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PHASES OF RESONANT AMPLITUDES: $\pi N \to \pi \Delta$

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A B S T R A C T

The phases of resonant amplitudes in $\pi N \to \pi \Delta$ are studied in a modified version of $SU(6)_W$ in which amplitudes involving different relative orbital angular momenta ℓ are uncoupled from one another. This form of $SU(6)_W$ is equivalent to one studied recently by Melosh, in which the set of selection rules for decays is extended to allow for more types of transition than in the original version of this symmetry.

The predictions are compared with a recent preliminary analysis by Herndon et al. Even the extended ("ℓ- broken") version of $SU(6)_W$ is found to disagree with the present experimental solution. If this solution persists, it constitutes the strongest present evidence against such a symmetry.

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DISCLAIMER

Elastic πN phase shift analyses have provided useful information on resonances for a number of years. Much more recently, the analysis of $\pi N \to \pi\pi N$ by two groups [1),2)] indicates that a wealth of resonances may also be observed in $\pi N \to \pi\Delta$ and $\pi N \to \rho N$.

The decays of resonances into particles with higher spin are extremely valuable in testing symmetry schemes higher than SU(3). For example, in $\pi N \to \pi\Delta$, while SU(3) does not specify the relative phases of contributions from different resonances, such higher symmetries as $SU(6)_W$ can do so.

Although $SU(6)_W$ has been applied to decays of resonances with non-zero internal quark angular momentum L [3)] it has recently been suggested [4)-10)] that the unbroken symmetry is too strong for such an application. Basically, this is because $SU(6)_W$ implies that decays of L excited hadrons to ones with $L=0$ (such as all πN and $\pi\Delta$ decays) must proceed from an initial $L_z=0$ state. This circumstance would correspond to neglecting the transverse momentum of quarks inside a hadron [11)], and is one which has been considered unreasonable [4)].

The particular $\Delta L_z=0$ aspect of $SU(6)_W$ appears whenever a decay can proceed via two final state orbital angular momenta ℓ. It has the effect of relating the amplitudes for different ℓ in a manner which for mesons is known to be incompatible with experiment. The most famous example is perhaps the $B \to \omega\pi$ process whose s- and d-wave amplitudes are linked in such a way as to forbid the clearly observed [12)] helicity ±1 omegas. If, on the other hand, $\Delta L_z = \pm 1$ transitions could be admitted, as suggested in Refs. 4)-6), 13) and motivated more recently by the work of Melosh [8)], this link between different ℓ would no longer appear, and the decay of the B meson would present no problem.

Forewarned by the $B \to \omega\pi$ situation, various authors have, in their treatment of the decays of baryon resonances, employed a version of $SU(6)_W$ in which amplitudes for different ℓ are taken as independent; Refs. 4), 5), 7), 14), 15). Although such fits yield a remarkably good understanding of partial decay rates, it has not been possible until now to check whether the relative sign of the couplings for different ℓ satisfies the $\Delta L_z=0$ constraint or not. With the appearance of Ref. 1) and particularly Ref. 2) such a test is now possible.

On the basis of the contributions of N and Δ resonances below 2 GeV (in mass) to $\pi N \rightarrow \pi\Delta$ we find that the present experimental situation is inconsistent with even the extended version of $SU(6)_W$ which admits both $\Delta L_z = 0$ and $\Delta L_z = \pm 1$ transitions.

We have calculated the sign of the amplitude at resonance in $\pi N \rightarrow \pi\Delta$ for a number of observed and predicted resonances below ~2 GeV in mass. These signs were predicted in Ref. 7) for the 70, L = 1 states. They may be obtained in general by using the formalism of Ref. 5) to calculate the helicity amplitudes, and then comparing them with the corresponding expression in Ref. 16). The reason that definite phase predictions are possible rests on the fact that both the nucleon and $\Delta(1236)$ belong to the same $SU(6)_W$ multiplet. A corresponding determination of phases in SU(3) for such processes as $\bar{K}N \rightarrow \pi\Lambda, \pi\Sigma, \ldots$ has been in use for some time [17)], so by adopting the commonly used "baryon-first" isospin convention we are able to add $\pi N \rightarrow \pi\Delta$ phase predictions to this list.

Apart from an arbitrary over-all phase there are two basic types of prediction that experiment must choose between: the "$SU(6)_W$-like" solution in which the relative S/D and P/F phases are constrained by the $\Delta L_z = 0$ condition, or the "anti-$SU(6)_W$" solution which has the opposite relative phases, as would hold if $\Delta L_z = \pm 1$ transitions were dominant [see e.g., Ref. 9)]. Note that this S/D and P/F phase ambiguity in no way affects amplitudes of the kind PP, DD, FF whose signs depend only upon the assumed particle classification. There are twelve experimental phases [2)] that we are able to compare with our predictions and they are indicated by crosses [*)] set against their corresponding theoretical clocks in Fig. 1. For definiteness and in order that the ensuing discussion should be easy to follow we have set the clocks for all $SU(6)_W$ multiplets to their anti-$SU(6)_W$ positions.

Let us discuss the results.

a. Firm predictions, likely predictions, guesses

The possibility of configuration mixing makes some predictions less firm than others. Accordingly, we have noted in Fig. 1 three classes of

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*) We thank R. Cashmore for discussions and correspondence regarding the conventions of Ref. 2), whose Argand circles are based on the isospin convention $\pi N \rightarrow (\text{Res.}) \rightarrow \Delta\pi$. Accordingly, we have reversed all signs of $I = \frac{1}{2}$ amplitudes from those of Ref. 2).

predictions. Those of "class 1" involve states which cannot mix within a given representation of $SU(6)_W \times O(3)$ and for which no nearby states with the same quantum numbers from other $SU(6)_W \times O(3)$ multiplets are expected. "Class 2" predictions involve states that can mix but for which the effect of mixing is considered to be reasonably well understood. The analyses of Ref. 7) for mixing of the states inside the $\underline{70}$, $L = 1$ and of Ref. 15) for mixing between $\underline{56}$, $L = 2$ and $\underline{70}$, $L = 2$ states indicate that for all our "class 2" resonances the predictions for the physical state phases are the same as those for the unmixed assignments shown in Fig. 1. These are the conventional ones [18]. "Class 3" predictions involve guesses as to the assignments of the states. These guesses are based on previous quark model and classification studies [18] but the results could in principle be altered by mixing for which we have no quantitative estimates as yet.

b. Predictions crucial for the validity of ℓ-broken $SU(6)_W$

There are two sets of predictions that must be verified if $SU(6)_W$ in either of its unbroken or ℓ-broken forms is to survive. They are:

i) DD15 and FF37 must have the same relative phase;
ii) SD31 and DS33 must have opposite relative phases.

These constitute all the "class 1" predictions in Fig. 1 and it will be noticed that they are in agreement with the solution of Ref. 2). Furthermore, from the observation that it is SD31 which is the odd-man-out among these four phases we infer that for the $\underline{70}$, $L = 1$ multiplet the "anti-$SU(6)_W$" solution must be chosen. An immediate consequence of this choice is that both of the (8,2) amplitudes DS13 and DD13 must have the same relative phase and that they must moreover be in phase with SD31. The solution of Ref. 2) does indeed show DS13 and DD13 with the same relative phase but they have the WRONG phase relative to SD31. Could mixing alter this? The answer is no, because in the presence of (8,2)-(8,4) mixing these two amplitudes become:

$$DS13 \rightarrow DS13\left(1 - \frac{\sqrt{10}}{20}\tan\Theta\right)\left(1 - \frac{\sqrt{10}}{2}\tan\Theta\right)$$

$$DD13 \rightarrow DD13\left(1 - \frac{\sqrt{10}}{20}\tan\Theta\right)\left(1 + \frac{2\sqrt{10}}{5}\tan\Theta\right)$$

where the mixing angle Θ is defined as in Ref. 7) and clearly no value of Θ can change the signs of both.

c. Need to fill gap in data before concluding that ℓ-broken $SU(6)_W$ fails

No data between $E_{cm} = 1540$ and 1650 MeV (dashed line, Fig. 1) were used in Ref. 2). This may be a weak point, especially since the conclusion of the failure of ℓ-broken $SU(6)_W$ rests to a large extent on knowing the phases on one side of the gap relative to those on the other. In fact, using data only above the gap, one cannot even conclude ℓ-broken $SU(6)_W$ fails at all. [FF35(1890) although not strongly resonant in Ref. 2) is a strong effect in Ref. 1) and in both cases the sign is consistent with our predicted phase; PP31(1910) is a "class 3" prediction which could be altered by mixing - in any event it is only "weakly resonant" in Ref. 2) and not resonant in Ref. 1); the DS13(1730) phase is hard to estimate from the Argand diagram of Ref. 2); PP11(1750), another "class 3" state might be mixed or perhaps misclassified and FP15(1690) the only apparent bad failure is possibly no failure at all since we have arbitrarily set the 56, $L = 2$ clocks to their "anti-$SU(6)_W$" positions along with the rest.] What such data would indicate would be the failure of universal assumptions regarding the dominant ΔL_z [9] and the failure of more explicit quark models [18),19)].

d. SU(3) related checks

Very few unambiguous SU(3) related checks can be made on account of the large amount of configuration mixing that is generally possible among the hyperons. One valuable and particularly clean test however would be to compare the relative phases of $\Sigma(1765)$ and $\Sigma(2030)$ in an analysis of $\bar{K}N \to \pi\Sigma(1385)$. No mixing complications would affect the D15 resonance, and the F17 is expected to be, at worst, a simple octet-decuplet mixture whose phase behaviour is completely predictable in $SU(6)_W \times O(2)_{L_z}$. Specifically, if $\Sigma(2030)$ is assumed to be some mixture of 56, $L = 2$ and 70, $L = 2$ [belonging in part to each of the SU(3) multiplets which contain $\Delta(1950,7/2^+)$ and $N(2024,7/2^+)$] it so happens that mixing would have precisely the same effect on each of its decay amplitudes into the $\pi\Lambda$, $\pi\Sigma$ and $\pi\Sigma(1385)$ channels. Since the phases in $\bar{K}N \to \pi\Lambda$ and $\pi\Sigma$ are known to be consistent with a decuplet assignment [20] the same must hold true for the $\bar{K}N \to \pi\Sigma(1385)$ phase of $\Sigma(2030)$. We can therefore predict that the DD15 and FF17 amplitudes in $\bar{K}N \to \pi\Sigma(1385)$ will be out of phase with each other.

e. Additional predictions

If ℓ-broken $SU(6)_W$ survives a change in the experimental situation there are a number of important additional $\pi\Delta$ predictions it has to offer. In the first place a re-analysis of the 1690 MeV mass region should reveal - in addition to those amplitudes discussed above - prominent SD11 and DD33 amplitudes [7] with the signs as indicated in Fig. 2. This Figure also shows predictions for all members of the $\underline{56}$, $L^P = 0^+, 2^+$ and $\underline{70}$, $L^P = 0^+, 1^-, 2^+$ multiplets. The states in $\underline{70}$, $L = 2$ are expected to lie around 2 GeV in mass [15]. One state we expect to be prominent when the energy range of Ref. 2) is extended slightly is FF17. Its $\pi\Delta$ coupling should be substantial [15),21)] and it would provide excellent confirmation of the existence of the $\underline{70}$, $L = 2$

f. Possibility of predictions for $\pi N \rightarrow \rho N$

In principle our method can be applied to the process $\pi N \rightarrow \rho N$, since the π and ρ are in the same $SU(6)_W$ multiplet. The absence of explicit phase conventions in the published literature [2),16)] is all that has prevented us from such a discussion at present.

To conclude we have shown that the phases in $\pi N \rightarrow \pi\Delta$ provide important information about "ℓ-broken" $SU(6)_W$ and related symmetries. Given the present experimental situation, these schemes fail dramatically, with no particular pattern discernible in the failure. A possible exception is that most of the failure corresponds to states to the left of the dashed line in Fig. 1, which indicates a gap in the data. If this gap is filled and the solution remains as it is at present, we will have unambiguous proof for the failure of these symmetries.

We thank Anne Kernan and Jacques Weyers for valuable conversations. Roger Cashmore has been of invaluable importance in informing us of a recent discovery regarding the experimental phase conventions which significantly modified the conclusions drawn in an earlier version of our work.

A recent preprint by Gilman et al. [22] comes to conclusions basically similar to ours.

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FIGURE CAPTIONS

Figure 1 : Resonant phases in $\pi N \to \pi\Delta$ for cases which can be compared with experiment [Refs. 1), 2)]. Vertical dashed line indicates a gap in data, $1540\ \mathrm{MeV} \leq E_{cm} \leq 1650\ \mathrm{MeV}$.

a) Mixing should not affect class 1 or 2 predictions. Assignments for class 3 predictions are educated guesses.

b) To obtain the "$SU(6)_W$-like" solution, reverse all double-handed clocks.

c) Large interference between the two DS13 resonances makes experimental comparison difficult for the N(1730).

d) Only weakly resonant in Ref. 2). Non-resonant in Ref. 1).

Figure 2 : Prediction of resonant phases in $\pi N \to \pi\Delta$ for all likely $L = 0, 1, 2$ states below ~ 2 GeV ["anti-$SU(6)_W$" solution]

a) $L = 0$ states have no f-wave couplings in $SU(6)_W$.

b) Mixing does not alter the $\underline{70}$, $L^P = 1^-$ predictions, c.f., Ref. 7).

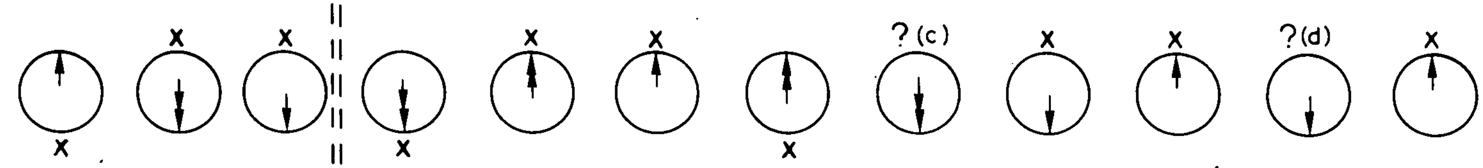
Partial wave (l_{in} l_{out} 2I 2J)	PP 11	DS 13	DD 13	SD 31	DS 33	DD 15	FP 15	DS 13	PP 11	FF 35	PP 31	FF 37
Associated resonance	N (1470)	N (1520)		Δ (1650)	Δ(1670)	N(1670)	N(1690)	N(1730)	N(1750)	Δ(1890)	Δ(1910)	Δ(1950)
Assignment[(a)]: $SU(6)_w$, L (SU(3), SU(2))	56 L=0 (8,2)	70 L=1 (8,2)		70 L=1 (10,2)	70 L=1 (10,2)	70 L=1 (8,4)	56 L=2 (8,2)	70 L=1 (8,4)	70 L=0 (8,2)	56 L=2 (10,4)	56 L=2 (10,4)	56 L=2 (10,4)
"anti-$SU(6)_w$" solution[(b)]												
Class of prediction (see text)	3	2	2	1	1	1	2	2	3	2	3	1

FIG. 1

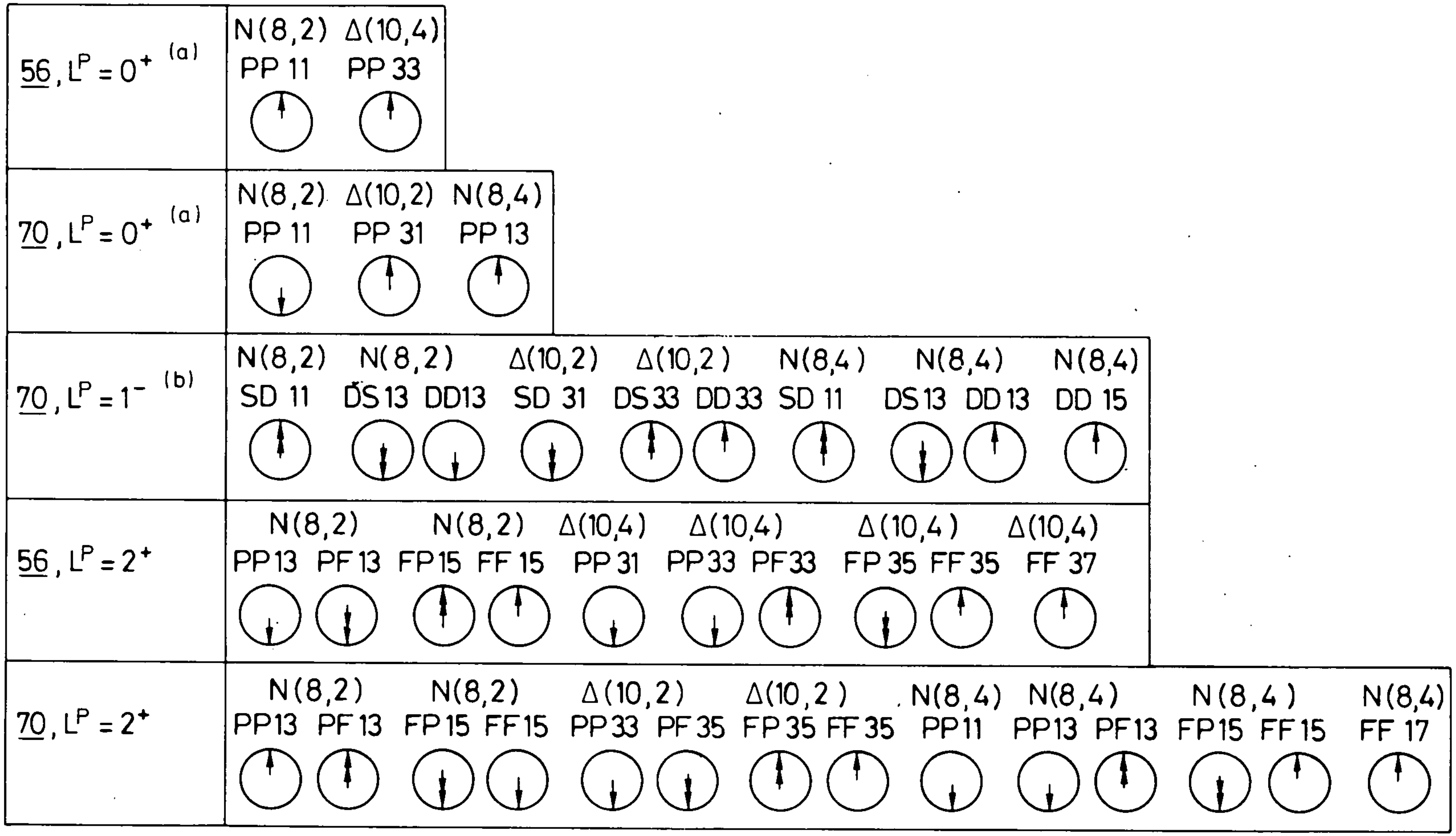

56, L^P = 0+ (a)
N(8,2) PP 11
Δ(10,4) PP 33
70, L^P = 0+ (a)
N(8,2) PP 11
Δ(10,2) PP 31
N(8,4) PP 13
70, L^P = 1− (b)
N(8,2) SD 11
N(8,2) DS 13 DD13
Δ(10,2) SD 31
Δ(10,2) DS 33 DD 33
N(8,4) SD 11
N(8,4) DS 13 DD 13
N(8,4) DD 15
56, L^P = 2+
N(8,2) PP 13 PF 13
N(8,2) FP 15 FF 15
Δ(10,4) PP 31
Δ(10,4) PP 33 PF 33
Δ(10,4) FP 35 FF 35
Δ(10,4) FF 37
70, L^P = 2+
N(8,2) PP 13 PF 13
N(8,2) FP 15 FF 15
Δ(10,2) PP 33 PF 35
Δ(10,2) FP 35 FF 35
N(8,4) PP 11
N(8,4) PP 13 PF 13
N(8,4) FP 15 FF 15
N(8,4) FF 17

FIG. 2