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Absorption Formalism for Multiparticle Production and the Inclusive Transverse Distribution*

by

Charles B. Chiu and Kuo-hsiang Wang

Center for Particle Theory
The University of Texas, Austin
Austin, Texas 78712

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ABSTRACT

We review and further motivate the need of absorption in simple multiperipheral-like models and present a general formalism for multiparticle production with absorption. With this formalism, we reproduce the well-known unitarity relation and derive new expressions for inclusive momentum distributions. We demonstrate that the parameter-free predictions based on our earlier solution to the elastic data reproduce the gross feature of the inclusive p_T-data of high energy pp collisions.

1. Introduction

This is a sequel to an earlier work[1] (hereafter referred to as I) on the absorbed multiperipheral-like (MP-like) model. The MP-like models are those models of multiparticle production which have the following three features:

(a) particles in the final state have sharp transverse momentum cutoff;

(b) the multiplicity distribution is a Poisson (or Poisson-like) distribution with a logarithmic increase in s for the average particle multiplicity; and

(c) the inelastic production cross section increases with a power law in s, s^c.

Theoretical models such as ABFST model,[2] multi-Regge model and their variations,[3] the uncorrelated jet model,[4] and the independent emission model[5] have all (or most) of these features. The data are compatible with these properties except perhaps (c), since $c>0$ implied by the rising total cross section would lead to the violation of Froissart bound. We shall return to this point below.

In connection with property (a), the sharp p_T-cutoff, there are mainly two proposals. On the one hand, among the various MP-models, this cutoff is achieved by having factorizable cutoff functions for the momentum-transfer dependence of the 2 to n+2 amplitude. In particular,

$$T_{n+2,2} \propto \prod_{i=1}^{n+1} f(t_i),$$

where t_i is the momentum-transfer-square of the ith rung along the multiperipheral chain and the $f(t_i)$ is the corresponding cutoff function. On the other hand, in the uncorrelated jet model or the independent emission model, particle emission is assumed to be essentially uncorrelated except for the constraints due to conservation laws such as the conservation of four momentum and the conservation of charge. In this class of models, the p_T-cutoff is built-in directly. Typically, one has

$$T_{n+2,2} \propto \prod_{i=0}^{n+1} f(p_{iT}) \quad .$$

In I, the first parameterization is referred to as the MP case, and the second is referred to as the IE case.

The need of absorption for the MP-like amplitude is based on phenomenological considerations and some intuitive arguments. There are essentially three points.

(1) If one were to assume that the observed rise of pp total cross section in the ISR region is a manifestation of the asymptotic behavior of the MP-like model, as mentioned it would imply $c>0$, in violation of Froissart bound. However, with the absorption effect included, the positive power behavior will be suppressed to a $\ln s$ or $\ln s^2$ behavior.[6]

(2) In I, it is demonstrated that both the IE- and MP- parameterizations for cutoff function are consistent with the elastic data. However, for the MP case, it is well known one cannot simultaneously explain the slope of the diffractive peak and inclusive p_T-distribution.[7,8,9] On the other hand, we shall show that for the IE case in the presence of absorption this simultaneous description is possible. It turns out that without absorption for the IE case, the slope parameter of the diffractive peak would essentially be independent of energy. Absorption effect together with $c>0$ is needed to provide an energy dependence for this parameter to agree with the ISR data.

(3) In section 2, we will argue that the inclusion of absorption is one of the key steps to systematically account for the non-productive type interactions during the process of multiparticle production.

Our aim in the present paper is two-fold. First, we review and motivate the need of absorption. In a manner similar to Henyey's approach,[8] we present a general formalism for multiparticle production with absorption. Secondly we apply this formalism to derive formulae quoted in I, and present the absorbed inclusive distribution predicted by the IE solution of I. Our plan of the remaining paper is as follows: In section 2, motivations for the introduction of the absorption mechanism are discussed. In section 3, we present a general formalism of absorption

models for multiparticle production which partly makes use of a helicity state formalism developed earlier by one of us.[10] In the context of the absorption models, the unitarity relation for the elastic amplitude and general formulae for inclusive momentum distributions are derived. In section 4, expressions for the inclusive transverse momentum distribution with the transverse momentum cutoff function of I are derived. Finally, in the same section, we also demonstrate that the parameter-free predictions of the pion- and the nucleon- inclusive p_T-distributions do indeed reproduce the gross features of the data.

2. Absorption--One of the nonproductive type interactions.

The application of absorption to a simple multiparticle-production amplitude is an old idea. It was first suggested by Caneschi[11] in the context of getting the sign of the pomeron-pomeron cut to be opposite to that obtained by the unitarity-rescattering correction. The importance of the inclusion of absorption effect was also recognized by Cheng and Wu in a different context.[12] Within their "QED model," this effect corresponds to including those "closed ladder" contributions in the multiparticle production amplitude. This is a crucial step, which eventually leads them to predict the indefinite rise of the total cross section and at the same time assures the Froissart bound. Subsequently, absorption effects were further elaborated in the context of the eikonal model based on perturbation theory[13] and expounded by Finkelstein and Zachariasen[6] and others in the context of absorbed MP models.[14] The basic idea amounts to that, analogous to the situation in two-body inelastic scattering, the physical multiparticle production amplitude should be described by the product of some simple unabsorbed amplitude and the elastic S-matrix S_{22}, or $S^{\frac{1}{2}}_{22}$, depending on respectively, whether both the initial and final state interactions or only the initial state interaction between the two nucleons are included.

We share the basic contention of these authors. In fact, we consider absorption to be one of the three basic nonproductive types of interactions in the multiparticle productions, which should all be taken into account. In particular, the high energy pion productions are described by the collective effects of the following two processes: a direct production process via some MP-like mechanism, and some remaining nonproductive-type processes. For the latter, there are those further interactions:

I. among these pions produced,
II. between pions and each of the nucleons, and
III. between the two nucleon-systems.

For interaction I, among the pions the interaction is expected to be most pronounced for those cases where the energies of subsystems of pions are low and the subsystems have the appropriate quantum numbers, so that meson-resonances can be formed. To take into account this effect, many authors have already suggested, to alter the original proposal, allowing meson resonances in addition to pions to be directly produced.[15,16] Similarly, the interaction II can also be approximately accounted for by allowing the presence of the nucleon resonances (or clusters). Some effects here have also been considered by various authors. For instance, the diffractive dissociation events could be identified as the type II events, in which pions tend to cluster with the nucleons forming a relatively low invariant mass system(s). Presumably, at higher energies, there could also be events with diffractive dissociation plus fireball productions, etc.[17] The type III interaction is related to those absorption ideas mentioned above. Up to this date, the details of absorption assumed still vary from author to author. For instance, in ref. 14, only the initial state interaction has been considered. On the other hand, it was suggested that various functions of S_{22} could be used for the absorption factor.[6] Below we discuss some kinematics, arguing in effect that *both the initial and final state interactions should be included*, and suggesting that the absorption for multiparticle production has the form of Eq. (4) below.

We take a geometric picture[18,19] for hadron collisions. For definiteness, let us first look at an n-pion production process, i.e., $pp \to pp + n\pi$. The two nucleons and the pions are labelled by o, n+1, and i=1,2,....,n respectively. In the c.m. frame, two protons with extended structure are passing through each other. Initially they are at some relative impact parameter b and c.m. longitudinal momentum q, c.m. energy squared s. After the passage, they emerge with c.m. longitudinal momentum p_{oL} and p_{n+1L}, their relative impact parameter $\underset{\sim}{b}' = \underset{\sim}{b}_0 - \underset{\sim}{b}_{n+1}$ and energy-square of the two nucleons, s'. High energy data show that on the average, p_{oL} and p_{n+1L} are not too different from q owing to the leading particle effect. For example at 102 GeV/c, the mean value

$\tilde{x}_N = |p_{oL}|/q = |p_{n+1L}|/q \sim 0.7$. (For definiteness, the zero-th particle is associated with $x > 0$, while the (n+1)-th is associated with $x < 0$.)

In the context of impact parameter formalism, denote the transverse coordinates of pions by b_i and the corresponding fraction of c.m. longitudinal momentum p_{iL} by $x_i = p_{iL}/q$, where $i = 1,\dots n$. After a 90 degree rotation around the positive z-axis, conservation of angular momentum leads to[8]

$$\underset{\sim}{b} = \sum_{i=1}^{n} x_i \underset{\sim}{b}_i + x_0 \underset{\sim}{b}_0 + x_{n+1} \underset{\sim}{b}_{n+1} \quad , \qquad (1)$$

where we have neglected the terms involving transverse momenta of pions and the two nucleons which are small compared to q. After averaging over the longitudinal distributions, this expression becomes

$$b' \equiv |\underset{\sim}{b}_0 - \underset{\sim}{b}_{n+1}| = |\underset{\sim}{b} - \sum_{i=1}^{n} \tilde{x}_\pi \underset{\sim}{b}_i|/\tilde{x}_N \quad . \qquad (2)$$

In the averaging process for pions, due to the random phases of b_i's, the signs of x_i's are absorbed into b_i's, $\widetilde{|x_i|} = \tilde{x}_\pi$. At high energies, $\tilde{x}_\pi$ is in general very small. For example, we have estimated[1] that $\tilde{x}_\pi \sim 0.08$ at 102 GeV/c. In the summation of Eq. (2), typical values of b_i's are bounded by the radius of the cutoff function which is smaller than that of b (see Eqs. (55) and (70)). The average pion multiplicity[20] is about 6.6 at 102 GeV/c. Note that $\tilde{x}_N \sim 0.7$, $\tilde{x}_\pi$ is small, and the phases of b_i's are random. These factors together with the bounded properties of b_i's imply that b' does not deviate substantially from b.

We assume that the phase shift due to the interaction between *the two nucleon-systems, in the presence of* production, is essentially the same as that for elastic scattering. In particular, for a given impact parameter b and energy s,

$$\tilde{S}^{(n)}_{22}(b',s') \approx \tilde{S}_{22}(b',s') \approx \tilde{S}_{22}(b,s) \ . \qquad (3)$$

For the approximation of the last step, we have used the experimental fact that the elastic S-matrix varies slowly as a function of energy and the fact that $b \approx b'$. This approximation is expected to be better in the intermediate stages of the production.

Taking into account the phase shift due to the interaction of the two nucleon-systems, we propose that the multiparticle production amplitude takes the form

$$\tilde{T}_{n+2,2}(b) = \tilde{T}^{B}_{n+2,2}(b)\ \tilde{S}_{22}(b), \qquad (4)$$

where $\tilde{T}^{B}_{n+2,2}(b)$ is the unabsorbed production amplitude to be specified later. In the arguments of $\tilde{T}_{n+2,2}(b)$ and $\tilde{T}^{B}_{n+2,2}(b)$, the energy dependence and detail specifications of the n+2 particles in the final state are suppressed. It turns out that the absorption effect is most relevant in the small b region. In this region, $\tilde{S}_{22}(b)$ varies slowly with b. This provides a further justification for our approximation in the last step of Eq. (3).

We stress that for the specific choice of Eq. (4), in addition to the assumption that the effect of the "remaining interaction" of the two nucleon-systems in the presence of production is the same as that in the absence of the production, we also relied on the following four features of the data: (1) the smallness of $\tilde{x}_\pi$, (2) the finite radius of the pion cutoff function, (3) the weak energy dependence of the elastic amplitude at high energies, and (4) the slowly-varying dependence for the elastic amplitude in the small b region. The absorption scheme discussed here is illustrated in Fig. 1a. All the vertical lines in Fig. 1a and all other figures in this paper are particles on mass shell. The scheme here is in reminiscence of the absorption models for two-body scattering[18,21] which are depicted in Fig. 1b. The absorption mechanism specified in Fig. 1a is also a feature of those models based on perturbation theory,[12,13] where typically the diffractive amplitude can be cast in the eikonal-model form (or the impact picture form). In these models, the unabsorbed multiparticle production amplitude is the s-channel iteration of the MP-like amplitude or its analog. The unabsorbed overlap function has the form $(e^{fs^c} - 1)$, where f is independent of s, to be compared to the present case, which has a power behavior, s^c for the corresponding function.

In the next section, we shall develop a general formalism applicable for any type of unabsorbed amplitude. Our formalism reproduces the well-known absorption form of the unitarity relations, Eq. (37) below, and it provides the framework for the calculation of the one-particle and multiparticle inclusive cross sections. The expressions for the one-particle inclusive cross section are detailed in Eqs. (48), (49), and (50). The reader who is not interested in the detail derivations could pick up a few defining formulae and the key results mentioned above, and move on to Sec. 4.

3. General Formalism

The schematic diagram of Fig. 1a is either in the impact parameter space or in the momentum space. It is advantageous, as we shall do, to introduce production amplitudes in the momentum space, since for this case formalisms for multiparticle production have been well investigated. We shall formulate the absorption model of multiparticle production in the helicity state formalism.[10,22] For simplicity, we shall deal with scalar nucleons and other scalar particles. It is easy to generalize the treatment here to cases of particles with spin.

As we shall see later, according to Eq. (4), the absorbed amplitude $T_{n+2,2}(p_i;p_a,p_b)$ for the production process, $p_a + p_b \to p_0 + \dots p_{n+1}$, is expressed in terms of the unabsorbed amplitude $T^B_{n+2,2}$ and the elastic S-matrix element S_{22}:

$$T_{n+2,2}(p_i;p_a,p_b) = \int d\Phi_2^{ab} T^B_{n+2,2}(p_i;k_a,k_b) S_{22}(k_a,k_b;p_a,p_b), \quad (5)$$

with

$$S_{22}(k_a,k_b;p_a,p_b)\,\delta^4(k_a+k_b-p_a-p_b) \equiv \langle k_a,k_b|S|p_a,p_b\rangle \ , \quad (6)$$

$$T_{n+2,2}(p_i;p_a,p_b)\,\delta^4(\sum_i p_i-p_a-p_b) \equiv \langle p_0,\dots,p_{n+1}|T|p_a,p_b\rangle \ , \quad (7)$$

$$T^B_{n+2,2}(p_i;k_a,k_b)\,\delta^4(\sum_i p_i-k_a-k_b) \equiv \langle p_0,\dots,p_{n+1}|T^B|k_a,k_b\rangle \ , \quad (8)$$

where the scattering operator S is related to the T operator by

$$S = 1 + iT \quad (9)$$

and the two-particle phase space volume element is given by

$$d\phi_2^{ab} = d^4k_a\delta(k_a^2-m_a^2)d^4k_b\delta(k_b^2-m_b^2)\delta^4(k_a+k_b-p_a-p_b) \quad . \tag{10}$$

We have suppressed and will suppress in phase space volume elements the step functions which ensure the positivity of the energy of particles.

The helicity state of a spinless particle is normalized with

$$\delta(p^2-m^2)\langle p|p'\rangle = \delta^4(p-p') \quad . \tag{11}$$

$$\int d^4k_a\delta(k_a^2-m_a^2)d^4k_b\delta(k_b^2-m_b^2)|k_a,k_b\rangle\langle k_a,k_b| \neq I \quad , \tag{12}$$

where I is the relevant identity operator. It follows that $T^BS \neq T$. In the <u>two-particle</u> <u>subspace</u>, we have a useful relation[10,23]

$$\int d^4k_a\delta(k_a^2-m_a^2)d^4k_b\delta(k_b^2-m_b^2)|k_a,k_b\rangle\langle k_a,k_b| = \sum_{\ell m}\int d^4Q|Q\ell m\rangle\langle Q\ell m| \quad , \tag{13}$$

where the two-particle angular momentum state is normalized by

$$\langle Q'\ell'm'|Q\ell m\rangle = \delta_{\ell\ell'}\delta_{mm'}\delta^4(Q-Q') \quad . \tag{14}$$

The angular momentum state is defined for an arbitrary four-momentum corresponding to the sum of the four-momenta of the two particles involved, not restricted to their c.m. system. Substituting Eqs. (6)-(9), and (13) into Eq. (5), we have

$$T_{n+2,2}(p_i;p_a,p_b) = \sum_{\ell m} T^{B\ell m}_{n+2,2}(p_i)S^{m}_{22}(p_a,p_b) \quad , \tag{15}$$

with

$$T^{B\ell m}_{n+2,2}(p_i)\delta^4(Q-\sum_i p_i) \equiv \langle p_0,\ldots,p_{n+1}|T^B|Q\ell m\rangle \quad , \tag{16}$$

and

$$S^{\ell m}_{22}(p_a,p_b)\delta^4(Q-p_a-p_b) \equiv \langle Q\ell m|S|p_ap_b\rangle \quad . \tag{17}$$

In the c.m. frame, we have

$$S^{\ell m}_{22}(p_a,p_b)\delta^4(Q-p_a-p_b) = \sum_{\ell'm'}\int d^4Q'\langle Q\ell m|S|Q'\ell'm'\rangle\langle Q'\ell'm'|p_a,p_b\rangle \tag{18}$$

$$= \delta^4(Q-p_a-p_b)\left(\frac{w}{\pi q}\right)^{1/2}(2\ell+1)^{1/2}S^{\ell m}_{22}\,D^{\ell}_{mo}(\theta,\phi) \quad , \tag{19}$$

with[10,23]

$$S^{\ell m}_{22}\delta^4(Q-Q')\delta_{\ell\ell'}\delta_{mm'} \equiv \langle Q\ell m|S|Q'\ell'm'\rangle \quad , \tag{20}$$

$$\langle Q\ell m|p_a,p_b\rangle = \left(\frac{w}{\pi q}\right)^{1/2}(2\ell+1)^{1/2}\delta(Q-p_a-p_b)D^{\ell}_{mo}(\theta,\phi) \quad , \tag{21}$$

$$D^{\ell}_{mo}(\theta,\phi) = e^{-im\theta}d^{\ell}_{mo}(\theta) \quad \text{and} \quad d^{\ell}_{m,o}(0) = \delta_{mo} \quad , \tag{22}$$

where $w = \sqrt{(p_a+p_b)^2} = \sqrt{s}$, $q = |\underset{\sim}{p}_a|$, and θ and ϕ are the polar and azimuthal angles of $\underset{\sim}{p}_a$ respectively. The function d^{ℓ}_{mo} is the standard rotation matrix element.[24] For convenience, we shall suppress throughout this paper the arguments which specify the energy-dependence of all quantities of interest. For example, $S^{\ell m}_{22}$ means $S^{\ell m}_{22}(s)$. Note the difference between $S^{\ell m}_{22}(p_a,p_b)$ and $S^{\ell m}_{22}$. Similarly, one can show that the unabsorbed amplitude for the production process, $k_a+k_b \rightarrow p_o+\ldots p_{n+1}$, is given by

$$T^B_{n+2,2}(p_i;k_a,k_b) = \left(\frac{w}{\pi q}\right)^{\frac{1}{2}} \sum_{\ell m} (2\ell+1)^{\frac{1}{2}} T^{B\ell m}_{n+2,2}(p_i) D^{\ell}_{mo}(\gamma,\zeta) \; ; \tag{23}$$

$$S_{22}(k_a,k_b;p_a,p_b) = \frac{w}{\pi q} \sum_{\ell m} (2\ell+1) D^{\ell *}_{mo}(\gamma,\zeta) S^{\ell m}_{22} D_{mo}(\theta,\phi) \quad , \tag{24}$$

where γ and ζ are the polar and azimuthal angles of $\underset{\sim}{k}_a$ respectively. From Eqs. (23) and (24), one can project out $T^{B\ell m}_{n+2,2}(p_i)$ and $S^{\ell m}_{22}$. However, we shall only project out $T^{B\ell o}_{n+2,2}(p_i)$ and $S^{\ell o}_{22}$, since $T^{B\ell m}_{n+2,2}(p_i)$ and $S^{\ell m}_{22}$ for $m \neq 0$ are not involved in final results of interest. In the standard c.m. frame, where the momentum $\underset{\sim}{p}_a$ is along the positive z-axis ($\theta=\phi=0$), one has

$$T^{B\ell o}_{n+2,2}(p_i) = (2\ell+1)^{\frac{1}{2}} \left(\frac{q}{16\pi w}\right)^{\frac{1}{2}} \int d\cos\gamma \, d\zeta \; T^B_{n+2,2}(p_i;k_a,k_b) P_\ell(\cos\gamma) , \tag{25}$$

$$S^{\ell o}_{22} = \frac{\pi q}{2w} \int d\cos\gamma \; S_{22}(k_a,k_b;p_a,p_b) P_\ell(\cos\gamma) \quad , \tag{26}$$

where $d_{oo}(\gamma) = P_\ell(\cos\gamma)$ and P_ℓ is the ℓth Legender function of the first kind.

From Eqs. (7) and (9), the unitarity relation of the scattering operator ($S^+S = I$) for the elastic scattering process $p'_a + p'_b \rightarrow p_a + p_b$, implies

$$2\mathrm{Im}\, T_{22}(\Delta;\Delta') = \sum_n \int d\Phi_{n+2} \; T^*_{n+2,2}(p_i;p_a,p_b) T_{n+2,2}(p_i;p'_a,p'_b) \; , \tag{27}$$

where $T_{22}(\Delta,\Delta') \equiv T_{22}(p_a,p_b;p'_a,p'_b)$, Δ and Δ' are the transverse components of $\underset{\sim}{p}_a$ and $\underset{\sim}{p}'_a$ respectively, and the (n+2)-particle phase

space volume element is given by

$$d\Phi_{n+2} = \prod_{i=0}^{n+1} [d^4 p_i \delta(p_i^2-m_i^2)] \delta^4(\sum_i p_i - p_a - p_b) \quad . \tag{28}$$

According to the absorption scheme discussed previously, as in Eq. (15), the amplitude $T_{n+2,2}(p_i;p'_a,p'_b)$ is the sum of products of the unabsorbed amplitude and the absorption factor. The integration of the elastic part on the right-hand side of Eq. (27) can be performed explicitly, with the aid of the partial-wave expansion of the elastic amplitude, Eq. (24). From Eqs. (15) and (27), one has in the standard c.m. frame,

$$2\,\mathrm{Im}\, T_{22}(\Delta,0) = \frac{w}{\pi q} \sum_\ell (2\ell+1) |T^{\ell o}_{22}|^2 d^\ell_{oo}(\theta)$$
$$+ \sum_{n=1}^{\infty} \sum_{\substack{\ell m \\ \ell' m'}} S^{\ell m}_{22}(p_a,p_b) H^{(n)}_{\ell m \ell' m'} \; S^{\ell' m'}_{2,2}(p'_a,p'_b) \; , \tag{29}$$

where

$$T^{\ell o}_{22} = +i(1-S^{\ell o}_{22})$$

and

$$H^{(n)}_{\ell m \ell' m'} = \int d\Phi_{n+2} \; T^{B\ell m *}_{n+2,2}(p_i) T^{B\ell' m'}_{n+2,2}(p_i) \quad . \tag{30}$$

From Eq. (16), one can rewrite Eq. (30):

$$H^{(n)}_{\ell m \ell' m'} = \int d^4Q \, d^4Q' \int d\Phi_{n+2} \; \langle Q\ell m | T^B | p_o,\ldots,p_{n+1} \rangle$$
$$\cdot \langle p_o,\ldots,p_{n+1} | T^B | Q'\ell' m' \rangle \quad . \tag{31}$$

The complete set of particle states can be replaced by[10] the complete set of angular momentum states in the (n+2)-particle subspace as in the case of the two-particle subspace (see Eq. (13)). The unabsorbed partial-wave overlap function $H^{(n)}_{\ell m \ell' m'}$ is thus diagonalized in angular momentum states. It follows from Eq. (31) that

$$H^{(n)}_{\ell m \ell' m'} = H^{(n)}_{\ell m}\ \delta_{\ell\ell'}\delta_{mm'} \equiv \delta_{\ell\ell'}\delta_{mm'}\int d\Phi_{n+2}|T^{B\ell m}_{n+2,2}(p_i)|^2 \ , \qquad (32)$$

a statement of conservation of angular momentum. Substituting Eq. (32) into Eq. (30) and the resulting equation and Eq. (19) into Eq. (29), one has

$$2\ \mathrm{Im}\ T_{22}(\Delta,0) = \frac{W}{\pi q}\sum_{\ell=0}^{\infty}(2\ell+1)[|T^{\ell o}_{22}|^2 + S^{\ell o *}_{22}H'_{\ell o}S^{\ell o}_{22}]d^{\ell}_{oo}(\theta) \ , \qquad (33)$$

where $H'_{\ell o} = \sum_{n=1}^{\infty} H^{(n)}_{\ell o \ell o}$. At high energies, it is customary to separate the elastic scattering contribution to $\mathrm{Im}T_{22}$ into two pieces: a piece which is a part of the dynamical input and corresponds to the contribution of $H^{(o)}_{\ell o}$, and the remaining piece which contains all the shadow effects. The latter is referred to as the "diffractive piece." And it is the dominating one. Its contribution to the elastic amplitude is predominantly imaginary. With this separation in mind, the partial wave unitary relation can be approximately written as

$$2\ \mathrm{Im}\ T^{\ell o}_{22} \approx |\mathrm{Im}\ T^{\ell o}_{22}|^2 + S^{\ell o *}_{22}H_{\ell o}S^{\ell o}_{22}, \text{ with } H_{\ell o} = H^{(o)}_{\ell o} + H'_{\ell o} \ , \qquad (34)$$

where the diffractive contribution has been approximated as

$|\mathrm{Im}\ T^{\ell o}_{22}|^2$, and $H_{\ell o}$ is the total unabsorbed overlap function. The unitarity relations in Eqs. (32) and (34) are valid for general absorption models of multiparticle production.

It is well known that at high energies, the partial-wave representation in small angle approximation can be transformed into the impact parameter representation[25] by the following replacements:

$$\ell(\ell+1) \to q^2b^2 \ , \quad \sum_{\ell}(2\ell+1) \to 2q^2\int_0^{\infty} b\,db \ , \qquad (35)$$

$$P_{\ell}(\cos\Theta) \to J_o(\delta b), \quad q^2\int_{-1}^{1} d\cos\Theta \to \int_0^{\infty}\delta\, d\delta \ , \qquad (36)$$

where Θ , q and δ are the polar angle, the magnitude and transverse component of the proper momentum and $\delta = q\sin\Theta \sim q\,\Theta$. The function J_o is the zero-th order Bessel function of the first kind. Defining $T^{\ell o}_{n+2,2}(p_i)\delta^4(Q-\Sigma p_i) \equiv \langle p_o,\ldots,p_{n+1}|T|Q\ell o\rangle$, one can easily show from Eqs. (5), (13), (16) and (20) that $T^{\ell o}_{n+2,2}(p_i) = T^{B\ell o}_{n+2,2}(p_i)S^{\ell o}_{22}$. One immediately sees that in the impact parameter space the corresponding expression is just Eq. (4).

In the impact parameter space, denote the amplitudes $T^{\ell o}_{22}$, $H^{(n)}_{\ell o}$ and $H_{\ell o}$ by $\tilde{T}_{22}(b)$, $\tilde{H}^{(n)}(b)$ and $\tilde{H}(b)$ respectively. From Eq. (34) the corresponding unitarity relation in the impact parameter space is given by

$$2\ \mathrm{Im}\ \tilde{T}_{22}(b) = |\mathrm{Im}\ \tilde{T}_{22}(b)|^2 + |1+i\ \tilde{T}_{22}(b)|^2\tilde{H}(b) \ . \qquad (37)$$

This is the starting point of I. At high energies, $\mathrm{Im}\tilde{T}_{22}(b)$

dominates over $\mathrm{Re}\tilde{T}_{22}$. Neglecting $\mathrm{Re}\tilde{T}_{22}$ in the last term, one can solve for $\mathrm{Im}\tilde{T}_{22}(b)$. From Eq. (33), one has

$$\mathrm{Im}\,\tilde{T}_{22}(b) = 1 - \frac{1}{\sqrt{1+\tilde{H}(b)}} \quad . \tag{38}$$

From Eqs. (26) and (36), the amplitude $T_{22}(\Delta,0)$ $(\equiv T_{22}(k_a,k_b;p_a,p_b))$ is given by

$$T_{22}(\Delta,0) = \frac{2wq}{\pi} \langle \tilde{T}_{22}(b) \rangle_\Delta \equiv T_{22}(\Delta) \quad , \tag{39}$$

where Δ is the transverse component of k_a in the standard c.m. frame and the symbol "<>" designates a Fourier-Bessel transform

$$\psi(y) = \langle \tilde{\psi}(x) \rangle_y = \int_0^\infty x dx \tilde{\psi}(x) J_o(xy) \quad . \tag{40}$$

With the present convention, the optical theorem implies that the total cross section is given by[26]

$$\sigma_T = \frac{2\pi^2}{wq} \mathrm{Im} T_{22}(o) = 4\pi\, \mathrm{Im} \langle \tilde{T}_{22}(b) \rangle_o \quad . \tag{41}$$

The elastic differential cross section is given by

$$\frac{d\sigma}{dt} = \frac{\pi^3}{4q^2w^2} |T_{22}(\Delta)|^2 = \pi |\langle \tilde{T}_{22}(b) \rangle_\Delta|^2 \quad , \tag{42}$$

where $t = (p_a'-p_a)^2 \sim -\Delta^2$, in the small angle approximation.

From Eqs. (23) and (31), in the c.m. frame, where k_a' is along the positive z-axis, one has

$$\int d\Phi_{n+2}\, T^{B*}_{n+2,2}(p_i;k_a,k_b) T^{B}_{n+2,2}(p_i;k_a',k_b') = \frac{w}{\pi q} \sum_\ell (2\ell+1) H^{(n)}_{\ell o} d^\ell_{oo}(\gamma) \quad . \tag{43}$$

Here, $T^B_{n+2,2}(p_i;k_a,k_b)$ is the unabsorbed amplitude for the process $k_a+k_b \to p_0+\dots p_{n+1}$, and γ is the polar angle of the momentum k_a. In the impact parameter space, one has from Eqs. (35), (36) and (43),

$$\tilde{H}(b) = \frac{\pi}{2wq} \langle \sum_n \int d\Phi_{n+2}\, T^{B*}_{n+2,2}(p_i;\delta) T^B_{n+2,2}(p_i;0) \rangle_b \quad , \tag{44}$$

$$= \frac{\pi}{2wq} \sum_n \int d\Phi_{n+2} \langle T^{B*}_{n+2,2}(p_i;\delta) \rangle_b T^B_{n+2,2}(p_i;0) \quad , \tag{45}$$

where $T^B_{n+2,2}(p_i;\delta) \equiv T^B_{n+2,2}(p_i;k_a,k_b)$, $T^B_{n+2,2}(p_i;0) \equiv T^B_{n+2,2}(p_i;k_a',k_b')$, and δ is the transverse component of the momentum k_a. From Eqs. (41), (42) and (44), one sees that the unabsorbed amplitude alone determines the elastic scattering amplitude completely. In I, the elastic amplitude $\tilde{T}_{22}$ is complexified by imposing the condition of crossing symmetry. Since this complexification does not change the elastic differential cross sections and inclusive predictions significantly, for simplicity we will not carry out this step below.

Multiparticle inclusive momentum distribution can also be calculated from the unabsorbed production amplitude. Undoing the phase space integral of the measured particle, one has from Eqs. (19), (29), (30) and (41),

$$2E \frac{d^3\sigma}{d^3p} = \frac{2\pi}{q^2} \sum_{n=1}^{\infty} n' \sum_{\ell,\ell'} (2\ell'+1)(2\ell+1) S^{\ell' 0}_{22} H^{(n)}_{\ell' o \ell o}(P) S^{\ell o}_{22} \quad , \tag{46}$$

where n' is the number of particles of the same species as the measured particle and

$$H'^{(n)}_{\ell' o \ell_o}(p) = [(2\ell'+1)(2\ell+1)]^{-\frac{1}{2}} \int d\Phi'_{n+1} T^{B\ell' 0*}_{n+2,2}(p_i,p) T^{B\ell 0}_{n+2,2}(p_i,p). \qquad (47)$$

The energy of the measured particle is denoted by E. In Eqs. (46) and (47), the four-momentum p of the measured particle in $T^{B\ell 0}_{n+2,2}(p_i,p)$ has been singled out from the set of p_i's. Also, the primed (n+1)-particle phase space volume element $d\Phi'_{n+1}$ is obtained from $d\Phi_{n+2}$ by removing the phase space volume element of the measured particle.

In the impact parameter space, denoting $H'^{(n)}_{\ell' o \ell m}(p)$ by $\tilde{H}'^{(n)}(b;p;b')$, one has

$$2E\frac{d^3\sigma}{d^3p} = 8\pi q^2 \sum_{n=1}^{\infty} n' \int_0^\infty bdb \int_0^\infty b'db' \, \tilde{S}^*_{2,2}(b) \, \tilde{H}'^{(n)}(b;p;b') \cdot \tilde{S}_{22}(b') \quad . \qquad (48)$$

The schematic illustration of this expression is given in Fig. 3. From Eqs. (25) and (47), one has

$$H'^{(n)}_{\ell' o \ell_o}(p) = \frac{q}{16\pi w} \int d\Phi'_{n+1} \int d\cos\gamma \int d\cos\zeta \; T^{B*}_{n+2,2}(p_i;k_a,k_b) P_{\ell'}(\cos\gamma) \cdot \int d\cos\gamma' \int d\cos\zeta' \; T^{B}_{n+2,2}(p_i;k'_a,k'_b) P_\ell(\cos\gamma'), \qquad (49)$$

where γ' and ζ' are the polar and azimuthal angles of the momentum $\underset{\sim}{k}'_a$. From Eqs. (35), (36), and (49), the expression corresponding to Eq. (49) in the impact parameter space is given by

$$\tilde{H}'^{(n)}(b;p;b') = \frac{1}{16\pi w q^2} \int d\Phi'_{n+1} \; \langle T^B_{n+2,2}(p_i,p;\underset{\sim}{\delta})\rangle'_b \langle T^B_{n+2,2}(p_i,p;\underset{\sim}{\delta}')\rangle'_b \; ,$$

$$\tilde{H}'(b;p;b') \equiv \sum_n \tilde{H}'^{(n)}(b;p;b') \quad , \qquad (50)$$

where $T^B_{n+2,2}(p_i,p;\underset{\sim}{\delta}) \equiv T^B_{n+2,2}(p_i;k_a,k_b)$, $T^B_{n+2,2}(p_i,p;\underset{\sim}{\delta}') = T^B_{n+2,2}(p_i;k'_a,k'_b)$, and $\underset{\sim}{\delta}$ and $\underset{\sim}{\delta}'$ are respectively the transverse components of the momenta $\underset{\sim}{k}_a$ and $\underset{\sim}{k}'_a$. Here the symbol "$\langle\ \rangle'$" denotes the modified "Fourier-Bessel transform,"

$$\langle\psi(\underset{\sim}{y})\rangle'_x = \int d^2\underset{\sim}{y}\,\psi(\underset{\sim}{y}) J_o(xy), \quad \text{with } x = |\underset{\sim}{x}| \quad . \qquad (51)$$

Note that it is a two-dimensional integration to be in contrast with the one-dimensional integral of Eq. (40). Substituting Eq. (50) into Eq. (48), one has

$$2E\,\frac{d^3\sigma}{d^3p} = \frac{2\pi^2}{wq} \sum_{n=1}^{\infty} n' \int d\Phi'_{n+1} \left| \int bdb \; \tilde{S}_{22}(b) \langle T^B_{n+2,2}(p_i,p;\underset{\sim}{\delta})\rangle'_b \right|^2 \; . \qquad (52)$$

Note that positivity of each multiparticle contribution to the inclusive distribution is ensured.

To sum up, the dynamical input to our multiparticle absorption model is the unabsorbed production amplitude $T^B_{n+2,2}(p_i;\underset{\sim}{\delta})$. Once this is given, quantities such as $\tilde{H}(b)$ and $\tilde{H}'(b;p;b')$ can be calculated from Eqs. (45) and (50). From Eq. (38) the unitarized elastic amplitude, and from Eq. (52) the inclusive distribution can be obtained. It is also straightforward to generalize our scheme to derive the expressions of multiparticle inclusive distributions. We will not detail them here.

4. Predictions of the absorptive multiparticle production model.

In this section, from the unabsorbed amplitude specified in I, we derive the expressions of the diffractive amplitude and the inclusive p_T-distribution of pion and proton in pp collision, and compare them to the data. We will look at the case where the unabsorbed amplitude is parameterized according to the independent emission parameterization for the cutoff function. In principle, here one should look at the case in which pions as well as meson resonances are independently emitted. For the calculation of the elastic differential cross section and the inclusive distribution, it turns out, as we shall see more explicitly below, that it is not important to identify these particles explicitly. Again, we shall continue to refer to the production process as $p+p \to p+p+n\pi$. We ignore the effect of the type II interaction and defer the consideration of this effect to future work.

The p_T-dependent part of the unabsorbed production amplitude is given by

$$T^B_{n+2,2}(p_{iT};0) \equiv T^B_{n+2,2}(p_{iT};k_{aT},k_{bT}) = \prod_{i=0}^{n+1} f(p_{iT}) \, , \qquad (53)$$

where

$$f(p_{iT}) = \langle \tilde{f}(b_i) \rangle_{p_{iT}} \, , \qquad (54)$$

with

$$\tilde{f}(b_i) = \left[\frac{B_1(1+\lambda_1 \sqrt{b_i^2+B_1^2})}{(b_i^2 + B_1^2)^{3/2}} \right]^{\frac{1}{2}} \exp\left[-\frac{\lambda_1}{2}\left(\sqrt{b_i^2+B_1^2} - B_1\right)\right] . \qquad (55)$$

Here, $k_{aT} = k_{bT} = 0$, $p_{iT} = |\underline{p}_{iT}|$, and $\underline{p}_{iT}$ is the transverse momentum of the i-th particle. For convenience, we have suppressed the arguments which specify the longitudinal-momentum dependence in Eq. (53). We shall do so for other quantities also unless otherwise specified. Recall that the two nucleons in the final state are the zero-th and the (n+1)-th particles. The cutoff functions for nucleon and pion have been chosen to be the same. This is mainly for simplicity, although the data do show crude resemblance of the corresponding inclusive p_T-distributions. The specific form for the cutoff function in Eq. (55) gives rise to a simple unabsorbed overlap function (see Eq. (70) below). The latter has a couple of nice properties. First, this overlap function has essentially a Gaussian shape in the small four-momentum transfer $\sqrt{-t}$ region and gradually turns into an exponential form. This crudely resembles the elastic data. Secondly, it possesses simple square-root branch point in t, which is expected from the t-channel unitarity. The position of the branch point there characterizes the long range nature of the exchange force involved, and it is responsible for the apparent break in the differential cross section in the small |t| region at high energies.

It is important to emphasize that $T_{n+2,2}(p_{iT};k_{aT},k_{bT})$ in Eq. (53) is only defined in the standard c.m. frame. When

$\underset{\sim}{\delta}(\equiv \underset{\sim}{k}_{aT})$ is not zero, the amplitude $T^B_{n+2,2}(p_{iT};\underset{\sim}{\delta})$ can be obtained by transforming it into the c.m. frame in which the transformed momentum of k_a is along the positive z-axis. This transformation is a rotation along the direction perpendicular to both the $\underset{\sim}{\delta}$ and the z-axis, and with an angle $(-\gamma)$. Recall that $\delta = |\underset{\sim}{\delta}| = q \sin\gamma$. At high energies, the ratio δ/q is small, so $\gamma \sim \delta/q$. Under this rotation, the following replacement[8] should be made:

$$\underset{\sim}{p}_{iT} \to \underset{\sim}{p}_{iT} + p_{iL}\,\underset{\sim}{\delta}/q = \underset{\sim}{p}_{iT} + x_i\,\underset{\sim}{\delta} \quad , \tag{56}$$

where $x_i = p_{iL}/q$. At high energies, x_i is the corresponding Feynman scaling variable. The longitudinal component p_{iL} does not change significantly under this rotation ($p_{iL} \to p_{iL} - \underset{\sim}{p}_{iT}\cdot\underset{\sim}{\delta}/q \approx p_{iL}$), due to the fact that the experimental average transverse momentum ($\sim$0.35 GeV/c) is much smaller than q (e.g., $\sim$7 GeV/c at 100 GeV/c). Rotational invariance of the amplitude gives

$$T^B_{n+2,2}(p_{iT};\underset{\sim}{\delta}) = T^B_{n+2,2}(\underset{\sim}{p}_{iT}+x_i\underset{\sim}{\delta};0)\approx \prod_{i=0}^{n+1} (|\underset{\sim}{p}_{iT}+x_i\underset{\sim}{\delta}|) . \tag{57}$$

When pion average multiplicity $\bar{n}$ is large, it can be shown that for a Gaussian cutoff function, the constraint due to the conservation of transverse momentum does not change the shape of $\tilde{H}(b;p;b')$ significantly. In fact, there is no change in $\tilde{H}(b)$. Although our cutoff function is not exactly a Gaussian, with a general sharp cutoff shape and the fact that the average pion multiplicity is large, we expect that the constraint of the

conservation of transverse momentum for the present case also should not play an important role for the shape of $\tilde{H}(b)$ and $\tilde{H}'(b;p;b')$. From Eqs. (45) and (57), one has

$$\tilde{H}(b) = \frac{\pi}{2wq} < \sum_{n=1}^{\infty} \int d\Phi_{n+2}\, T^B_{n+2,2}(\underset{\sim}{p}_{iT};\underset{\sim}{\delta})T^B_{n+2,2}(\underset{\sim}{p}_{iT};0)>_b ,$$
$$\equiv \frac{\pi}{2wq} <\Big\{\prod_{i=0}^{n+1} \int d^2\underset{\sim}{p}_{iT}\; f^*(|\underset{\sim}{p}_{iT}+x_i\underset{\sim}{\delta}|)f(p_{iT})\Big\}_{L,n} >_b , \tag{58}$$

where the symbol "$\{\ \}_{L,n}$" designates the operation of integrating over the longitudinal phase space and of summing over n. From Eqs. (53) - (55) and (58), one has

$$\tilde{H}(b) = \frac{\pi}{2wq} <\Big\{\exp[-\sum_{i=0}^{n+1} B_1(\sqrt{x_i^2\delta^2+\lambda_1^2} - \lambda_1)]\Big\}_{L,n} >_b . \tag{59}$$

For the derivation of Eq. (59), the following formulae were used:

$$J_0(|\underset{\sim}{p}_T+x\underset{\sim}{\delta}|b) = J_0(p_Tb)J_0(\delta b) + 2\sum_{n=1}^{\infty} J_n(p_Tb)J_n(\delta b)\cos n\gamma , \tag{60}$$

$$\int_0^{\infty} y\,dy\, J_m(xy)J_n(x'y) = \frac{1}{x}\,\delta_{m\,n}\,\delta(x-x') \quad , \tag{61}$$

with γ being the angle between $\underset{\sim}{p}_T$ and $\underset{\sim}{\delta}$. The function J_n is the nth Bessel function of the first kind. Performing the integration over the longitudinal momentum phase space and summing over n, one has approximately

$$\tilde{H}(b) = FE^C <\exp[-2B_1(\sqrt{\bar{x}_N^2\delta^2+\lambda_1^2} - \lambda_1) - \bar{n}_\pi B_1(\sqrt{\bar{x}_\pi^2\delta^2+\lambda_1^2} - \lambda_1)]>_b , \tag{62}$$

where $\bar{x}_N$ and $\bar{x}_\pi$ are the rms x-moments of the normalized inclusive rapidity distributions for nucleon and pion respectively, $\bar{n}_\pi$ is the average pion multiplicity, and E is the lab energy of the incident nucleon. The parameters F and c are to be determined. The power behavior E^c is due to the property (c) of the MP-like model. For completeness, we include the corresponding unabsorbed overlap function for the multiperipheral model cutoff function in ref. 27.

We proceed now to discuss the approximation involved in arriving at Eq. (62). For small δ^2, we expand the exponentials in Eqs. (59) and (62) in power series of δ^2. Comparing the corresponding coefficients of the δ^2-powers, one gets for the first few terms

$$\frac{\pi}{2wq}\{1\}_{L,n} \sim FE^c \quad , \tag{63}$$

$$\frac{\pi}{2wq}\{\sum_{i=0}^{n+1} x_i^2\}_{L,n} \sim FE^c(2\bar{x}_N^2 + \bar{n}_\pi \bar{x}_\pi^2) \quad , \tag{64}$$

$$\frac{\pi}{2wq}\{-3\sum_{i=0}^{n+1} x_i^4 + \lambda_1 B_1(\sum_{i=0}^{n+1} x_i^2)^2\}_{L,n} \sim FE^c[-3(2\bar{x}_N^4 + \bar{n}_\pi \bar{x}_\pi^4) + \lambda_1 B_1(2\bar{x}_N^2 + \bar{n}_\pi \bar{x}_\pi^2)^2] \quad . \tag{65}$$

The coefficients of the zero-th power of δ^2 relate the overall factor to the unabsorbed total inelastic cross section, $\sigma_{in}^B = 2\pi FE^c$ (see Eqs (39) and (41) for normalization). In Eq. (59) the left-hand side reduces to $\{2x_N^2 + nx_\pi^2\}_{L,n}$, since there are n identical pions and two identical nucleons. Hence, from the

definitions of $\bar{x}_N$ and $\bar{x}_\pi$, the coefficients of the first δ^2-power are equal.

It is estimated[1] from the data that at 102 GeV/c,

$$\bar{x}_\pi \simeq 0.04 \quad \text{and} \quad \bar{x}_N > 0.7 \tag{66}$$

with $\bar{n}_\pi$ = 6.6. We then neglect the pion contribution on both sides of Eq. (65). Relation (65) reduces to

$$\frac{\pi}{2wq}\{-3(x_o^4+x_{n+1}^4) + \lambda_1 B_1(x_o^2 + x_{n+1}^2)^2\}_{L,n} \sim 2FE^c(-3+2\lambda_1 B_1)\bar{x}_N^4 \quad . \tag{67}$$

It is well known that high energy data show a leading particle effect. To the extent that one can ignore those events where the final nucleons are travelling along the same longitudinal direction in the c.m. system, and the fact that one can assume the two proton x-spectra are not strongly correlated, we can approximately write,

$$\pi\{x_o^2 x_{n+1}^2\}_{L,n} \simeq \pi\{x_o^2\}_{L,n}\{x_{n+1}^2\}_{L,n}/\{1\}_{L,n} \simeq 2wq\ FE^c\ \bar{x}_N^4 \quad , \tag{68}$$

$$\pi\{x_o^4\}_{L,n} = \pi\{x_{n+1}^4\}_{L,n} \approx 2wq\ FE^c\ \bar{x}_N^4 \quad . \tag{69}$$

From Eqs. (67) - (69), the coefficients of the second δ^2power are approximately equal. As δ^2 increases, to get a reasonable approximation, more precise equalities of the corresponding coefficients of higher δ^2-powers are required. It is important to note that the exponents in Eqs. (59) and (62) are negative. The overestimate or underestimate of one moment may be balanced by the discrepancies of opposite sign of the alternate one. For the pion contribution, since $\bar{x}_\pi^2\delta^2$ is always much smaller than λ_1^2 in the δ^2-region of interest, we neglect the δ^2-dependence here. For the nucleon contribution, higher moments are expected to be close to the peaks of the x-spectra near $x = \pm 1$. So the approximation for the higher moments is better. In I, with our simple parametrization, the differential cross section for high energy pp collisions can only be fitted for $-t \lesssim 0.8$ GeV^2. This could partly be that the approximation is inadequate at around this δ^2 value.

Ignoring the pion contribution, one has, from Eq. (62),

$$\tilde{H}(b) = \frac{FE^c\, b_{\bar{n}}(1+\lambda\sqrt{b^2+b_{\bar{n}}})}{(b^2+b_{\bar{n}}^2)^{3/2}} \exp[-\lambda(\sqrt{b^2+b_{\bar{n}}^2} - b_{\bar{n}})] \quad , \tag{70}$$

with

$$\lambda = \lambda_1/\bar{x}_N \quad \text{and} \quad b_{\bar{n}} = 2B_1\bar{x}_N \quad . \tag{71}$$

The corresponding unabsorbed overlap function in the t-space is given by

$$H(t) = FE^c \exp[-b_{\bar{n}}(\sqrt{\delta^2+\lambda^2} - \lambda)] \quad . \tag{62a}$$

The dependence of $\bar{x}_N$ on energy in high energy region is weak. We take $\bar{x}_N$ to be a constant.

Thus far we have treated the production process of the type $pp \to pp + n\pi$, with all pions being directly emitted. Allowing

some of the pions to be from the decay of meson resonances, the pion contribution to the exponent of Eq. (62) should be replaced by

$$-\bar{n}_\pi\, B_1(\sqrt{\bar{x}_\pi^2\delta^2 + \lambda_1^2} - \lambda_1) + -\sum_j \bar{n}_j\, B_1(\sqrt{\bar{x}_j^2\delta^2 + \lambda_1^2} - \lambda_1)$$

with $\bar{n}_\pi = \Sigma\bar{n}_j N_j$, where N_j is the number of decayed pions from the j-type resonance and $\bar{n}_j$ the corresponding average multiplicity. Let us estimate the contribution of this term compared to the nucleon contribution. For instance, from our previous study of the multiplicity distribution,[16] at 102 GeV/c, with the emission of π and an effective B meson, we found $\bar{n}_B = 0.7$. On the other hand, for B with $N_j = 4$, we expect $\bar{x}_B < N_j\bar{x}_\pi = 0.16$. The ratio of the coefficients of the δ^2-term coming from the corresponding nucleon and B contributions is given by

$$2\bar{x}_N^2/\bar{n}_B\bar{x}_B^2 > 40 \quad .$$

So the t-dependence from the B is much weaker than that from the nucleon, at least near $\delta^2 = 0$. Thus the inclusion of the effects from meson resonances essentially does not affect our approximation of Eq. (70).

In Eq. (62), the lower bound of the rms value $\bar{x}_N$ given is the mean value $\tilde{x}_N$. From the shape of the nucleon x-spectra, one deduces $\bar{x}_N > \tilde{x}_N$. In I, the mean value for nucleon was estimated from that for pion together with the asymptotic relation of energy conservation. From the two sharp peaks near the ends of the nucleon x-spectra, we estimated a ratio $\bar{x}_N/\tilde{x}_N = 1.3$, suggesting

$$\bar{x}_N \simeq 0.9 \quad . \tag{72}$$

with Eqs. (38), (41), (42) and (70), in I it is shown that the amplitude T_{22} in its complexified version together with an added proper-Regge-pole term gives a reasonable description to the pp elastic scattering data for $p_{Lab} \geq 10$ GeV/c and $-t \leq 0.8$ GeV^2. The elastic data included are the total cross section, the Re/Im ratio at t=0, the slope parameters of differential cross sections at various t values, and some sample differential cross sections at 12.8, 19.2 and 1500 GeV/c. A typical solution has the parameters

$$F = 24.3 \text{ GeV}^{-2} \;, \qquad \lambda = 0.43 \text{ GeV} \;, \tag{73}$$

$$b_{\bar{n}} = 2.97 \;, \qquad c = 0.09 \;. \tag{74}$$

From Eqs. (71) - (74), one deduces

$$\lambda_1 = 0.39 \qquad \text{and} \qquad B_1 = 1.65. \tag{75}$$

The inclusive p_T-distributions for pion and nucleon can be calculated from Eqs. (48) and (50). Consider first the nucleon inclusive p_T-distribution. Ignoring again the constraint of the transverse momentum conservation and the pion contribution to the

δ^2-dependence, from Eqs. (48), (50) and (57), one arrives at

$$\tilde{H}(b;p_T;b') \propto \int d^2p_T' \, \langle f^*(|p_T+\bar{x}_N\delta|)f^*(|\tilde{p}_T'+\bar{x}_N\delta|)\rangle_b' \cdot \langle f(|p_T + \bar{x}_N\delta'|)f(|p_T'+\bar{x}_N\delta'|)\rangle_{b'}' \;. \tag{76}$$

Using the relation

$$J_o(\Delta b) = \frac{1}{2\pi}\int d^2b e^{-i\Delta\cdot b} \;, \tag{77}$$

one has

$$\int d^2p_T' \, f^*(|p_T+\bar{x}_N\delta|)f(|p_T+\bar{x}_N\delta'|) = \int d^2b|\tilde{f}(b)|^2 e^{-i\bar{x}_N b\cdot(\delta'-\delta)} \;. \tag{78}$$

From Eqs. (76) and (78), it follows that

$$\tilde{H}(b;p_T;b') \propto \int d^2b d^2\delta d^2\delta' J_o(\delta b)J_o(\delta b')f^*(|p_T+\bar{x}_N\delta|)f(|p_T+\bar{x}_N\delta'|) \cdot e^{-i\bar{x}_N b\cdot(\delta'-\delta)}|\tilde{f}(b)|^2 \;. \tag{79}$$

Substituting Eq. (79) into Eq. (48), after some calculations, we have for the nucleon inclusive momentum distribution,

$$\frac{d^2\sigma}{d^2p_T} \propto \int d^2b \int d^2b' S_{22}^*(b)S_{22}(b') \int d^2b''|\tilde{f}(b)|^2\tilde{f}^*(|b'/\bar{x}_N-b''|) \cdot f(|b/\bar{x}_N-b''|)\exp[-i(b-b')\cdot p_T/\bar{x}_N] \;, \tag{80}$$

where from Eq. (38), $\tilde{S}_{22}(b) = 1/\sqrt{1+\tilde{H}(b)}$. Under a similar approximation, the pion inclusive p_T-distribution is given by

$$\frac{d^2\sigma}{d^2p_T} \propto |f(p_T)|^2 \int d^2b|\tilde{S}_{22}(b)|^2 \int d^2b|f(b)|^2|f(|b/\bar{x}_N-\tilde{b}|)|^2 \quad ,$$

$$\propto |f(p_T)|^2 = |\int bdb\tilde{f}(b)J_o(p_Tb)|^2 \quad . \qquad (81)$$

This is the same as the unabsorbed nucleon p_T-distribution, owing to our assumption of universality of the unabsorbed p_T-cutoff function for the different particles.

One can easily calculate $d\sigma/dp_T^2$ by integrating $d^2\sigma/d^2p_T$ over the azimuthal angle of p_T. The expression in Eq. (80) is quite involved. It needs at least a triple integration. In practice, one may approximate the obtained $\tilde{S}_{22}(b)$ with its parameters given in Eqs. (72) - (74), by two Gaussian functions:

$$\tilde{S}_{22}(b) \simeq 1 + \sum_{i=2}^{3} D_i\, e^{-d_ib^2} \equiv \sum_{i=1}^{3} D_i\, e^{-d_ib^2} \quad . \qquad (82)$$

The parameters d_i and D_i are listed in Table I. The error is $\sim$2% for $b^2 < 30$ GeV^{-2} and is $\sim$8% for $b^2 \leq 150$ GeV^{-2}. We have also checked that the inclusive p_T-distribution obtained within this approximation is not sensitive to the variation of $\tilde{S}_{22}(b)$ within the range of errors quoted. Using the relation,

$$\int bdb\, e^{-z_1^2b^2} J_n(z_2b)J_n(z_3b) = \frac{1}{2z_1^2}\exp[-\frac{z_2^2+z_3^2}{4z_1^2}]\, I_n(\frac{z_2z_3}{2z_1^2}) \quad , \qquad (83)$$

and substituting Eq. (82) into Eq. (80), one has after some calculations,

$$\frac{d\sigma}{dp_T^2} \propto \int bdb|\tilde{f}(b)|^2[\sum_{i,j} D_iD_j e^{-(d_i+d_j)\bar{x}_N^2b^2}(F_o^*(\bar{x}_Nd_i;b)F_o(\bar{x}_Nd_j;b)$$

$$+ 2\sum_{n=1}^{\infty} F_n^*(\bar{x}_Nd_i;b)F_n(\bar{x}_Nd_j;b))] \quad , (84)$$

with

$$F_n(\bar{x}d;b) = \int b'db'f(b'/\bar{x})J_n(kb')I_n(2d\bar{x}bb') \quad , \qquad (85)$$

where I_n is the n-th order Bessel function of the second kind. This expression needs a double integration. Furthermore, we approximate $\tilde{f}(b)$ by three Gaussian functions:

$$\tilde{f}(b) \simeq \sum_{i=1}^{3} C_i \exp(-c_ib^2) \quad , \qquad (86)$$

where the parameters c_i and C_i are also listed in Table I. The percentage error is within $\sim$1% for $b^2 \leq 20$ GeV^{-2} and $\sim$3% for $b^2 \geq 150$ GeV^{-2}. We have checked that this approximation of $\tilde{f}(b)$ gives an error of 14% for the unabsorbed p_T-distribution which can be directly calculated from Eq. (81). Substituting Eqs. (82) and (86) into Eq. (80), and using Eq. (60) and the relation, $\exp(z) = I_0(z) + 2\sum_{n=1}^{\infty} I_n(z)$, we have after some calculation,

$$\frac{d\sigma}{dp_T^2} \propto \sum_{\substack{ijk\\ \ell mn}} D_iD_jC_kC_\ell C_mC_n a_{ik}^{-1}a_{j\ell}^{-1}L_{ijk\ell mn}^{-1}$$

$$\cdot \exp[-p_T^2(M_{ijk\ell mn}-N_{ijk\ell mn})/\bar{x}_N^2] \qquad (87)$$

with

$$a_k = c_k/\bar{x}_N, \quad a_{ik} = d_i + c_k/\bar{x}_N^2, \quad a_{k\ell mn} = c_k + c_\ell + c_m + c_n, \qquad (88)$$

$$L_{ijk\ell mn} = a_{k\ell mn} - (a_{ik}^{-1}a_k^2 + a_{j\ell}^{-1}a_\ell^2) \qquad (89)$$

$$N_{ijk\ell mn} = [a_{k\ell mn}(a_{ik}^{-1}+a_{j\ell}^{-1}) - a_{ik}^{-1}a_{j\ell}^{-1}(a_k^2+a_\ell^2)]/4L_{ijk\ell mn} \quad . \tag{90}$$

$$N_{ijk\ell mn} = a_k a_\ell / 2a_{ik}a_{j\ell} \; L_{ijk\ell mn} \quad . \tag{91}$$

We have used this expression for numerical calculations.

With the parameters of I, the pion and nucleon inclusive p_T-distributions are calculated. The theoretical predictions and the data are shown in Fig. 3. Since the p_T-distribution data integrated over all x are not available, we illustrate the data at some fixed x-values. For proton, the experimental x-spectra peak close to x=1. We choose the data at x=0.8 (0.7 - 0.9) and from 100 GeV/c up. For the pions, the x-spectra peak near x=0. So the data are taken at x=0. The predicted proton-curve is shown as the solid curve. It is to be compared with the data points illustrated. Notice that although in the small p_T-region, the curve is somewhat sharper than the trend indicated by the data; in view of the fact that it is a parameter-free prediction, the curve does reproduce the gross feature of the data rather well. The pion distribution predicted from the assumed universal shape of the $f(p_T)$ function is shown as the dashed curve. It is to be compared with the data presented as the curve with crosses. The agreement here is reasonable.

In our calculation, as mentioned earlier, for simplicity we did not consider the "type II" nonproductive interactions which correspond to events of the type: $pp \to p^*p + \ldots$ and $pp \to p^*p^* + \ldots$,

where p^* could be the nucleon resonances and also the nucleon-clusters (or fragmentations). If we identify these events as the diffractive dissociation events, the data show that they could be of the order of 15%, say at 100 GeV/c. It is conceivable that with the inclusion of these events the predicted inclusive distribution could be altered, if the cutoff structure for these events turns out to be markedly different from what we considered here. Also in our calculations, so far as the longitudinal momentum information is concerned, only the moments $\bar{x}_N$ and $\bar{x}_\pi$ are included. Details of the longitudinal exclusive distribution have not been specified. Therefore, further formulation of the model with the inclusion of the diffractive dissociation type of events and proper account for the exclusive p_L-distributions, etc., are still needed to provide a more refined description of the data.

Table I

Parameters of Gaussian functions for the approximations of $\tilde{H}(b)$ and $\tilde{f}(b)$.

$\tilde{H}(b)$		$\tilde{f}(b)$	
b_i	d_i	C_i	c_i
1.	0.	0.423	0.246
-0.678	0.048	0.138	0.050
-0.033	0.016	0.043	0.013

Figure Captions

Fig. 1a. The production amplitude $T_{n+2,2}$ approximated as the product of the unabsorbed production amplitude and the elastic S-matrix. This can either be in the impact parameter space or in the momentum space.

Fig. 1b. A schematic illustration of 2 to 2 inelastic scattering amplitude, $\tilde{T}_{cd,ab}(b) = \tilde{S}_{ab}(b)\tilde{T}^{B}_{cd,ab}(b) \approx \sqrt{\tilde{S}_{cd}(b)}\tilde{T}^{B}_{cd,ab}(b) \cdot \sqrt{\tilde{S}_{ab}(b)}$.

Fig. 2. Diagrammatic representation of inclusive momentum distributions in the impact parameter space. The open line could be either pion or nucleon.

Fig. 3. Pion and proton transverse momentum distributions. Solid curve is the predicted proton p_T-distributions and the dashed curve is the predicted pion p_T-distribution. Data points of nucleon distribution at a mean value $x = 0.8$ (0.7 to 0.9): ● NAL 303 GeV/c; x, 0, □ are at 11.8 + 11.8, 15.4 + 15.4 and 22.5 + 22.5 GeV ISR energies by CHLM collaboration. The data points Δ and ∇ are at 1060 GeV/c and the x interval: 0.85 - 0.95, also by CHLM collaboration. The curve with crosses shows the experimental pion inclusive p_T-distribution at 23.2 +23.2 GeV ISR energy and $x=0$, from Saclay-Strasbourg collaboration. For the data, see Refs. 28 and 29.

Footnotes and References

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26. One (KHW) of the authors takes the advantage to correct a mistake in ref. 10. The differential cross section in Eq. (94) should be $d\sigma = \pi^2 q_{b'} |T_{\nu_a'\nu_b'\nu_a\nu_b}(s_b,u_{bb'})|^2 du_{b'}/4w_b^2 q_b$. Similar correction should also be made in Eq. (95).

27. For completeness, we give here the derivation for the unabsorbed overlap function with the multiperipheral parameterization. Define the momentum transfer dependence of the n-particle production amplitude to be

$$T^B_{n+2,2} \propto \prod_{i=0}^{n} f(q_{iT})$$

where $q_{iT} = \sum_{j=0}^{i} p_{jT}$. Following a formalism similar to Henyey's of ref. 8, we replace the transverse phase space $\prod_{i=0}^{n+1} d^2p_{iT} \cdot \delta^2(\sum_i p_{iT} - k_{aT} - k_{bT})$ by $\prod_{i=0}^{n} d^2q_{iT}$. The unabsorbed overlap function is then given by

$$\tilde{H}(b) \propto \sum_n \langle \{\int \prod_{i=0}^{n} d^2q_{iT} f(|q_{iT} + v_i\delta|) f(q_{iT})\}_{L,n} \rangle_b, \text{ with } v_i = \sum_{j=0}^{i} x_j .$$

Comparing this expression with Eq. (58) in the text, one sees that the only difference is the upper limit of i; the former is n and the latter n+1. In the MP case, the conservation of transverse momentum is automatically satisfied. Following the treatment leading to Eq. (62), one has

$$\tilde{H}(b) = FE^c \langle \exp[-2B_1(\sqrt{\bar{x}_N^2\delta^2+\lambda_1^2} - \lambda_1) - (\bar{n}_\pi - 1)(\sqrt{\bar{v}^2\delta^2+\lambda_1^2} - \lambda_1)] \rangle_b ,$$

where $\bar{v}$ is the rms value of $\sum_{i=1}^{n-1} v_i/(\bar{n}_\pi - 1)$ over the longitudinal distributions. Note that $\bar{v}$ is bounded by $\bar{x}_N$; i.e., $\bar{v} >> \bar{x}_N$. This leads to, as mentioned in the introduction, the difficulty for the MP-parameterization (see refs. 7,8 and 9), not present in the IE case. For further discussion, see ref. 1.

28. The proton inclusive p_T-data: F. T. Dao, et al., pp Interactions at 303 GeV/c, in Proceedings on Experiments on High Energy Particle Collisions, edited by R. S. Panvini (AIP, New York, 1973), p. 54, Fig. 11. The CHLM data quoted here are from CERN-Holland-Lancaster-Manchester collaboration.

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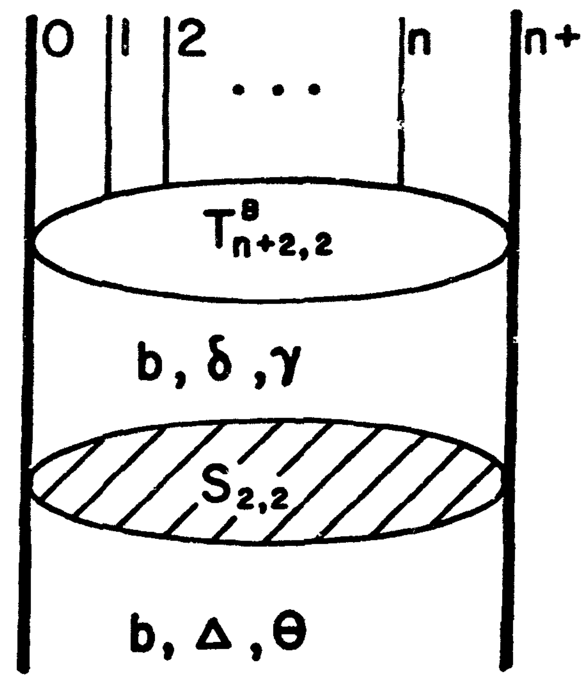


Figure 1a

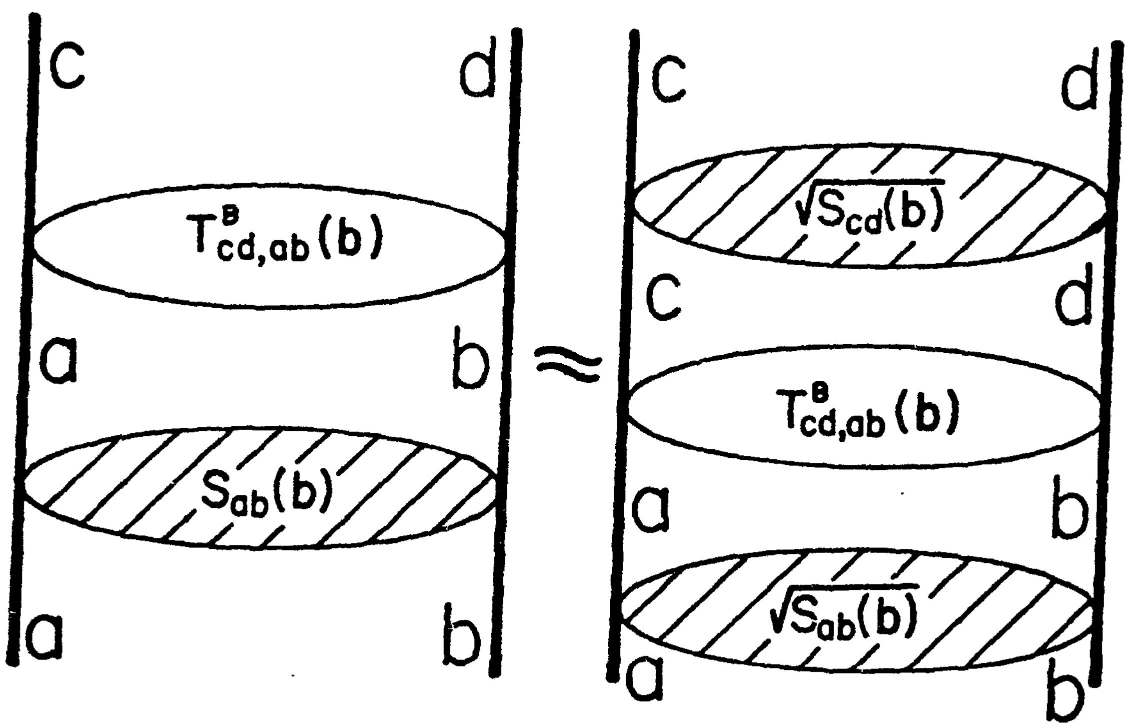


Figure 1b

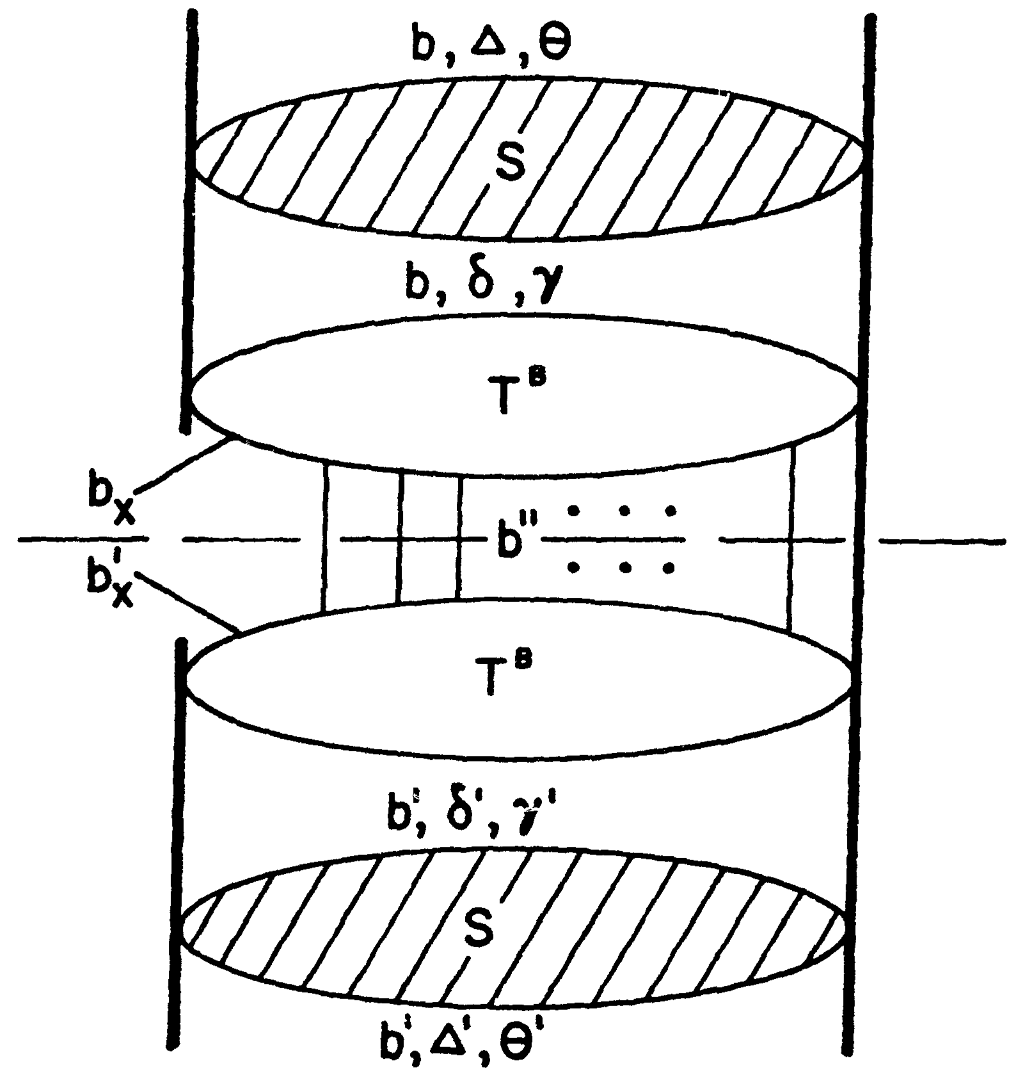
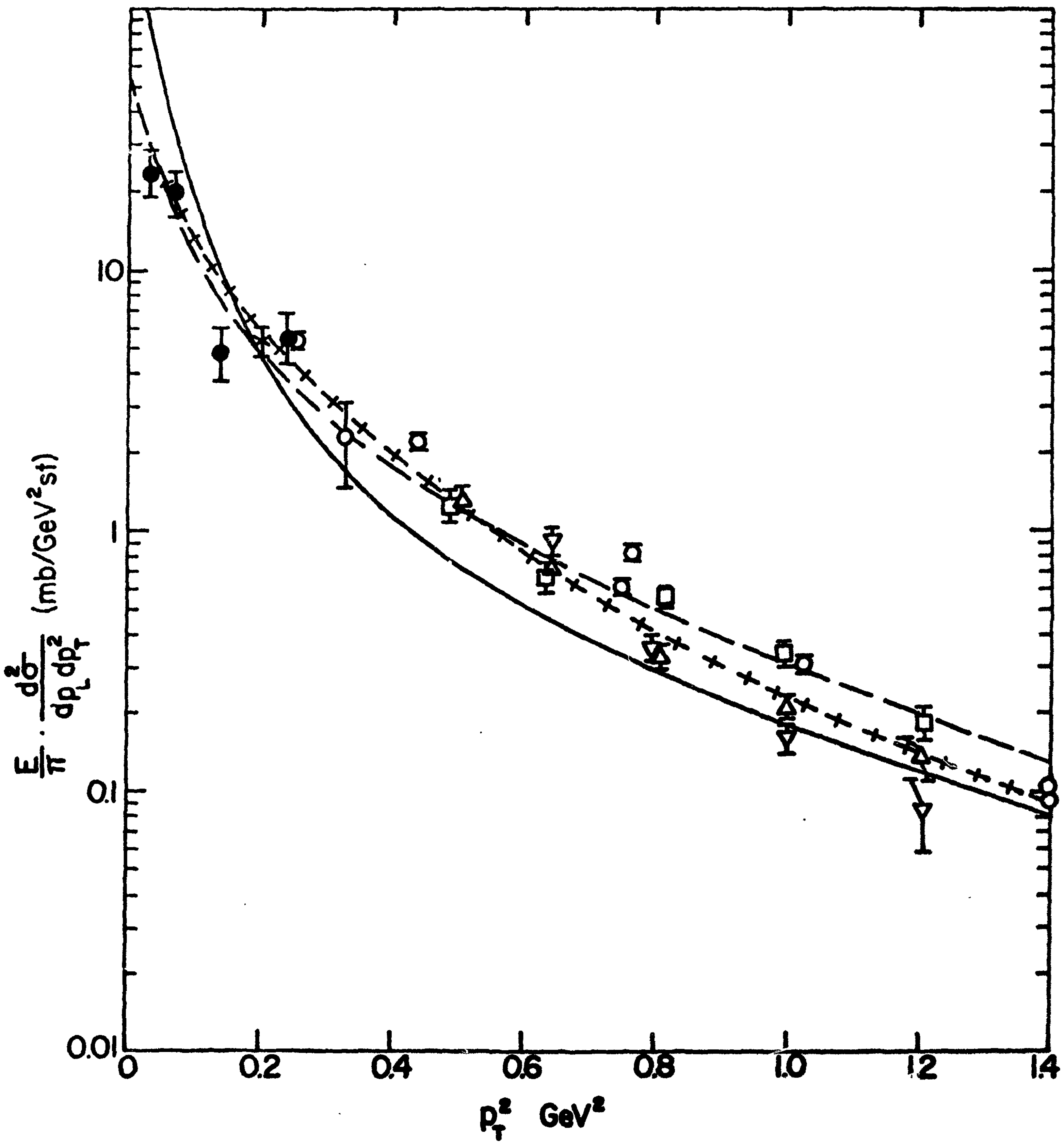


Figure 2



$\frac{E}{\pi} \cdot \frac{d^2\sigma}{dp_L dp_T^2}$ (mb/GeV^{2}st)
10
1
0.1
0.01
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
p_T^2 GeV^2

Figure 3