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A THEORETICAL ANALYSIS OF HEAT TRANSFER IN TURBULENT CONVECTION

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ABSTRACT

This paper is an extension of a previous one (1). The two cases of fully developed turbulent flow passing a flat plate at zero incidence, and through a tube are considered. The laminar sublayer whose thickness is usually considered as constant at a given Reynolds number is postulated, in effect, to vary with the heat flow. The effect of natural convection is taken into account, despite its minor importance in predicting the heat transfer by forced convection in the turbulent regime. A general formula of Nusselt number is obtained as a function of Prandtl, Reynolds, and Grashof number. The heat transfer by natural convection alone becomes only a particular case and the Nusselt number is readily found by dropping out the term containing the Reynolds number. Calculated results agree excellently with experiments as conducted by previous investigators.

NOMENCLATURE

The following nomenclature is used in the paper:

A = a function of x

a = maximum amplitude of wave, ft

B = a function of x

c_p = specific heat at constant pressure Btu/(lb) (deg. F)

d = diameter of tube, ft

e = base of natural logarithms

F = force per unit mass ft/(hr)^2

f = friction factor, $\tau_o/(\rho J^2/2)$ for plate

g = gravitational acceleration ft/(hr)^2

Gr_x = Grashof number with respect to x

h = average film coefficient of heat transfer from a plate, Btu/(hr)(ft^2)(deg F)

h_x = local film coefficient of heat transfer from a plate, Btu/(hr)(ft^2)(deg F)

h_d = film coefficient of heat transfer from a tube Btu/(hr)(ft^2)(deg F)

j = Colburn's factor of heat transfer

k = thermal conductivity, Btu/(hr)(ft^2)(deg F/ft)

L = total length of the plate, ft

p = pressure, lb/ft^2

Pr = Prandtl number

q = rate of heat flow, Btu/(hr)(ft^2)

St = Stanton number, Nu/(RePr)

T = temperature, deg. F

t = time, hr

u = turbulent velocity in x-direction, ft./hr

u_1 = maximum turbulent velocity in x-direction, ft/hr

$\bar{u}$ = velocity of mean flow in x-direction, ft./hr

$\bar{u}_1$ = $u_1 + \bar{u}$, maximum instantaneous velocity in x-direction, ft/hr

u' = convective velocity in x-direction, ft./hr

U = free stream velocity in x-direction, ft/hr

U_m = mean velocity of flow through tube ft/hr

$\bar{v}$ = velocity of mean flow in y-direction ft/hr

x,y = coordinates

α = thermal diffusivity $(ft)^2/(hr)$

β = thermal coefficient of expansion

φ = velocity ratio

ν = kinematic viscosity $(ft)^2/(hr)$

ρ = density, pcf

θ = temperature of difference, deg. F

δ = thickness of boundary film, ft.

δ_b = thickness of laminar sublayer, ft.

τ_o = shear stress at the boundary wall, $lb/(ft^2)$

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INTRODUCTION

In a previous paper, reference (1), the concept of wave motion has been used for the prediction of the heat transfer coefficient of natural convection, with and without boiling, over a horizontal surface. In the present paper this concept is extended in an attempt to obtain a general formula for heat transfer for both natural and forced convection in the turbulent regime. A two-dimensional flow over a smooth flat plate at zero incidence is considered, and its result is extended immediately to flow through a smooth tube.

When fluid originally at rest is heated from below, Benard (2), Rayleigh (3) and others have shown that a stable cellular wave exists in a layer adjacent to the heating surface. Many researchers have considered that this simple form of wave is also present in a fluid heated from above (4). Ostrach (5) opines that the cellular wave would represent only the second or final stage of motion development for fluid heated from below. With the first phase of motion development still unknown, a hydrodynamic wave was assumed, in reference (1), to represent this first phase and a satisfactory result was obtained for natural convection and for boiling.

In laminar flow at comparatively low speed this kind of cellular structure has also been found by experiments (6), (7).

Turbulent flow in tubes has been examined experimentally by Fage and Towneed (8). The maximum values, u_1, v_1, w_1, of u, v, w, the three components of the turbulent velocity in axial (x), normal(y) and tangential (z) directions of the tube were measured. It was found that though u_1, v_1, w_1 become zero at the wall, yet $u_1/\bar{u}$, where $\bar{u}$ is the mean velocity, tends to become constant, $v_1/\bar{u}$ decreases to zero and $w_1/\bar{u}$ increases to a maximum as the wall surface is approached. While the flow tends to the laminar type at the wall, the motions of the particles in the laminae are sinuous. No particle is seen to move in a rectilinear path. The constancy of $u_1/\bar{u}$ and the sinuous motion of the particles near the wall are very striking features. Though it may be a good premise that a wave motion may exist in the so called laminar sublayer, its nature is still unknown, and, therefore, for convenience it will be only referred as "wave motion" in the following paragraphs.

In viscous flow, vorticity must arise from the wall and sheets of vortices are formed. A vortex sheet, in fact, is unstable and will roll up in the manner shown in Fig. 1, as a result of the joint effects of the natural convection and of the mean flow.

The universally accepted physical picture of heat transfer in turbulent flow is that of pure conduction in the laminar sublayer, conduction and convective mixing in the transition zone and a predominance of turbulent convective mixing in the turbulent core. In this paper this picture is also accepted, except that in the laminar sublayer the motion is postulated to consist of three parts, a mean part which is laminar, a turbulent part and a convective part which are periodic. In the present analysis, the effective thickness of the laminar sublayer, which is usually assumed constant at a given Reynolds number, will vary with the heat flow. Therefore, in the following paragraphs the term "sublayer" refers only to that of isothermal flow, while the effective sublayer is called "boundary film" and is designated by δ . Inside this film and somewhere above it, a periodic motion is postulated to exist. At a certain distance from the wall this motion will be so much deformed that rolls of vortex starts to form. The region between this distance and the boundary film is here termed the "wave layer" and is denoted by a. Following the same reasoning in reference (1), the magnitude of a should be approximately equal to that of δ. The combined boundary film and wave layer is called the "gross boundary film".

The method of approach used in this paper is a little different from that in the previous one, although the fundamental concept remains unaltered. Of course, if the method used in the previous paper is followed, the same results can also be obtained. The author feels, however, the present approach has the following advantage: The analysis will be based simply on a layer of vortex rolls in the wave layer and will not require the postulation of a particular type of wave motion. In this paper only two-dimensional flow at moderate velocity is considered, and thus the effects of compressibility and the energy of dissipation can be neglected. The model postulated may be summarized as follows:

1. The motion of fluid near the wall is assumed as to follow a sequence of development: laminar, sinuous, vortex rolls and turbulent. The heat flow will destabilize the laminar motion and, thus, will shorten proportionally the distance of this transition. To facilitate the analysis, the sinuous motion as well as the vortex rolls are assumed as regularly, but not necessarily uniformly, distributed along the direction of flow.
2. The rotating speed of the vortex rolls depends on the velocity gradient of the mean flow which is affected by the heat being transferred. The higher this rotating speed, or the vorticity, the higher will be the velocity gradient, and also the temperature drop across the boundary film. Thus, a direct portionality between the velocity and the temperature drop at the boundary film will be established.

It should be noted that engineering science so far has not achieved a complete understanding of the mechanism of turbulent motion, especially with heat transfer, because of its extremely complicated nature. All that can be done is to make plausible assumptions about the velocity distribution in the boundary film and thus to estimate a corresponding temperature distribution. If the calculated results of heat transfer are comparable with experimental data, the assumptions would be justified.

VELOCITY DISTRIBUTION ACROSS THE GROSS BOUNDARY FILM

The von Karman universal velocity distribution for turbulent flow is generally considered as the best available today, but its validity in the sub-layer remains uncertain, especially when there is heat being transferred.

Lees and Lin (9) have found that if heat is being transferred from the wall to the fluid, the stability of the laminar boundary layer in compressible flow is decreased, that is, heat transfer has a destabilizing effect compared with the case of no heat transfer. It can be taken for granted that this conclusion applies also to incompressible flow.

The von Karman velocity fields in the transition zone and in the laminar sublayer are, respectively as follows:

$$\bar{u}^* = 5\left[1 + \ln(y^*/5)\right]$$

$$\bar{u}^* = y^*$$

where $\bar{u}^* = \bar{u}/\sqrt{(\tau_0/\rho)}$ and $y^* = (y\sqrt{\tau_0/\rho})/\nu$, with $\bar{u}$ denoting the mean velocity, τ_0 the shear stress at the wall, ν the kinematic viscosity, and ρ the density of the fluid.

The mean flow in the laminar sublayer is by definition laminar and $\bar{u}$, $\bar{v}$, the components of velocity in the x-direction along the wall and in the y-direction normal to the wall will satisfy the following equation, since $\partial^2\bar{u}/\partial x^2$ is negligibly small in comparison with $\partial^2\bar{u}/\partial y^2$. In current practice

$$\nu\frac{\partial^2\bar{u}}{\partial y^2} = \bar{u}\frac{\partial\bar{u}}{\partial x} + \bar{v}\frac{\partial\bar{u}}{\partial y}$$

the velocity gradient is considered as constant, or, in other words, the terms in the right side of the above equation are omitted. This approximation gives a discontinuous point at the border between the laminar sublayer and the transition zone. Since a parabolic curve generally represents the velocity distribution in laminar motion, a function of x may be assumed as a substitute for these non-linear terms. This assumption is justified in view of the fact that the velocity profiles at different positions along x-direction are congruent curves. Along this line of reasoning, the velocity distribution across the laminar sublayer may be expressed by the form of

$$\bar{u} = (A\,\delta_b - By)\,y \tag{1}$$

where A and B are functions of x only and are to be determined by proper boundary conditions. The velocity gradient at $y = 2\delta_b$ will be

$$\left.\frac{\partial \bar{u}}{\partial y}\right]_{y=2\delta_b} = \frac{5}{2}\sqrt{\frac{\tau_0}{\rho}}\,\frac{1}{\delta_b} \tag{2}$$

and at the boundary surface, that is at $y = 0$,

$$\left.\frac{\partial \bar{u}}{\partial y}\right]_{y=0} = \frac{\tau_0}{\rho\nu} \tag{3}$$

Applying equations (2) and (3) to equation (1), the functions A and B are obtained as follows:

$$A = \frac{\tau_0}{\rho\nu}\,\frac{1}{\delta_b} \;; \qquad B = \frac{1}{4}\sqrt{\frac{\tau_0}{\rho}}\left(\sqrt{\frac{\tau_0}{\rho}}\,\frac{1}{\nu} - \frac{5}{2\delta_b}\right)\frac{1}{\delta_b} \tag{4}$$

substituting this into equation (1) yields

$$\bar{u} = \frac{\tau_0}{\rho\nu}\,y - \frac{1}{4\delta_b}\sqrt{\frac{\tau_0}{\rho}}\left(\sqrt{\frac{\tau_0}{\rho}}\,\frac{1}{\nu} - \frac{5}{2\delta_b}\right)y^2 \tag{5}$$

By the basic assumption 2, the component u_1 of the turbulent velocity in the sublayer should be of similar form as equation (5). Since, however, u_1 has to vary with the heat transfer, the effective thickness should be substituted by $(\delta + a)$, the thickness of the gross boundary film, which will vary with the heat flow and be determined later. Therefore, at the upper surface of the gross boundary film where $y = \delta + a = 2\delta$, the mean velocity may be written as

$$\bar{u}\Big]_{y=2\delta} = \frac{\tau_0}{\rho\nu}\delta + \frac{5}{2}\sqrt{\frac{\tau_0}{\rho}} \qquad (6)$$

and at the upper surface of the boundary film the velocity is

$$\bar{u}\Big]_{y=\delta} = \frac{3}{4}\frac{\tau_0}{\rho\nu}\delta + \frac{5}{8}\sqrt{\frac{\tau_0}{\rho}} \qquad (7)$$

Now, consider the convective motion due to the bouyant force. Since the average thickness of the gross boundary film is represented by the distance from the wave node to the wall, Figure 1, and since the velocity across the plane perpendicular to the wall through this point can be easily determined, the study of motion is, therefore, centered in this vertical plane. Following the same reasoning as given in reference (1) and considering the wave node as a relative singular point of m otion, the horizontal component of convective velocity at $y = \delta$ is given by

$$u'\Big]_{y=\delta} = \pm\frac{3}{2\pi}\frac{g\beta\,\theta_{ac}\,\delta^2}{\nu} \qquad (8)$$

where β denotes the thermal expansion coefficient of the fluid at the mean temperature of the transition zone and the gross boundary film, and $\theta_{ac} = T_a - T_c$, the mean temperature difference between these two regions.

THICKNESS OF BOUNDARY FILM

The thickness of the boundary film can be similarly obtained by following the procedure given in reference (1), that is by adding equations (6) and (8) to get the resultant relative velocity and equating it to the velocity of a fluid particle induced by the wave, if such a wave were assumed to exist. An alternate method, however, is introduced here which is not only simpler but also obviates the arguments in connection with the exact type of wave motion, as mentioned previously.

It is obvious that the heat flows into the gross boundary film mainly by conduction and then is transported by lumps of fluid, or the vortex rolls, to the so called transition zone. Therefore, the fluid is actually heated suddenly and periodically within each wave length, and detailed heat-flow in each wave length is at an unsteady condition. When the whole system is considered, the average penetrating depth of heat by conduction will be a definite value for a given set of heating and flow conditions. The size and the angular velocity of the vortex rolls must be of such magnitudes as to be just capable of carrying away the heat transmitted by conduction through the boundary film. This statement will be true if the thermal conductivity of the fluid is not so large as the conduction heat would penetrate beyond the layer of vortex rolls. Thus, it will not apply to the case of liquid metals which have very large thermal conductivities.

By following the same procedure of reference (1), the time required to heat up the fluid to a depth of the gross boundary film at temperature T_0 is,

$$t = \frac{\delta^2}{4\alpha}$$

where α is the thermal diffusivity of the fluid at the mean temperature of the gross boundary film. This should be the time required to displace the heated fluid inside the wave layer by the cooler fluid in the transition zone.

The vortex rolls would have to make a complete rotation within this period of time. It is obvious that the lumps of fluid will tend to form rolls around the wave node downstream.

Then the angular velocity of the vortex rolls is

$$\omega = \frac{1}{2\delta}\left[2(\bar{u}_{,2\delta} - \bar{u}_{,\delta}) + u'_{\delta}\right] \tag{9}$$

where the subscripts 2δ and δ indicate the positions of the level. By the foregoing reasoning the time required for a complete circulation must be equal to the time given by equation (9).

$$\frac{2\pi\delta}{2(\bar{u}_{,2\delta} - \bar{u}_{,\delta}) + u'_{\delta}} = \frac{\delta^2}{4\alpha} \tag{10}$$

Substituting values of $\bar{u}_{,2\delta}$, $\bar{u}_{,\delta}$ and u'_{δ} from equations (6), (7) and (8) into equation (10) yields an expression by which the thickness of the boundary film can be calculated. Equation (10), then, becomes

$$\frac{\pi\delta}{\frac{1}{4}\left(\frac{\tau_0}{\rho\nu}\delta + \frac{15}{2}\sqrt{\frac{\tau_0}{\rho}}\right) + \frac{3}{4\pi}\frac{g\beta\theta_{ac}}{\nu}\delta^2} = \frac{\delta^2}{4\alpha} \tag{11}$$

However, a simple expression for the thickness δ can not be obtained very easily from this cubic algebraic equation. Assuming the view that in the regime of turbulent flow, the effect of convective motion is usually insignificant in comparison with the steady motion, the second term in the denominator of the left side of equation (11) may be temporarily disregarded. With this simplification, the thickness of the boundary film is obtained as

$$\delta = \frac{15}{4}\nu\sqrt{\frac{\rho}{\tau_0}}\left(\sqrt{1 + \frac{3.57}{Pr}} - 1\right) \tag{12}$$

The general practice is to employ a dimensionless quantity, f, called the "Fanning" friction factor, instead of using the wall shear stress. It represents the resistance force of the plate divided by the area of the surface and the dynamic pressure of the flow, that is $f = \tau_0/(\rho U^2/2)$. Thus, equation (12) can be expressed alternatively by

$$\delta = \frac{15}{4}\frac{\nu}{U}\sqrt{\frac{2}{f}}\left(\sqrt{1+\frac{3.57}{Pr}}-1\right) \qquad (13)$$

Now, by the aid of equations (7) and (13) the velocity gradient in the boundary film can be obtained with good approximation as

$$\frac{\bar{u}_{\delta}}{\delta} = \frac{\tau_0}{\rho\nu}\left(\frac{3}{4}+\frac{1}{6}\frac{1}{\sqrt{1+\frac{3.57}{Pr}}-1}\right) \qquad (14)$$

It is seen that the velocity gradient near the heated wall increases with an increase of the Prandtl number. This result is very encouraging, because a complete similarity between the temperature and velocity ratios may be obtained in the boundary film as will be seen later.

Equations (12) and (13) were obtained under the condition that the convective velocity can be neglected. This simplification, however, may lead to some excessive error when the temperature difference between the heating surface and the fluid is very high and the Reynolds number is not sufficiently large. If this is the case, the value of δ may be conveiniently obtained from equation (11) by the method of iteration. To show this procedure, equation (11) is written in the following form:

$$\frac{\pi}{\frac{1}{4}\left\{\sqrt{\frac{\tau_0}{\rho}}\left(\sqrt{\frac{\tau_0}{\rho}}\frac{1}{\nu\delta}+\frac{15}{2}\frac{1}{\delta^2}\right)+\frac{3}{\pi}\frac{g\beta\theta_{ac}}{\nu}\right.} = \frac{\delta^3}{4\alpha} \qquad (15)$$

To determine the value of δ an approximate value of δ is substituted into the right side of equation (15) and the new value of δ is evaluated. This new value of δ is then substituted into equation (15) and the process is repeated until the values of δ in both sides of equation (15) do not change appreciably. In fact, if the value of δ in equation (13) is used as a first approximation and inserted into the right side of equation (15) the calculated value of δ

should be quite close to the exact value, because of the minor role of the convective velocity in turbulent flow. Thus

$$\frac{\delta}{x} = \frac{(16\,\pi)^{\frac{1}{3}}}{\left[\frac{f}{15}\left(\frac{f}{2}\right)^{\frac{1}{2}} \frac{Re_x^3}{\sqrt{1+\frac{3.57}{Pr}}-1}\left(1+\frac{1}{\sqrt{1+\frac{3.57}{Pr}}-1}\right)+\frac{3}{\pi}Gr_x\right]^{\frac{1}{3}} Pr^{1/3}} \tag{16}$$

In the case of natural convection, the Reynolds number vanishes and equation (16) reduces to that given in reference (1).

HEAT TRANSFER COEFFICIENT

When the thickness of the boundary film is known, the heat transfer rate can be predicted by considering it as pure conduction through the boundary film; for this film is very thin and, therefore, the heat-flow through it will change by only a negligible amount due to the fluid motion. Comparing the Fourier's equation of heat conduction and "Newton's law of cooling" the local film coefficient will be

$$h_x = \frac{k}{\delta}\,\frac{\theta_{oa}}{\theta_{os}} \tag{17}$$

where the subscripts s, and a indicate the conditions of free stream and of the boundary film, respectively. Since the influence of heating has already been taken into account in the determination of the velocity, u_δ, at the boundary film it would be advisible, by the basic assumption 2, to take θ_{oa}/θ_{os} as equal to $\bar{u}_\delta/U$. This can be evidenced by the fact that $\bar{u}_\delta/_{\delta U}$ is approximately equal to $\theta_{oa}/\theta_{os\delta}$ as given by Martinelli (10) for considerable ranges of Reynolds and Prandtl numbers. Substituting u_δ/U for θ_{oa}/θ_{os} in eg. (17) will yield the local heat transfer coefficient for flow over a flat plate at zero incidence with a turbulent boundary layer,

$$h_x = k_{oa} \frac{\tau_0}{\rho} \frac{1}{\nu U} \left(\frac{3}{4} + \frac{1}{6} \frac{1}{\sqrt{1+\frac{3.57}{Pr}} - 1} \right) \tag{18}$$

and the local Nusselt mumger is

$$Nu_x = \frac{x h_x}{k_{oa}} = \frac{f}{2} Re_x \left(\frac{3}{4} + \frac{1}{6} \frac{1}{\sqrt{1+\frac{3.57}{Pr}} - 1} \right) \tag{19}$$

For a smooth plate the Blasius formula (11) for shear stress at the wall may be used,

$$\tau_0 = 0.0296 \frac{\rho U^2}{Re_x^{0.2}}$$

Eq. (18) and (19), then, become

$$h_x = \frac{k_{oa}}{x} \left(0.0221 + \frac{0.00495}{\sqrt{1+\frac{3.57}{Pr}} - 1} \right) Re_x^{0.8} \tag{20}$$

$$Nu_x = \left(0.0221 + \frac{0.00495}{\sqrt{1+\frac{3.57}{Pr}} - 1} \right) Re_x^{0.8} \tag{21}$$

The average heat transfer coefficient for a flat plate of length L should be determined as follows:

$$h(\theta_{os})_{mean} = \frac{1}{L} \int_0^L h_x (\theta_{os})_x \, dx$$

When the wall temperature is constant, however, the average heat transfer coefficient is obtained by simply integrating Eq. (20) with respect to x and dividing by L.

$$h = \frac{1}{L} \int_0^L h_x \, dx = \frac{5}{4} h_L \tag{22}$$

and the average Nusselt number is

$$Nu = \left(0.0273 + \frac{0.0061}{\sqrt{1+\frac{3.57}{Pr}} - 1} \right) Re_L^{0.8} \tag{23}$$

Eq. (21) and von Karman's equation (12) for a flat plate are plotted in Fig. 2. They agree fairly well for Prandtl numbers from 1.0 to 10. All physical properties are evaluated on the basis of the mean bulk temperature of the fluid, except that the thermal diffusivity, α , is to be estimated according to the mean film temperature,

Eq. (19) can be readily adapted to flow through tubes. However, in dealing with heat transfer from tubes, it is generally referred to the bulk temperature of the flowing fluid while in Eq. (19) the temperature of the free stream corresponds to the temperature at the tube axis. Moreover, in practical problems the Reynolds number is calculated generally in terms of mean velocity, U_m, instead of the axial velocity. This, however, can be easily corrected by introducing the velocity and temperature ratios as given by the following relations

$$\varphi = U_m / U_s \quad ; \qquad \vartheta' = (T_s - T_B)/(T_s - T_o)$$

Then the Nusselt number for tubes is

$$Nu_d = \frac{dh}{k_{oa}} = \frac{f}{2}\frac{\varphi}{\vartheta'}\left(\frac{3}{4} + \frac{1}{6}\frac{1}{\sqrt{1+\frac{3.57}{Pr}} - 1}\right) Re_d \qquad (24)$$

The velocity ratio, φ , is obtained from a logarithmic velocity distribution and theoretically equals 4/5, reference (13). Experiments (14), however, show that it is a function of Reynolds number and is represented by Fig. 3.

For constant wall temperature, Boelter, Martinelli and Jonassen (15) calculated the temperature difference ratio, ϑ' , and showed that it varies very little with the Reynolds number but increases with an increase of Prandtl number, as is reproduced in Fig. 4.

Using values of φ and ϑ' in Figs. 3 and 4, and taking values of f from the friction factor chart, such as that given by reference (16), the calculated results of Eq. (24) are shown in Fig. 5. Von Karman's equation and Colburn's formula (17) are also plotted for comparison. It is seen that Eq. (24) agrees pretty well with Colburn data for Prandtl numbers from 1.0 to 40, but yields a slightly lower value of Stanton's number. However, further improvement of Eq. (24) can be readily made.

It should be recalled that Eq. (24) was obtained from Eq. (15) which was a first approximation in the determination of the boundary film thickness by neglecting the effect of natural convection. In order to get a closer approximation, Eq. (17) should be used. In the case of flow passing a flat plate, the Nusselt number is

$$Nu_x = \frac{3}{4}\frac{f}{2}Re_x + \frac{5}{8}\frac{1}{(16\pi)^{1/3}}\sqrt{\frac{f}{2}}\left[\frac{8}{15}\left(\frac{f}{2}\right)^{\frac{3}{2}}\frac{Re_x^3}{\sqrt{1+\frac{3.57}{Pr}}-1}\left(1+\frac{1}{\sqrt{1+\frac{3.57}{Pr}}-1}\right)+\frac{3}{\pi}Gr_x\right]^{\frac{1}{3}}Pr^{\frac{1}{3}} \qquad (25)$$

Employing the coefficients, φ and ϑ' will transform Eq. (25) into an equation for flow through tubes,

$$Nu_d = \frac{3}{4}\frac{\varphi}{\vartheta'}\frac{f}{2}Re_d + \frac{5}{8}\frac{1}{(16\pi)^{1/3}}\frac{1}{\vartheta'}\sqrt{\frac{f}{2}}\left[\frac{8}{15}\left(\frac{f}{2}\right)^{\frac{3}{2}}\frac{\varphi^3 Re_d^3}{\sqrt{1+\frac{3.57}{Pr}}-1}\left(1+\frac{1}{\sqrt{1+\frac{3.57}{Pr}}-1}\right)+\frac{3}{\pi}Gr_d\right]^{\frac{1}{3}}Pr^{\frac{1}{3}} \qquad (26)$$

If, now, Grashof's number is neglected in Eq. (26), a simple formula similar to Colburn's (17) is obtained:

$$Nu_d = \frac{3}{4}\frac{\varphi}{\vartheta'}\psi\frac{f}{2}Re_d\, Pr^{\frac{1}{3}} \qquad (27)$$

where ψ is a function of Prandtl number and is equal to

$$\psi = \frac{1}{Pr^{1/3}} + \frac{0.182}{\left(\sqrt{1+\frac{3.57}{Pr}}-1\right)^{1/3}}\left(1+\frac{1}{\sqrt{1+\frac{3.57}{Pr}}-1}\right)^{\frac{1}{3}} \qquad (28)$$

Using Colburn j factor of heat transfer, an alternative form of Eq. (29) is

$$j = St\, Pr^{\frac{2}{3}} = C\frac{f}{8} \qquad (29)$$

where f is the friction factor for tubes and is defined, as usual, equal to

$\tau_0/(\rho U_m^2/8)$, and $C = \frac{3}{4}\varphi\psi/\vartheta'$. It is seen that C is not a constant of unity as generally adopted in emperical formulas, but it varies very little from unity in the practical ranges of Reynolds and Prnadtl numbers. The factor, $(3\psi)/(4\vartheta')$ is plotted against Pr. in Fig. 6. Therefore, in practical design Eq. (27) should be used in conjunction with Figs. 3 and 6. Predicted magnitudes of Nusselt number from Eq. (29) are plotted against experimental magnitudes of Eagle and Ferguson (18) in Fig. 7; they agree extremely well.

CONCLUSION

The present analysis has restricted the application of Eq. (27) and others to fluids with Prandtl numbers within the range of 0.60 to 40, in comparison with the Colburn's equation. Unfortunately, experimental data are very limited for higher Prandtl numbers, and it is, therefore, difficult to conclude definitely the upper limit of application of Eq. (27).

It is to be noted that as long as the thickness of the gross boundary film is less than that of the hydrodynamic laminar sublayer, the original postulated model will hold true. In other words, the validity of the above analysis applies only for fluids whose Prandtl number is larger than a certain value which is calculated below:

When the condition that the thickness of the gross boundary film is to be smaller than that of the laminar sublayer, the following inequality should hold,

$$43.6\left(\sqrt{1+\frac{3.57}{Pr}}-1\right)\frac{x}{Re_x^{0.9}} \leq \delta_b$$

To make a rough estimation for the hydrodynamic laminar sublayer, the von Karman seventh root law may be used, and $\delta_b \cong 72/Re_x^{0.9}$ (11) can be obtained for a smooth plate. It follows, therefore, that the Prandtl number should not be less than 0.60, beyond which the above analysis will not apply.

Calculation from Eq. (17) for a Prandtl number of 50 and a Reynolds number of 10^4 will show that the boundary film has become too thin. Further decrease of this thickness will be intolerable to reality. It may be that the boundary film thickness has reached the minimum magnitude at Prandtl numbers higher than 50. The increase of heat transfer with an increase of Reynolds number is probably due to the increase of angular velocity of the vortices, that is the assumption of $\delta = a$ would lose its generality at very high Prandtl numbers.

Eq. (25) is a general formula of heat transfer in turbulent convection. When there is no forcing flow, the term consisting of the Reynolds number will vanish and Eq. (19) reduces to that of natural convection over a horizontal plate as given in reference (1).

This method of analysis has been extended to the solution of heat transfer problems in forced convection with boiling. It was pointed out in reference (1) that heat transfer in film boiling can be readily solved through the application of wave motion in the boundary surface of liquid and vapor, provided an equivalent thermal diffusivity is obtained. These, however, will not be discussed here and will be presented by separate papers.

During the preparation of this paper for publication, Mr. S. de Soto of the University of California, Los Angeles, has informed the author that he has witnessed tests in North America Aviation Corporation involving high heat transfer rates across tube walls which created a very strong, high-frequency vibration. "These vibrations were common and occurred consistantly when similar test conditions were repeated over and over again. The fluid flowing through the tubes was a hydrocarbon flowing at a pressure far above its critical pressure, and hence the vibration phenomenon could not have been due to effects of local boiling. The vibrations occurred every time the heat flux was such that the tube wall temperature (for this particular fluid) reached 850° F or higher. Although the cause of the vibration was not known in detail, it was definitely established that it was not affected by the flow system and seemed to be a function of the temperature and the type of fluid being heated. Under prolonged exposure to these vibrations, tubes developed small longitudinal splits. Another fluid, which was tested, produced these vibrations whenever the wall temperature reached 700° F. When the velocity of the fluid into the heated section was changed, the vibrations occurred again at the same wall temperature but at a different value for the heat flux."

While the author has not found time to follow up these tests and to investigate them in detail, he is inclined to believe that this phenomenon of strong vibration should be closely related to that of resonance, the natural frequency of the tube wall being the same as the average frequency of the oscillating fluid in the boundary film.

As a matter of interest, a rough calculation of this frequency is made here according to the limited data given by Mr. de Soto: Measured frequency is in the order of thousands cycles per second, from 2,000 to 8,000. $Pr = 23 \sim 24$, $Re = 5 \times 10^4$, $C_p = 0.50$ Btu/(lb)(°F), $k = 0.87$ Btu/(hr)(ft^2)(deg F/ft), $T_o = 850°$ F, $T_B = 150°$ F Sp. gravity = 0.81. If these data are used in Eq. (9) of this paper, the calculated frequency will be in the order of 4000 cycles per second, which is pretty close to the average value of the test data.

ACKNOWLEDGEMENT

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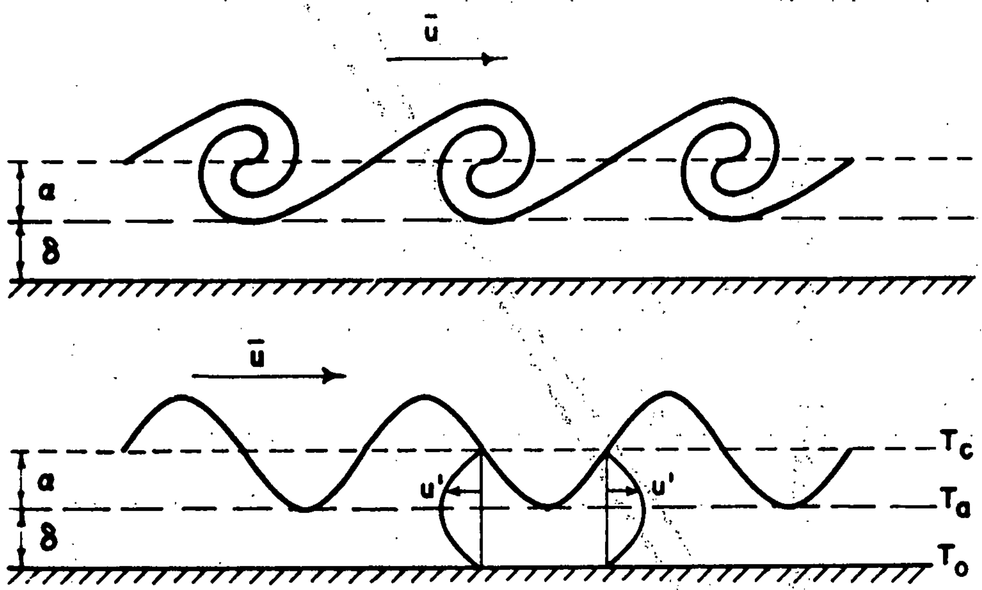

Fig. 1 Vortex Rolls in the Gross Boundary Film.

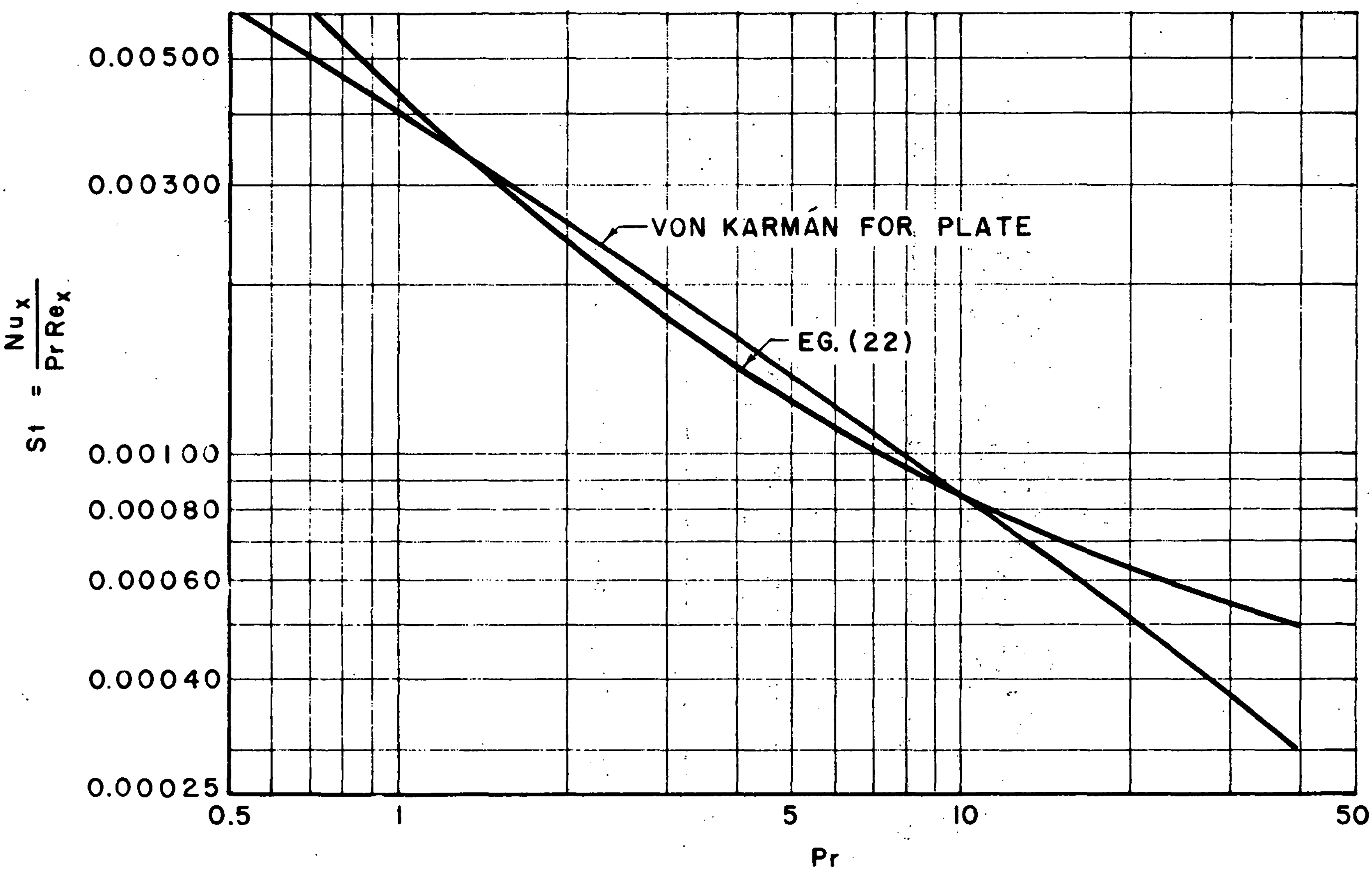

Fig. 2 Comparison of von Karman Equation and Eq. (21) for Various Magnitudes of Prandtl Numbers for Reynolds Number = 10^4.

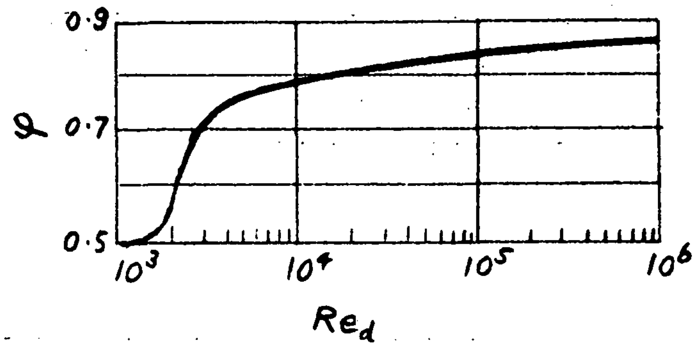

Fig. 3 Ratio of Mean Velocity to Axial Velocity as a Function of Reynolds Number.

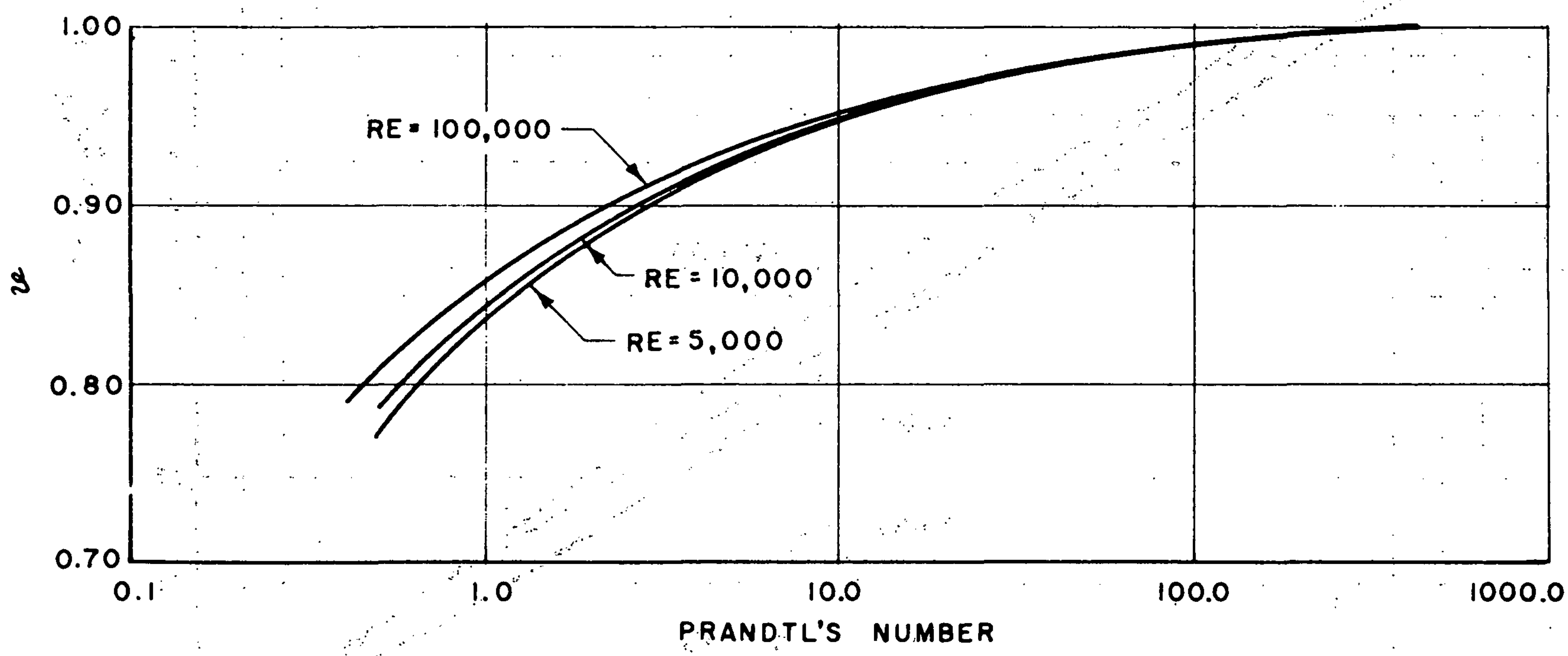

Fig. 4 Ratio of Mean to Maximum Temperature Differences as a Function of Prandtl and Reynolds Numbers.

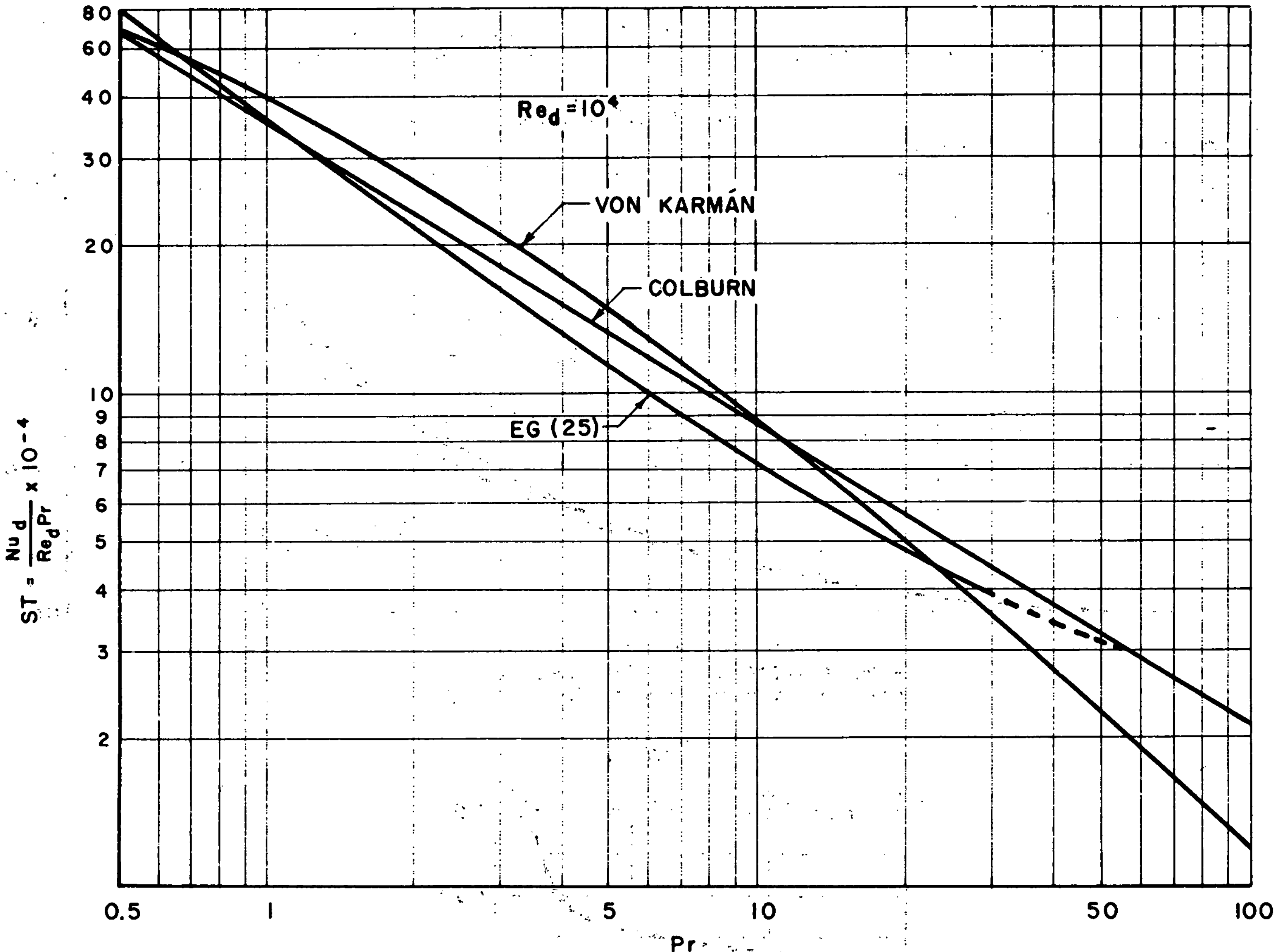

Fig. 5 Comparison of von Karman Equation, Colburn Formula and Eq. (24) for Nusselt Number.

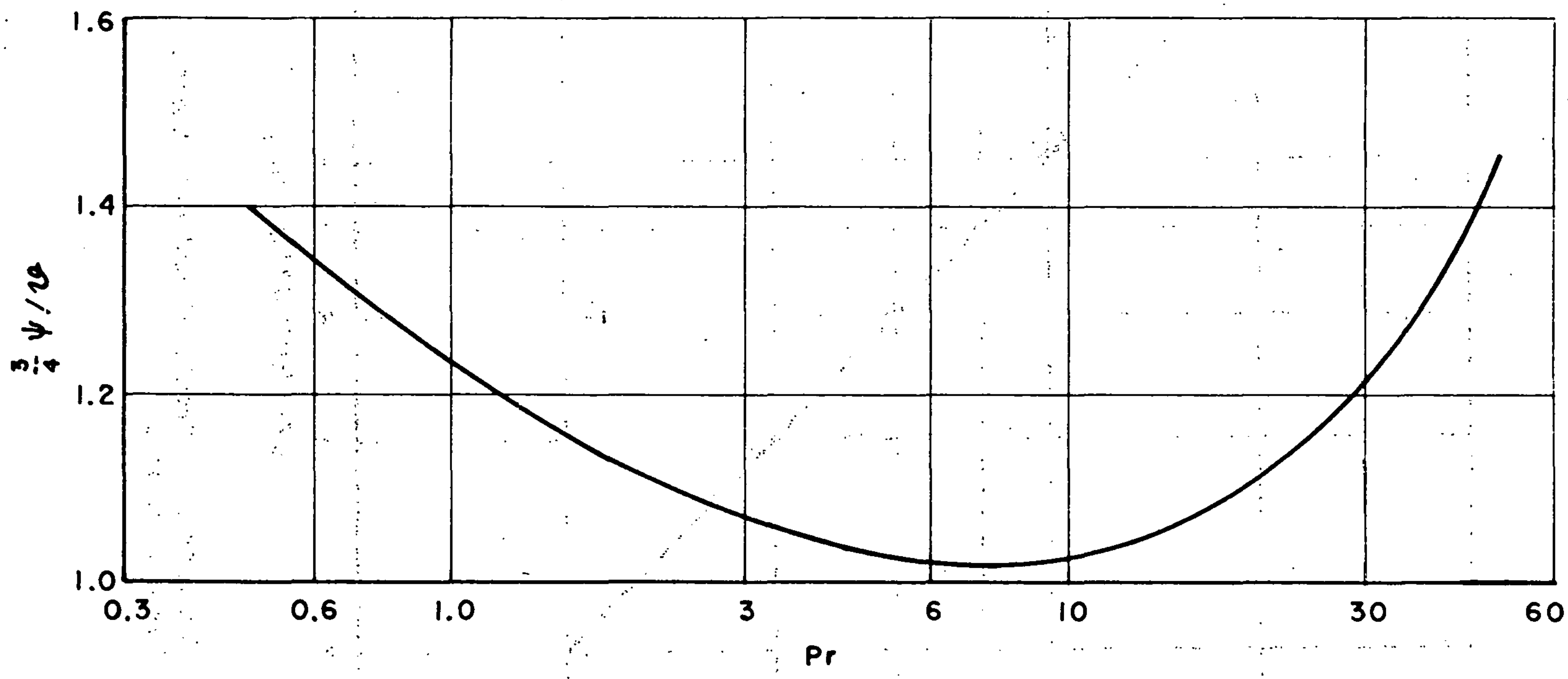

Fig. 6 Relation Between the Coefficient $3/4\,\psi/\varphi'$ and Prandtl Number.

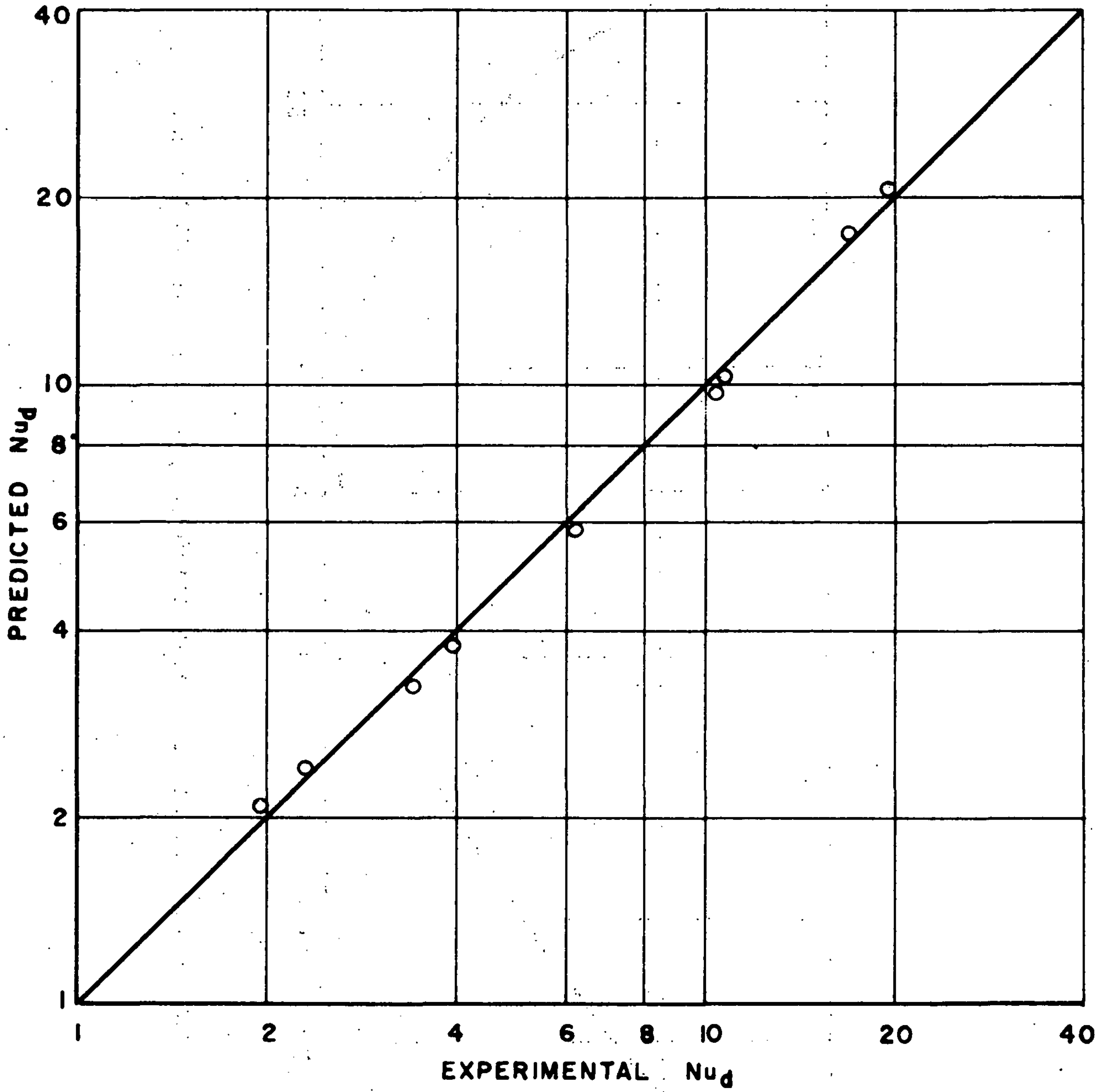

Fig. 7 Comparison of Predicted and Experimental Magnitudes of Nusselt Number.

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