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SUBJECT: TEST OF APPR TYPE CONTROL ROD IN THE MTR

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CONFIDENTIAL COVER SHEET

TEST OF APPR TYPE CONTROL ROD IN THE MTR

I. Introduction

In order to better evaluate the absorber material proposed for the APPR-1 control rods, it has been proposed that one of the MTR cadmium rods be replaced with an APPR type rod. The APPR control rod consists of enriched boron-10 dispersed in electrolytic iron and clad with 304L type stainless steel.

The initial worth of the control rod can be determined experimentally. This report is concerned with the change in the worth of the control rod as a function of time during operation for both the MTR cadmium rod and the proposed APPR boron rod. The change in worth, or "blackness" of the rod, is determined in terms of the change in thermal neutron attenuation with burn-up of the absorber atoms.

II. Specifications

For the purpose of this report the following values have been assumed:

MTR Cadmium Control Rod

Thickness of cadmium per side	$t_{Cd} = 0.1016$ cm
Neutron cross section of Cd-113*	$\sigma_a^{Cd} = 19{,}500$ b
Density of cadmium	$d_{Cd} = 8.65$ g/cc
Abundance of Cd-113 isotope	12.3%
Density of Cd-113	$(d)_{Cd\text{-}113} = 1.064$ g/cc
Number of Cd-113 atoms per cc	$(N)_{Cd\text{-}113} = 5.67 \times 10^{21}$/cc

*It is assumed that the absorption cross section of all other isotopes of Cd are negligible compared to Cd-113.

APPR Boron Type Control Rod

Thickness of boron-iron matrix per side	$t_B = 0.2413$ cm
Neutron cross section of B-10	$\sigma_a^B = 3990$ b
Density of boron	$d_B = 2.3$ g/cc
Density of iron	$d_{Fe} = 6.9$ g/cc
Density of matrix (assuming 14% voids)	$d_M = 5.81$
Density of B-10	$(d)_{B-10} = 0.2001$ g/cc
Number of B-10 atoms per cc	$(N)_{B-10} = 1.206 \times 10^{22}$/cc

III. Analysis

A measure of a control rod's ability to absorb thermal neutrons can, for a given rod geometry, be obtained by calculating the attenuation of thermal neutrons through the absorber material. Equation 1 shows that for a given geometrical configuration the worth of a rod is a function of its thickness, number of absorber atoms per unit volume and the neutron cross section of the absorber.

$$R = \frac{I(t)}{I_o} = \frac{1}{1+\alpha} \left[\alpha e^{-\alpha} + (2 - \alpha^2 \int_0^1 e^{-\alpha/x} xdx \right] \qquad (1)$$

where: R = thermal neutron attenuation

$I(t)$ = number of neutrons passing through a unit area control rod.

I_o = number of incident neutrons per unit area of control rod.

$\alpha = N\sigma_a a = a\Sigma_a$

N = number of absorber atoms per unit volume of absorber material.

a = 2t

The derivation of Equation 1 is shown in Appendix I and the solution is plotted in Figure 1.

In order to compare the neutron attenuation of two different absorber materials in rods of different thickness after a given time in the reactor, it is necessary to make the comparison on the basis of equal number of absorber atoms destroyed rather than equal atom percent burn-up.

IV. Calculations

The thermal neutron attenuation through a new MTR control rod is determined by calculating the value of α and substituting into Equation (1).

Hence,

$$(\alpha)_{Cd-113} = \left[N \sigma_a a \right]_{Cd-113} = \frac{A_o d_{Cd-113} \sigma_a^{Cd} a_{Cd}}{A}$$

$$= \frac{(0.602 \times 10^{24})(1.064)(19{,}500 \times 10^{-24})(0.2032)}{113}$$

$$= 22.4$$

As the Cd-113 atoms are destroyed during operation the value of α will vary as shown in Table I.

TABLE I

VARIATION OF α WITH BURN-UP

Atom Percent Burn-Up of Cd-113	$(\alpha)_{Cd}$
0	22.40
10	20.16
20	17.92
30	15.68

The value for α for the APPR type control rod can be calculated in like manner.

$$(\alpha)_{B-10} = \frac{(0.602 \times 10^{24})(0.2001)(3990 \times 10^{-24})(0.4826)}{10}$$

$$= 23.2$$

The equivalent atom percent burn-up of the APPR rod for the same total number of absorber atoms destroyed is shown in Table II.

TABLE II

EQUIVALENT BURN-UP OF BORON ROD FOR SAME TOTAL NUMBER OF ABSORBER ATOMS DESTROYED

Atom Percent of Cd-113 Burn-up	Equivalent Atom Percent of B-10 Burn-up	$(\alpha)_{B-10}$
0	0	23.20
10	1.96	22.74
20	3.92	22.30
30	5.88	21.84

By substituting the values of α obtained in Tables I and II into Equation (1) a curve can be plotted which indicates the change in thermal neutron attenuation as a function of burn-up.

V. Discussion

Figure II indicates the change in thermal neutron attenuation as a function of Cd-113 burn-up. It can be seen that the rate of change in the MTR type rod is greater than in the APPR type rod.

Control rods of this type also exhibit the ability to absorb fast neutrons by a process of thermalization in the central region of the rod and absorption of the resulting thermal neutron in the rod matrix. As this is a function of the available water volume along the center of the rod and the rod's ability to absorb thermal neutrons, the APPR type rod can be expected to be equal to the MTR rod.

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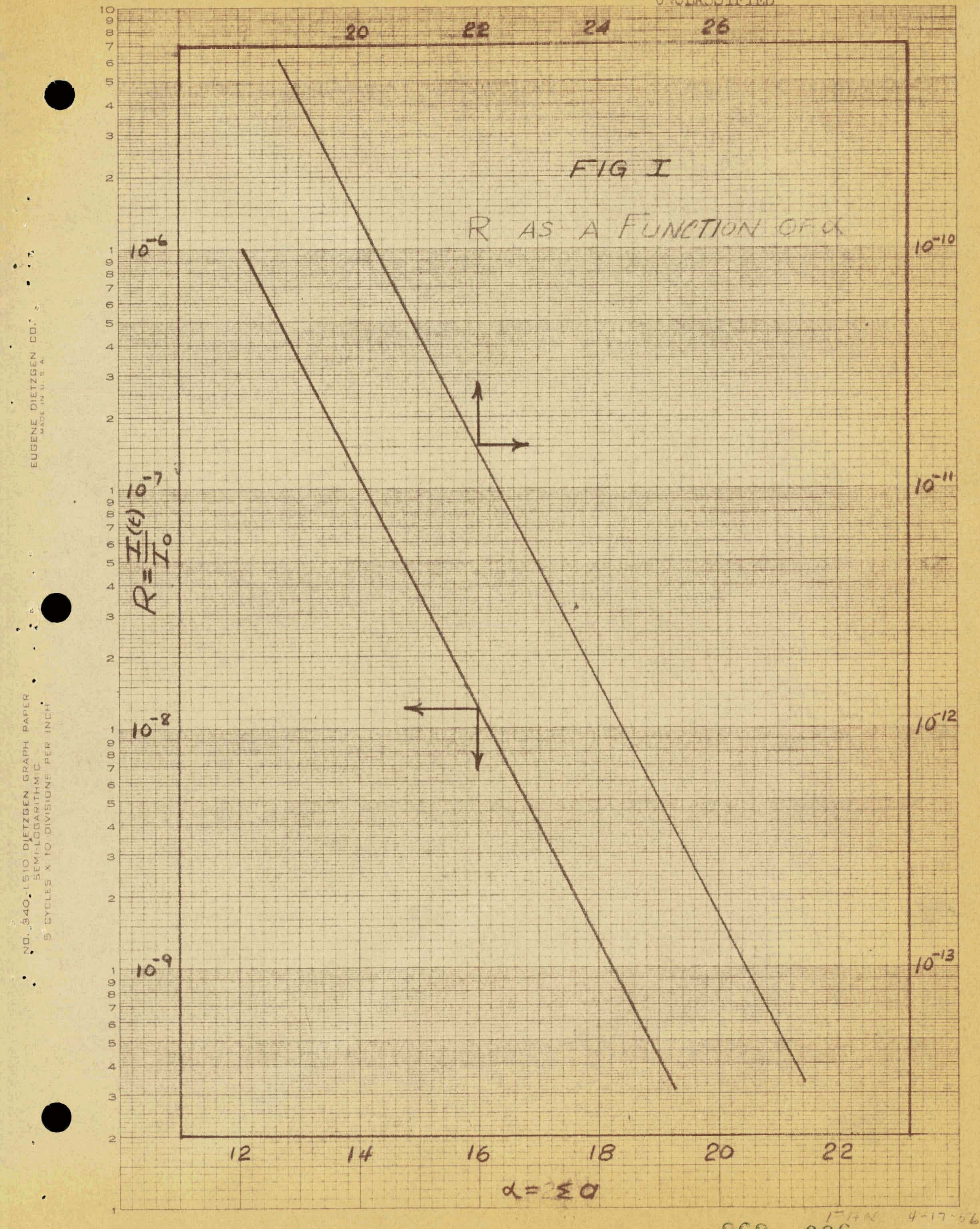

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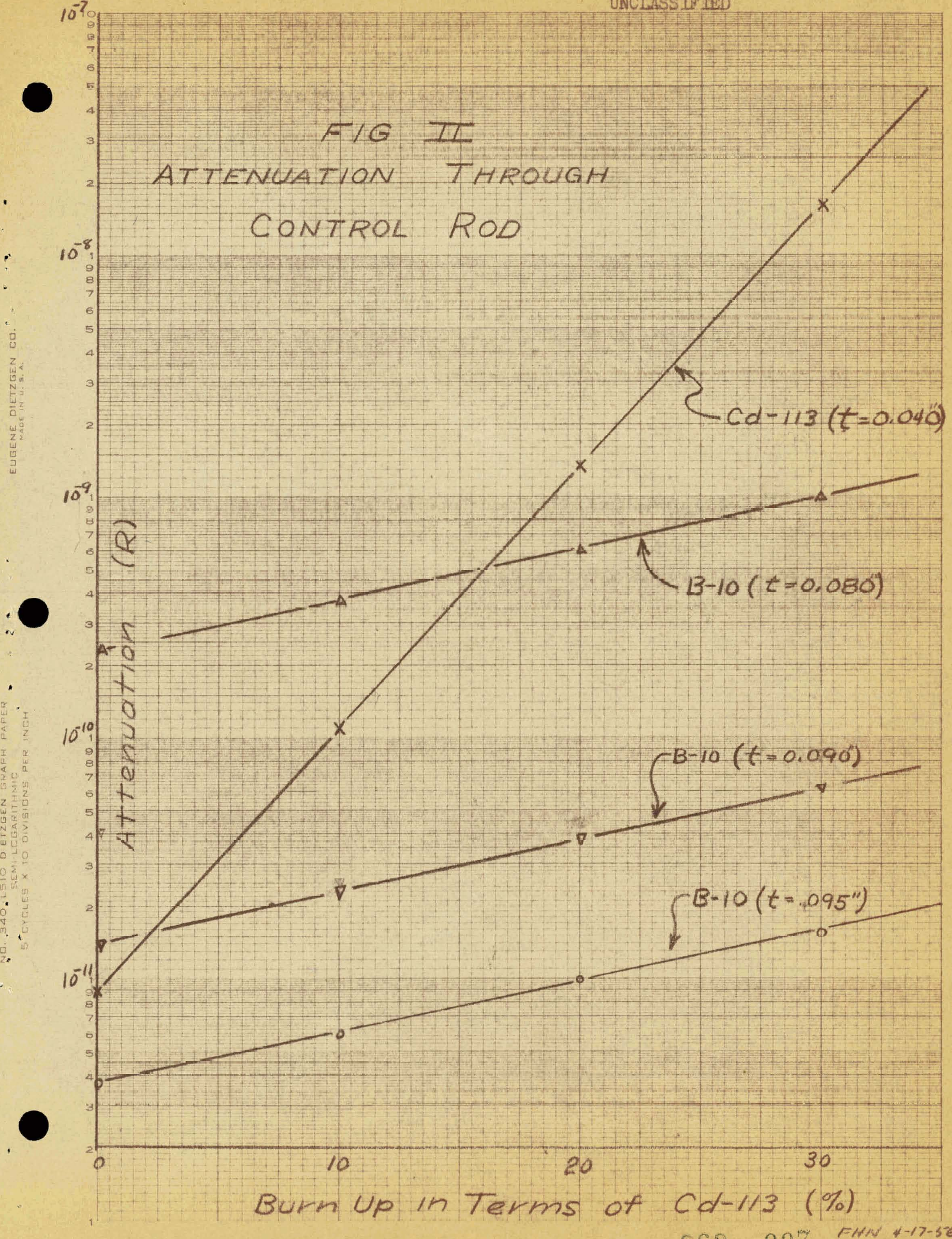

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APPENDIX I

TRANSMISSION OF NEUTRONS THROUGH ABSORBING MATERIAL IN A REACTOR

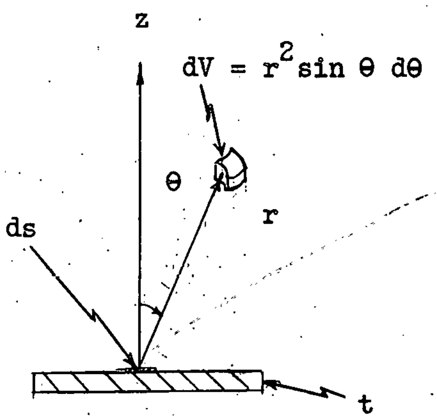

Figure III

Consider an absorber of thickness, t, within a reactor. One surface of the absorber is chosen at z = 0 (see Figure III). The flux near the absorber may be expressed as:

$$\phi_{(z)} = \phi_o + \left(\frac{\partial \phi}{\partial z}\right)_o z + \left(\frac{\partial^2 \phi}{\partial z^2}\right) z^2 + \ldots. \tag{1}$$

In the following treatment we will neglect terms of higher order than the first order and assume that the flux can be represented by,

$$\phi_{(z)} = \phi_o + \left(\frac{\partial \phi}{\partial z}\right)_o z = \phi_o + \left(\frac{\partial \phi}{\partial z}\right)_o r \cos \theta \tag{1}$$

If Σ_s is the macroscopic scattering cross section in the reactor material then $\Sigma_s \phi_{(z)} dV$ is the number of scattering events in the element of volume $dV = r^2 \sin \theta \, d\theta \, d\phi \, dr$. Assuming the scattering to be spherically symmetric, the number of neutrons scattered into an element of area of absorber, ds, will be proportional to the fraction of solid angle subtended by ds which is $\frac{ds \cos\theta}{4\pi r^2}$. The number of neutrons scattered towards ds by the element of volume $r^2 \sin\theta\, d\theta\, d\phi\, dr$ is then = $\frac{\Sigma_s \phi(z) \sin\theta \cos\theta\, d\theta\, d\phi\, dr ds}{4\pi r^2}$. However, this number is attenuated by the factor $e^{-\Sigma_s r}$ before reaching ds where the total cross section in the reactor material is assumed to be entirely scattering cross section. The number of neutrons arriving at the absorber per unit area per unit time between the angles θ and θ + dθ is then given by:

$$N(\theta)d\theta = \int_0^{\infty} \int_0^{2\pi} \Sigma_s e^{-\Sigma_s r} \frac{\phi(z) dr d\phi \sin\theta \cos\theta\, d\theta}{4\pi}$$

$$= 1/2\, \Sigma_s \int_0^{\infty} e^{-\Sigma_s r} \left[\phi_o + \left(\frac{\delta \phi}{\delta z}\right)_o r \cos\theta \right] dr \sin\theta \cos\theta\, d\theta$$

$$= 1/2 \left[\phi_o + \frac{1}{\Sigma_s} \left(\frac{\delta \phi}{\delta z}\right)_o \cos\theta \right] \sin\theta \cos\theta\, d\theta \qquad (2)$$

The net current into the absorber is given by:

$$\frac{1}{3\Sigma_s} \left(\frac{\delta \phi}{\delta z}\right)_o = \frac{A}{2} \qquad (3)$$

The quantity A is the number of neutrons absorbed per second per unit area of absorber and is given by:

$$A = \int_0^{-t} \Sigma_a \phi(z)\, dz \qquad (4)$$

Where Σ_a is the macroscopic absorption cross section of the absorbing material whose thickness is t.

In this treatment, the flux gradient term, $\left(\frac{\delta \phi}{\delta z}\right)_o$, is regarded as a correction term in Equation 2. The following approximation for the flux gradient is therefore used:

$$\left(\frac{\delta \phi}{dz}\right)_o = \frac{3}{2} \Sigma_s \int_o^{-t} \Sigma_a \phi(z)\, dz \approx \frac{3}{2} \Sigma_s \Sigma_a\, t\, \phi_o \qquad (5)$$

Substituting Equation (5) into Equation (2), the angular flux incident on the absorber becomes:

$$N(\theta) d\theta = \frac{1}{2} \left[\phi_o + \frac{3}{2} \Sigma_a t\, \phi_o \cos \theta \right] \sin \theta \cos \theta\, d\theta \qquad (6)$$

Assuming no scattering in the absorber, these neutrons would have a path length of $t/\cos\theta$ in the absorber. The flux incident on the absorber at the angle θ will then be attenuated by the amount $e^{-\Sigma_a t/\cos\theta}$. The ratio of the total transmitted flux to the total incident flux is then:

$$R(\alpha) = \frac{\int_o^1 \left[1 + \frac{3}{2}\alpha x\right] e^{-\alpha/x}\, x dx}{\int_o^1 \left[1 + \frac{3}{2}\alpha x\right] x dx} \qquad (7)$$

where $x = \cos\theta$ and $\alpha = \Sigma_a t$, Equation (7) may be reduced to the form:

$$R(\alpha) = \frac{1}{1+\alpha} \left[\alpha e^{-\alpha} + (2 - \alpha^2) \int_o^1 e^{-\alpha/x}\, x dx \right] \qquad (7')$$

The transmission as defined by Equation (7') is plotted against $\alpha = \Sigma_a t$ in Figure I.

May 1, 1956
mvj