}$ $T_2=\frac{1}{B}$	$A=\frac{1}{T_1}$ $B=\frac{1}{T_2}$ $C=\frac{1}{T_1T_2}$
8		$\frac{KP^2}{(T_1P+1)(T_2P+1)}$	$T_1=\frac{1}{A}$ $T_2=\frac{1}{B}$ $K=\frac{C}{AB}$	$A=\frac{1}{T_1}$ $B=\frac{1}{T_2}$ $C=\frac{K}{T_1T_2}$



Item No.	Bode Plot	Transfer Function	Time Constants	Gains
9		$\frac{T_2P+1}{T_1P+1}$	$T_1=\frac{1}{A-BC}$ $T_2=\frac{1}{A-B}$	$A=B+\frac{1}{T_2}$ $C=\frac{T_1-T_2}{BT_1T_2}+1$ $D=\frac{T_2}{T_1}$
10		$\frac{K(T_2P+1)}{T_1P+1}$	$T_1=\frac{1}{A-BC}$ $T_2=\frac{1}{A-B}$ $K=\frac{T_1D}{T_2}$	$A=B+\frac{1}{T_2}$ $C=\frac{T_1-T_2}{BT_1T_2}+1$ $D=\frac{KT_2}{T_1}$
11		$\frac{K(T_1P+1)}{T_2P+1}$	$T_1=\frac{1}{A-B}$ $T_2=\frac{1}{A}$ $K=\frac{T_2}{T_1}$	$A=\frac{1}{T_2}$ $B=\frac{T_2-T_1}{T_1T_2}$ $C=0$ $D=1$
12		$\frac{K(T_1P+1)}{T_2P+1}$	$T_1=\frac{1}{A-B}$ $T_2=\frac{1}{A}$ $K=\frac{T_2D}{T_1}$	$A=\frac{1}{T_2}$ $B=\frac{T_2-T_1}{T_1T_2}$ $C=0$ $D=\frac{KT_1}{T_2}$

*The transfer functions given here are those derived by L. E. Heizer and S. J. Abraham, "Transfer Functions Simulation by Means of Amplifiers and Potentiometers," *J. Assoc. Computing Machinery* 3(3), 186 (1956).

Table 5. Transfer Functions – Items 13–17

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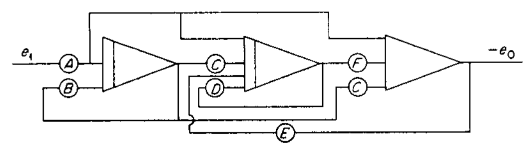
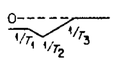
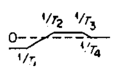
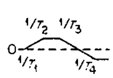
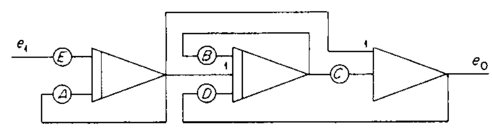
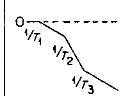
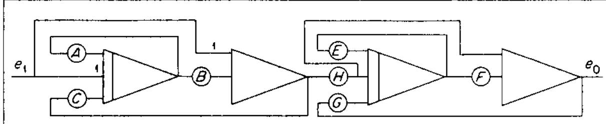


Item No.	Bode Plot	Transfer Function	Time Constants	Gains
13		$\frac{K(T_2P+1)^2}{(T_1P+1)(T_3P+1)}$	$T_3=\frac{1}{B}$ $T_2=\frac{1}{B-C}=\frac{1}{D-F}$ $T_1=\frac{1}{D-EF}$ $K=\frac{T_1T_3}{T_2^2}$	$A=1$ $B=\frac{1}{T_3}$ $B-C=D-F=\frac{1}{T_2}$ $D-EF=\frac{1}{T_1}$
14		$\frac{K(T_1P+1)(T_4P+1)}{(T_2P+1)(T_3P+1)}$	$T_1=\frac{1}{B-C}$ $T_2=\frac{1}{B}$ $T_3=\frac{1}{D-EF}$ $T_4=\frac{1}{D-F}$ $K=\frac{T_2T_3}{T_1T_4}$	$A=1$ $B=\frac{1}{T_2}$ $B-C=\frac{1}{T_1}$ $D-F=\frac{1}{T_4}$ $D-EF=\frac{1}{T_3}$
15		$\frac{(T_1P+1)(T_4P+1)}{(T_2P+1)(T_3P+1)}$	$T_1=\frac{1}{B-C}$ $T_2=\frac{1}{B}$ $T_3=\frac{1}{D-EF}$ $T_4=\frac{1}{D-F}$	$A=\frac{T_1T_4}{T_2T_3}$ $B=\frac{1}{T_2}$ $B-C=\frac{1}{T_1}$ $D-F=\frac{1}{T_4}$ $D-EF=\frac{1}{T_3}$



Item No.	Bode Plot	Transfer Function	Time Constants	Gains
16		$\frac{T_3P+1}{(T_1P+1)(T_2P+1)}$	$T_1=\frac{1}{A}$ $T_2=\frac{1}{B-CD}$ $T_3=\frac{1}{B-C}$	$A=\frac{1}{T_1}$ $B=C+\frac{1}{T_3}$ $D=\frac{1}{CT_3}-\frac{1}{CT_2}+1$ $E=\frac{T_3}{T_1T_2}$



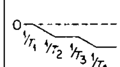
Item No.	Bode Plot	Transfer Function	Time Constants	Gains
17		$\frac{(T_2P+1)(T_4P+1)}{(T_1P+1)(T_3P+1)}$	$T_1=\frac{1}{A-BC}$ $T_2=\frac{1}{A-B}$ $T_3=\frac{1}{E-FG}$ $T_4=\frac{1}{E-F}$	$A=B+\frac{1}{T_2}$ $C=\frac{T_1-T_2}{BT_1T_2}+1$ $E=F+\frac{1}{T_4}$ $G=\frac{T_3-T_4}{FT_3T_4}$ $H=\frac{T_2T_4}{T_1T_3}$

Table 5. Transfer Functions – Items 18–24

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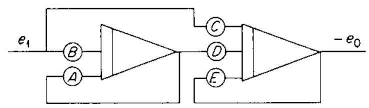
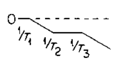
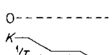
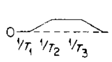
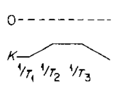
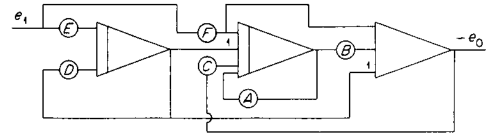
ORNL-LR-DWG 30801



Item No.	Bode Plot	Transfer Function	Time Constants	Gains
18		$\dfrac{T_2P+1}{(T_1P+1)(T_3P+1)}$	$T_1=\dfrac{1}{E}$ $T_2=\dfrac{E}{EA-D}$ $T_3=\dfrac{1}{A}$	$A=\dfrac{1}{T_3}$ $B=AT_2$ $C=BE$ $D=EA-\dfrac{E}{T_2}$ $E=\dfrac{1}{T_1}$
19		$\dfrac{K(T_2P+1)}{(T_1P+1)(T_3P+1)}$	$T_1=\dfrac{1}{E}$ $T_2=\dfrac{E}{EA-D}$ $T_3=\dfrac{1}{A}$ $K=\dfrac{T_3B}{T_2}$	$A=\dfrac{1}{T_3}$ $B=KAT_2$ $C=BE$ $D=EA-\dfrac{E}{T_2}$ $E=\dfrac{1}{T_1}$
20		$\dfrac{T_1P+1}{(T_2P+1)(T_3P+1)}$	$T_1=\dfrac{E}{EA-D}$ $T_2=\dfrac{1}{E}$ $T_3=\dfrac{1}{A}$	$A=\dfrac{1}{T_3}$ $B=AT_1$ $C=BE$ $D=EA-\dfrac{E}{T_1}$ $E=\dfrac{1}{T_2}$
21		$\dfrac{K(T_1P+1)}{(T_2P+1)(T_3P+1)}$	$T_1=\dfrac{E}{EA-D}$ $T_2=\dfrac{1}{E}$ $T_3=\dfrac{1}{A}$ $K=\dfrac{T_3B}{T_1}$	$A=\dfrac{1}{T_3}$ $B=KAT_1$ $C=BE$ $D=EA-\dfrac{E}{T_1}$ $E=\dfrac{1}{T_2}$



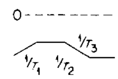
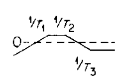
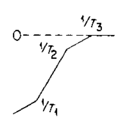
Item No.	Bode Plot	Transfer Function	Time Constants	Gains
22	 $\dfrac{K}{T_1}<1$	$\dfrac{KP(T_3P+1)}{(T_1P+1)(T_2P+1)}$	$T_1=\dfrac{1}{D}$ $T_2=\dfrac{1}{A-BC}$ $T_3=\dfrac{1}{A-B}$ $K=\dfrac{T_2FT_1}{T_3}$	$A=B+\dfrac{1}{T_3}$ $C=\dfrac{T_2-T_3}{BT_2T_3}+1$ $D=\dfrac{1}{T_1}$ $E=DF$ $F=\dfrac{T_3K}{T_2T_1}$
23	 $\dfrac{K}{T_1}>1$	$\dfrac{KP(T_3P+1)}{(1+T_1P)(T_2P+1)}$	$T_1=\dfrac{1}{D}$ $T_2=\dfrac{1}{A-BC}$ $T_3=\dfrac{1}{A-B}$ $K=\dfrac{T_2T_1F}{T_3}$	$A=B+\dfrac{1}{T_3}$ $C=\dfrac{T_2-T_3}{BT_2T_3}+1$ $D=\dfrac{1}{T_1}$ $E=DF$ $F=\dfrac{T_3K}{T_2T_1}$
24		$\dfrac{KP(T_1P+1)}{(T_2P+1)(T_3P+1)}$	$T_1=\dfrac{1}{A-B}$ $T_2=\dfrac{1}{A}$ $T_3=\dfrac{1}{D}$	$A=\dfrac{1}{T_2}$ $B=\dfrac{T_2-T_1}{T_1T_2}$ $D=\dfrac{1}{T_3}$ $E=F=\dfrac{T_1K}{T_2T_3}=1$

Table 5. Transfer Functions – Items 25–29

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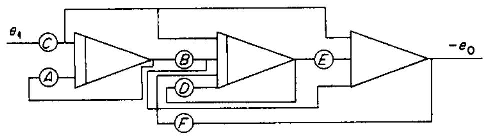
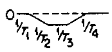
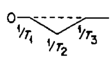
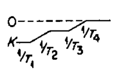
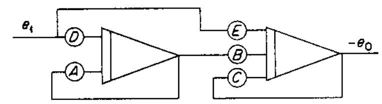
ORNL-LR-DWG 30802



Item No.	Bode Plot	Transfer Function	Time Constants	Gains
25		$\frac{(T_2P+1)(T_3P+1)}{(T_1P+1)(T_4P+1)}$	$T_1=\frac{1}{D-EF}$ $T_2=\frac{1}{D-E}$ $T_3=\frac{1}{A-B}$ $T_4=\frac{1}{A}$	$A=\frac{1}{T_4}$ $B=\frac{T_3-T_4}{T_3T_4}$ $C=\frac{T_2T_3}{T_1T_4}-1$ $D=E+\frac{1}{T_2}$ $F=\frac{T_1-T_2}{ET_1T_2}+1$
26		$\frac{(T_2P+1)^2}{(T_1P+1)(T_3P+1)}$	$T_1=\frac{1}{D-EF}$ $T_2=\frac{1}{D-E}$ $=\frac{1}{A-B}$ $T_3=\frac{1}{A}$	$A=\frac{1}{T_3}$ $B=\frac{T_2-T_3}{T_2T_3}$ $C=1$ $D=E+\frac{1}{T_2}$ $F=\frac{T_1-T_2}{ET_1T_2}+1$
27		$\frac{K(T_1P+1)(T_3P+1)}{(T_2P+1)(T_4P+1)}$	$T_1=\frac{1}{D-E}$ $T_2=\frac{1}{D}$ $T_3=\frac{1}{A-B}$ $T_4=\frac{1}{A}$ $K=\frac{T_2T_4}{T_1T_3}$	$A=\frac{1}{T_4}$ $B=\frac{T_3-T_4}{T_3T_4}$ $C=1$ $D=\frac{1}{T_2}$ $E=\frac{T_1-T_2}{T_1T_2}$ $F=0$



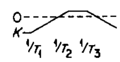
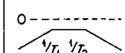
Item No.	Bode Plot	Transfer Function	Time Constants	Gains
28		$\frac{K(T_1P+1)}{(T_2P+1)(T_3P+1)}$	$T_1=\frac{1}{A-B}$ $T_2=\frac{1}{A}$ $T_3=\frac{1}{C}$ $K=\frac{DT_2T_3}{T_1}$	$A=\frac{1}{T_2}$ $B=\frac{T_1-T_2}{T_1T_2}$ $C=\frac{1}{T_3}$ $D=E=\frac{KT_1}{T_2T_3}$
29		$\frac{KP}{(T_1P+1)(T_2P+1)}$	$T_1=\frac{1}{A}$ $=\left(\frac{K}{D}\right)^{1/2}$ $T_2=\frac{1}{B}$ $=\frac{1}{C}$ $K=ET_2T_1$	$A=\frac{1}{T_1}$ $B=C=\frac{1}{T_2}$ $D=\frac{K}{T_1^2}$ $E=\frac{K}{T_2T_1}$

Table 5. Transfer Functions – Items 30–35

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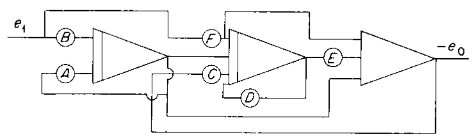
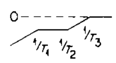
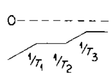
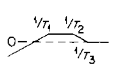
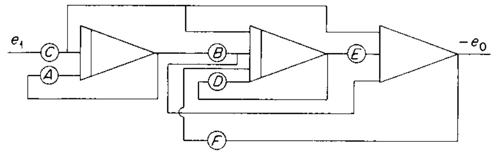
ORNL-LR-DWG 30803



Item No.	Bode Plot	Transfer Function	Time Constants	Gains
30		$\frac{KP(T_2P+1)}{(T_1P+1)(T_3P+1)}$	$T_1 = \frac{1}{A}$ $T_2 = \frac{1}{D-E}$ $T_3 = \frac{1}{D}$ $K = \frac{T_3T_1}{T_2}$	$A = \frac{1}{T_1}$ $B = AF$ $C = 0$ $D = \frac{1}{T_3}$ $E = \frac{T_2-T_3}{T_2T_3}$ $F = 1$
31		$\frac{KP(T_2P+1)}{(T_1P+1)(T_3P+1)}$	$T_1 = \frac{1}{A}$ $T_2 = \frac{1}{D-E}$ $T_3 = \frac{1}{D}$ $K = \frac{T_3FT_1}{T_2}$	$A = \frac{1}{T_1}$ $B = AF$ $C = 0$ $D = \frac{1}{T_3}$ $E = \frac{T_2-T_3}{T_2T_3}$ $F = \frac{T_2K}{T_3T_1}$
32		$\frac{KP(T_3P+1)}{(T_1P+1)(T_2P+1)}$	$T_1 = \frac{1}{A}$ $T_2 = \frac{1}{D-EC}$ $T_3 = \frac{1}{D-E}$ $K = \frac{T_2FT_1}{T_3}$	$A = \frac{1}{T_1}$ $B = AF$ $C = \frac{T_2-T_3}{ET_2T_3}+1$ $D = E+\frac{1}{T_3}$ $F = \frac{T_3K}{T_2T_1} = 1$



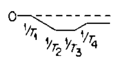
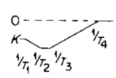
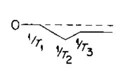
Item No.	Bode Plot	Transfer Function	Time Constants	Gains
33		$\frac{(T_2P+1)(T_3P+1)}{(T_1P+1)(T_4P+1)}$	$T_1 = \frac{1}{D-EF}$ $T_2 = \frac{1}{D-E}$ $T_3 = \frac{1}{A-B}$ $T_4 = \frac{1}{A}$	$A = \frac{1}{T_4}$ $B = \frac{T_3-T_4}{T_3T_4}$ $C = \frac{T_2T_3}{T_1T_4}$ $D = E+\frac{1}{T_2}$ $F = \frac{T_1-T_2}{ET_1T_2}+1$
34		$\frac{K(T_2P+1)(T_3P+1)}{(T_1P+1)(T_4P+1)}$	$T_1 = \frac{1}{D-EF}$ $T_2 = \frac{1}{D-E}$ $T_3 = \frac{1}{A-B}$ $T_4 = \frac{1}{A}$ $K = \frac{T_1T_4}{T_2T_3}$	$A = \frac{1}{T_4}$ $B = \frac{T_3-T_4}{T_3T_4}$ $C = 1$ $D = E+\frac{1}{T_2}$ $F = \frac{T_1-T_2}{ET_1T_2}+1$
35		$\frac{(T_2P+1)^2}{(T_1P+1)(T_3P+1)}$	$T_1 = \frac{1}{D-EF}$ $T_2 = \frac{1}{D-E}$ $= \frac{1}{A-B}$ $T_3 = \frac{1}{A}$ $T_2^2 < T_2T_3$	$A = \frac{1}{T_3}$ $B = \frac{T_2-T_3}{T_2T_3}$ $C = \frac{T_2^2}{T_1T_3}$ $D = E+\frac{1}{T_2}$ $F = \frac{T_1-T_2}{ET_1T_2}+1$

Table 6. Second-Order Linear Systems

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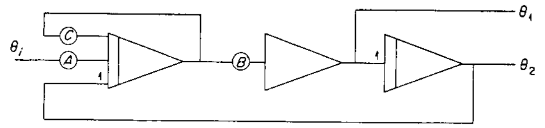
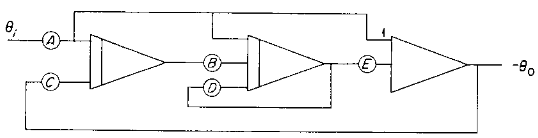
ORNL-LR-DWG 30804



$$\frac{\theta_1}{\theta_i} = \frac{KP}{T^2P^2 + 2bTP + 1} \qquad K = A \qquad A = K$$

$$T = \frac{1}{\sqrt{B}} \qquad B = \frac{1}{T^2}$$

$$\frac{\theta_2}{\theta_i} = \frac{-K}{T^2P^2 + 2bTP + 1} \qquad b = \frac{C}{2\sqrt{B}} \qquad C = \frac{2b}{T}$$



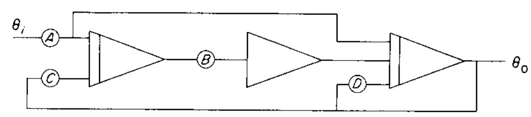
$$K = \frac{A}{C} \qquad A = CK$$

$$T_1 = \frac{1}{\sqrt{BE}} \qquad B = \frac{T_2}{2T_1(b_2T_1 - b_1T_2)}$$

$$\frac{\theta_0}{\theta_i} = \frac{K(T_1^2P^2 + 2bT_1P + 1)}{T_2^2P^2 + 2bT_2P + 1} \qquad b_1 = \frac{D - E}{2\sqrt{BE}} \qquad C = \frac{T_1^2}{T_2^2}$$

$$T_2 = \frac{1}{\sqrt{BCE}} \qquad D = \frac{2b_2}{T_2}$$

$$b_2 = \frac{D}{2\sqrt{BCE}} \qquad E = 2\left(\frac{b_2}{T_2} - \frac{b_1}{T_1}\right)$$

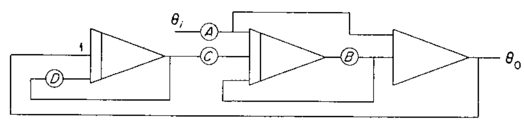


$$K = \frac{A}{C} \qquad A = CK$$

$$T_1 = \frac{1}{B} \qquad B = \frac{1}{T_1}$$

$$\frac{\theta_0}{\theta_i} = \frac{-K(T_1P + 1)}{T_2^2P^2 + 2bT_2P + 1} \qquad T_2 = \frac{1}{\sqrt{BC}} \qquad C = \frac{T_1}{T_2^2}$$

$$b = \frac{DT_2}{2} \qquad D = \frac{2b}{T_2}$$



$$K = \frac{AD}{B(D + C)} \qquad A = \frac{KB(D + C)}{D}$$

$$T_1 = \frac{1}{D} \qquad B = \frac{2b}{T_2} - \frac{1}{T_1}$$

$$\frac{\theta_0}{\theta_i} = \frac{KP(T_1P + 1)}{T_2^2P^2 + 2bT_2P + 1} \qquad T_2 = \frac{1}{\sqrt{B(D + C)}} \qquad C = \frac{T_2^2 + T_1^2 - 2bT_1T_2}{T_1T_2(2bT_1 - T_2)}$$

$$b = \frac{B + D}{2\sqrt{B(D + C)}} \qquad D = \frac{1}{T_1}$$

MULTIPLICATION

As shown in the section "The Operational Amplifier," a scalar relates the output to the input for an operational amplifier employing pure resistance in the input and feedback circuits. Furthermore, a scaling factor accompanies each of the linear operations described, and this scalar depends upon the circuit parameters. A product of a time-dependent variable times a scalar is inherent in all operational amplifier applications. Each of these scalars is the result of a choice of circuit parameters and is a fixed quantity, that is, a circuit and problem constant. A change of its value requires a change in the value of one or several of the circuit parameters.

Multiplication, as considered in this section, deals with those products in which both the multiplier and the multiplicand are time dependent and will vary during the course of a single problem. A time-dependent variation of the scalar mentioned in the preceding paragraph is implied in such a multiplication. The manner by which this variation may be accomplished depends upon the nature of the time behavior. A step change in the scalar may be made by opening or closing a switch appropriately located in the circuit. Certain arbitrary variations may be provided by manually manipulating a potentiometer setting or a capacitor value. Such methods are frequently used for determining the behavior of a system which is operator-controlled. In general, however, much more extensive control over this scalar must be exercised than the human operator is capable of inserting manually.

Servocontrol of the circuit elements extends the range of multiplication beyond that provided by manual operation. Servomechanisms convert analog quantities to shaft rotations which, in turn, alter the value of circuit parameters. The response times of these systems are much shorter than those of a human operator; yet the motion of mass in such systems limits their ultimate response times also.

For even shorter response times an electronic system of multiplication must be employed. Several such systems have been developed. For some of these the inherent nonlinearities of circuit elements, such as vacuum tubes, are utilized; for others the effective value of the elements is modified by gating methods. Use of such methods might be considered a direct approach. There are other electronic means for multiplying two time-dependent variables. One example is commonly referred to as a pulse-time multiplier. Another example is represented by an amplitude-frequency multiplier. The method of multiplication by one of the latter devices may be considered as an indirect approach to the solution of the problem.

In the following discussion only those methods of obtaining a product which are employed in the simulator and described in this report will be considered. Other methods are described elsewhere (5).

Manual and Servo

The schematic diagram, Fig. 13, illustrates electromechanical means for multiplying two time-varying functions. For this circuit,

(1) $$e_2 = i_{21} R_{21} \; ,$$

(2) $$i_{21} = -i_1 \; .$$

If R_{11} and R_1 are fixed, then i_1 is directly proportional to e_{11}, and e_2 will be directly and independently proportional to both e_{11} and R_{21}; the output, e_2, will be proportional to the negative product of e_{11} and the value of R_{21}; R_{21} is proportional to the gain factor of the operational amplifier. Hence time variation of the gain factor can be provided by means of a time variation in the feedback impedance, R_{21}.

Next, in Fig. 13, let R_{21} and R_{11} be fixed and let R_1 be varied in some predetermined fashion.

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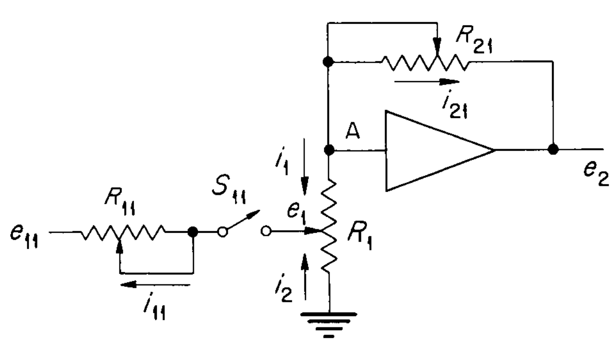

Fig. 13. General Scaling Circuit.

For the purpose of this discussion let

(3) $$R_1 = R_{11}$$

so that

(4) $$i_{11} = \frac{e_{11}}{R_{11}}$$

to within the limits of the precision desired. For this case the current delivered to the sliding contact of R_1 may be considered to be independent of the position of the sliding contact and to be a function of only e_1 since R_{11} is fixed. Furthermore,

(5) $$i_{21} = -i_1$$

so that the output will be a direct function of i_1.

The point A in Fig. 13 will maintain nearly zero potential with reference to ground through the action of the coupled amplifier so that the equivalent circuit of Fig. 14 may be used to represent the resistance R_1. The impedance to ground at the sliding contact appears as the parallel combination of the two sides of the potentiometer. Thus,

(6) $$R_{\text{eff}} = \frac{(1-\alpha)\,R_1(\alpha R_1)}{(1-\alpha)\,R_1 + \alpha R_1} = \frac{\alpha(1-\alpha)\,R_1^2}{R_1}$$
$$= \alpha(1-\alpha)\,R_1 \;,$$

which has a minimum value when α is zero or 1 and a maximum value when α is $\frac{1}{2}$. For a maximum,

(7) $$R_{\text{eff}} = \frac{R_1}{4} \;,$$

and, for the conditions imposed at the start to apply, this must be small in comparison with R_{11}.

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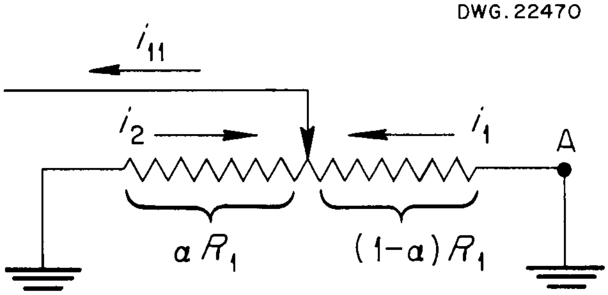

Fig. 14. Input Potentiometer.

The voltage developed across this network will be

(8) $$e_1 = i_{11} R_{\text{eff}} = i_{11}\,\alpha(1-\alpha)\,R_1 \;;$$

however, since

(9) $$i_1 = \frac{e_1}{(1-\alpha)\,R_1} \;,$$

(10) $$i_1 = \frac{i_{11}\,\alpha(1-\alpha)\,R_1}{(1-\alpha)\,R_1} = i_{11}\,\alpha \;.$$

Since the output of the amplifier is directly proportional to i_1, then it is a function of e_1 (which determines i_{11}) and of α. When these functions are introduced in Eq. 10, which includes all elements of the input circuit,

(11) $$i_1 = \frac{e_{11}}{R_{11}}\,\alpha \; / \;,$$

it may be seen that varying R_{11} provides means for multiplying. In this case the multiplying or gain factor is proportional to the reciprocal of R_{11}. The unit function symbol $/$, Eq. 11, represents the switch shown in Fig. 13 and indicates that i_1 may be "step-changed" by its closure. Each of these variations modifies the input circuit of the amplifier and, in general, can be classified as a means for multiplying by variation of the input conductance, or admittance.

A potentiometer between point A and ground can likewise be employed in the feedback circuit. If the attenuation of the current i_{21} is β, then for the general circuit,

(12) $$e_2 = -e_{11}\left(\frac{\alpha}{R_{11}}\,\frac{R_{21}}{\beta}\,/\right) \;,$$

and a product between e_{11} and another factor may be obtained by servo or manual control of any of the elements shown within the parentheses.

From Eq. 12 it follows that the effective input resistance is R_{11}/α. The conditions for which Eq. 12 holds require R_1 to be small but not small enough to cause malfunction of the amplifier. Common values of R_1 range from 20,000 to 100,000 ohms. This range of resistance values permits use of a low-resistance, high-precision potentiometer for varying the effective resistance over very wide limits.

It is worth noting that in multipliers utilizing admittance variations the input current to the amplifier is proportional to the product. Consequently, a fixed feedback network can be used to integrate this product or operate upon it in some other manner without affecting the process of multiplication. Likewise, the product of one input network may be independent of the operations occurring in other parallel inputs so that two or more different products may be simultaneously performed in a single stage. The combinations and permutations of operations are quite extensive in the use of operational amplifiers.

Descriptions and theory given above have to do only with the variations required on certain component parameters to provide an electromechanical multiplier. Means for varying these parameters will be given no further consideration than an indication that they may be either by hand, by relays, or by servomechanism.

Servomechanical Multiplier

The Electronic Associates servo multipliers have four linear, center-tapped cups or slide-wire potentiometers. Where only one polarity of the input variable e_1 occurs, it is possible to eliminate the inverting amplifiers shown in Fig. 15 and other figures which follow. Either side of the servo cup, marked "+ CT –," may be grounded, thereby permitting use of the full length of the potentiometer. If +REF is used, it is put on terminal +, and terminal – is grounded; if –REF is used, it is put on terminal –, and terminal + is grounded. When these connections are used, it is necessary to readjust the GAIN control.

Multiplier Circuits. – Figure 15 is a simplified circuit diagram of the multiplier circuit provided. Variables e_2, e_4, and e_6 are each multiplied by the common variable e_1, their respective products being $-e_B$, $-e_C$, and $-e_D$. The common variable is brought into the input side of the servo amplifier. Compared with this is the feedback or follow-up voltage supplied from the first, or servo, cup of the ganged potentiometer. The output of the amplifier drives a two-phase servo motor which rotates the potentiometer to complete the loop. This servo loop is designed in such a manner that the potentiometer shaft will be positioned so that the cup 1 output voltage, e_A, is always identical in magnitude and sign to the common variable input voltage e_1.

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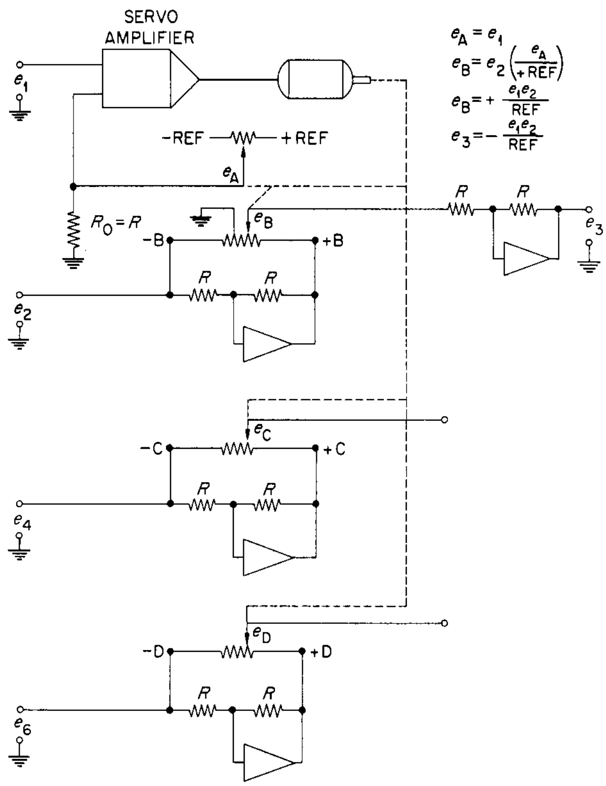

Fig. 15. Simplified Schematic Diagram of Multiplying Circuit.

If a second variable, e_2, is applied across a second winding of the potentiometer, the voltage e_B will appear at this wiper. The voltage e_B will be the product of e_1 and e_2 divided by a constant (the absolute value of the reference voltage).

The voltage e_3, indicated as the final output of the multiplying cup, is equal to the product

$$e_B = \frac{-e_1 e_2}{REF} . \qquad (13)$$

To derive the positive value $e_1 e_2$, it is necessary to reverse the polarity of one of the input signals or to connect the input to the right section of the potentiometer and the inverted output to the left section of the potentiometer.

Divider Circuits. – Figures 16–19 show suggested divider circuits for normal computational use. These dividing circuits utilize precisely the same elements used in the multiplier circuits, the only difference being the method of connecting the various units. Figure 20 shows a suggested

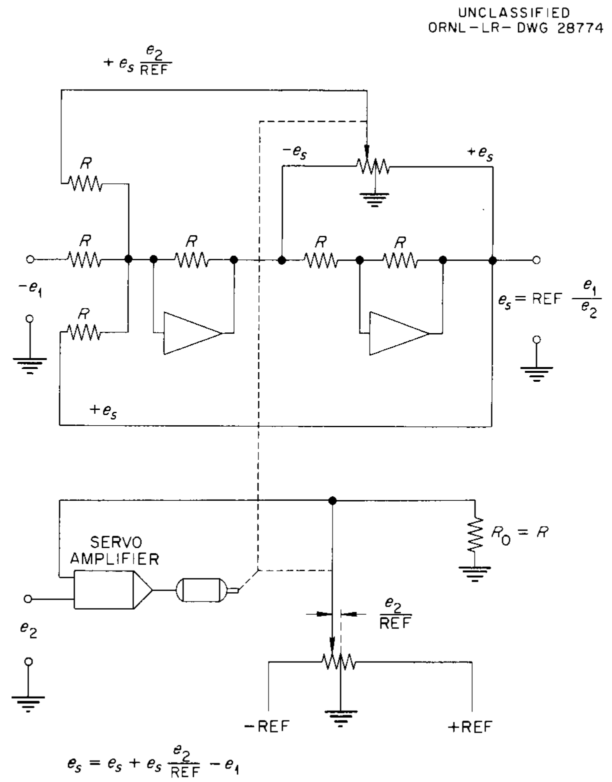

Fig. 16. Method 1 – Dividing Circuit.

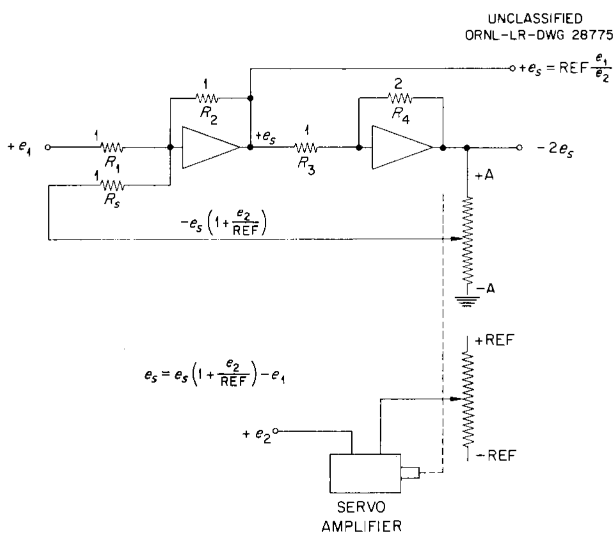

Fig. 17. Method 2 – Dividing Circuit.

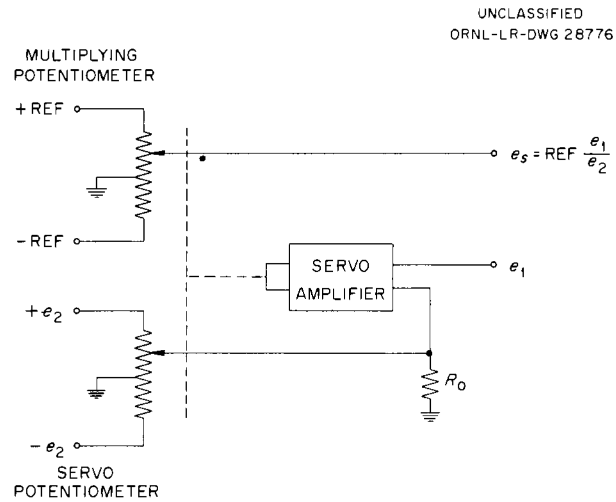

Fig. 18. Method 3 – Dividing Circuit.

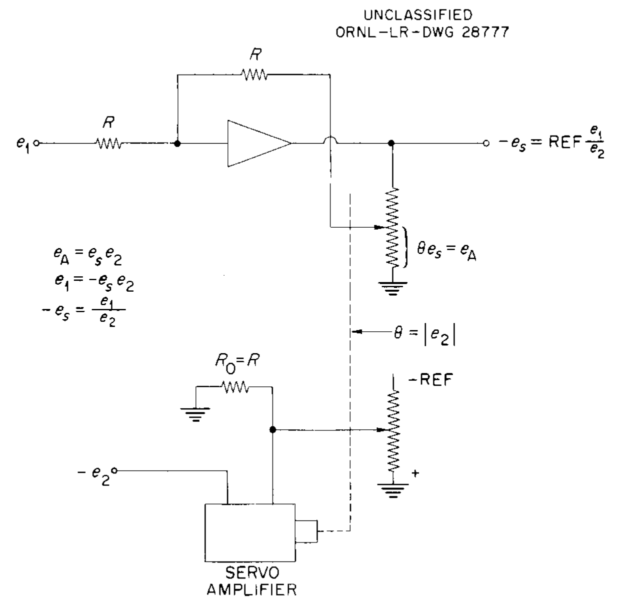

Fig. 19. Method 4 – Dividing Circuit.

method of connecting the various elements to derive square roots. For illustrative purposes the problem

$$e_s = \frac{e_1}{e_2} \tag{14}$$

has been included in the following circuit explanations.

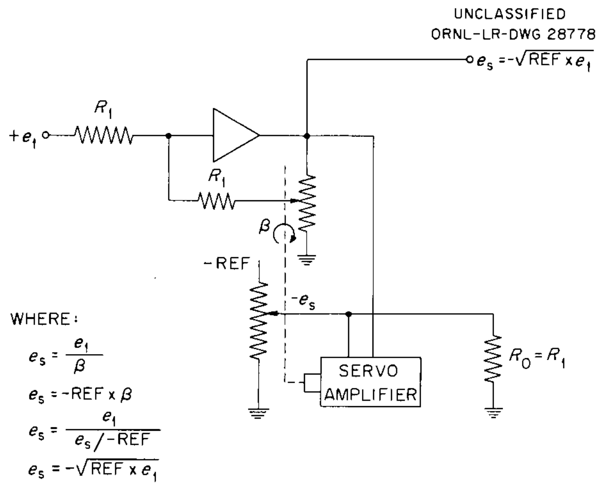

Fig. 20. Square-Root Derivation.

The mathematical manipulation

$$(15) \qquad e_s + e_s e_2 - e_1 = e_s$$

can be solved by using the circuit shown as Method 1 (Fig. 16). This is the preferred form of division equivalent.

An alternate computer circuit that can be utilized to solve the mathematical variation

$$(16) \qquad e_s (1 + e_2) - e_1 = e_s$$

of the equation above is demonstrated as Method 2 (Fig. 17). This method provides another means of producing quotients. It is quite obvious that by the simple expedient of varying the circuit values from

$$R_4 = 2$$

$$R_5 = 1$$

to

$$R_4 = 1$$

$$R_5 = \tfrac{1}{2}$$

Method 2 can be modified to provide a voltage equivalent to $-e_s$ instead of $-2e_s$ at the second amplifier output. This revision is often useful in further computations.

Figure 18 illustrates a third method of connecting a circuit for division by means of a servo loop. The solution of the mathematical problem

$$(17) \qquad e_s = \text{REF}\frac{e_1}{e_2}$$

is demonstrated as Method 3.

Another mathematical variation,

$$(18) \qquad -e_s = \frac{e_1}{e_2} ,$$

can be solved by the circuit illustrated in Method 4 (Fig. 19). If $+e_2$ is available, apply +REF to the servo cup of the potentiometer and ground the "–" end.

The design features incorporated in the computer elements under discussion here enable the mathematical derivation $e_s = -\sqrt{\text{REF} \times e_1}$ to be obtained by making the simple circuit connections shown in Fig. 20.

In the division methods discussed above, the limitations of value for e_1 and e_2 are that e_1 may be positive or negative but cannot exceed the value of the reference voltage. If e_2 goes through zero, an incorrect value will be derived.

Method 2 (Fig. 17) is not recommended, because the potentiometer loading error can be canceled through the use of a load compensating resistor only where asymmetric connections of the potentiometer cups exist.

Method 3 (Fig. 18) is recommended only when the variance in value of e_2 can be closely limited. Since e_2 is a part of the servo loop, any wide variation of value will be reflected by a wide variation in loop gain, and operational stability cannot be maintained.

In square-root derivations (Fig. 20), limiting values apply to both e_1 and the reference voltage. Stabilization is obtained when e_1 and REF are always of opposite sign.

Electronic Quarter-Square Multiplier

Figure 21 shows the block diagram of the electronic multiplier. Amplifier 1 adds $y/2$ and $-x/2$ to give $-(x + y)/2$. Amplifier 2 adds $-(x + y)/2$ and x to give $(y - x)/2$. Function generator A operates on $-(x + y)/2$ to give $-(x + y) + (x + y)^2/400$. Function generator B operates on $(y - x)/2$ to give $(y - x) - (y - x)^2/400$. The outputs from the two

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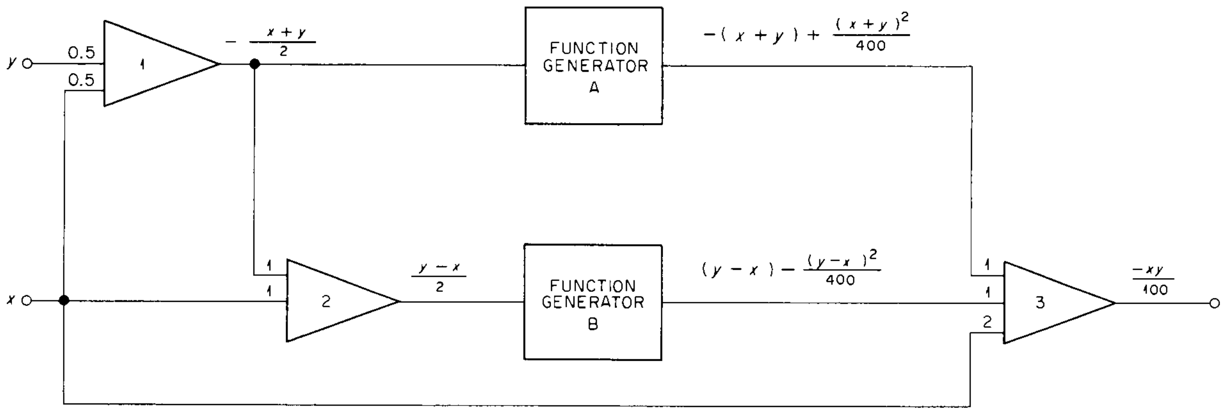

Fig. 21. Block Diagram of Electronic Multiplier.

function generators are added to $2x$ in amplifier 3 to give

$$(19)\quad (x+y)-\frac{(x^2+2xy+y^2)}{400}+(x-y)+\frac{(x^2-2xy+y^2)}{400}-2x\ ,$$

which reduces to

$$(20)\qquad -\frac{xy}{100}\ .$$

Each diode in function generators A and B generates a portion of a parabola for which $dy/dx = 0$. Each parabola passes through the origin but not at the point where the derivative equals zero. Each parabola is divided into 20 line segments in such a manner that the algebraic difference between the line segment and the curve never exceeds ±0.1 v. In order to reduce the overall probable error, the areas above and below the curves are made equal.

Calibration of the multiplier is accomplished by establishing the break points of each of the diodes by 1%-precision resistors. In order that the slopes of each segment may be adjusted, a string of resistors generates precise values of 10 v, 20 v, and so on to 100 v, which occur at the mid-points of each line segment. Another string of resistors generates the square of this voltage divided by 100, plus 0.1 v. This voltage is used to null the output of the multiplier when it is correctly calibrated. A simplified diagram of the circuit set up for calibration of network A is shown in Fig. 22. Since the input to network B is grounded, the output of the multiplier will be $-x^2/100$. The voltage from the $x^2/100$ string is fed in through a gain of 1 in such phase as to cancel the output. To calibrate network B, the input to A is grounded and the output of the calibration amplifier is fed into B.

DIODE FUNCTION GENERATOR

Function Generator Calibration. – Each function generator has eight diodes, basically, which can generate eight independent line segments. The quadrant in which a line segment falls is determined by means of a four-position selector switch labeled QUAD. The total input to a function generator channel is the algebraic sum of the outputs of all diode segments permanently assigned to it and several other inputs which may be applied. These other inputs are usually removed during calibration. Since curves of varying complexity are encountered, provision is made for allocating diode segments in different combinations by means of switches I through IV at the control panel. Each function generator has several diodes permanently assigned to it: Channel A, two; Channel B, three; Channel C, four; and Channel D, five. There are unassigned diodes to fill out the string of eight. The $+x$ and $-x$ inputs and the segment outputs of the unassigned diodes of each channel are brought to the control panel. The Group I switch on the control panel permits paralleling of the unassigned diodes on Channel A with the assigned diodes of any channel,

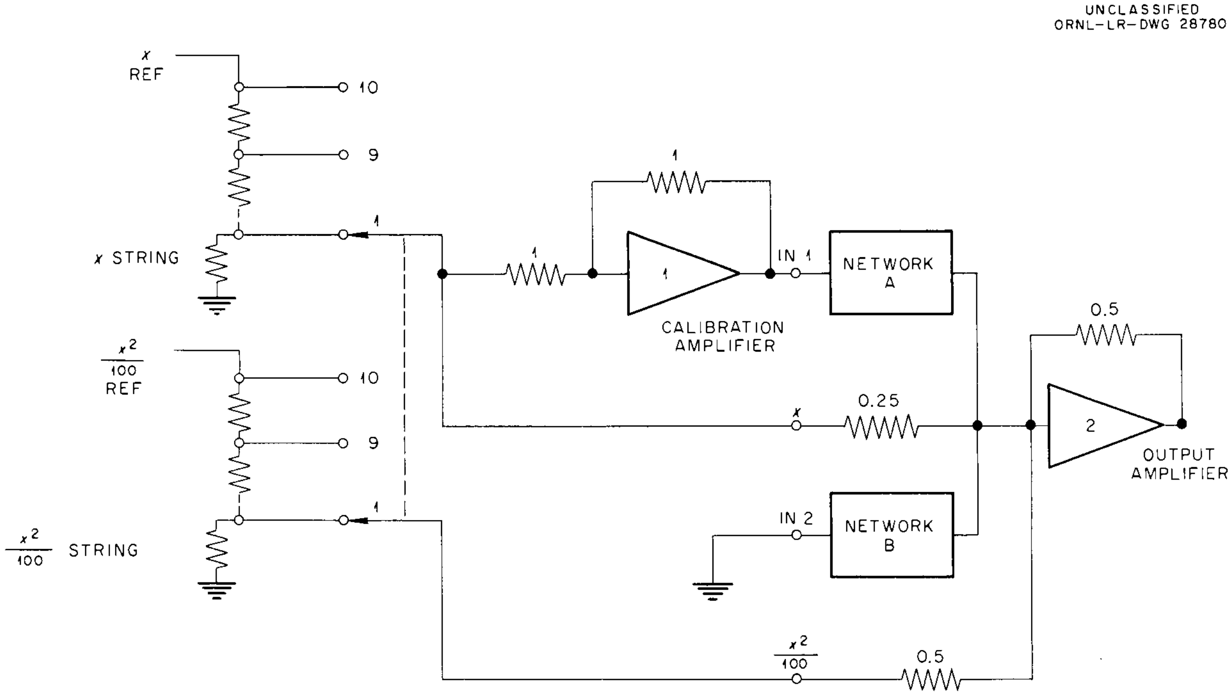

Fig. 22. Simplified Diagram of Circuit for Calibrating Network A.

and so on through Group IV and Channel D. Thus the maximum number of segments available to a given channel equals the number of assigned diodes, plus the total number of unassigned diodes, 18. The control panel also has provision for an additional line segment, the LINEAR TERM, in each of the four channels. This segment has four possible forms: $y = Kx$, $y = -Kx$, $y = A$, and $y = -A$, where y is the dependent variable and x is the independent variable. The magnitude of K may be from zero to 20 v, and that of A may be from zero to 100 v, any one of which is set with the LINEAR TERM SCALE FACTOR potentiometer. The particular form of this segment is determined by the four-position LINEAR TERM selector switch which selects, in the "+100" position, a negative constant added to the amplifier output; in the "−100" position, a positive constant added to the amplifier output; in the "$+x$" position, a line through the origin with negative slope added to the amplifier output; and in the "$-x$" position, a line through the origin with a positive slope added to the amplifier output.

There is one other input to each function generator channel, a miscellaneous input, w. This is a 1-megohm input resistor to the output amplifier. Thus, the total output of the function generator is $f(x) - 2w$, where $f(x)$ is the output resulting from all diodes assigned to the channel and the linear term segment.

Figure 23 is a simplified circuit of a basic diode building block for the generation of one line segment. In quadrant 1, +100 v is applied to the BREAK-POINT potentiometer. The arm of the potentiometer and $-x$ are summed into the cathode of the diode. For negative values of x the diode

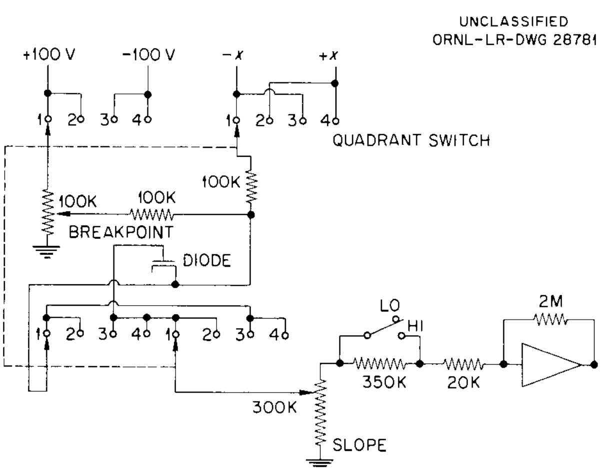

Fig. 23. Diode Building Block for Function Generator.

does not conduct. When x becomes positive and equal in magnitude to the voltage on the arm of the BREAK-POINT potentiometer, the diode begins to conduct. It should be noted that $-x$ does not refer to polarity but only to an inversion of the independent-variable signal. The output of the amplifier is shown in Fig. 24.

The slope of the segment is determined by the setting of the SLOPE potentiometer and by a HI–LO SLOPE switch.

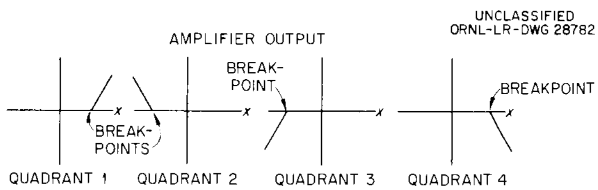

Fig. 24. Basic Line Segments Generated by Diode Building Block.

IMPLICIT FUNCTIONS

The technique of equation solution in implicit form is frequently applied throughout this manual and is often a powerful technique when a direct solution would be more complicated or even impossible. When y is some expressed function of x, it is expressed as $y = f(x)$; for example, $y = x/(1 - x)$ is an explicit function. When $f(x,y) = 0$, for example, $x + xy - y = 0$, there is an implicit function in x and y, in which the relation is implied only. To obtain the explicit function from the implicit function, y is usually solved for in terms of x. However, in many cases this is difficult, and in some cases impossible, to do.

It is usually possible to simulate a known implicit functional relation without solving it for the unknown explicit relation. When this is done, a device for multiplication or integration frequently replaces one for division or differentiation, which is desirable because multipliers and integrators are better performers than dividers and differentiators. Figure 25 illustrates the simulation of the above example in three ways: Case 1, in explicit form and Cases 2 and 3, in implicit form. The procedure in the third case exhibits the use of feedback, y being fed back to assist in finding its own value. The need to know an answer before it is found is only an apparent paradox. However, it does point up that a proper solution of the problem must exist and that the computer setup

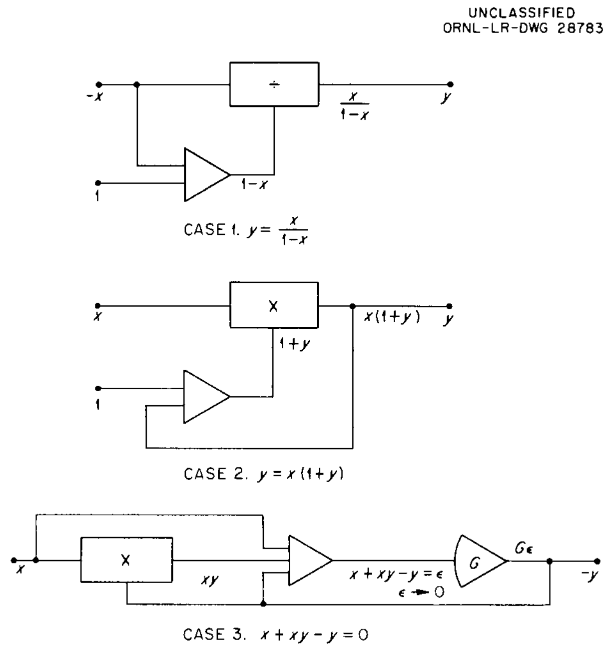

Fig. 25. Explicit and Implicit Function Simulation.

must be stable. Another example of the advantage of implicit function solution application occurs when the explicit form solution appears as an infinite series.

LOGARITHMIC RATE

In a system whose variables have a wide range of level, for example, as in reactor operation, the most informative forms of power or flux indication are logarithmic, $\ln \phi$, and logarithmic rate, $\dfrac{d(\ln \phi)}{dt}, \dfrac{d\phi/dt}{\phi}, \dfrac{1}{T}$, or inverse period. The logarithmic rate can be simulated directly by using a servo divider and a differentiator to produce $\dfrac{d\phi/dt}{\phi}$, but integrating this quantity to produce the $\ln \phi$ leaves much to be desired over long-time periods because of divider error and integrator drift. A preferred system is shown in Fig. 26. A commercial crystal-diode attenuator network is used which yields a 10-mv output change for a factor-of-2 input change over a range of more than two decades. Potentiometer 1 is set empirically to give a 0.25-v output per decade flux rise. The differentiator is fed through a 30-K series potentiometer to improve the amplifier high-frequency stability, and amplifier 3 is given a 0.25-sec time

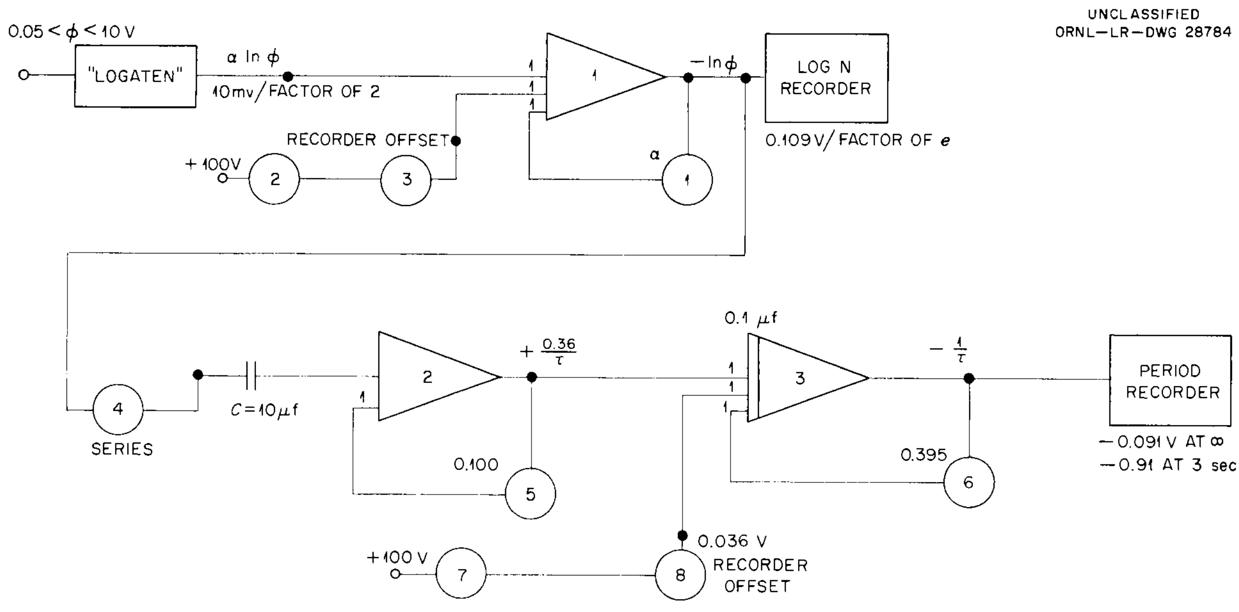

Fig. 26. Logarithmic Rate Simulation To Produce Reactor "Period."

constant to further cut the high-frequency response at low ϕ levels. Special chart paper is supplied for the Brown strip-chart recorders, a 4-decade logarithmic scale for the log ϕ record and a −30, ∞, +3 sec scale for the period record.

LOW-FREQUENCY OSCILLATOR

A sinusoid is a frequently required input and test function for system simulation. A low-frequency oscillator, which can produce frequencies up to 3.18 cps by use of commercial analog equipment, is illustrated in Fig. 27. Its upper frequency limit is $f = \omega/2\pi RC$, which, for the coefficient potentiometers, ω, set at 1.00 and for the integrator gain, $1/RC$, set at 1.00, is $1/2\pi$, or 0.159 cps. The amplitude is set with potentiometer A, and if zero damping is desired, series potentiometer D is set at mid-scale.

TIME-CONSTANT MODIFICATION

The equation for amplifier 1 without the amplifier 2 feedback is, in Fig. 28,

$$(21) \qquad e_1 = \frac{R_2}{R_1} e - R_2 C_1 \frac{de_1}{dt} \; .$$

Here, the steady-state gain $= R_2/R_1$, and the time constant $= R_2 C_1$. With the feedback loop in place,

$$(22) \qquad e_1 = \frac{1}{R_1 (1/R_2 - G/R_3)} e - \frac{C_1}{(1/R_2 - G/R_3)} \frac{de_1}{dt} \; .$$

In practice, $R_2 = R_3$, so that

$$(23) \qquad e_1 = \frac{R_2}{R_1 (1 - G)} e - \frac{R_2 C_1}{(1 - G)} \frac{de_1}{dt} \; ,$$

which shows that the effect of such positive feedback is to divide both the time constant and gain by $(1 - G)$. Where G is less than unity, for example, G_1, both the time constant and the gain are effectively increased, and time constants as large as 1000 sec are made possible without the need for impractically large values of R_2 or C_1. Where G exceeds unity, for example, G_2, negative time constants are created, corresponding to regenerative equations. The absolute value may be larger or smaller than in the case without feedback, depending upon whether G_2 is less than or greater than 2.0.

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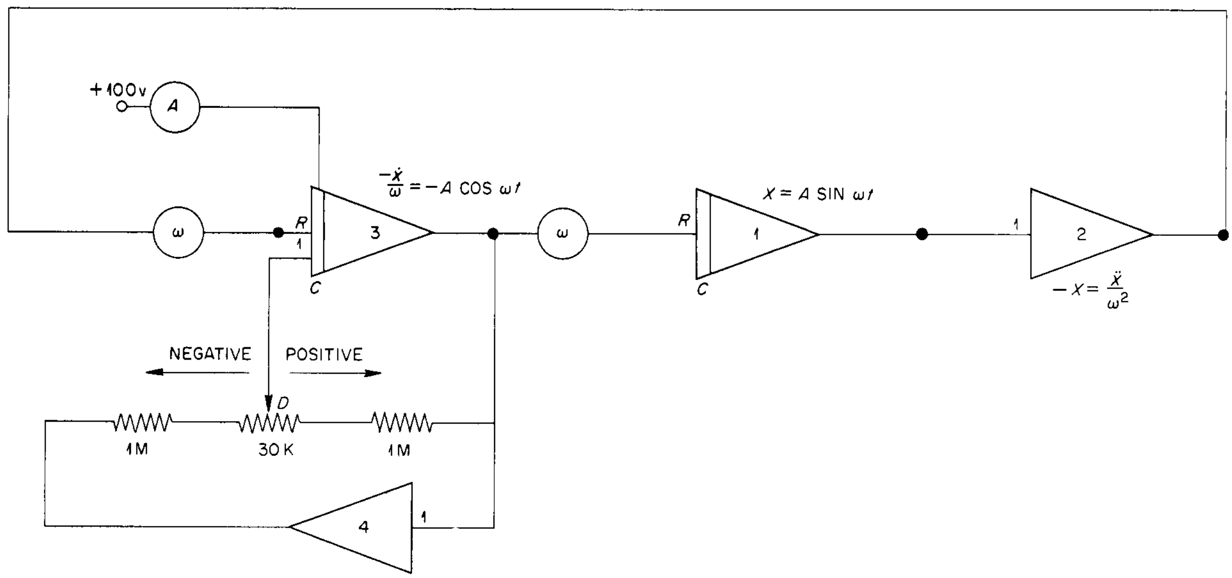

Fig. 27. Low-Frequency Oscillator.

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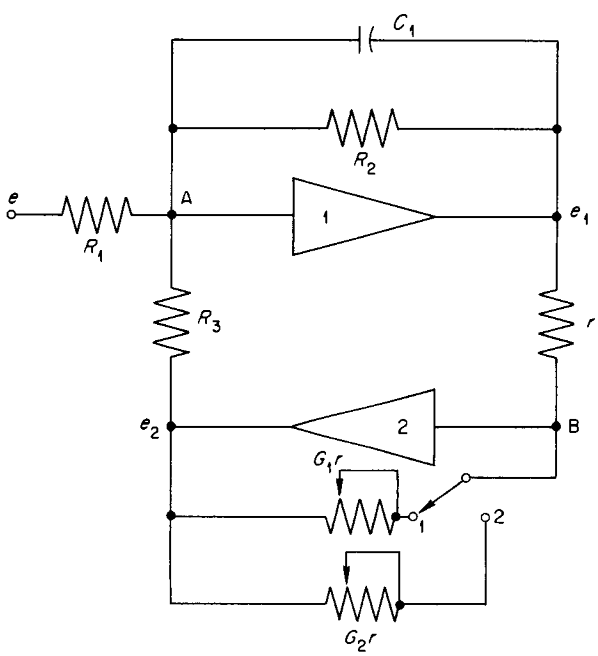

Fig. 28. Wide-Range, Linear, Time-Constant Modification.

Another case of interest is that in which resistor R_3 is replaced by a capacitor, $C_2 = C_1$. The Kirchhoff nodal equation at point A and the feedback are

$$\frac{e}{R_1} - \frac{e_1}{R_2} - C_1\frac{de_1}{dt} + C_1\frac{de_2}{dt} = 0 \ , \tag{24}$$

$$e_2 = Ge_1 \ , \tag{25}$$

from which

$$e_1 = \frac{R_2}{R_1}e - R_2C_1(1 - G)\frac{de_1}{dt} \ . \tag{26}$$

Comparison of Eq. 23 with Eq. 26 demonstrates that in the case of capacitive positive feedback the sole effect is to multiply the time constant by $(1 - G)$. Where G_1 is less than unity, this effectively decreases R_2C_1 and gives access to a range of very short time constants.

The potentiometers may be calibrated in such a circuit, and a linear plot of potentiometer setting vs λ (or τ) can be drawn for resistive or capacitive feedback. For positive time constants this is done for two or more settings of Gr by applying a constant potential e, allowing the circuit to reach equilibrium, and then disconnecting the driving voltage and measuring the e-folding time, τ, of the decaying function e_1.

For negative time constants this could be done for two or more settings of Gr by applying a constant potential e, differentiating the output e_1, and measuring the rise time constant, τ, of the expanding derivative. A typical plot of λ vs loop gain is shown in Fig. 29.

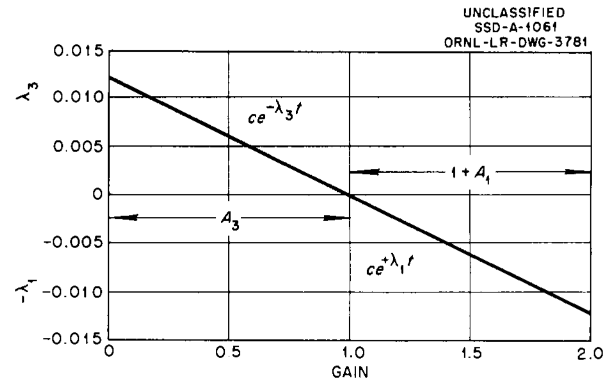

Fig. 29. Typical Plot of λ vs Loop Gain.

A technician is available for the following types of assistance:

1. checking and replacing components, vacuum tubes, and fuses,
2. recorder chart loading, chart speed changing, pen refilling, and scale calibration,
3. constructing and installing special nonlinear and control hardware.

PRELIMINARY ADJUSTMENTS

Before operation is attempted, a study should be made of the descriptions and initial positions of the EA (Electronic Associates) Control Panel switches to provide a better understanding of the system operation. All control designations are capitalized and used as they appear on the panels.

Normally, the computer is left from the previous day's operation with all vacuum tube heaters on and with the EA control switch placed on STAND-BY. Also, all a-c and d-c power supplies can initially be assumed to be in proper adjustment. However, these can be checked by a new operator while he is becoming familiar with the equipment.

Clear a prepatch panel of all plugs and cords, except for the orange plugs, which are placed to cover INT. *If the patch panel is not in its operating position, raise the patch-bay lever arm and, holding the prepatch panel by the handles, carefully push the panel into position. When the panel is properly seated, slowly lower the lever arm to lock the panel. Place all attenuator panel switches in the neutral (center) position, and see that the* PAPER DRIVE *switch is* OFF.

START Switch. – This is a normally open switch which energizes the filament power relay, applying power from line No. 1. It illuminates the FILAMENT green indicator light, operates the fans, and, after a 30-sec delay, applies power from line No. 2 to the control switch for application of plate power through interlocked circuits. *If the* FILAMENT *indicator is out, depress the* START *switch.*

STOP Switch. – This normally closed switch is used when power is to be removed from all circuits except the oven heater, motor, and service outlet located under the EA desk. The OVEN indicator light may or may not be illuminated.

Control Switch. – *Throw this switch to* REF OFF, *which will apply plate voltages to the* EA *amplifiers and to the* VTVM. In this position all power supplies and amplifiers can be checked. The PLATE red indicator will light.

Monitoring Controls. – The following six controls on the panel are associated with the VTVM used to monitor the computer.

BALANCE Control. – *Adjust to electrically center the meter.*

VOLTS Switch. – *Place the voltmeter range switch on 500.*

SELECTOR Switch. – *Place this switch, which picks up the output of a particular deck of the* AMPLIFIER *or* ATTENUATOR *switch, on* POWER VOLTAGES.

VTVM-DIGITAL READOUT Switch. – *Turn this three-position control to* VTVM *to pick up the output of the* SELECTOR *switch for application to the* VTVM, *turn to* DIGITAL-READOUT *for application to the* SERVO VOLTMETER, *or turn to center position to pick up whichever voltages are plugged into the patch-board holes marked* VM *and* DIG.

AMPLIFIER Switch. – *Use this switch to select any one of the a-c and d-c rack potentials or any one of amplifiers' 1 through 20 output potentials. Check the following voltages in the order indicated and adjust if necessary:*

Potential	Control	Unit
–300 v	VOLTAGE ADJUST	16-10P
+300 v	VOLTAGE ADJUST	16-10P
–500 v	200-v ADJ	16-10Q
+110 v	110-v ADJ	16-10Q
VIB (18.5 v)	AMPLITUDE	16-21C
REL (60 v)	None	713-10E

Until the supplies have been adjusted, the overload alarm may "cry" and the amplifier OVERLOAD BALANCE INDICATORS may be illuminated, due to voltage unbalances.

ATTENUATOR Switch. – This control makes available for measurement the brush voltage of attenuators 1 through 80.

Zero Check. – *To balance each amplifier, depress the* BALANCE *switch (on each amplifier) and slowly rotate the* BALANCE *potentiometer until the* OVERLOAD BALANCE INDICATOR *is extinguished. Balance all amplifiers (in use) at the* *beginning of each workday and once every 4 hr throughout the day.* Overload indication occurs whenever the amplifier offset exceeds 10 mv.

PAPER DRIVE Switch. – This switch is wired to apply operating voltages to the chart drive and pen motors of the recorders.

FUNCTION Switches. – These switches are dpdt center-off transfer type of units, labeled 1, 2, 3, with terminals connected to function switch terminals F1, F2, and F3 on the prepatch panel, with UP-DOWN positions correlated.

Warmup Procedure. – *After completing the preliminary checks and adjustments, place the control switch on* STAND-BY. *Allow at least 1 hr for heater warmup before attempting computation.*

OPERATIONAL STEPS

Prior to operating the system, patch the prepatch panel for the specific problem and install it on the console.

> **Caution:** *Use extreme caution when patching after the prepatch panel has been installed in the bay. Many of the prepatch terminations, such as an attenuator brush, have reference potentials on them. If the patch is first made to the potentiometer brush, inadvertent grounding of the other end of the patch cord will blow the potentiometer fuse. Considering any patch as a connection from an output to an input, always patch the input first, then the output, that is, to and then from. In removing a cord, remove the output, then the input.*

When the panel is removed, an interlock switch operates the plate power control circuit, removing all voltages from the springs. With the panel in place, reference voltages can momentarily be removed from the equipment by pressing downward on the lever arm which operates a microswitch to open the reference relays. This feature aids in the elimination of attenuator fuse burnout.

Running operational problems through the computer system is based upon use of the control switch to select the particular function desired:

REF OFF permits the reference voltage to be removed and the reset and hold relays to be operated. Patching may be accomplished at the panel, and all computational circuits will be held in a stable condition.

POT SET turns on the reference and plate voltages and operates the reset and potentiometer set relays. This position permits the attenuators, loaded by the impedance that they are connected to during operation, to be set.

RESET operates the network reset and hold relays. The amplifiers are thereby restored to initial condition.

HOLD operates the hold relays, which remove the input voltages to the amplifiers. This position permits momentary interruption of integration.

OPERATE permits the amplifiers to begin integration.

Recorder Activation. – Normally, recorder amplifiers 1–7 are on, and amplifier 8 is off. The switches to activate all Brown recorders are located inside the cases, behind the chart mechanism, and are labeled INSTRUMENT POWER. *Also, turn on the* CHART DRIVE *switches, located on the chart mechanism front, governing those recorders to be used on a particular problem, since these are in series with the* EA CHART DRIVE *switch.*

BROWN RECORDER CALIBRATION

Mechanical Full-Scale Pen-Chart–Scale Alignment

With the amplifier OFF, *mechanically run the pen to the full-scale stop. If the full-scale chart line does not line up with the full-scale scale line, loosen the scale, align it, and then retighten it. If the pen with its pointer is not aligned with the full-scale line, loosen the pen carriage screws and slide the pen along the cable to the proper postion; and tighten the screws.*

Zero Adjustment

With the amplifier ON *and with the d-c input signal at zero, check the pen alignment with the scale zero, not the chart zero; if it is off, refer to Sec 4.3.1.1 in the "Service Manual for Class 15 Electronik Instruments" for zero adjustment.*

Span Adjustment

With a ±100-v reference signal on the input, check the scale reading, and see that the pen rides without mechanical oscillation along the "100" chart line. If the mechanical growl which accompanies off-scale inputs is present, have an assistant manipulate the screwdriver adjustment on the back of the recorder control panel until the growl just disappears.

PATCHING INSTRUCTIONS FOR EA EQUIPMENT

The patch cords and plugs furnished as part of the system are used to interconnect terminals on the prepatch panel. Three types of plugs are used to interconnect grouped terminals: four adjacent terminals (two vertical pairs); two adjacent horizontal terminals ($^{7}/_{16}$-in.-spaced contacts); and two adjacent vertical terminals ($^{9}/_{16}$-in.-spaced contacts). Coaxial patch cords are provided in four lengths, in multiples of 6 in.

This section is concerned primarily with the functional use of the individual operational circuits, such as integrator, summer, limiter, etc. No attempt is made to cover the combined operation of these circuits, since there are too many possible combinations and problems.

Attenuators

The attenuators have three general uses: as a divisible source of regulated d-c voltage (±100 v max) for the setting of initial conditions in integrator circuits, as a variable voltage source to simulate mathematical parameters being inserted as voltage inputs to computing equipment, and as pads. *Use the loading correction chart shown in Fig. 30 when the accuracy of solution desired will not tolerate the loading on the attenuator by the input resistor. Curves shown are drawn for amplifier gains of 1, 5, and 10 and for a tandem pair of attenuators. To obtain the corrected attenuator setting under loaded conditions, (1) locate the desired setting on Fig. 30 baseline; (2) pick the curve corresponding to the amplifier gain used; (3) note the intersection of the desired setting and curve; (4) project this point horizontally to the left or right until it intersects the "Add Dial Divisions" line; (5) add the dial divisions noted to the attenuator setting.*

Combination Amplifiers 1–20

Summing Amplifier. – *To use the circuit for summing, insert a 4-prong patch plug into the terminals marked* INT, *leaving* SUM *visible (Fig. 31).* The patch panel provides two terminals for a gain of 10, two terminals for a gain of 5, and three terminals for a gain of 1, all colored green for easy identification. Six identical output terminals, all identified by a red color, permit application of the amplifier output to six other terminals. By using the input and output terminals and leaving out the 4-prong bottle plug, it is possible to make use of the amplifier with external input and feedback networks. *When these networks are removed, be sure that stability is maintained in the amplifier.*

Summing Integrator. – *To use the combination amplifiers as a summing integrator, insert a 4-prong bottle plug to leave the terminals marked* INT *exposed (Fig. 31).* The same inputs of 1, 1, 1, 5, 5, 10, 10 are available, in addition to an IC input which has unity gain. The integrating capacitor will charge to within 0.01% in approximately 0.92 sec.

Limiters (Shunt)

Eight diode limiters are provided to control the output of amplifiers. *Patch these diodes as follows: Remove the 4-prong bottle plug from the patch-board terminations of the amplifier to be used. Connect the cathode (negative limiting) or plate (positive limiting) of the diode to the two center bottle-plug terminals by connecting two patch cords between these terminals and the diode, each diode having multiple terminals for this purpose. Connect the other terminal of the diode via the biasing potentiometer to the output of the amplifier. Establish the desired value of the feedback resistor by placing a red bottle plug between an output and an input terminal.*

Function Switches

Three double-pole, three-position switches located on the control panel are made available at the patch panel. The position markings of UP and DOWN at the patch panel correspond to the actual position of the switch arm.

Boost Resistors

Twelve boost resistors, all tied at one end to the −300-v supply, are available at the patch panel to permit operation of the d-c amplifiers into loads that are heavier than normal. *In operation, connect the boost resistor to one output of the operational amplifier where it parallels the amplifier cathode load resistor.* Without boost, the amplifier is capable of a +100-v output into resistances as low as 7500 ohms. The following tabulation shows the number of boost resistors required to give a voltage swing of ±100 v under varying loads:

Load	Number of Boost Resistors
7500	0
4500	1
3000	2

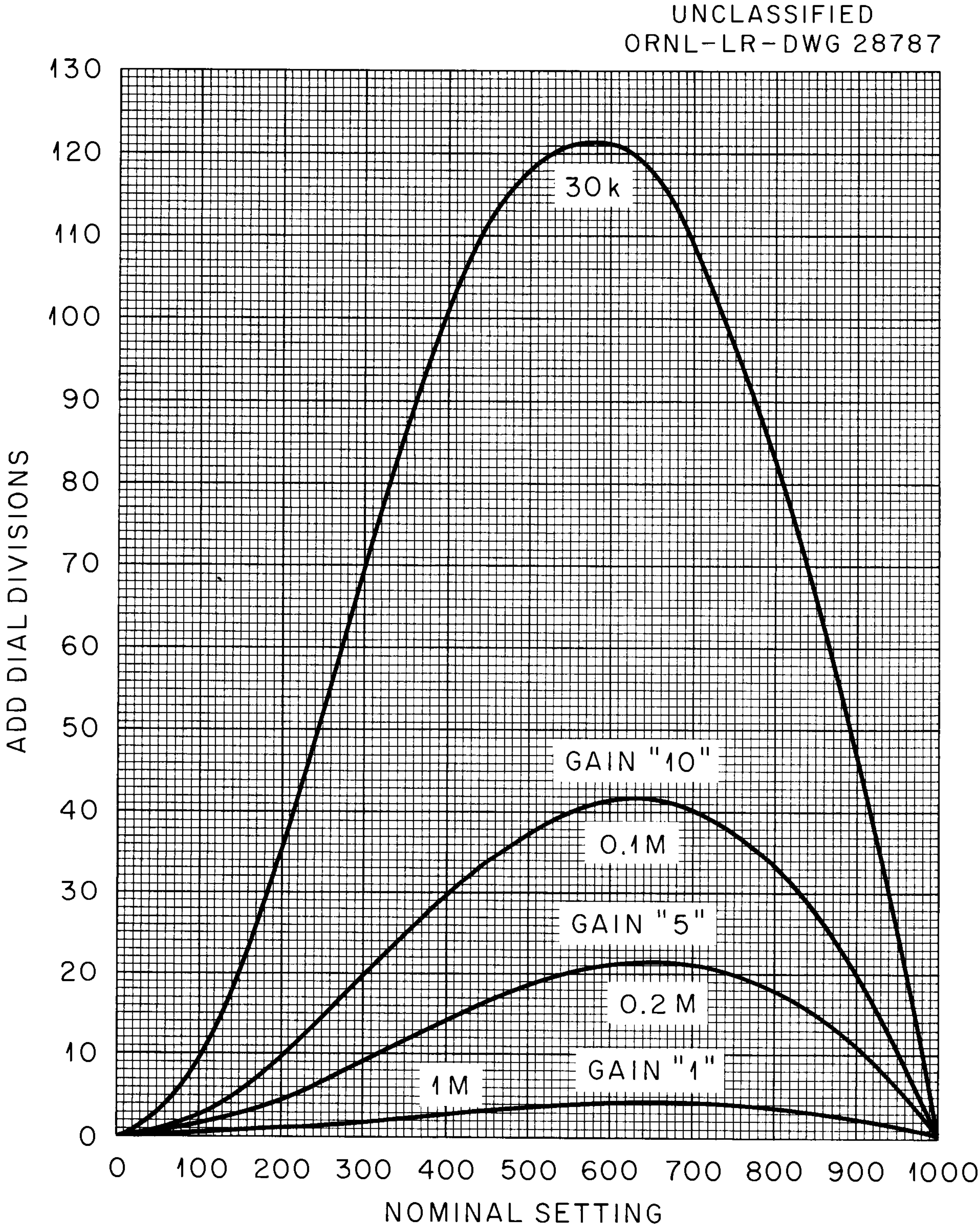

Fig. 30. Attenuator Loading Correction Chart for EA Analog Computer (30-K Potentiometer).

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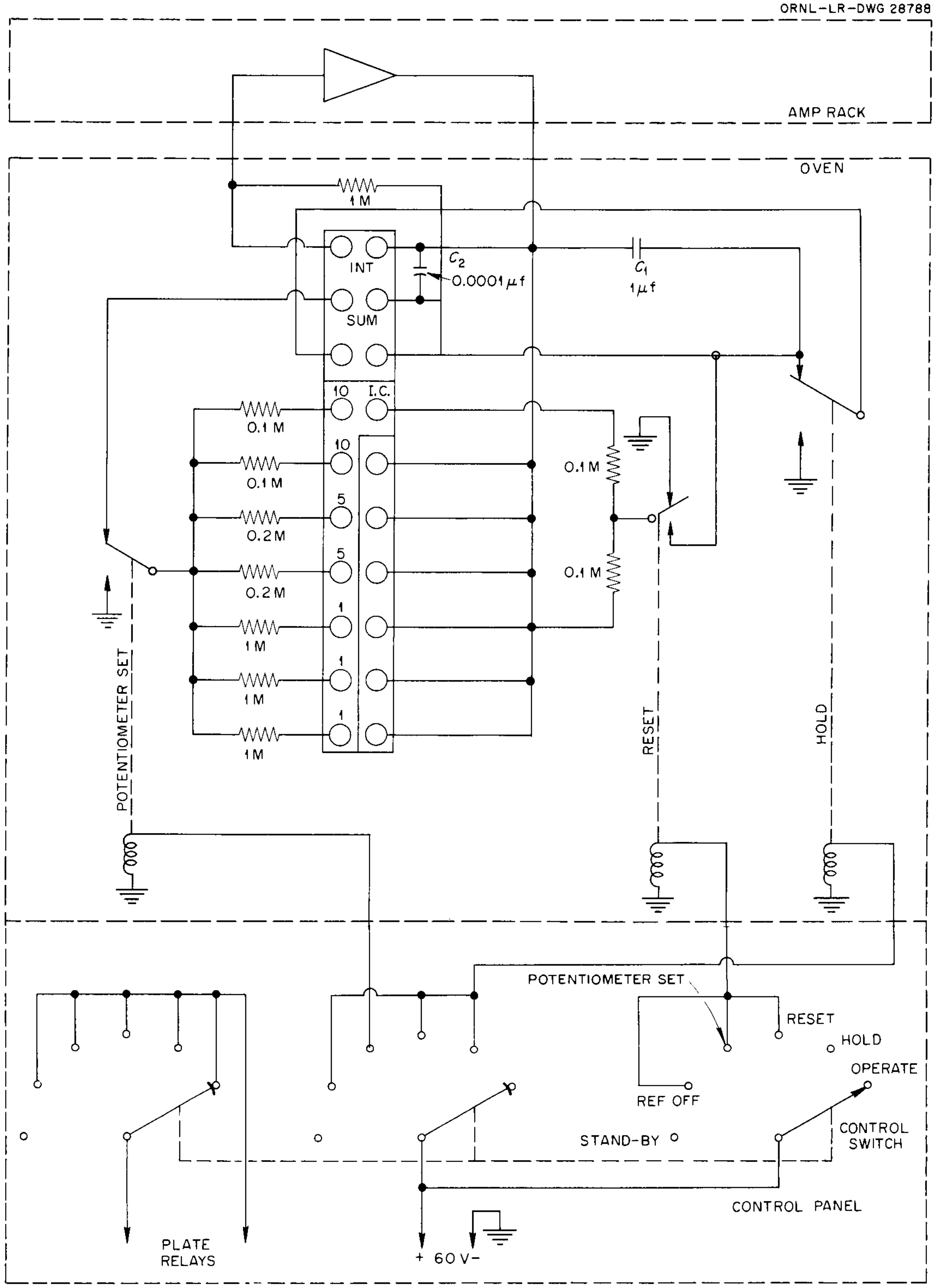

Fig. 31. Summer-Integrator Patching.

Multiples

The patch panel area designated MULTIPLES and marked as tie points contains ten groups of six terminals each. In addition, five groups of six terminals each appear to the left of each fast multiplier section.

Grounds

The patch panel provides 49 terminals marked in black that can be used for connections to ground.

Trunks

The patch panel presents 40 terminals in the strip designated TRUNKS, which are available at two Jones type of strips located on the plug mounting plate at the rear of the computer group. The panel designations correspond with the Jones strip designations.

Patching to External Equipment

The section at the bottom of the patch panel is illustrated in Fig. 32. A section of the computer group control switch S5A has its terminals brought out to the prepatch panel to provide a means of synchronizing external equipment switching with the positioning of the control switch. *To utilize these circuits, make a patch between the* COM *and +60-v terminations on the panel. The selector switch controls the signal to the terminations indicated in Fig. 32 to which an oscilloscope or meter can be attached. To put the integrators in* HOLD *condition, patch them to an external switch, providing external control of the computer* HOLD RELAYS. *Patch the inputs that cannot be selected by the control panel metering switches, such as the amplifiers 21–40 outputs, to the servo voltmeter* DIG *or* VM *prepatch panel terminals. Put the meter selector in the* neutral *position so that both*

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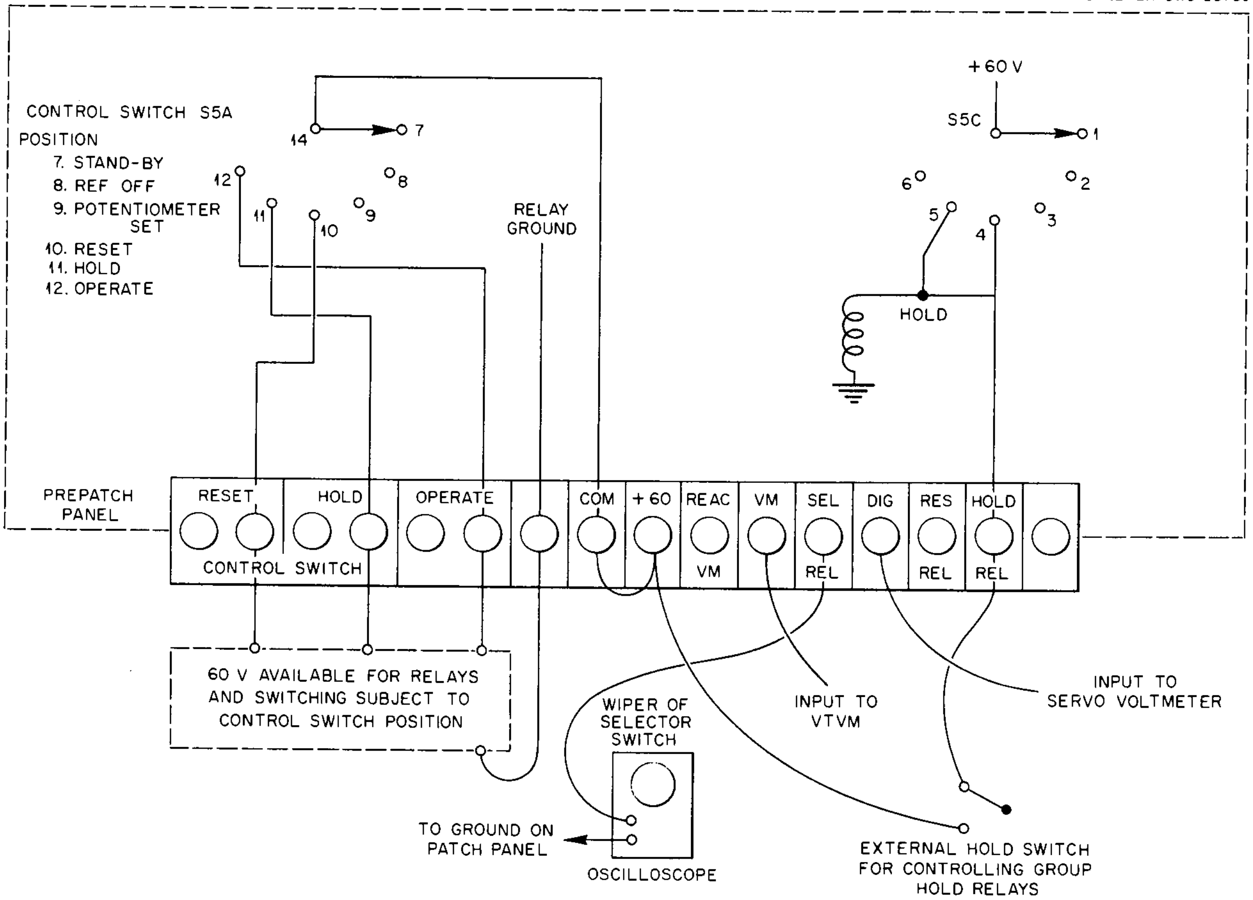


Fig. 32. Patching to External Equipment.

the VTVM *and the servo voltmeter are available at the panel.*

Plate Relays

Two polarized plate relays are available with terminations on a blue area of the prepatch panel, each having two, single-pole, double-throw contacts, normally open, which are actuated by approximately 1 v or 10 ma on the relay coils.

Servo Multipliers

The two servo multipliers available on the patch-board are Nos. 3 and 4, and have servo amplifier inputs marked 3*x* and 4*x*. *Normally, patch the servo slide wire into the +100-v and −100-v references with vertical plugs and ground the slide-wire center tap. Then feed the follower slide wires with one to three multiplicands of both polarities ±A, B, and C so that the multiplier outputs will come out at A***x***, B***x***, and C***x***. These slide-wire center taps are usually grounded. Compensate on the servo slide wire for the potentiometer loading errors produced by one follower slide wire by plugging the resultant load impedance to ground into the hole marked* LD.

Servo Voltmeter

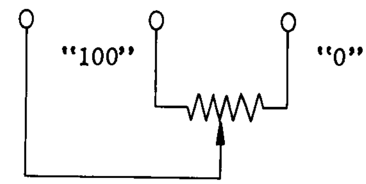
The servo voltmeter, located in servo multiplier position No. 1, provides the most accurate means of reading output voltages in the range ±100 v to within 0.05%. In addition to the three-digit voltage designation, polarity indication and a servo gain control are located on its panel. *Advance the* GAIN *potentiometer to the point of optimum sensitivity just below the region of instability. To provide potentials for reference in the servo device, insert vertical bottle plugs between the minus, ground, and plus terminals and the reference terminals of the 1x servo multiplier position on the patch-board.*

Apply the voltages to be measured to the servo voltmeter either from the control panel, via the SELECTOR *switch being set on* DIGITAL READ-OUT, *or from the patch-board by patching the output voltage into the terminal marked* DIG *and throwing the voltmeter* SELECTOR *switch to neutral. Observe that the servo voltmeter has a mechanical stop in its counter set at (1)025.*

Auxiliary Servo Multipliers — Recorders' Helipot Slide Wires

Four additional servo multipliers are available to the facility, associated with Recorders 1, 2, 3, and 4. They are attached to the patch-bay TRUNKS according to the tabulation shown below. Due to the gear arrangement involved, only 0.886 of the full potentiometer 50,000-ohm range can be used; at the recorder "100" position, the potentiometer is at 0.886.

Recorder No.	TRUNK Numbers		
1	38	39	40
2	35	36	37
3	32	33	34
4	29	30	31



Recorders

Patch the recorder inputs into 1 through 8 of the INTERCONSOLE *holes,* not *into the* RECORDER *holes.*

PATCHING INSTRUCTIONS FOR NONLINEAR EQUIPMENT

REAC Electronic Multipliers

The terminals for the five fast multipliers appear on the patch-board on the area marked initially for SERVO MULTIPLIERS 4*x* through 9*x*. Associated with each multiplier are two amplifiers: AMP 49 and 54, AMP 50 and 55, AMP 51 and 56, AMP 52 and 57, AMP 53 and 58. When the particular multiplier is not in use, any of these amplifiers can be converted to inverters by plugging together the holes marked +26 and CONV. Additional MULTIPLIERS 11 through 15 appear to the left of each multiplier grouping. *It is not necessary to patch any portion of the board for multiplier calibration.*

Electronic Multiplier Calibration

To calibrate panels, for example, Panel 1 of the Diode Electronic Multiplier, check the null indications of each of the 40 adjustments on the face of Panel 1. First, with the METER SCALE *switch on* OFF, *set the microammeter to zero by use of the* ZERO ADJ *screwdriver adjustment. Then, with the switch on ".1", place the* CALIBRATE CHANNEL *selector on "1" and place the* OPERATE *switch on* CAL A, *with the* EA *control switch on* REF OFF. *This operation turns on the* CALIBRATE A *light to the left of the upper two rows of 10 potentiometer knobs and indicates the correspondence between these potentiometer numbers and the numbers at the* SEGMENT SELECTOR *switch on the control panel. Now, to calibrate channel A, zero the meter for each segment sequentially, 1 through 20, since the setting of each successive segment depends upon the preceding segments. Zero to within one division on the ".1" scale. For the second half of the calibration, throw the* OPERATE *switch to* CAL B *and repeat the zeroing on the lower two rows of potentiometers on Panel 1, starting with segment 1. To return to the operating regime, set the* METER SCALE *switch to "100" and set the* OPERATE *switch to* OPERATE, *at which time the* READY *light on the* REAC *control panel will come back on.*

REAC Function Generators

The terminals for the four function generators appear on the patch-board on the area initially marked RESOLVERS. Associated with each function generator are two amplifiers: AMP 59 and 63, AMP 60 and 64, AMP 61 and 65, and AMP 62 and 66. When the particular function generator is not in use, any of these amplifiers can be used directly as inverters. The four groups of holes numbered 1 through 8 to the left of each amplifier grouping permit each respective function generator diode to be fed a variable bias for generation of a function of two variables. The prepatching necessary for introducing a function into any function generator is diagramed in Fig. 33. The x input to function generator A, for instance, is plugged into the IN hole of AMP 59, and the output appears at the holes under A of the column marked $f(x) - 2w$. (Inadvertently, some of the overlays have had the unused space where "$-2w$" was written punched out.) The w hole provides an additional input to the function generator output amplifier for applying an $f(x)$ other than zero at the independent variable zero.

Function Generator Calibration

For the most convenient method of inserting a function, use the plotted curve with the dependent

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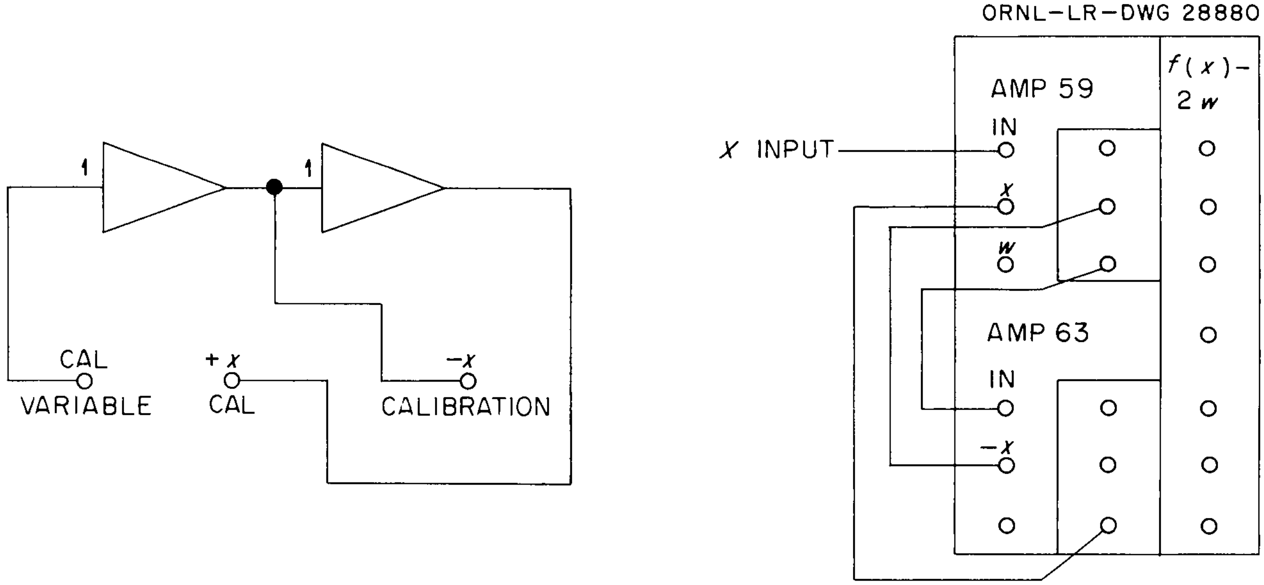

Fig. 33. Function Generator Calibration Patching.

variable scaled in volts and the minimum increments of the straight-line segments which are selected greater than 0.5 v. Determine the break points by the intersections of the straight-line approximations. Set the quadrant switches as required by inspection of the straight-line segments. When the OPERATE—CALIBRATE *switch on the control panel is in* CALIBRATE *position, disconnect the normal inputs and insert the known inputs from the* CALIBRATION *section of the panel. See that all* NORM—f(y) *switches are in* NORM *position. All* HI—LO *slope switches should usually be in the* LO *position (2 v/volt maximum); if, however, the incremental slope of any segment is greater than the maximum, throw to the* HI *position (12 v/volt maximum). The calibration circuit in simplified form is shown in Fig. 34.*

A null method in which the VTVM is used as an indicator compares the calibration function breakpoint settings with the function generator segment settings. Function generator A output is observed on Reeves amplifier 18; B, on amplifier 17; C, on amplifier 16; and D, on amplifier 15. The FUNCTION VARIABLE potentiometer supplies the independent variable x, and the FUNCTION VALUE potentiometer supplies the dependent variable y, both essentially direct reading in volts, 0 to 100.

Assume that it is desired to set up the function shown in Fig. 35. At the start of the calibration,

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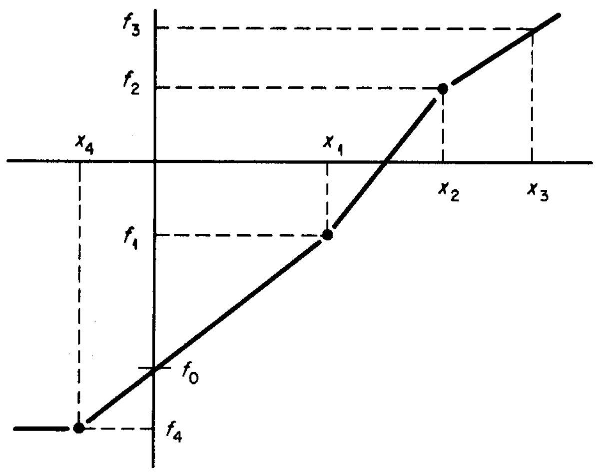

Fig. 35. Typical Arbitrary Function.

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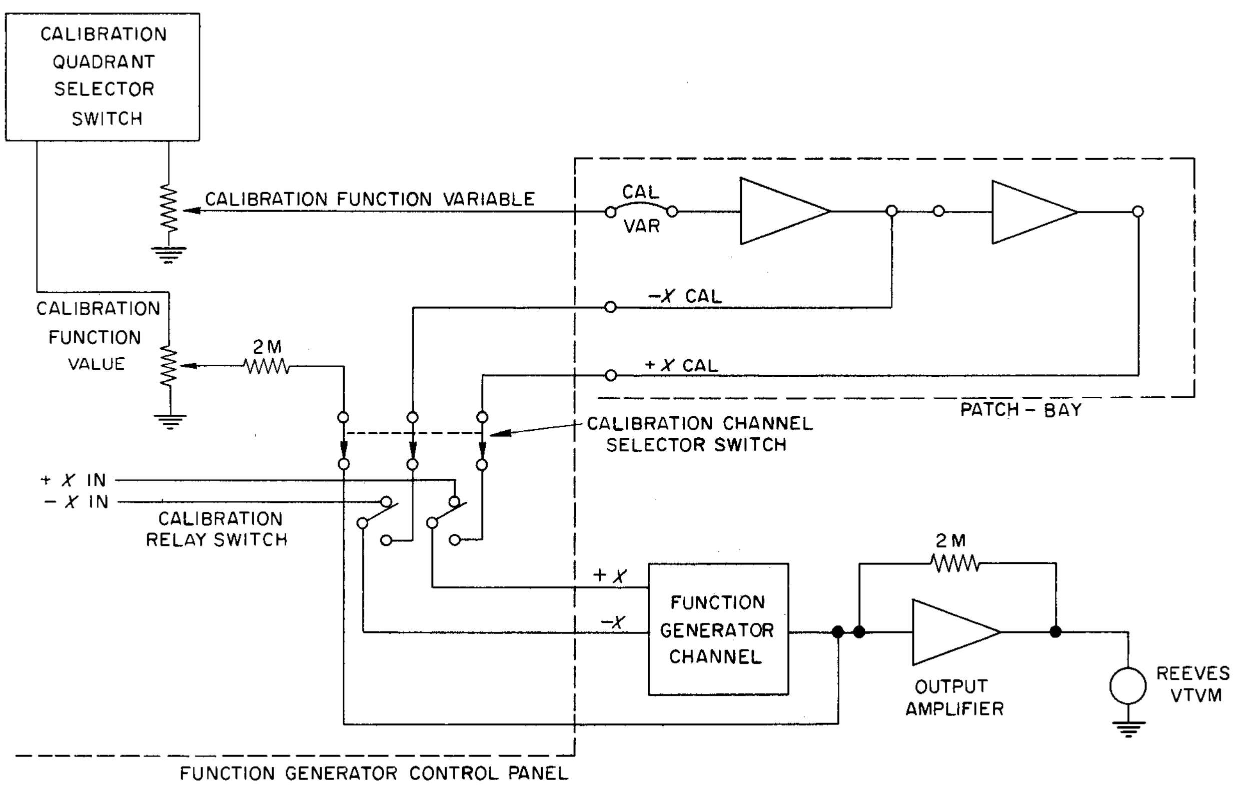

Fig. 34. Simplified Diagram of Function Generator Calibration Circuit.

see that all segments are nonconducting: break points at "1000", linear term at zero, slopes greater than "100", and group switches I through IV not *assigned to the channel in use but set off that channel. Also, if* EA *amplifiers are used for calibration, throw the* EA *control switch to* RESET. *For this function, it would be desirable to use the linear term segment between x_4 and x_1; therefore a constant term should be inserted through the miscellaneous input* w. *If this constant was derived from the* LINEAR TERM *potentiometer, determine the exact setting by setting the calibrate* QUADRANT *selector to quadrant 3 or 4 and the* FUNCTION VALUE *potentiometer to f_0. Adjust the* LINEAR TERM *potentiometer until a null is obtained on the* VTVM. *Then set the* FUNCTION VARIABLE *to x_1, the* FUNCTION VALUE *to f_1, and the calibrate* QUADRANT *selector to quadrant 4. Adjust the* LINEAR TERM *until a null is obtained on the* VTVM. *To set the next line segment, from x_1 to x_2, set the* QUAD *switch for the first diode segment in quadrant 1, since a positive incremental slope is needed. With the* FUNCTION VARIABLE *at x_1 and the* FUNCTION VALUE *at f_1, adjust the* BREAK POINT *until a deflection of 0.5 to 1.0 division is observed on the* VTVM *when on the ".1" scale. To set the slope for this segment, turn the calibrate* QUADRANT *selector to quadrant 1, the* FUNCTION VARIABLE *to x_2, and the* FUNCTION VALUE *to f_2. Adjust the* SLOPE *potentiometer for the first diode segment until a null is obtained. With the quadrant switch of the next diode segment in quadrant 4 (negative incremental slope), adjust the* BREAK-POINT *potentiometer until a deflection is just observed. Then go to the next point, x_3, f_3, to set the proper slope. After completing calibration of the right half plane, repeat the process for the left half plane. The linear term as set for the line segment to x_1, f_1 extends to x_4, f_4. At this point, introduce a quadrant 2 slope in a manner similar to that described previously.*

Functions of Two Variables

An extension of the method of function generation for functions of one variable to functions of more than one variable is possible by making the break points a function, or functions, of a second variable, y. A family of curves may be completely described by the lateral shift of the y intercept with y, the shift of the break-point voltage with y, and the initial slope and slope increments. The lateral shift and the shift of the break points may be linear or nonlinear functions of y. The bias functions that govern the break-point shift are common to all diode segments for a large variety of curves, but in the most general cases are completely independent of each other. For this reason the high side of break-point potentiometers for all segments are brought out individually when the NORMAL-$f(y)$ switches on the function generator panels are put in the $f(y)$ position. For a systematic procedure of diode network synthesis and determination of lateral shift and break-point shift functions, reference should be made to the article, "An Electronic Circuit for the Generation of Functions of Several Variables" by H. F. Meissinger, which is presented in Appendix 4 in the *REAC Computer 400-2 Manual*, vol 1.

Partial Differential Equations

The most detailed presentation found to date on the use of analog computers for the solution of partial differential equations is in the chapter on "Miscellaneous Applications" of C. L. Johnson's text, *Analog Computer Techniques* (9).

PATCHING INSTRUCTIONS FOR AUXILIARY EQUIPMENT

The connector panel on the special-components rack serves exclusively to tie in ORNL amplifiers and special-components signals into the EA patch-board section marked AMPLIFIERS 21 through 40, corresponding respectively to ORNL amplifiers 1 through 20. Figure 36 illustrates the internal computer connections. *In cases when an ORNL amplifier is not in use and the input resistor terminals on the* EA *patch-board are being used for special-networks interconnection, tie the upper right pair of terminals together, putting the 1-megohm resistor directly across the tie lines, to prevent ORNL amplifier saturation due to lack of feedback.* As many as 12 amplifiers may be converted to integrators by patching into the four-terminal networks panel, either by capacitor feedback or by a "T" input network. Nine combination decade resistor–potentiometer, plug-in units are supplied on the panel above the capacitors.

Transport Lag Devices

Incorporation of transport lag or dead time in a computation usually requires the use of two amplifiers, one associated directly with the device and one to handle phase inversion and gain loss. The

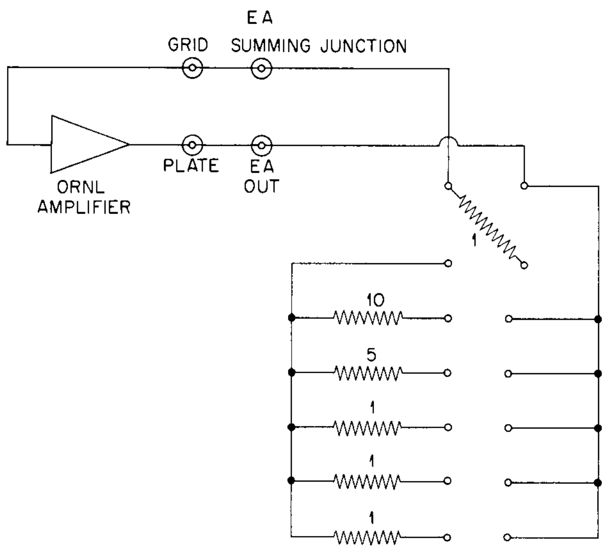

Fig. 36. Internal Computer Connections for ORNL Amplifier.

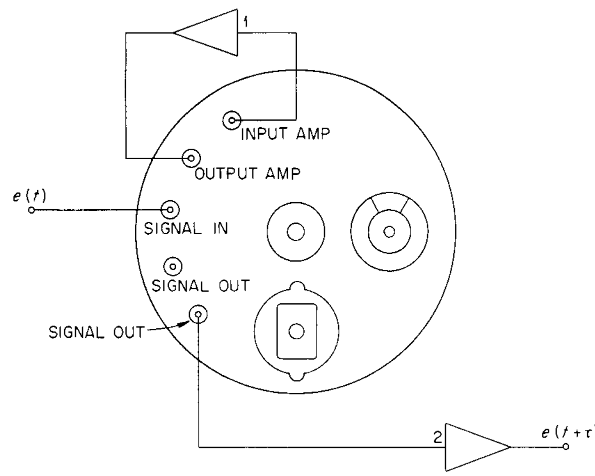

Fig. 37. Transport Lag Circuit Patching.

connections necessary for operation are shown in Fig. 37. The motor drive is so arranged that the Bodine motors can be easily exchanged and a 25-, 10-, or 3-rpm unit selected to permit delays up to 2.4, 6, or 20 sec, respectively. Since this unit is a data sampling device with 20 samples per revolution, resolution is limited to less than 0.12, 0.30, or 1 sec per step, respectively. Two concentric shafts appearing on the front rotate clockwise when the unit is in operation. The angular separation between the colored arrows is adjustable in discrete steps indicated on the panel. The black arrow is the "read" arm and is a reference position, and the red arrow is the "write" arm. *Position the "write" arm by pulling out slightly to extricate the index; rotate it to the desired angular position to produce the desired delay and then reseat. Set the output potentiometer near mid-scale; however, to get the optimum linearity, compensate for its loading by using an oscilloscope and the sawtooth generator located at the top of the rack.*

To calibrate the transport lag output potentiometer, replace the six-terminal Jones shorting plug with the TRANSPORT LAG CHECK *cable, connect the* INPUT *and* OUTPUT *cables to the transport lag* SIGNAL *input and output, connect the* OSCILLOGRAPH *cable to the* x *input and* y *input terminals of the oscilloscope, and ground the shield strap. Adjust the oscilloscope to display a stair-step array in which one gap is probably much larger or smaller than the rest. Adjust the output potentiometer to remedy this irregularity. Replace the shorting plug and remove the generator input and output before using in the computer.*

The transport lag device has a 14-msec writing time constant. At the slowest reading speed the voltage loss during reading is less than 5 mv, and the switching spill-over due to the filtering capacitor, *C*, is less than 25 mv.

Nuclear Reactor Kinetics Panel

This panel, lowest in the special-equipment rack, is described in detail in "Analysis of a Reactor."

In the preceding sections only operations employing a single or very limited number of operational amplifiers have been described. Simulation of a reactor system requires many such operations and, accordingly, a network of operational amplifiers. Each amplifier operates as described previously and contributes to the performance of the entire network.

Each new reactor has similarities and differences compared with those that preceded it. The concept of "an engineering tool for the development of reactor control equipment" implies a flexibility sufficient to follow changes in a particular reactor design and in the design of reactors in general. Flexibility depends upon the availability of a number of amplifiers and upon the ability to quickly modify the operations which each amplifier performs. Since no built-in system exists, the engineer who uses the simulator must determine the operations which suit his needs and must design the appropriate feedback networks.

A set of differential equations which describe the nuclear kinetics of one reactor may, with minor modifications of the coefficients, be used to describe the nuclear kinetics of another. A second-order differential equation which describes, in an idealized manner, the operation of a servo system in one reactor may be used to describe the operation of a servo system in another by minor modifications of the coefficients.

The principal differences between reactors are in the thermodynamic properties, in the manner in which the heat generated by nuclear processes is transformed to the load, and how the reactor is controlled by the load demand. To represent these properties, the circuits employed in the simulator will be basically different, and it is these circuits which the control engineer must design.

In the following section those feedback circuits which are permanently available in the computer will be described; in addition, a sample reactor will be described to illustrate the thermodynamic properties of a system. These circuits are not unique. Many others exist with which the same results may be achieved. The examples are intended only as an aid to acquaint the engineer with design principles.

The same set of equations describes the general functional behavior of both the simulator and the system being simulated. An exact determination of the kinetics of a reactor and load is not possible. First, many of the functions necessary for an exact solution are unknown prior to actual operation of the reactor; second, space and time variables are not generally suitable for analog solutions. Thus the system simulated differs from the actual reactor and is a simplified model. Mean values replace space variables. Transport lags are replaced by linear approximations or by special transport lag devices. The degree of correlation between the simulator results and the reactor kinetics depends upon the severity of the assumptions, and they must be analyzed with care in each system.

The design of a network of operational amplifiers may accompany or follow the formulation of a set of equations which describe its behavior. In the first case a circuit is "formulated" in the same way that an equation is "formulated." It is this method that will be employed in the following examples, except for the nuclear kinetics. The second approach deals with the simulator as a computer for the solution of a set of equations.

ANALYSIS OF A SERVO SYSTEM

The following analysis of a servo system illustrates the method of formulating a circuit. As a simplified model, it is assumed that all inertia and mass in the system may be combined as a single unit. This mass, perhaps a control rod, moves under the influence of an accelerating force composed of three parts. First, an error in the position or velocity of this mass is amplified and applied as a force; second, a frictional force which is proportional to the velocity of the mass retards its motion; and third, a restoring force which is a function of its deviation from equilibrium may act upon the system. The total force acting upon the mass is the sum of these three components. The ratio of total force to mass equals acceleration so that

$$(1) \qquad a = \frac{F_t}{M} = \frac{F(E)}{M} - \frac{B_5 V}{M} - \frac{B_6}{M}(S - S_0) ,$$

where

a = acceleration of mass M,

F_t = total force acting upon mass M,

$F(E)$ = component of force derived from system error,

B_5 = coefficient of friction,

V = velocity of mass,

B_6 = stiffness coefficient,

S = position of mass,

S_0 = equilibrium position of mass.

As shown in Fig. 38, a circuit with three inputs, each associated with one of the terms on the right of Eq. 1, supplies a current to the input grid of amplifier 1 which is proportional to the total acceleration of the mass, M. The reciprocal of the coefficient in Eq. 1 equals the resistance of the input circuit, and the variables are equal to the input voltages.

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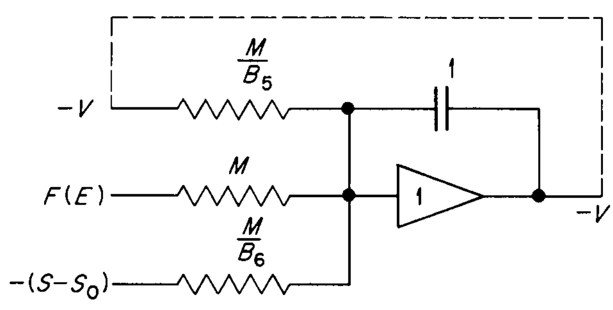

Fig. 38. Force Integrator.

Since

(2) $$V = \int a\, dt \quad ,$$

an integrator circuit, integrating the input current of the circuit of Fig. 38, produces an output proportional to velocity. Thus

(3) $$-V = -\int a\, dt \quad .$$

Due to the polarity reversal of the amplifier, the output is proportional to the negative of the velocity. This is just the value of voltage required for one of the inputs; so, as shown by the dotted line in Fig. 38, the output may be coupled to the input.

A current of one unit flows in a resistance of one unit when 1 v is applied across it. Further, a unit current results in a circuit containing a unit capacity when a rate of change of 1 v/sec is applied across it. The practical units of current, resistance, and capacity are not the most convenient to use in analog circuits. Currents encountered are normally measured in microamperes. If 1 μa is defined as the unit of current, it follows that 1 megohm is the unit of resistance and 1 μf is the unit of capacity. This establishes the ohmic and faradic values appearing in Fig. 38 and in subsequent circuits to be described. It is not necessary that the microampere be chosen as the unit of current. Many cases occur in which this unit of current leads to impractical resistance and capacity values. In such cases some other unit of current may be defined and the unit of capacity and resistance scaled accordingly. Furthermore, the unit of voltage used above was 1 v, and it was tacitly implied that 1 v was in 1:1 correspondence with the function it represents. Since this frequently does not occur, it may become necessary to redefine the unit of voltage to agree with the functions represented. This modifies the unit of resistance and capacity, as does the current. The units employed in one amplifier for current, resistance, and capacity do not affect the choice of units in the other amplifiers. The unit of voltage, however, does affect the other amplifiers since it is via this quantity that the amplifiers are coupled.

Since the output of the circuit of Fig. 38 equals the velocity of the mass M, integration of this output yields the displacement, S. Figure 39 shows the addition of an integrator to the circuit of Fig. 38.

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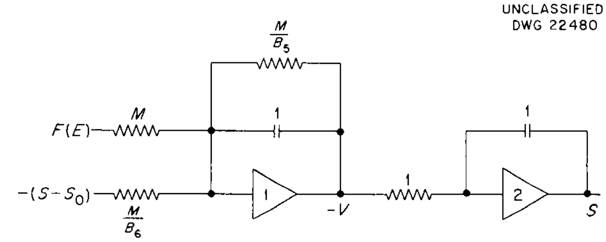


Fig. 39. Force and Velocity Integrators.

In Fig. 40, an adder follows the integrator for the purpose of obtaining the difference between S and some equilibrium value of position, S_0. A negative S_0 causes the difference between S and S_0 to appear at the output of amplifier 3. This difference equals the third input to amplifier 1 and may be so connected, as shown in Fig. 40 by a dotted line. Amplifier 4 is a differentiator circuit added to obtain the acceleration of the body being simulated.

Although no equation of motion was employed directly in the description of the circuit in the preceding paragraphs, it follows from Eqs. 1 and 2 that

$$(4) \qquad M\frac{d^2S}{dt^2} + B_5\frac{dS}{dt} + B_6(S - S_0) = F(E) \ ,$$

and this is the equation which relates the model and the simulator. The same result could be obtained by starting with the model formulating Eq. 4 and designing an analog circuit for its solution. This might prove useful where an artificial time base is to be employed.

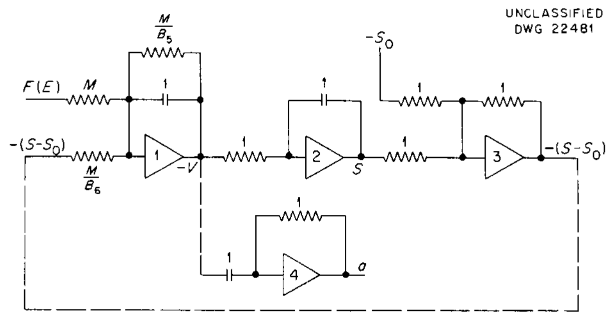


Fig. 40. Servo Simulator.

Figure 41 shows the feedback circuits for an idealized servo system which the simulator provides. The circuit is the same as that shown in Fig. 40, with the addition of means to vary the effective values of each of the parameters.

Figure 42 shows three typical solutions obtained from the simulator using the circuit of Fig. 41. No $F(E)$ was used, and the response of the system is to a step change at zero time of the value of S_0. This is a solution of

$$(5) \qquad \frac{d^2S}{dt^2} + B_5\frac{dS}{dt} + B_6(S - S_0) = 0 \ .$$

Solution (a) in Fig. 42 represents the case where B_5 equals zero and, as a result, the system is undamped. Solutions (b) and (c) represent nonzero values of B_5. In (b) the system is underdamped and in (c) is critically damped. The period of oscillation depends upon the coefficients of Eq. 5 and, in the circuit, may be varied over wide limits. The upper limit of simulator frequency response exceeds that possible with the present techniques of servo control.

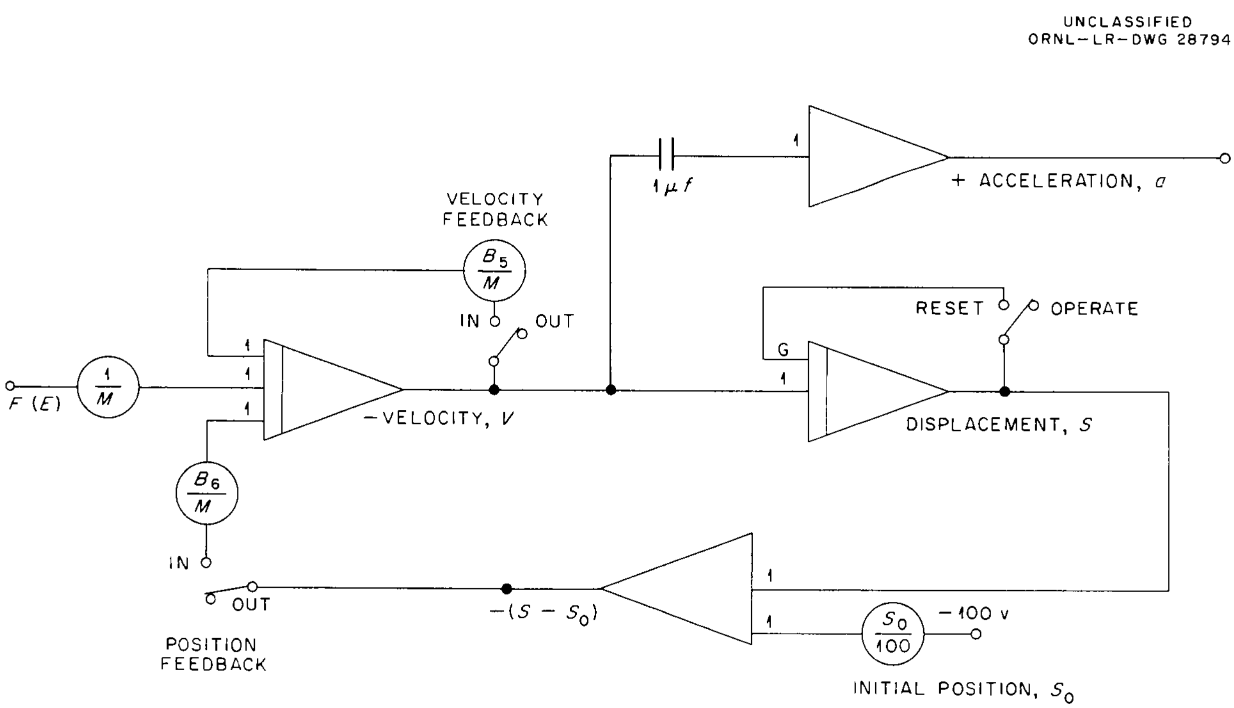


Fig. 41. Schematic Diagram of Servo Simulator.

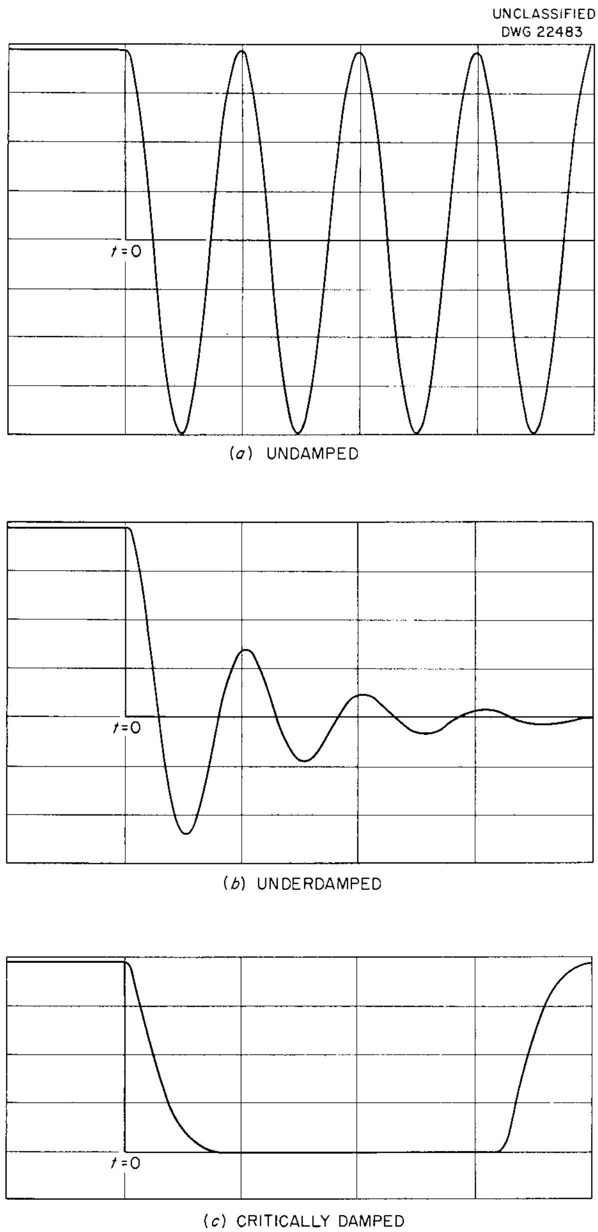


Fig. 42. Typical Responses of Servo Simulator.

ANALYSIS OF A REACTOR

The nuclear kinetic equations commonly employed to describe a reactor may be written as follows:

$$(6)\quad l^* \frac{dP}{dt} = (1 - \beta)\, kP - P + \sum_{i=1}^{i=6} \lambda_i X_i + S \ ,$$

and

$$(7)\quad \frac{dX_i}{dt} = -\lambda_i X_i + \beta_i \alpha_i kP \ ,$$

where

l^* = mean lifetime of prompt neutrons from generation to absorption,

P = neutron concentration,

k = effective multiplication factor,

X_i = ith delayed-neutron emitter concentration times l^*,

λ_i = decay rate of ith delayed-neutron emitter,

S = neutron source in neutrons per l^* seconds,

β_i = fraction of total neutron production which is delayed by the ith delay group,

β = total fraction of neutron production which is delayed by all delay groups,

α_i = factor employed to account for the reduction of the effective concentration of delayed neutrons due to fuel circulation.

Although the variables employed in Eqs. 6 and 7 were defined in terms of neutron concentration, little error results from the assumption that the power produced by the reactor is directly proportional to the neutron level. Thus P may be considered as the power production in Btu/sec, and X_i would be the potential power stored in the delayed-neutron emitter. The definition of units and terms depends upon the purpose of the reactor. For research reactors the neutron flux is important and the terms would be as defined. For power-producing reactors, units of heat production are more conveniently employed. Since power reactors will be described in the following paragraphs, the units employed will be power units.

Since, for the circuit of Fig. 43,

$$(8)\quad i_1 + i_2 = 0$$

and

$$(9)\quad i_2 = l^* \frac{dP}{dt} \ ,$$

then it follows that

$$(10)\quad i_1 = -l^* \frac{dP}{dt} \ .$$

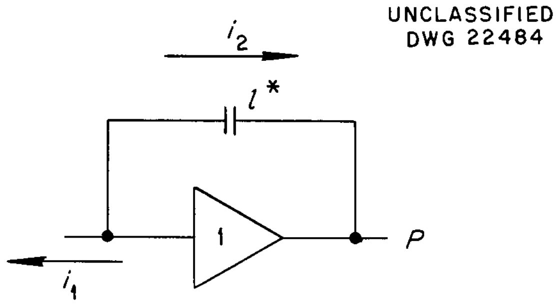


Fig. 43. Power Integrator Feedback Circuit.

Current i_1 may be produced by multiple inputs to amplifier 1. Each input equals the negative of one term on the right side of Eq. 6. Three input currents may be produced directly and are shown in Fig. 44. Since P is both an input and an output, the dotted connection may be used to couple them.

The currents proportional to the delayed emitters require further consideration. It is required that currents be produced at the input to amplifier 1 which equal $\Sigma\, \lambda_i X_i$. This involves six inputs, since six classes of delayed emitters are utilized in the simulator. The current, or contribution, from each group is described by Eq. 7. A steady-state solution of these equations yields

$$\lambda_i X_{i0} = \beta_i \alpha_i k P_0 \ , \tag{11}$$

which terms are proportional to kP_0. During transient conditions, however, each current differs from that of Eq. 11 and is delayed by a time constant equal to $1/\lambda_i$. The source term, or driving function, in Eq. 7 is proportional to kP.

The input circuit shown in Fig. 45 was described previously. The input current, here denoted by D_i, may be described by the following:

$$C_i \frac{d(2R_i D_i)}{dt} + D_i = \frac{e_{11} - 2R_i D_i}{2R_i} \ , \tag{12}$$

or, by rearranging,

$$\dot{D}_i = -\frac{1}{R_i C_i} D_i + \frac{e_{11}}{4R_i^2 C_i} \ . \tag{13}$$

From Fig. 45

$$e_{11} = \gamma_i kP \ , \tag{14}$$

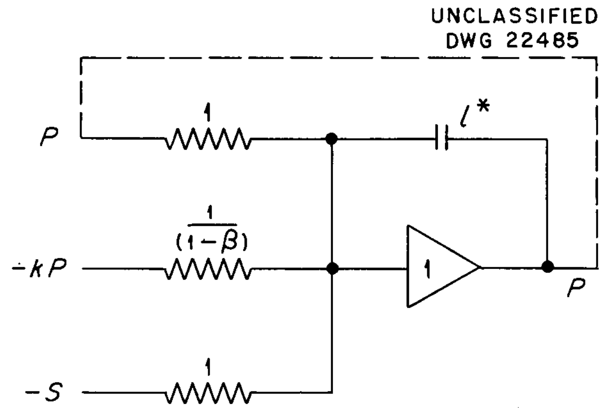


Fig. 44. Power Integrator with Prompt Inputs.

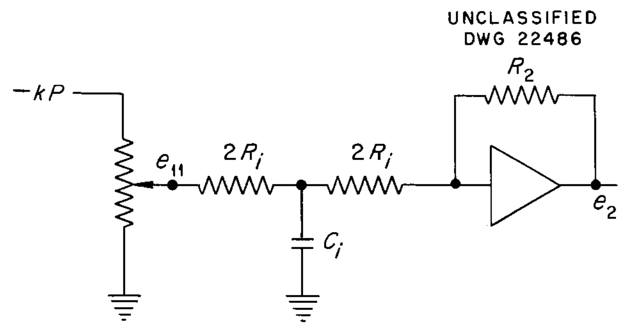


Fig. 45. First-Order Delay in Input Circuit.

where γ_i = gain of input potentiometer. Then Eq. 13 becomes

$$\dot{D}_i = -\frac{1}{R_i C_i} D_i + \frac{\gamma_i}{4R_i^2 C_i} kP \ . \tag{15}$$

The necessary input current to amplifier 1 equals $\lambda_i X_i$. Therefore, let

$$D_i = \lambda_i X_i \ . \tag{16}$$

Substituting this in Eq. 7 yields

$$\dot{D}_i = -\lambda_i D_i + \beta_i \alpha_i \lambda_i kP \ , \tag{17}$$

which equals Eq. 15 if

$$\lambda_i = \frac{1}{R_i C_i} \tag{18}$$

and

$$\beta_i \alpha_i = \frac{\gamma_i}{4R_i} \ . \tag{19}$$

Figure 46 shows the circuit of Fig. 44 after six networks were added to furnish the necessary delayed contribution. Amplifier 2 performs as a scalar to produce $-kP$, and closes the loop. The scalar will be described in more detail near the conclusion of this section.

A schematic diagram, Fig. 47, shows the feedback network of amplifier 1 as employed in the simulator. This functions in the manner described in the preceding paragraphs and provides, in addition to means of varying appropriate coefficients, a way of setting the initial level of each group of delayed emitters. The unit of current employed is 2 μa, so that the unit of resistance becomes 500,000 ohms and the unit of capacity is 2 μf.

Table 7 lists the controls which are located on the front panel and the range of values obtainable with each. Variable components at appropriate points permit modification of all coefficients except the λ_i values. The λ_i values are associated with the decay rates of nuclear fission products and are the same in all reactors.

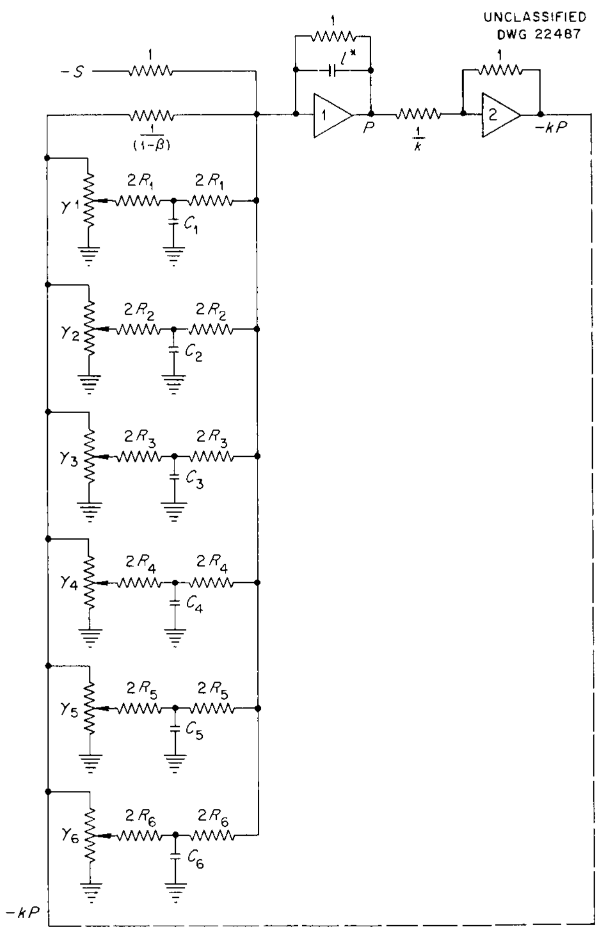


Fig. 46. Nuclear Kinetic Simulator.

In determining the γ_i settings, values of α_i are obtained from Fig. 48. The parameter characterizing this curve family is the ratio of the fuel passage time outside the critical lattice, T_2, divided by the fuel passage time through the critical lattice, T_1.

Figure 49 illustrates in a more complete manner the reactor nuclear loop. The feedback circuits associated with amplifier 1 of Fig. 46 are represented as a single block. The circuit shows the means of generating $-kP$ from P; k is the effective multiplication factor and is variable. It equals the effective scalar product of P when comparing the output of amplifier 2 with that of amplifier 1. Six inputs furnish current to amplifier 2. Each current is proportional to P, and the output of amplifier 2 is proportional to the sum of all input currents. Consider, therefore, each input separately.

The upper three inputs furnish currents which are

(20) $$i_{11} = 0.95P \quad ,$$

(21) $$i_{12} = 0.10P\beta_1 \quad ,$$

(22) $$i_{13} = 0.05P\beta_2 \, \mathbb{1} \quad .$$

The currents i_{12} and i_{13} may be controlled manually by potentiometers. The "unit function" symbol in Eq. 22 indicates that closure of S1 permits application of i_{13} as a "step."

The attenuation of P through an external potentiometer is β_5. This potentiometer, when employed, may be either manually or servo controlled as desired. The current is, therefore,

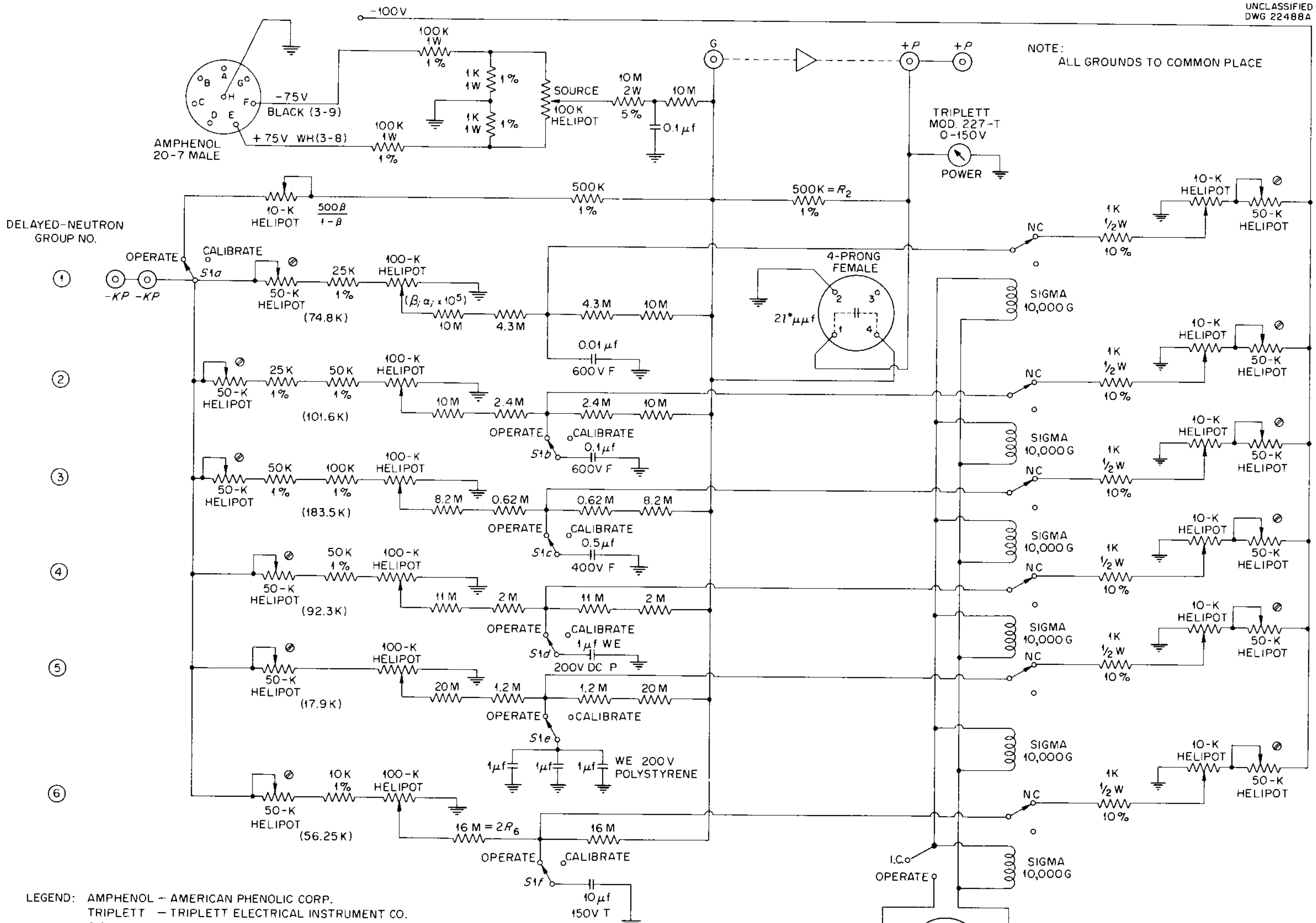
(23) $$i_{14} = 0.1\beta_5 \beta_3 P \quad .$$

The input, θ, to the multiplier includes all other factors which affect the value of k:

(24) $$i_{15} = -\frac{P\theta\beta_3}{1000} \quad .$$

The sum of all input currents is, therefore,

(25) $$i_{total} = 0.95P + 0.10P\beta_1 + 0.05P\beta_2 \, \mathbb{1} + $$
$$+ 0.10P\beta_3\beta_5 - 0.001P\theta\beta_3 \quad .$$



LEGEND:
AMPHENOL – AMERICAN PHENOLIC CORP.
TRIPLETT – TRIPLETT ELECTRICAL INSTRUMENT CO.
SIGMA – SIGMA INSTRUMENTS INC.
F – FAST AND CO.
WE – WESTINGHOUSE ELECTRIC CORP.
T – TOBE DEUTSCHMANN CORP.
P – POLYSTYRENE

Fig. 47. Schematic Diagram of Feedback Circuits for Nuclear Kinetic Equations.

Table 7. Panel Controls

Control Number	Coefficient	Means of Adjustment	Range
1–6	$\beta_i a_i$	Potentiometer	0 – 0.01
7	l^*	Plug in condenser	10^{-5} sec
8	S	Potentiometer	Uncalibrated
9–14	$\lambda_i X_{i0}$ (initial condition)	Potentiometer	0 – $0.01P_0$

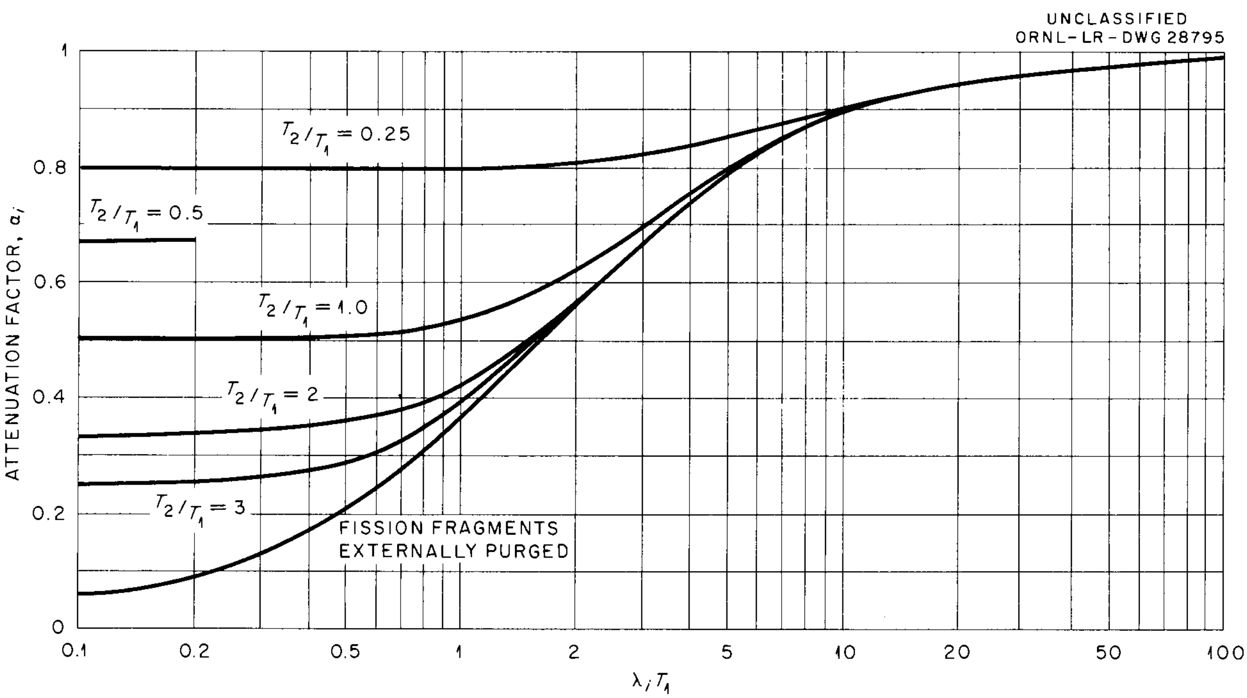


Fig. 48. Delayed-Neutron Attenuation Factor for Circulating-Fuel Systems.

Factoring out P from the terms on the right side of Eq. 25 yields k:

$$(26)\quad k = 0.95 + 0.10\beta_1 + 0.05\beta_2 7 + $$
$$+ 0.10\beta_3\beta_5 - 0.001\theta\beta_3 \ .$$

ANALYSIS OF A THERMAL SYSTEM

The energy released by the nuclear processes of a reactor appears as heat. This heat energy must be extracted from the reactor and transformed into the form of energy required by the load. Extraction of heat from the reactor may be accomplished by circulating the nuclear fuel through the reactor and then through an external heat exchanger; passing a coolant over the fuel plates within a reactor and then to an external heat exchanger; boiling of a coolant within the reactor and using the heat of vaporization; or by a combination of these methods.

The exchange of heat between point A and point B of a stationary system requires a difference of temperature between A and B, provided that no change of state of the material at either point is involved. Heat flows from the higher temperature point to the lower one. The rate of flow of heat, or power, depends upon the thermal conductivity and the difference of temperature between the points. An interface modifies the thermal conductivity between two points. Within and between

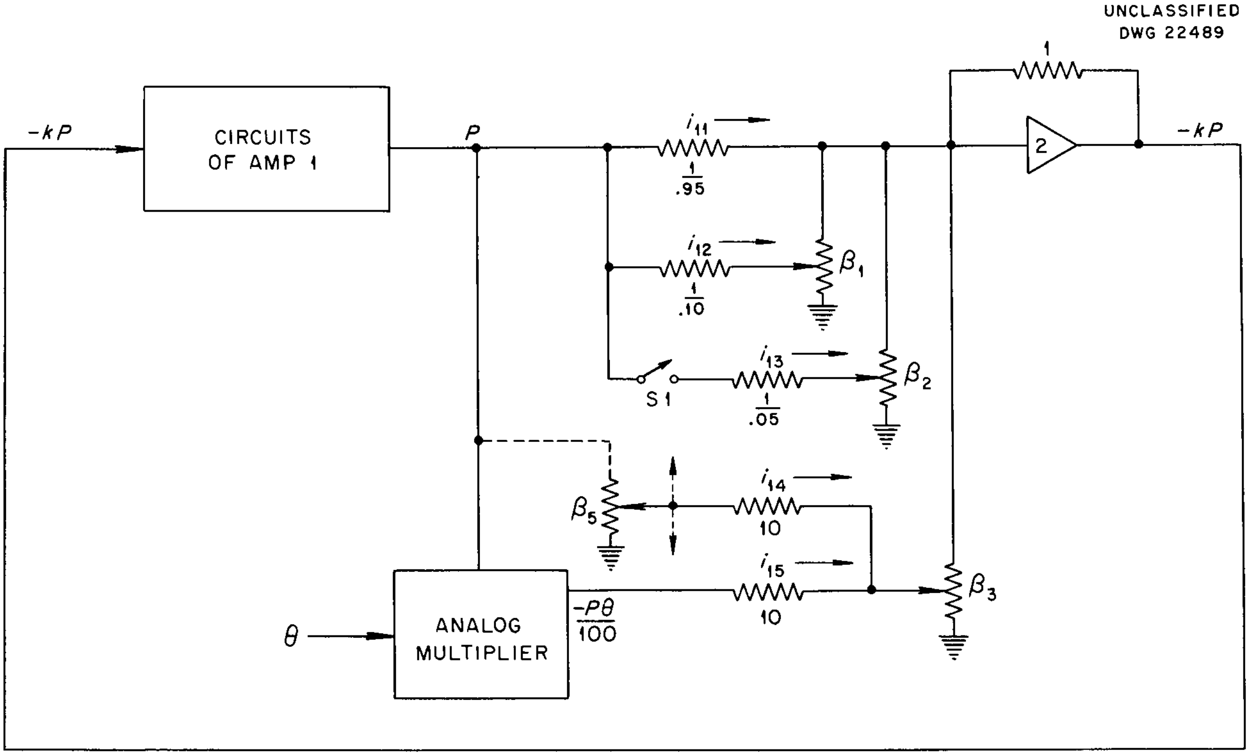


Fig. 49. Nuclear Kinetic Simulator Circuit with Means of Controlling k.

the mediums employed for heat conduction or heat transfer, temperature variations exist. A complete solution of the thermodynamic properties, therefore, involves both spatial and time variations. This solution is not suited for electronic analog computation. The mean temperature of each medium is employed in the simulator, and the average differences in temperature between two mediums are used. Where heat energy is transferred by a circulating fluid, linear approximations replace the time difference and spatial equations.

As a typical reactor, consider one in which the fuel is stationary and a coolant circulates past the fuel, carrying the heat to an external heat exchanger where it is cooled by a gas. The inlet temperature of the gas is fixed by the load and is not affected by the reactor. Figure 50 diagrammatically illustrates such a system.

Consider, first, the fuel. If no coolant is supplied, the temperature of the fuel, θ_f, at design point power, rises at a mean rate of $\dot{\theta}_f$ degrees per second. This is obtained by dividing the design

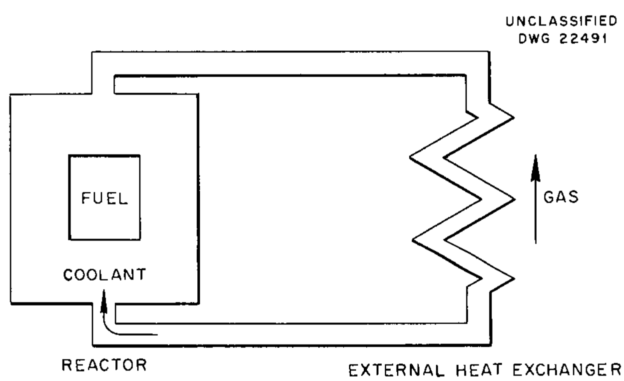


Fig. 50. Typical Nuclear Reactor.

point power, P_0, by the total heat capacity, C_f, of the fuel. Thus,

$$\dot{\theta}_f = \frac{P_0}{C_f} \quad . \tag{27}$$

By an integrator, such as that shown in Fig. 51, P_0 determines θ_f according to Eq. 27. The negative sign is due to the polarity reversal of amplifier 1.

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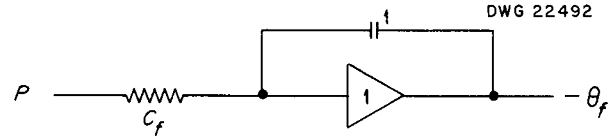

Fig. 51. Heat Integrator.

Without cooling, however, the fuel would melt. Heat is removed from the fuel at a rate proportional to the difference in temperature between the fuel and the coolant and is a function of the geometry of the physical system. At design point, however, Z_1 degrees exist between the two means, and this, by definition, is the difference required to transfer heat from the fuel to the coolant at the rate at which it is produced. Coolant flow removes heat from the reactor at this rate so that at design point the coolant mean temperature, θ_c, is constant.

Figure 52 shows the circuit of Fig. 51, with the addition of two inputs whose sum is proportional to the difference between the fuel and coolant mean temperatures. The dotted line denotes the connection of the output of amplifier 1 to one of these two inputs.

Since the mean fuel temperature is constant at design point power, P_0, the sum of all input currents to amplifier 1 must be zero at this power. Therefore

(28) $$\frac{P_0}{C_f} + \frac{\theta_c}{R} - \frac{\theta_f}{R} = 0 \ ,$$

but

(29) $$\theta_f - \theta_c = Z_1$$

so that

(30) $$\frac{P_0}{C_f} - \frac{Z_1}{R} = 0 \ ;$$

therefore

(31) $$R = \frac{Z_1 C_f}{P_0} \ .$$

The difference between the power introduced and the power removed from the coolant, within the reactor, determines the time behavior of the mean coolant temperature. Power is removed by the circulation of a coolant. The net heat flow causes the coolant to leave the reactor at a higher temperature than its entrance temperature. The difference between the inlet temperature, θ_{1c}, and the outlet temperature, θ_{2c}, is proportional to the power extracted, P_X. In general

(32) $$P_X = VC_c\,(\theta_{2c} - \theta_{1c}) \ ,$$

where V = volume flow of coolant per second and C_c = heat capacity of coolant per unit volume.

Power insertion into the coolant is proportional to the difference between the fuel and the coolant mean temperatures. The input power raises the coolant temperature at a rate determined by the total heat capacity of coolant within the reactor. Proceeding as with the fuel, let the integrator of Fig. 53 be used to generate an output proportional to θ_c. The upper two inputs are proportional to the power inserted, and the lower two are proportional to the power extracted. In the absence of coolant flow all the power input will be utilized to raise the coolant temperature. The rate of rise, θ_{10}, at design point may be computed as the ratio of design point power to the total coolant heat capacity within the reactor. Since Z_1 is the design point difference between θ_c and θ_f, it

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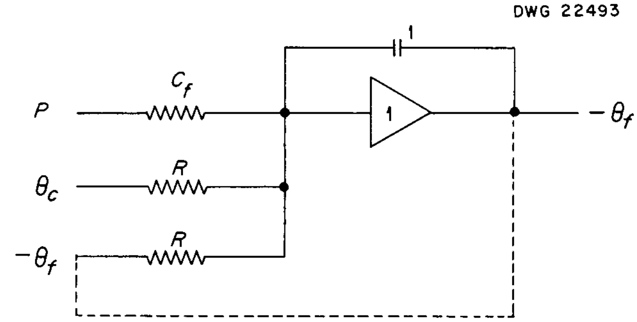

Fig. 52. Fuel Temperature Integrator with Heat Extraction.

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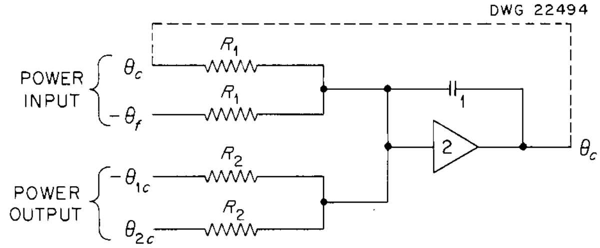

Fig. 53. Coolant Mean Temperature Integrator.

follows that

(33) $$\dot{\theta}_{10} = \frac{Z_1}{R_1} = \frac{P_0}{V_{rc} C_c} ,$$

where V_{rc} = volume of coolant within the reactor.

When coolant flow exists at design point, the temperature of each elementary volume of the coolant rises as it passes through the reactor. The mean rate of rise, during the time spent in the reactor, equals the rate given in Eq. 33. The total rise in temperature, Z_2, equals the product of the time spent in the reactor and the mean rate of rise in temperature. Thus

(34) $$Z_2 = \dot{\theta}_{10} \tau_1 = \frac{P_0 \tau_1}{V_{rc} C_c} ,$$

where τ_1 = mean transit time of coolant in the reactor. It follows from Eqs. 33 and 34 that

(35) $$R_1 = \frac{\tau_1 Z_1}{Z_2} .$$

At design point, with coolant flow, θ_c is constant. From the circuit of Fig. 53 it follows that, for θ_c to be constant,

(36) $$\frac{Z_1}{R_1} - \frac{Z_2}{R_2} = 0 .$$

Combining Eqs. 35 and 36 yields

(37) $$R_2 = \tau_1 .$$

The external heat exchanger determines the inlet coolant temperature to the reactor. During the passage of the coolant through the reactor, heat is added and the temperature is raised. The outlet temperature of the coolant leaving the reactor depends upon the inlet temperature at a time preceding the outlet time by the transit time of the coolant through the reactor. Furthermore, the heat added during its passage may vary in level so that the total heat depends upon a time integral of the heat transferred over the transit time. Linear approximations may be used to represent each of these effects.

The transport lag of the coolant is approximated by cascading two linear first-order delays. This may be mathematically expressed as

(38) $$(\tau_1/2)\dot{\theta}_1 + \theta_1 = \theta_{1c} ,$$

and

(39) $$(\tau_1/2)\dot{\theta}_{2c} + \theta_{2c} = \theta_1 .$$

As was explained in "Linear Operations," a solution of Eqs. 38 and 39 may be obtained by the circuit of Fig. 54. The upper two inputs in the figure represent the heat added during the passage and are delayed by the feedback network only. The delay equals $\tau_1/2$.

Since Z_2 is the design point rise of temperature of the coolant as it passes through the reactor and since Z_1 is the design point difference of temperature between the fuel and the coolant, it follows that, in Fig. 54,

(40) $$\frac{Z_1}{R} = \frac{Z_2}{\tau_1/2}$$

or that

(41) $$R = \frac{\tau_1 Z_1}{2Z_2} .$$

The external loop is simulated, step by step, in a manner similar to that just described for the reactor. Design point conditions of temperature and flow rates determine the resistance and capacity values of the input and feedback networks. Completion of the individual amplifier circuits permits each to be connected, as denoted by like symbols, and the resulting simulator network is shown in Fig. 55. Two inputs drive this network. These are the nuclear power developed in the fuel of the reactor and the gas inlet temperature of the external heat exchanger. The power may be obtained from the output of amplifier 1 of the nuclear kinetic simulator described in "Analysis of a Reactor."

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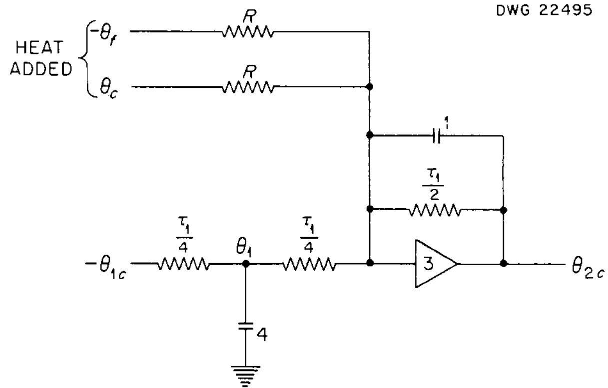
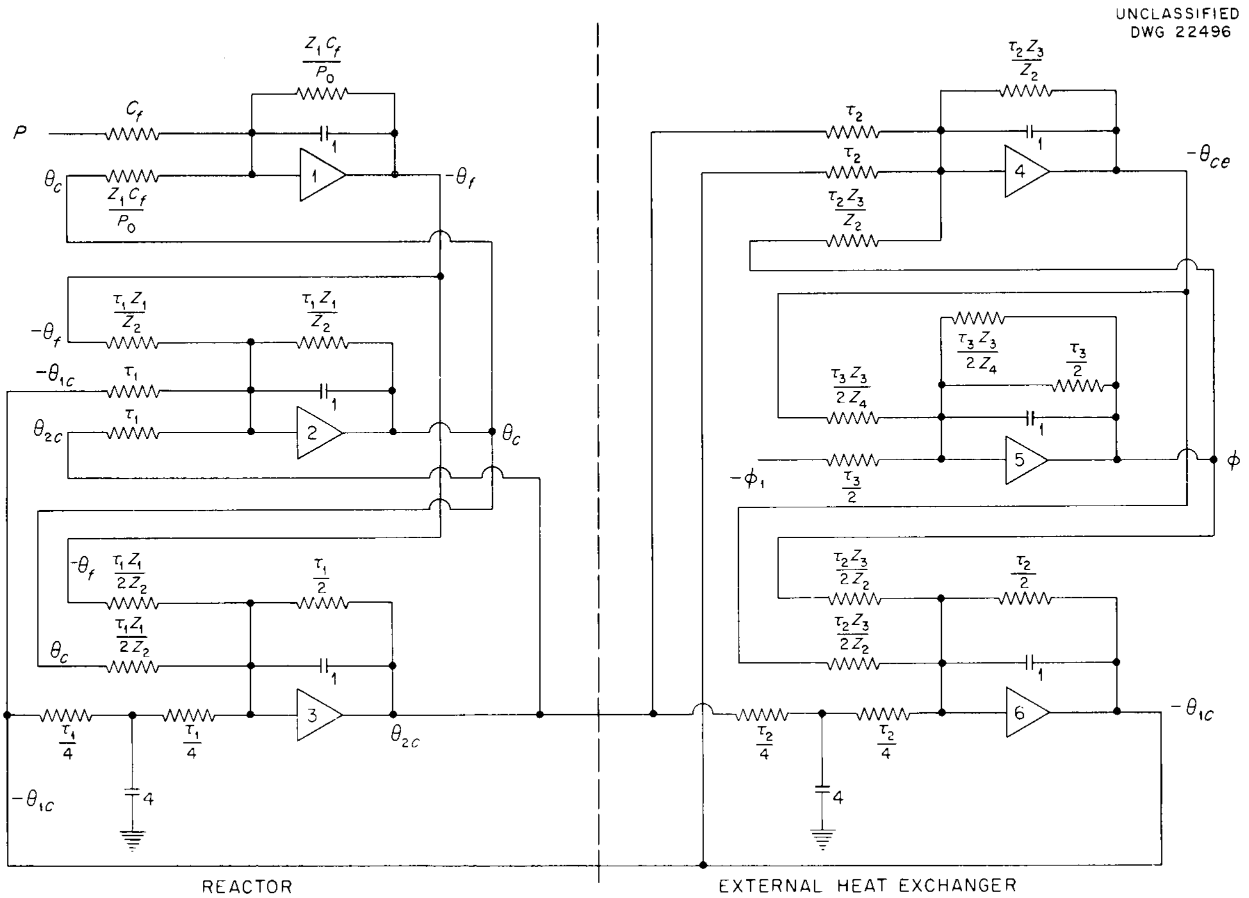


Fig. 54. Coolant Outlet Temperature Integrator.



P = nuclear power level

C_f = total heat capacity of fuel

θ_f = fuel mean temperature

θ_c = coolant mean temperature in reactor

Z_1 = design point difference in mean temperatures of fuel and coolant in reactor

Z_2 = design point difference between outlet and inlet coolant temperatures; this equals the rise in the reactor and drop in the external heat exchanger

τ_1 = mean transit time of coolant through reactor

θ_{1c} = coolant inlet temperature to reactor, also coolant outlet temperature from heat exchanger

θ_{2c} = coolant outlet temperature from reactor, also inlet coolant temperature to heat exchanger

τ_2 = mean coolant transit time through heat exchanger

τ_3 = mean gas transit time through heat exchanger

Z_3 = difference between coolant and gas mean temperature at design point

Z_4 = design point difference between gas mean and inlet temperatures

P_0 = design power level

θ_{ce} = coolant mean temperature in exchanger

ϕ = gas mean temperature in exchanger

ϕ_1 = gas inlet temperature to exchanger

Fig. 55. Reactor Thermodynamic Simulator.

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