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FROM: C. S. Walker

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AN ELEMENTARY APPROACH TO ANALOG COMPUTING

This material is supplementary to that in CF 57-1-1, Reactor Controls, by C. S. Walker, January 5, 1957.

The analog computer, or simulator, to be discussed here is described in more detail in ORNL-2405, ORNL Reactor Controls Analog Facility, Operation Manual by F. P. Green. Also, refer to CF 57-1-1.

An analog computer is made up of components which basically function as operators in the mathematical sense. For example; a variable which is a function of time is to be added to another function of time, or multiplied by another variable, or integrated with respect to time. Only a few of the operations that can be performed by an analog facility will be described.

I. Solution of a Given Mathematical Expression

As a specific example, the following relationship is to be considered:

$$y = 5x + 3\frac{dx}{dt} - 0.6\int x\,dt$$

The variable y is an independent function of time, the variable x is to be determined as y varies. A block diagram to specify the operators required to solve for x is to be set up. The first step is to solve for the highest ordered differential.

$$3\frac{dx}{dt} = y - 5x + 0.6\int x\,dt$$

The operator may be drawn as a block:

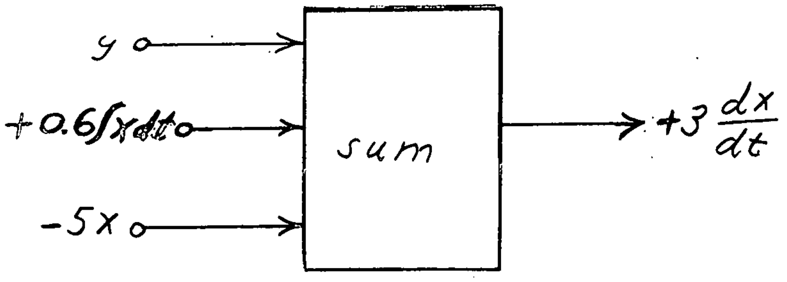

Fig. 1

This summing operator has three input variables, which when algebraically added provide an output of $+3\frac{dx}{dt}$.

Next, if the proper scale change is performed on $+3\frac{dx}{dt}$, $+\frac{dx}{dt}$ is obtained. This $+\frac{dx}{dt}$ is integrated to obtain $+x$, and a scale change with a sign inversion is performed to obtain $-5x$.

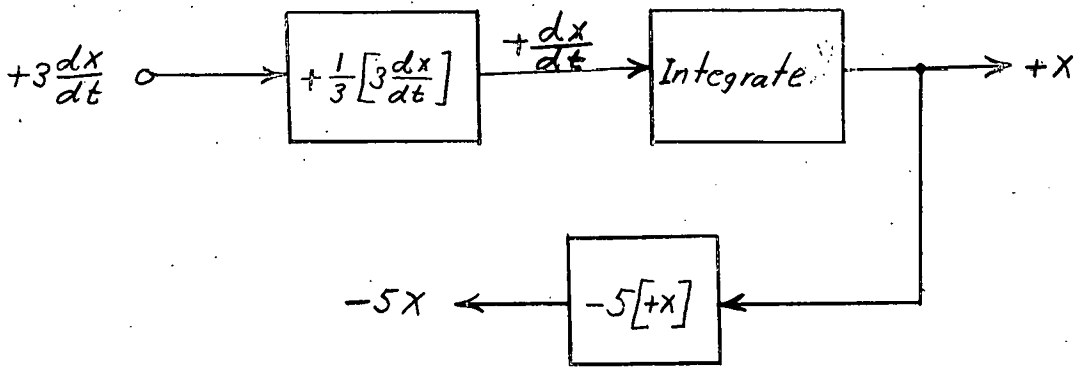

Fig. 2

Also, $+x$ may be integrated again and a scale change taken to obtain $+0.6\int x\,dt$.

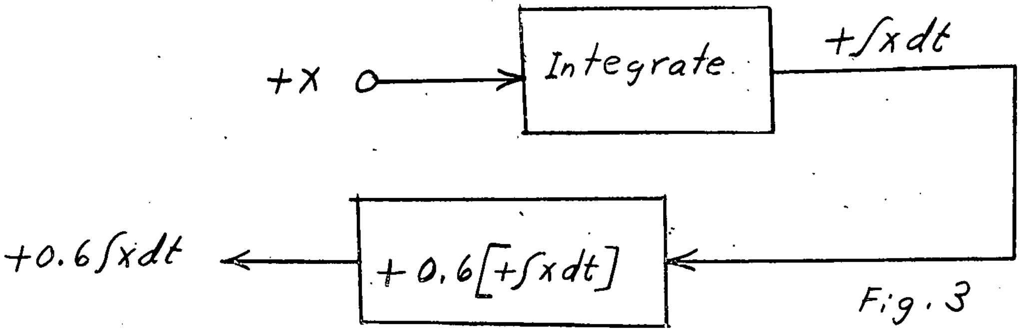

Fig. 3

Now, all the inputs to the first summing operator have been obtained, and the dependent variable x has been determined.

The complete solution by means of the operators is drawn below:

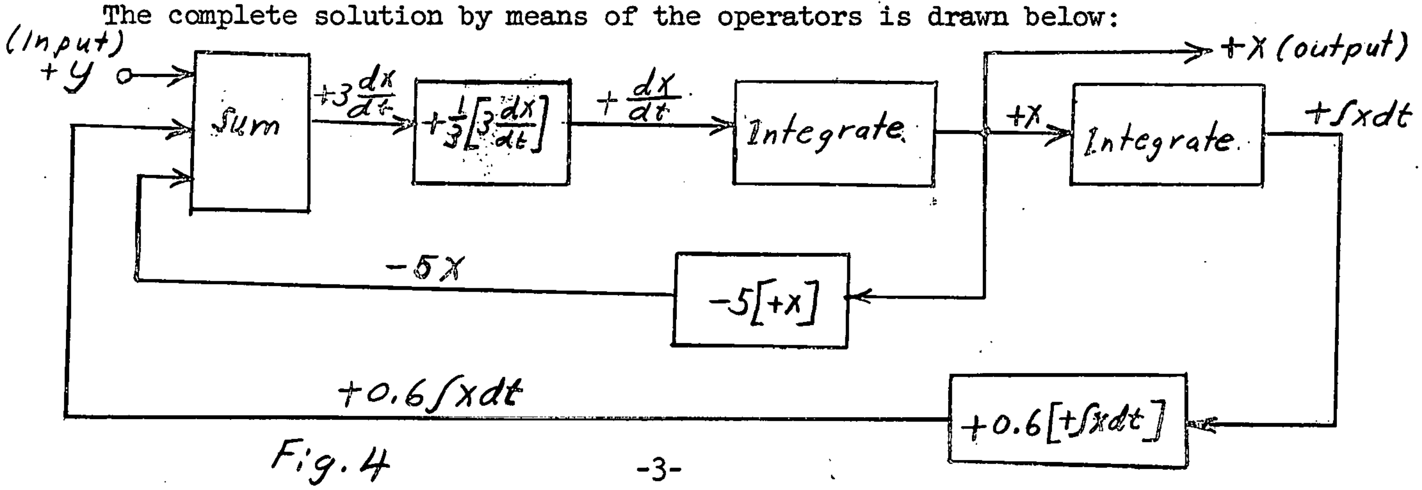

Fig. 4

In order to obtain the solution on the computer, the initial conditions must be known. In other words, the values of X and of $\int X dt$ must be established and set into the facility and clamped at the condition for $t = 0$. In general, the initial conditions following any integration must be known. The clamps are then released and y is varied as required. The output will be X as the dependent variable. Naturally, there are certain physical limitations on the magnitudes of the variables.

Another point that must be considered is that a device to vary y as a function of time must be provided.

In the ORNL analog facility, the input and output variables are voltages, the maximum allowable value being 100 volts, either positive or negative. These voltages are measured with the frame of the machine being a common point. The voltages are then said to be measured with respect to ground. Also, in the ORNL computer, each operation on a variable results in a change of sign. This means that the integration of $+\frac{dx}{dt}$ will result in $-X$, and the sum $+y + 0.6\int x dt - 5X$ will produce $-3\frac{dx}{dt}$ rather than $+3\frac{dx}{dt}$. The actual block diagram used in setting up the computer is called a road map, and the road map to solve the specific example taken above will be developed.

First, it will be necessary to study the symbols used on the road map to represent the operations to be performed.

II. Simple Operators

It should be noted that each of the operators to be described employ an electronic amplifier as the basic unit. An elementary description of the operation of the amplifier is given in CF 57-1-1, pages 77-80, with more information in ORNL-2405.

A. Summation (Inverter and Scale Changer)

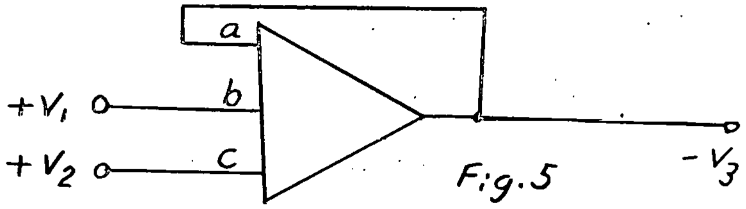

Fig.5

The basic mathematical relation is

$$+bV_1+CV_2-aV_3=0$$

$$\text{or,}\quad V_3=\frac{b}{a}V_1+\frac{c}{a}V_2 \quad .$$

Here, it has been assumed that the two input voltages are V_1 and V_2, and that both are positive with respect to ground. Since the operator inverts the polarity, the output voltage is negative. The letters a, b, and c indicate the numerical value of the coefficient of the scale change obtained in this summation. The numerical values of these coefficients available are generally 1, 5, and 10, although 2 and 20 can usually also be obtained. It should be noted that there are actually three input voltages, namely $+V_1$, $+V_2$, and $-V_3$. One useful way of thinking of the action involved here is to consider the device as a mechanism which adjusts the output voltage V_3, in such a manner that the sum $+bV_1+CV_2-aV_3$ is equal to zero. Note that this is the sum of the three voltages, each voltage having a coefficient to provide a scale change.

More than two variables may be added, if desired.

B. Integrator

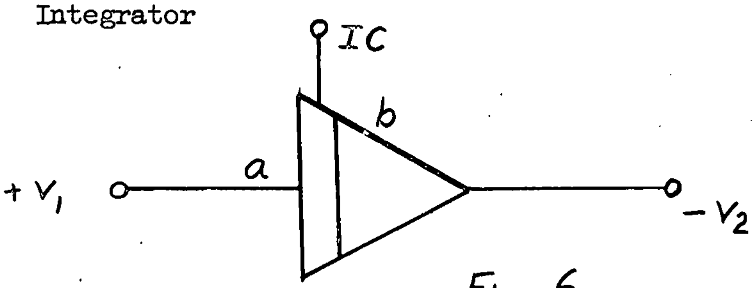

Fig.6

The basic relation is

$$+aV_1 - b\frac{dV_2}{dt} = 0$$

$$\text{or,} \quad V_2 = \frac{a}{b}\int V_1\,dt$$

Note the reversal in sign as in the summation operation. The coefficient a on the input is available as in the summing amplifier and may have the numerical values given above. The coefficient b written outside the triangle may have the values of 0.1 or 1, with 1 being the customary value. The terminal marked *IC* is used in case an initial condition other than zero volts is required from the integrator. In other words, at the instant in time that the system operation is to commence, the voltage V_2 is to have a certain specific value other than zero, and this value with sign reversed as in the case of an input voltage is written at the place indicated by the symbols *IC*. In many applications, an initial condition is zero, and the terminal marked *IC* is then omitted.

It is possible to algebraically sum as well as integrate in one operation. Refer to Fig. 7, in which a second input voltage V_3 is applied.

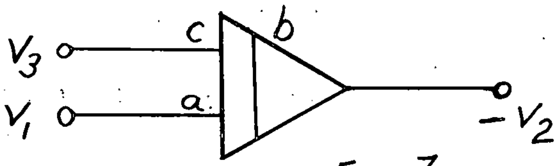

Fig. 7

The basic relation is

$$+aV_1 + cV_3 - b\frac{dV_2}{dt} = 0$$

$$\text{Or,} \quad V_2 = \frac{1}{b}\int (aV_1 + cV_3)\,dt$$

C. First Order Linear Operator
(First Order Lag)

Actually, this operation is identical to the combination of summation and integration shown in Fig. 7. If $-V_2$ in Fig. 7 were connected to the terminal where V_3 is applied, the operator would appear as in Fig. 8; and V_3 is replaced by $-V_2$ in the mathematical relations.

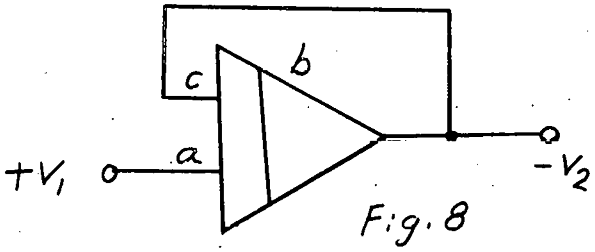

Fig. 8

The basic relation is

$$+aV_1 - cV_2 - b\frac{dV_2}{dt} = 0$$

$$\text{or,} \quad V_1 = \frac{c}{a}V_2 + \frac{b}{a}\frac{dV_2}{dt}$$

$$\text{or,} \quad V_2 = \frac{1}{b}\int(aV_1 - cV_2)\,dt$$

The basic relation is that of the first order linear differential equation having constant coefficients. The output voltage V_2 does not instantaneously follow a change of the input voltage V_1; in other words, a lag is introduced. Due to the fact that the differential equation is of first order, this operation is sometimes called first order lag. When steady-state is reached with V_1 being constant, the voltage V_2 will also become constant.

III. <u>Analog Calculation</u>

A. Road Map for Equation

For simplicity, it will be assumed that any desired constant value of scale change is available. The road map of each operator will be developed in the same manner as were the required functions previously drawn in block form in Figs. 1, 2, 3, and 4.

The first operator is to have input voltages of $+y$, $+0.6\int x\,dt$, and $-5X$. The output voltage is to be proportional to $\frac{dx}{dt}$.

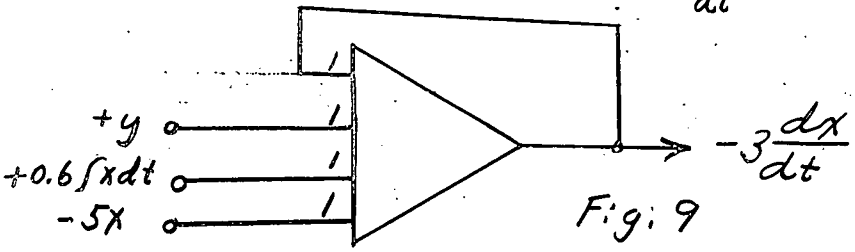

Fig. 9

The mathematical expression for the sum being taken is

$$3\frac{dx}{dt} = +y + 0.6\int x\,dt - 5X,$$

consequently the output of the summation operator is $-3\frac{dx}{dt}$ because of the inherent reversal in sign. The basic relation for the operation is

$$+y + 0.6\int x\,dt - 5X - 3\frac{dx}{dt} = 0$$

which is the desired equation.

The next operator is to have an input of $-3\frac{dx}{dt}$ and an output voltage of $+X$. Clearly, the operator must be an integrator.

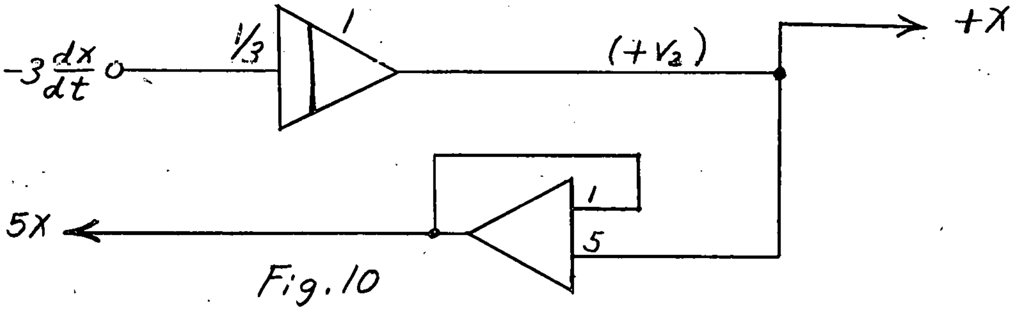

Fig. 10

Using the output voltage as $+V_2$, the basic relation for the integrator follows:

$$\frac{1}{3}\left(-3\frac{dX}{dt}\right) + V_2 = 0$$

$$\text{Or,} \quad V_2 = \int \frac{dX}{dt}\,dt = X$$

143 008

Note the scale factor of $1/3$ on the input voltage.

In addition, an operator to change the sign X and also raise the value of X by a factor of 5 is drawn in Fig. 10.

Next, the quantity $+0.6\int X dt$ is to be obtained by first integrating with a unity scale, then raising this integrated output by a factor of 0.6 and reversing the sign. Refer to Fig. 11.

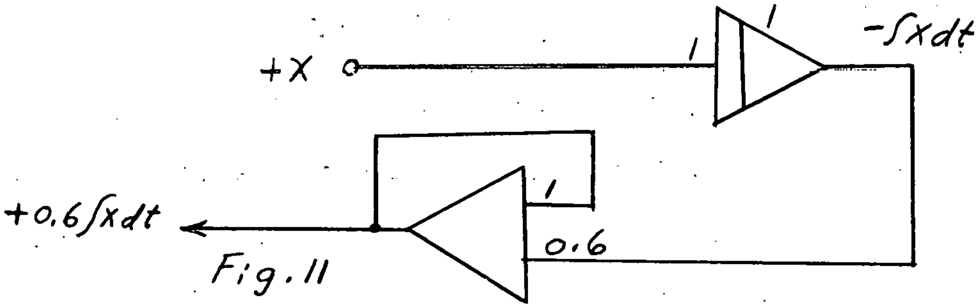

Fig. 11

All of the dependent variables required by the summation operator in Fig. 9 have now been generated. The complete road map is drawn in Fig. 12.

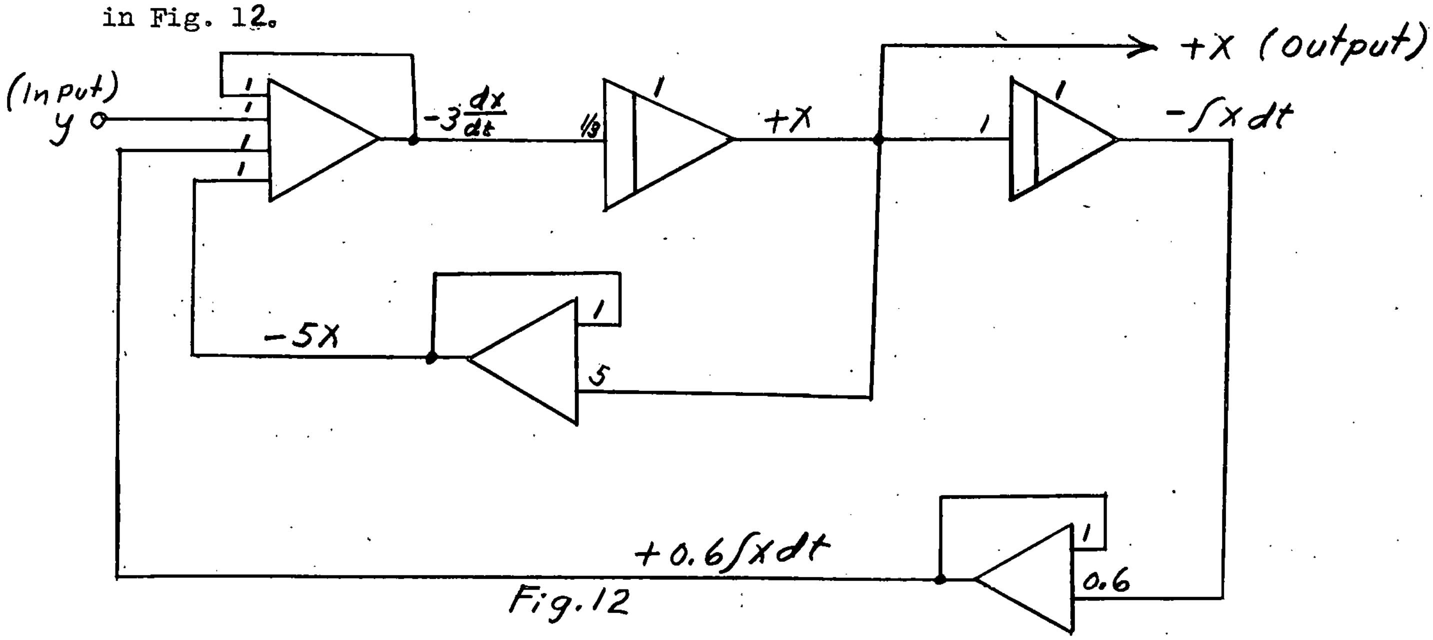

Fig. 12

The road map of Fig. 12 requires five amplifiers; it is possible to reduce this number by combining the operations of summation and integration. If the integration of Fig. 10 is combined with the summation of Fig. 9, it should be possible to obtain an output proportional to X ; however, $\frac{dX}{dt}$ cannot be measured.

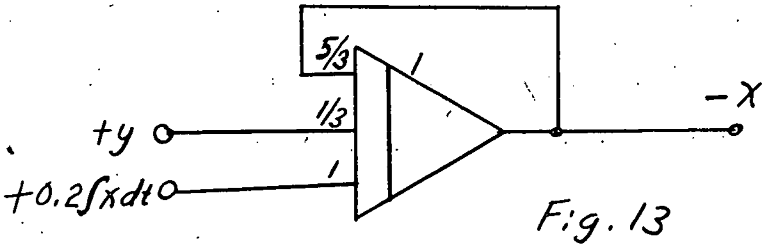

Fig. 13

It has been assumed that $+\int X dt$ has been reduced by a factor of 5 by another circuit. The coefficient associated directly with the integration is unity because values other than 1 or 0.1 are not available. The summation coefficients are those which are required to satisfy the equation. The basic relation of Fig. 13 follows:

$$+\tfrac{1}{3}y - \tfrac{5}{3}X + 0.2\int X dt - 1\frac{dX}{dt} = 0$$

Rearrange, and multiply by 3:

$$y = 5X + 3\frac{dX}{dt} - 0.6\int X dt$$

This is the original equation.

There is only one dependent variable needed in Fig. 13 to complete the solution. A reduction of $-X$ by a factor of 5, followed by a unity integration of $-0.2X$ together with the inherent reversal of sign will produce $+0.2\int X dt$. A unity inverter may be used to obtain $+X$ from $-X$. The road map is drawn in Fig. 14.

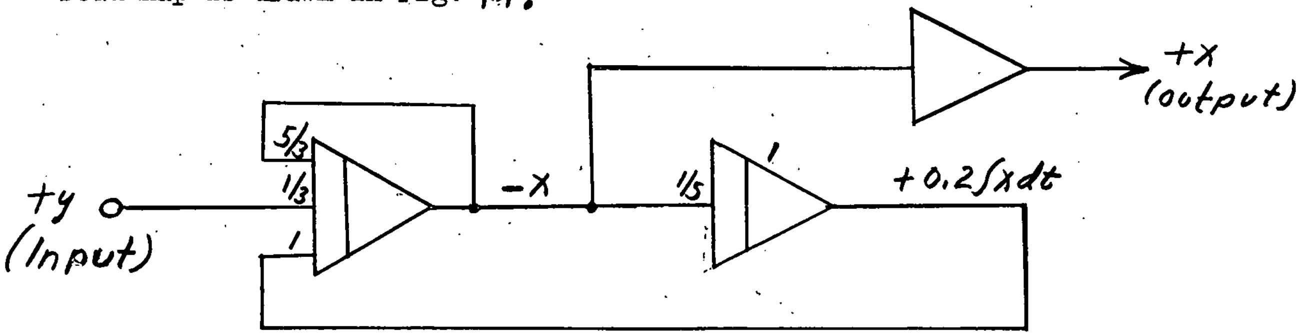

Fig. 14

143 010

Note that the factor of 1/5 is introduced in the input to the second integrator, rather than in the output. This reduction before integration is to limit the amplifier output to a maximum value less than 100 volts. It should also be noted that the connection from the output of an operator back to the input is often omitted on the road map when the summation scale factor on the output voltage is unity. This connection has been omitted in the inverter to obtain $+X$ in Fig. 14. In addition, the omission of the value of the summation scale factor on any input is interpreted to mean unity. Again, refer to the inverter in Fig. 14.

The values of the scale coefficients of the summation in Fig. 14 are not available directly. Obviously, the fixed values previously stated are hardly suitable for a wide variety of applications, consequently a continuously adjustable element is available to be added into the circuit. These elements are potentiometers, sometimes called "pots", which can be adjusted to reduce the respective voltage to any desired fraction of the input voltage to the pot. The phrase "voltage divider" aptly describes the function of one of these devices. When a pot is added to the input circuit of a summation amplifier, the actual scale change is equal to the product of the voltage ratio due to the pot and the coefficient of the amplifier input. Note that the pot ratio cannot exceed unity. In order to achieve an input coefficient of 5/3, a pot ratio of 0.333 together with a fixed coefficient of 5 could be used.

The road map of Fig. 14 is redrawn in Fig. 15, showing the pots as circles, with the pot voltage ratios written inside the circles.

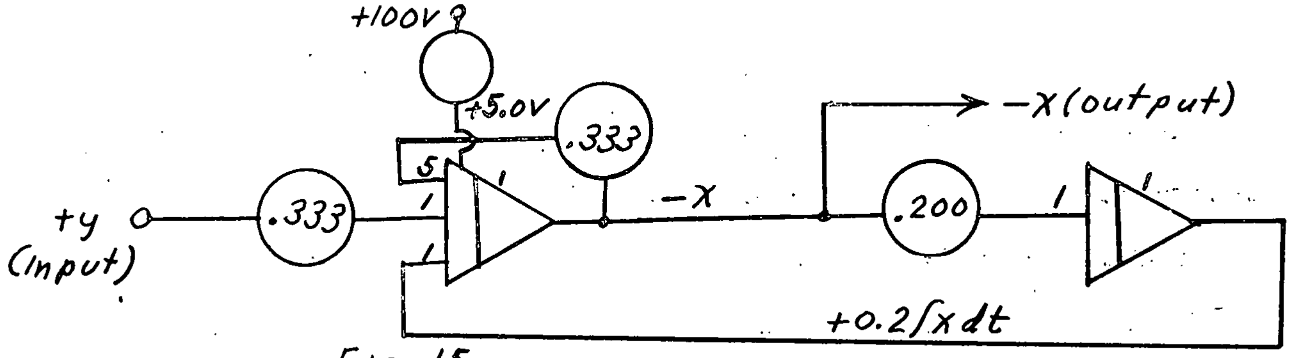

Fig. 15

The initial conditions are indicated in Fig. 15. No initial condition is shown for $\int x\,dt$ because this quantity is to have an initial value of zero volts. A pot is to supply + 5.0 volts to the integrator to produce -5.0 volts as the initial value of $-x$. Note the reversal in sign between the output and the initial condition as in the case of the relation between the output and input voltages.

The source of + 100 volts is provided by the power supply which is part of the analog facility. A source of - 100 volts is also provided.

The voltage $-x$ is not converted to $+x$ because the indicator of this voltage is equipped with a polarity reversing switch in order that either positive or negative voltages may be indicated in the up-scale direction.

It should be pointed out that the pot settings read on the potentiometer dials are not identical to the pot voltage ratios obtained. The current which flows through the pot resistance will reduce the voltage ratio to a value lower than the actual setting of the dial. Consequently, the pots are adjusted to give the desired voltage ratio, as determined by voltage measurements, after the circuits are connected. The setting of the pot to supply the initial condition of x in Fig. 15 is not shown because this pot is to be set to supply + 5.0 volts, rather than to provide a voltage ratio from a varying voltage source.

B. Actual Solution

The actual process followed in obtaining a solution of the differential equation is described in the succeeding paragraphs. The solution is shown in ORNL-LR-DWG 26123, together with the variation in y for which this solution was obtained. This particular variation of the independent variable was taken in order that it could be expressed mathematically, if desired.

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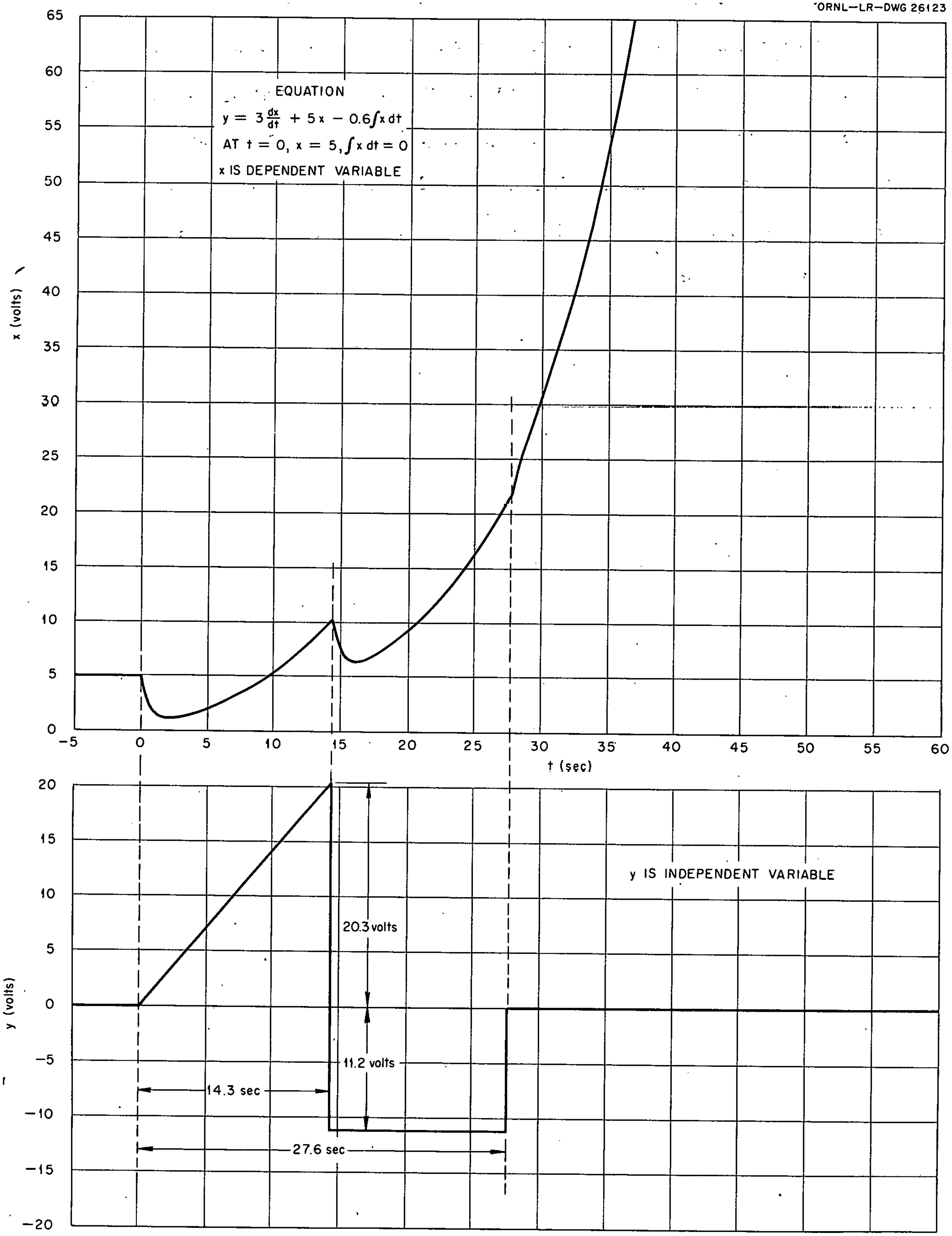
ORNL—LR—DWG 26123



Analog Solution of a Mathematical Equation.

143 014

The initial linear increase in y was obtained by integrating a constant voltage of 1.42 volts, as indicated in Fig. 16. After this voltage had been applied to the integrator for 14.3 seconds, the switch in the output of the integrating amplifier was thrown to obtain a constant value of -11.2 volts. After 13.3 seconds, the switch was thrown to the center position, which held y at zero volts.

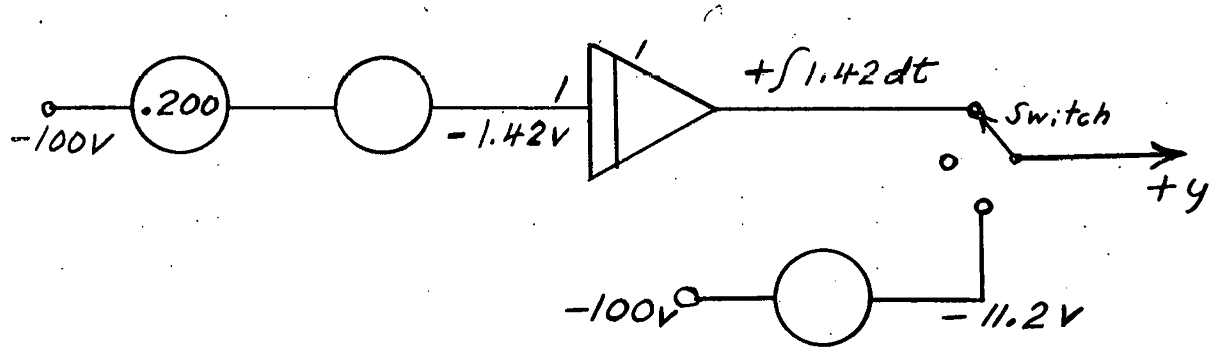


Fig. 16

Two of the pot voltage ratios in Fig. 16 are blank because the actual settings were unimportant, the desired voltage being the important consideration. Two potentiometers in cascade, or a "pot on a pot" were used in obtaining the low value of -1.42 volts from the source of -100 volts in order to avoid taking this rather large reduction on one pot..

The voltage y out of the road map of Fig. 16 was applied as the input of Fig. 15. The voltages y and X were recorder on strip-chart recorders and redrawn for ORNL-LR-DWG 26123. The initial conditions taken were + 5.0 volts for $+X$ and zero volts for $\int X\,dt$, as previously stated.

IV. Complex Operators

A. Function Generators

It is often necessary to produce a voltage V_2 which is dependent directly upon the value of V_1, but is not time dependent. For example, the variable $W^{0.8}$ may be needed, with W known. In other cases, the relationship between the desired variable and the independent variable is available only as a graph.

The function generator produces the desired variable with the known independent variable as the input. Refer to Fig. 17. The computer operator sets up the function generator to obtain the desired relation between the output and input voltages.

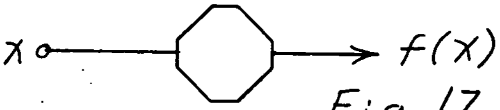


Fig. 17

B. Multiplication

1. Electronic Multiplier

The electronic multipliers in the ORNL Analog Facility depend upon function generators for their operation. First, the sum and the difference of the two variables in question are taken by means of summing and inverting operators. Next, the square of this sum and of this difference are individually obtained by function generators. The difference of the squares, obtained by another summation, provides a voltage proportional to the product. More information may be found in Fig. 21 of ORNL-2405. The road map symbol is given in Fig. 18. The output is $-\frac{xy}{100}$ when the inputs are x and y. The multiplier operates with either polarity input.

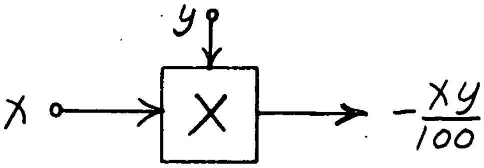


Fig. 18

2. Servo Multiplier

The basic element in the servo multiplier is a potentiometer, or pot, used as a voltage divider. Refer to Fig. 19.

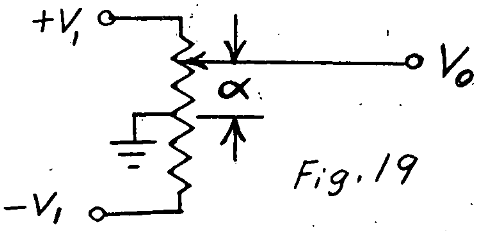


Fig. 19

One of the input variables is the voltage V_1, the other is the position α of the sliding contact. The output voltage V_0 will be proportional to the product $V_1\alpha$, α being expressed as a fraction of the total distance from the common ground point to the upper end of the resistance. Both polarities of V_1 are required if V_0 is to change sign; α may be either positive or negative to place the sliding contact in either the upper or lower half of the resistance, producing either positive or negative signs of V_0.

Since voltages are to be multiplied, it is necessary that α be continuously directly proportional to one of the input voltages impressed upon the multiplier. This proportionality is produced by an electromechanical servo system utilizing an electric motor to position the potentiometer shaft. The input X in Fig. 20 is the voltage to the servo which positions the sliding contact.

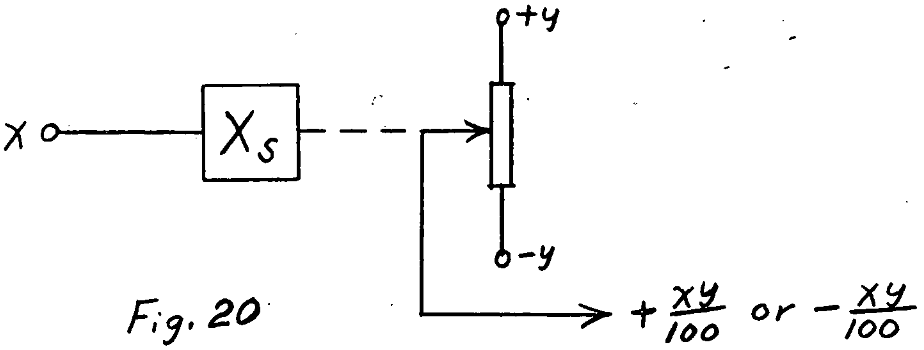


Fig. 20

Either polarity of output voltage is available by interchanging the polarities of the voltages $+y$ and $-y$ applied to the potentiometer terminals. Also, the servo motor positions the shafts of several pots; consequently additional voltages may be multiplied by x. This is useful when the products of x and z as well as of x and y must be obtained. The ability of the servo positioner to follow a rapidly varying input voltage limits the frequency of x to a maximum of approximately two cycles per second. There is no such limitation on the rate of change of the voltage applied to the potentiometer resistance. In cases where x does not change sign, it is possible to operate the multiplier with only one polarity of voltage applied to the potentiometer resistance.

C. Division

The process of division will be considered as the inverse of multiplication. The voltage z is to be obtained.

If, $z = \frac{x}{y}$

then $x = zy$

In this last equation, x could be found by multiplying z by y; however, z is not known but is desired, and both x and y are known. Then, the problem is to force the computer to generate a voltage v that when multiplied by y will produce a voltage equal to the known value of x. If this voltage v can be generated such that

$$vy = x$$

then, obviously,

$$z = v$$

and the operation of division is accomplished.

In this application, it is necessary to use an amplifier to obtain the maximum voltage amplification possible. Refer to Fig. 21.

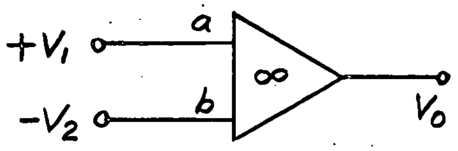


Fig.21a

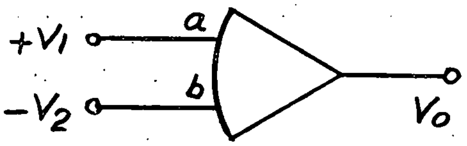


Fig.21b

Either Fig. 21a or 21b may represent this application. With the amplification available, the actual net input must be near zero in order to maintain an output between + 100 and - 100 volts. The only practical mathematical relation possible is

$$+aV_1 - bV_2 = 0$$

The value of V_0 is indeterminable as shown here; therefore, the circuit external to that indicated in Fig. 21 must be used to make this mathematical relation possible by making use of the output voltage V_0. The two input voltages must be opposite in sign.

The road map for the basic operation of division is drawn in Fig. 22.

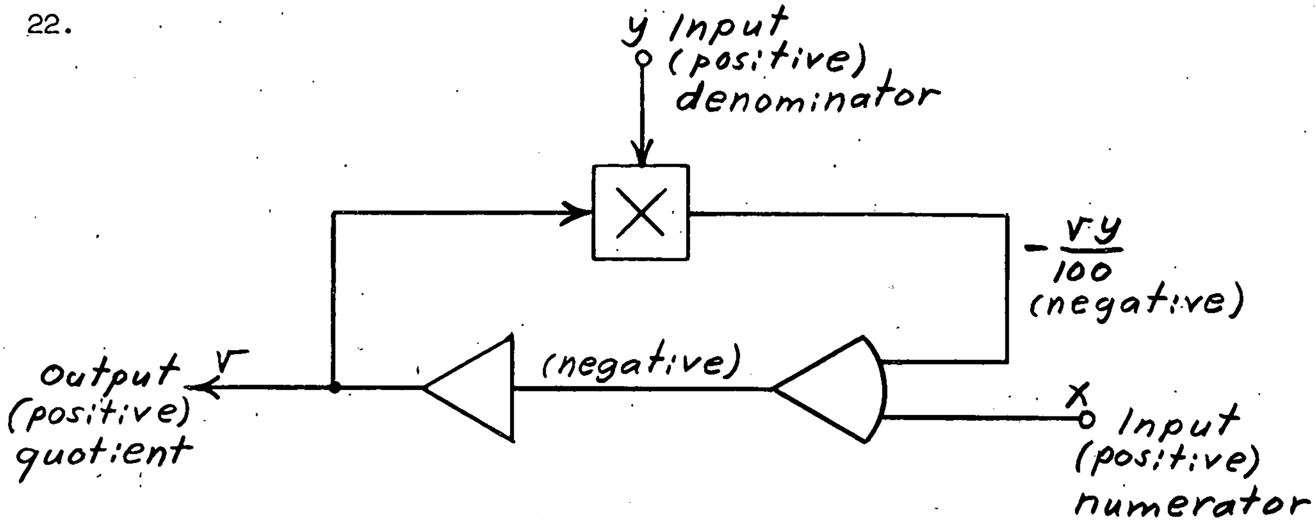


Fig. 22

Assume that both the voltages y and x have positive values. Further, assume that the voltage v is available as a positive quantity to be applied as one input to the multiplier. The output of the multiplier is $-\frac{vy}{100}$ in this case since the multiplier produces a change in voltage sign. This negative product is compared to the input voltage x by means of the extremely high gain amplifier illustrated in Fig. 21. The output of this comparison amplifier is reversed in sign by the inverter amplifier which supplies the positive voltage v back to the multiplier. The high gain comparison amplifier requires that

$$x - \frac{vy}{100} = 0$$

in order that the voltage v is finite. Therefore, $v = 100\frac{x}{y}$. The desired voltage was the quotient $\frac{x}{y}$; therefore, the division operation has been produced. All voltages must be limited to a maximum value of ± 100. It has been assumed that both input coefficients of the comparison amplifier were unity.

Another way of considering the operation of this circuit is to imagine that both x and y are simultaneously applied. The comparison amplifier and sign inverter will produce a positive voltage v. This positive voltage, together with the other positive input y to the multiplier will produce a negative voltage as one of the inputs to the comparison amplifier. This last negative input will reduce the numerical value of v until the output of the multiplier is sufficient to balance the voltage x.

The preceding road map is applicable for either positive or negative values of the numerator of the quotient, which is x in this example.

It is applicable only for positive values of the denominator y. The road map of Fig. 23 is for negative denominators, and for either positive or negative numerators.

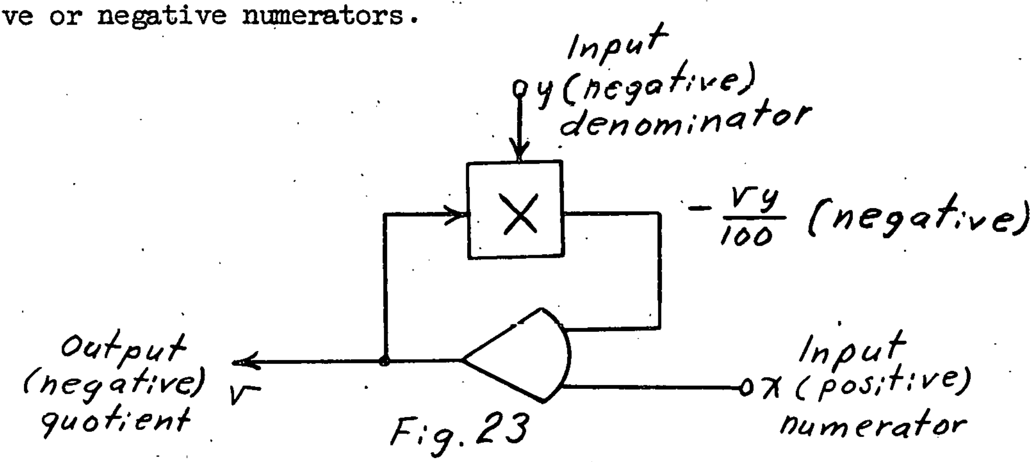


Fig. 23

This inverse operation of multiplication to obtain division can be applied by means of either the electronic multiplier or the servo multiplier. Note the signs of the voltages associated with the multiplier. However, other methods of obtaining quotients by use of the servo multiplier are described in ORNL-2405.

D. Function Delay

One commonly encountered situation is analog computing involves transport lag. For example, consider a pipe carrying a fluid. At equilibrium, the temperature of the fluid leaving the downstream end of the pipe is the same as that entering at the other end. However, if the temperature of the fluid entering the pipe changes, there will be a time lapse before this temperature change can be observed at the downstream end.

The symbol for a function delay is drawn in Fig. 24.

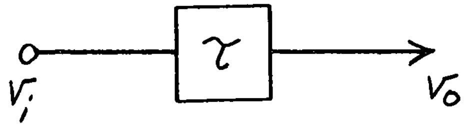


Fig. 24

The symbol τ represents the time delay. The output voltage V_o is delayed τ seconds later than the input voltage V_i. In addition, the sign of V_o will be reversed, and the magnitude reduced to one-half in the ORNL facility.

$$V_o(t) = -\frac{1}{2} V_i(t-\tau)$$

The above expression indicates the mathematical relation. The symbol (t) means "a function of time", and $(t-\tau)$ means "a function of time delayed by the time interval τ". The delay time is adjustable, but is not variable while the circuit is in operation.

It is a relatively simple matter to employ an inverter with a voltage gain of two following the function delay device in order to obtain the original voltage polarity as well as the original voltage magnitude at steady state. In the usual case, this extra amplifier and circuit is included in the symbol for the function delay in Fig. 24.

E. Explicit Circuits

The road map symbols do not always suffice to prescribe the computer operations. It is sometimes necessary to draw the explicit circuit, showing the electrical circuit elements in detail. An outstanding example of this situation is the simulation of that portion of a nuclear reactor system represented by the kinetic equations relating nuclear power and multiplication. Refer to CF 57-1-1, page 36, Eqns. (29) and (10), and to pages 77 through 83 for the explicit circuits.

In cases in which neither the explicit circuit nor the general symbol for an operation is known, one can specify the operation to be performed by drawing a box and stating the necessary operation. The operation specified in Figs. 1, 2, 3 and 4 are examples of this method, which may be especially useful when complicated operations must be carried out.

F. Differentiation

The process of differentiation with respect to time should be attempted only in cases of absolute necessity. In spite of the excellence of the amplifiers and circuit components of the Analog Facility, there will be a certain amount of noise in the voltage to be differentiated. A source of these small, fast, and undesired voltage variations may be a multiplier or a function generator. This noise may be especially troublesome at low voltage levels. The differentiation of this noisy voltage will yield a much noisier voltage which may be almost unusable. The example of the solution to the mathematical expression intentionally avoided the process of differentiation.

1. Direct Differentiation

A relatively simple way of obtaining the time-derivative of a voltage is described in CF 57-1-1, page 80.

2. Differentiation as the Inverse of Integration

An integrator and comparison amplifier can be used to produce the time-derivative in much the same manner as the division process was carried out as the inverse of multiplication. The road map of Fig. 25 illustrates this procedure.

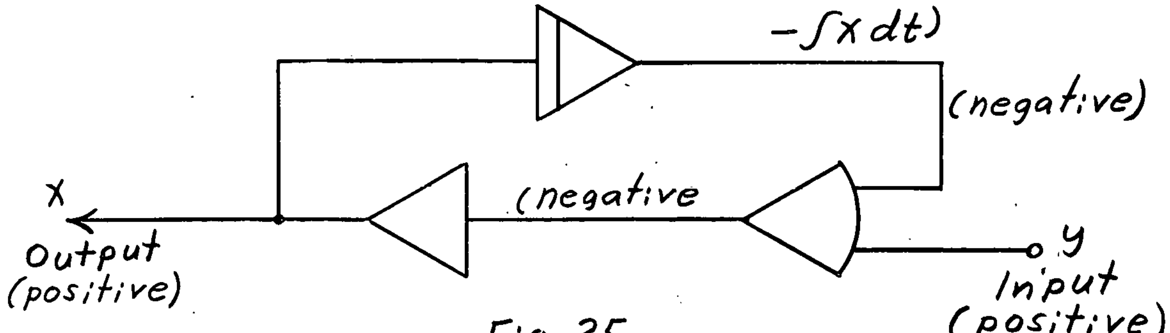


Fig. 25

Assume, for the moment, that the input voltage y is positive in sign. When this positive voltage is applied to the comparison amplifier, the output voltage is negative. The inverter then applies a positive voltage to the integrator input, which in turn supplies a negative voltage to the comparison amplifier. The difference between the two voltages to the input of the comparison amplifier is very nearly zero; consequently, the output of the integrator is taken as equal to the original input voltage y . Then,

$$y = \int x \, dt$$

or,

$$x = \frac{dy}{dt}$$

Thus, the time-derivative is taken.

This type of differentiation does not eliminate the problem of differentiating noisy voltages.

V. Simulation of a Complete Reactor System

The simulation of a reactor system should help define the control problem. The possibility of controlling certain variables can be studied, and the performance required of a control system can be examined. In some instances, it may be found impossible to obtain control through a method which seemed appropriate until carefully examined. There is very little to be gained by using the simulator to exhibit variations which are already known.

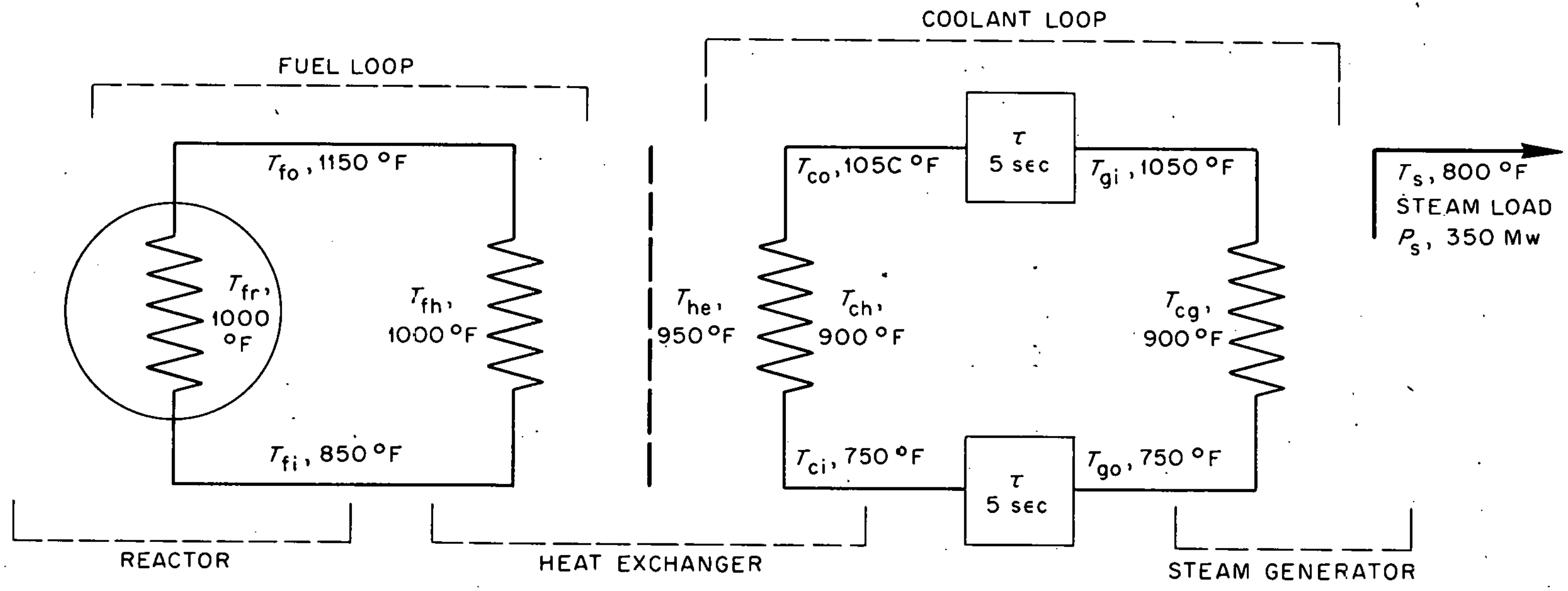
The simulation of the relatively simple system to be described here was undertaken for two reasons; (1) the process of setting up the simulator would serve as an example of analog computation, and (2) the results would indicate the actual behavior to be expected of a certain type of power reactor system.

A. Reactor System

The simplified flow diagram of the reactor system which was simulated is drawn in ORNL-LR-DWG 26402. The system was composed of a circulating fuel reactor, a simple counter-flow heat exchanger coupling the fuel and coolant loops, and a single coolant loop from which the heat was removed by a steam generator. The steam was produced at 800°F, the design point heat load was 350 megawatts. The fluids were referred to as "buttermilk" to indicate that this was obviously a hypothetical reactor system, simulated as a particular example of analog computing. The delayed neutrons were those of uranium - 235.

The symbols in the discussion are defined as follows:

UNCLASSIFIED
ORNL-LR-DWG 26402



	REACTOR	FUEL SIDE, HEAT EXCHANGER	METAL, HEAT EXCHANGER	COOLANT SIDE, HEAT EXCHANGER	COOLANT SIDE, STEAM GENERATOR
TRANSIT TIME	5.0 sec	6.67 sec		10.0 sec	12.5 sec, COOLANT 15.0 sec, EQUIVALENT
RATE OF TEMPERATURE CHANGE AT 350 Mw	60 °F/sec	45 °F/sec	150 °F/sec	30 °F/sec	24 °F/sec, COOLANT ONLY 20 °F/sec, COOLANT AND STEAM GENERATOR METAL

TEMPERATURE COEFFICIENT OF REACTIVITY = -2.89×10^{-5}/°F.

Elementary Flow Diagram and Data; ORSORT Buttermilk Reactor.

T_{fi} - reactor fuel inlet temperature or heat exchanger fuel outlet temperature

T_{fr} - reactor fuel mean temperature

T_{fo} - reactor fuel outlet temperature or heat exchanger fuel inlet temperature

T_{fh} - heat exchanger fuel mean temperature

T_{he} - heat exchanger metal mean temperature

T_{ci} - heat exchanger coolant inlet temperature

T_{ch} - heat exchanger coolant mean temperature

T_{co} - heat exchanger coolant outlet temperature

T_{gi} - steam generator coolant inlet temperature

T_{cg} - steam generator coolant mean temperature

T_{go} - steam generator coolant outlet temperature

T_s - steam temperature

P_r - reactor heat production rate

P_f - fuel to heat exchanger metal heat transfer rate

P_c - heat exchanger metal to coolant heat transfer rate

P_s - steam heat removal rate

τ - transit time of coolant in piping from heat exchanger to steam generator and in return from steam generator to heat exchanger

There was no transport lag of the fuel between the reactor and heat exchanger because these pipe lines would probably be short to limit the volume to be shielded as well as to hold down the fuel inventory. Consequently, T_{fi} is both the heat exchanger outlet and reactor inlet temperature, and T_{fo} is the heat exchanger inlet as well as the reactor outlet temperature.

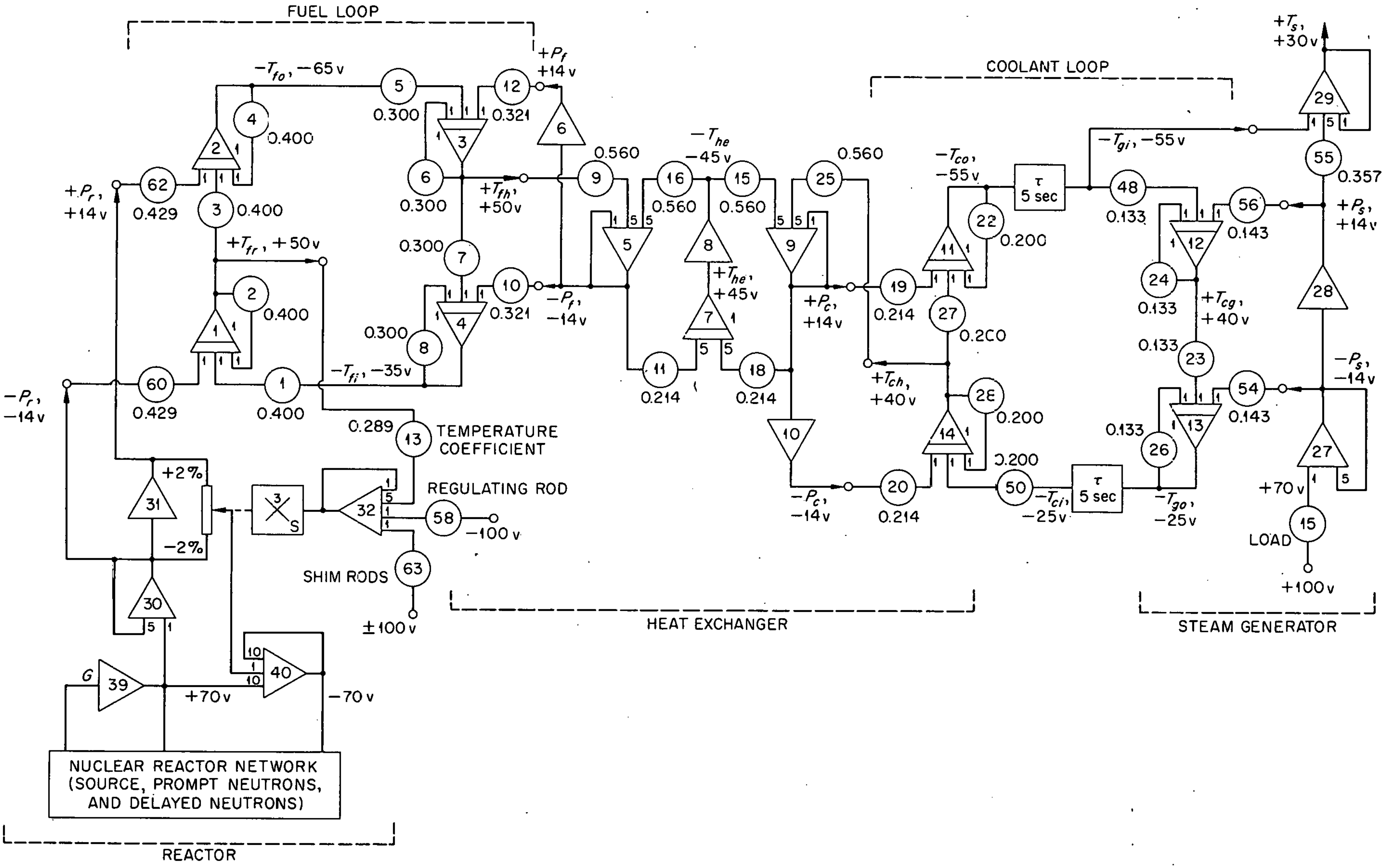
The design point temperatures are listed on the flow diagram. The average rate of change of the temperature of the fluid passing through a region where heat is added or removed is obtained by dividing the temperature change by the transit time through this region. The temperature change is the difference between the inlet and outlet temperatures. The average rate of change of the temperature of a solid is determined on the basis that the entire design point power is being stored in the solid as heat energy.

The negative temperature coefficient of reactivity was that of the fuel alone. It was assumed that the temperature coefficient contributed by any moderator would be small, and that in addition, the temperature change of such a moderator would be small during a transient.

The fluids were assumed to circulate at constant velocity. The energy put into the system by any necessary pumps was neglected. Heat loss by convection or radiation was also neglected.

The road map for simulating the reactor system is drawn in ORNL-LR-DWG 26401. A power of 350 megawatts was represented by 70 volts of potential from the reactor, and by 14 volts in the heat transfer portions of the system. Zero volts meant zero power in every case. A temperature of 500°F was analagous to zero volts, and 1500°F was represented by 100 volts, giving a ratio of 10°F per volt. A suppressed temperature scale was possible because the temperature scale itself is arbitrary. The time scale was unity, one second of time on the reactor being also one second of time on the computer.

UNCLASSIFIED
ORNL–LR–DWG 26401



Elementary Road Map for Simulation of ORSORT Buttermilk Reactor.

The identification numbers of the amplifiers are written inside the triangles which represent the amplifiers in ORNL-LR-DWG 24601. The potentiometers are identified by the numbers inside the circles, the desired voltage ratios being written outside the circles. The symbols for the variables of power and temperature with their design point voltage values are shown in the drawing.

B. Nuclear Reactor

The process of simulating the nuclear portion of a circulating fuel reactor having a negative temperature coefficient of reactivity was carried out in a manner similar to that described in CF 57-1-1, pages 77 through 89. Amplifiers 39 and 40 together with the circuit elements which included the souce, the prompt neutrons, and the delayed neutrons constitute the heart of the reactor. The symbol G on the input of amplifier 39 means that the reactor network is connected directly to the grid of the amplifier input tube, and that no input coefficients other than those in the reactor network were used. The contributions of the precursors of the delayed neutrons were computed and adjusted for the circulating fuel as described in CF 57-1-1, pages 61 to 69, and page 83. Once the circuit was in operation, the source voltage was removed to eliminate the effect of a source on the transients.

A five-to-one reduction of the voltage representing nuclear power was obtained by amplifier 30, and a polarity reversal of this voltage was produced by amplifier 31. The positive and negative values of P_r were applied to the potentiometer resistance of servo multiplier 3. The maximum output voltage of this multiplier was one-fifth of that supplied amplifier 40 directly by amplifier 39. Since the coefficient of the input of amplifier 40 from the multiplier was one-tenth of that of the input from amplifier 39, the actual maximum excess nuclear multiplication of the reactor available from the potentiometer of multiplier 3 was one-fiftieth, or 2%.

The output of amplifier 32 was the summation of the effects on reactivity due to shim rods, the regulating rod, and the mean temperature of the reactor fuel. A change in nuclear multiplication of 1% could be obtained with a 50 volt change of the voltage from pot 58 because 100 volts applied to the input of multiplier 3 produced the maximum output of the multiplier into amplifier 40. Thus, a change of 50 volts of the regulating rod was equivalent to 1% change in k. An increase in T_{fr} by 346°F, or 34.6 volts, would conteract this 1% change in k by the rod, as mathematically computed below:

$$(0.289)(5)\Delta T_{fr} = (1)(50)$$

$$\Delta T_{fr} = \frac{50}{(5)(0.289)} = 34.6$$

The symbol ΔT_{fr} means change in T_{fr}. It should be noted that the voltage ratio of 0.289 of pot 13 bore a direct numerical relation to the temperature coefficient. A temperature change of 346°F to produce a 1% change in k means a temperature coefficient of reactivity of $2.89 \times 10^{-5}/^{\circ}\mathrm{F}$. The negative sign of this coefficient was obtained from the polarities of the voltages representing temperature and rod position. Pot 63, representing the shim rods, was used in order that the regulating rod pot could be set at 50 volts at design point.

Since this is a circulating fuel reactor, the value of k must exceed unity to hold the reactor at steady state. There was an output voltage from the multiplier at steady state; therefore, the voltage output of amplifier 40 was a slight amount larger than the output of amplifier 39. This small difference in voltage was neglected. Actually, the excess k from the rods was computed to be 0.321% to hold the reactor at steady state and overcome the loss of the delayed neutrons. The measured excess k was 0.326%.

C. Fuel Loop

The fuel loop was simulated by amplifiers 1, 2, 3, and 4. These four amplifiers were connected as first order linear operators. Refer to Fig. 8. The relations concerning amplifier 1 will be studied in detail.

Amplifier 1 and its associated circuit represent the lower half of the reactor, with the fuel entering at the temperature T_{fi}, heat being added at the rate of P_r, and the fuel reaching a mean temperature of T_{fr} before leaving this portion of the reactor. If P_r were zero, then obviously T_{fi} and T_{fr} would be equal. Next, if P_r were suddenly step increased to the design point value, with T_{fi} remaining constant at its original value, the initial rate of increase in the temperature T_{fr}, measured half-way up the reactor core, would be 60°F/sec. This value of 60°F/sec. is determined by the relationship between the temperatures T_{fi} and T_{fr} at design point conditions. The transit time of the fuel in the first half of the reactor is 2.5 seconds, and the temperature rises 150° F; therefore, the average rate of rise of the moving fuel in transit is 60°F/sec. This means that a step rise in reactor power from zero to design point would produce an initial rate of rise of 60°F/sec. of the temperature measured at a stationary point, such as at the point where T_{fr} is measured. If the fuel continued to enter the reactor at a constant temperature, the temperature T_{fr}, measured at a stationary point, would asymptatically approach a value 150°F higher than T_{fi}.

The simulator provided an initial rate of increase in T_{fr} of 6 volts/sec., corresponding to a temperature rise of 60°F/sec. At steady state design point, T_{fr} was 50 volts, corresponding to 1000°F, with T_{fi} being 35 volts, or 850°F. The mathematical relation pertaining to amplifier 1 is stated as follows:

$$-(0.429)(1)P_r-(0.400)(1)T_{fi}+(0.400)(1)T_{fr}+(1)\frac{dT_{fr}}{dt}=0$$

Note that the coefficient of the rate of change of T_{fr} is unity. Assuming that T_{fi} and T_{fr} are equal, and that P_r makes a step increase from zero to 14 volts,

$$(0.429)(14)+\frac{dT_{fr}}{dt}=0$$

and,

$$\frac{dT_{fr}}{dt}=6 \text{ volts/sec.}$$

at steady state,

$$-(0.429)(14)-(0.400)T_{fi}+(0.400)T_{fr}=0$$

$$T_{fr}-T_{fi}=\frac{(0.429)(14)}{0.400}=\frac{6}{0.4}$$

$$=15 \text{ volts}$$

The preceding mathematics merely indicates that the pot voltage ratios were correct. The necessary ratios were computed by making use of the initial rise in T_{fr} of 6 volts/sec., together with the value of P_r of 14 volts to compute the ratio of 0.429 for pot 60. Once this ratio was known, the ratio of 0.400 of each of pots 1 and 2 was computed to produce a 15 volt difference between T_{fi} and T_{fr}. The ratios of pots 1 and 2 must be identical in order that the temperatures T_{fi} and T_{fr} be equal at steady state if P_r were zero.

The circuit associated with amplifier 2 was the same as that with amplifier 1 except for the reversal in signs of the voltages. The initial rates of increase in T_{fo} and T_{fr} will be identical following a step increase in P_r; however, T_{fr} will rise in value if T_{fi} remains constant, eventually forcing T_{fo} to rise 30 volts above T_{fi} at design point.

The situation regarding amplifiers 3 and 4 was similar to that of amplifiers 1 and 2, except that the heat was being removed, thus lowering the fluid temperature. A step increase in P_f of 14 volts will produce an initial rate of decrease in T_{fh} of 4.5 volts/sec. if T_{fo} remains constant. The ratio of pot 12 was computed to be 0.321 from this data. The ratio 0.300 of both pots 5 and 6 was computed from the requirement that T_{fh} must be 15 volts lower than T_{fo} at the steady state design point condition.

The circuit of amplifier 4 was identical to that of amplifier 3 except for the voltage signs.

There is an interesting relationship in these circuits. In the case of amplifier 1, the quotient obtained by dividing the coefficient of the rate of change of T_{fr} by the summation coefficient of T_{fr} is 2.5, which is the time in seconds for the fuel to pass half way through the reactor. The same situation applies to the coefficients on the output voltage of amplifier 2. In the case of amplifiers 3 or 4, this quotient is 3.33, again, the time in seconds for the fuel to pass through the section of the heat exchanger represented by the circuit. If the coefficient of the integrator had been 0.1 rather than 1 for amplifier 1, pots 60, 2,and 1 would have ratios of 0.0429, 0.0400, and 0.0400, respectively. The quotient obtained by dividing the coefficient of the integrator by the summation coefficient of the output will be equal to the transit time of the fluid in that portion of the system represented by the circuit.

D. Heat Exchanger

The rate of transfer of heat between a fluid and the metal of the heat exchanger was taken as being directly proportional to the difference of the mean temperatures of the metal and the fluid. The mean temperature of the metal was taken to be proportional to the integral of the difference between the power being absorbed by the metal and the power being removed from the metal. Thus:

$$P_f \sim T_{fh} - T_{he}$$

$$P_c \sim T_{he} - T_{ch}$$

$$T_{he} \sim \int (P_f - P_c)\,dt$$

or,

$$\frac{dT_{he}}{dt} \sim P_f - P_c$$

The sumbol $\sim$ means "proportional to".

The metal of the heat exchanger was assumed to rise at the rate of 150°F/sec. if all of the 350 megawatts of power were being stored in the metal. Thus, a difference between P_c and P_f of 14 volts was to cause T_{he} to rise at the rate of 15 volts/sec. The equation for amplifier 7 may be stated as follows:

$$-(0.241)(5)P_f + (0.241)(5)P_c + (1)\frac{dT_{he}}{dt} = 0$$

or, $$\frac{dT_{he}}{dt} = 10.7(P_f - P_c)$$

Now, if $$P_f - P_c = 14 \text{ volts}$$

$$\frac{dT_{he}}{dt} = (10.7)(14) = 15 \text{ volts/sec.}$$

The ratio of 0.214 on pots 11 and 18 to obtain a total summation coefficient of 10.7 for P_f and P_c was determined by setting $\frac{dT_{he}}{dt}$ at 15 volts/sec. and computing the coefficient for the difference of P_f and P_c.

The difference of 50°F between the mean temperature of the fuel and the mean temperature of the metal was to force 350 megawatts of power into the heat exchanger metal from the fuel. The 5 volt difference between T_{fh} and T_{he} at design point was to produce 14 volts as the value of P_f. The expression for the conditions of amplifier 5 are written as follows:

$$+(0.560)(5)T_{fh} - (0.560)(5)T_{he} - (1)P_f = 0$$

or, $$P_f = 2.8(T_{fh} - T_{he})$$

Obviously, the coefficient of the difference between T_{fh} and T_{he} had to be 2.8 if a 5 volt difference is to produce 14 volts as P_f. Thus, the ratios of pots 9 and 16 were 0.560. A similar relation was applied to amplifier 9 which computed P_c, requiring the ratios of pots 15 and 25 to be 0.560, also.

On the basis that $-P_f$ and $+P_c$ were available, amplifier 7 computed T_{he} by integrating the difference between the heat input and output rates of the metal. Amplifier 8 then produced $-T_{he}$. Amplifier 5 computed $-P_f$ from $+T_{fh}$ and $-T_{he}$; amplifier 6 then gave $+P_f$. Assuming that $+T_{ch}$ was available, amplifier 9 computed $+P_c$ from $+T_{ch}$ and $-T_{he}$, then amplifier 10 produced $-P_c$.

An approximation of the somewhat complex heat exchanger may be set up using only two amplifiers. The values of P_f and P_c are almost identical except during fast transients. If they are taken as being equal and called P, then this value of P is proportional to the difference between the temperatures T_{fh} and T_{ch}. A circuit to perform these operations was set up as drawn in Fig. 26.

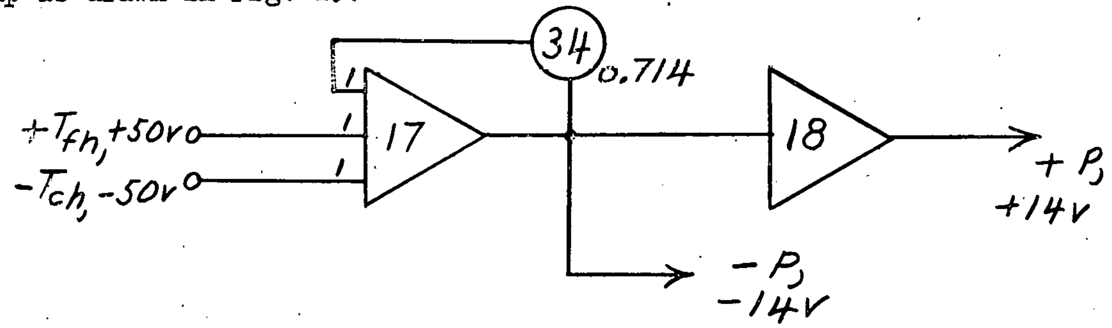


Fig. 26

In order to obtain a negative value of T_{ch}, the polarities of the voltages in the coolant loop and steam generator were reversed by reversing the polarity of the steam load voltage.

The reactor transients in the simulation using the simplified approximate heat exchanger were practically identical to those using the system drawn in ORNL-LR-DWG 26401. The variables of reactor power and temperatures, and of steam power and temperature differed only slightly from those in ORNL-LR-DWGS 26403, 26405, 26407, 26421, and 26406, which were obtained using the more complete heat exchanger. No heat storage in the metal was considered. The effect of heat storage may be at least partially simulated by introducing first order lag into the circuit of amplifier 17 in Fig. 26.

E. Coolant Loop

The circuit for simulating the coolant loop was set up in a manner similar to that for the fuel loop. The initial rate of increase of T_{ch} from amplifier 14 at the rate of 3 volts/sec. when a step change in P_c of 14 volts was applied required that pot 20 have a voltage ratio of 0.214. In order that the steady state value of T_{ch} was 15 volts higher than T_{ci} at design point, and equal when P_c was zero, the ratios of pots 50 and 28 were 0.200. Pots 19, 27, and 22 of amplifier 11 were set with the same ratios as pots 20, 50, and 28, respectively.

A function delay of 5 sec. was connected to simulate the transport lag in each section of the piping between the heat exchanger and the steam generator. An extra amplifier was included in each function delay to give the proper sign and value to the output voltage. Refer to the description of Function Delay in the section on <u>Complex Operators</u>, pages 21 and 22.

The heat removal from the coolant in the steam generator was simulated by the circuits of amplifiers 12 and 13. The actual transit time of the coolant through the steam generator was 12.5 sec., giving a coolant rate of decrease in temperature of 24°F/sec. However, in an attempt to account for the heat capacity of the steam generator, the simulator was set up with a 20°F/sec. rate of decrease in temperature. The heat capacity of the steam generator was not set up as a separate circuit. Pot 56, with a voltage ratio of 0.143, produced the initial rate of decrease in T_{cg} of 2 volts/sec. when P_s was step increased to 14 volts. The ratios of 0.133 in pots 48 and 24 set up the steady state conditions. Pots 54, 23 and 26 were identical to pots 56, 48, and 24.

F. Steam Generator

A complete simulation of the steam generator was not attempted. The voltage from pot 15 determined the steam load. This load was assumed to be constant at the value set by pot 15 regardless of the steam temperature T_S. The steam load P_S from amplifiers 27 and 28 produced the required temperature drop in the coolant passing through the steam generator at the design point condition.

The steam temperature was assumed to be associated with the inlet temperature of the coolant into the steam generator. This assumption was reasonable on the basis of counter flow of the steam and coolant. At design point, the difference between the steam temperature and the coolant inlet temperature was 250°F, or 25 volts. This difference was taken as proportional to the steam load. Pot 55 was set for a voltage ratio of 0.357 in order that amplifier 29 would compute the steam temperature in accord with these assumptions.

G. Simulation Results

The results of varying the steam load and rod position are shown in ORNL-LR-DWGS 26403, 26405, 26407, 26421, and 26406.

Actually the ramp changes in load and rod reactivity were probably faster than would be employed in a power plant of this type. Note that a change in the rod δk of 0.5% produced a change of 173°F in the reactor mean temperature, which agrees with the specified temperature coefficient. The step drop in load was simulated to approximate the variation caused by the loss of electrical load in case the circuit breaker connecting an electric generator to the transmission line were to open.

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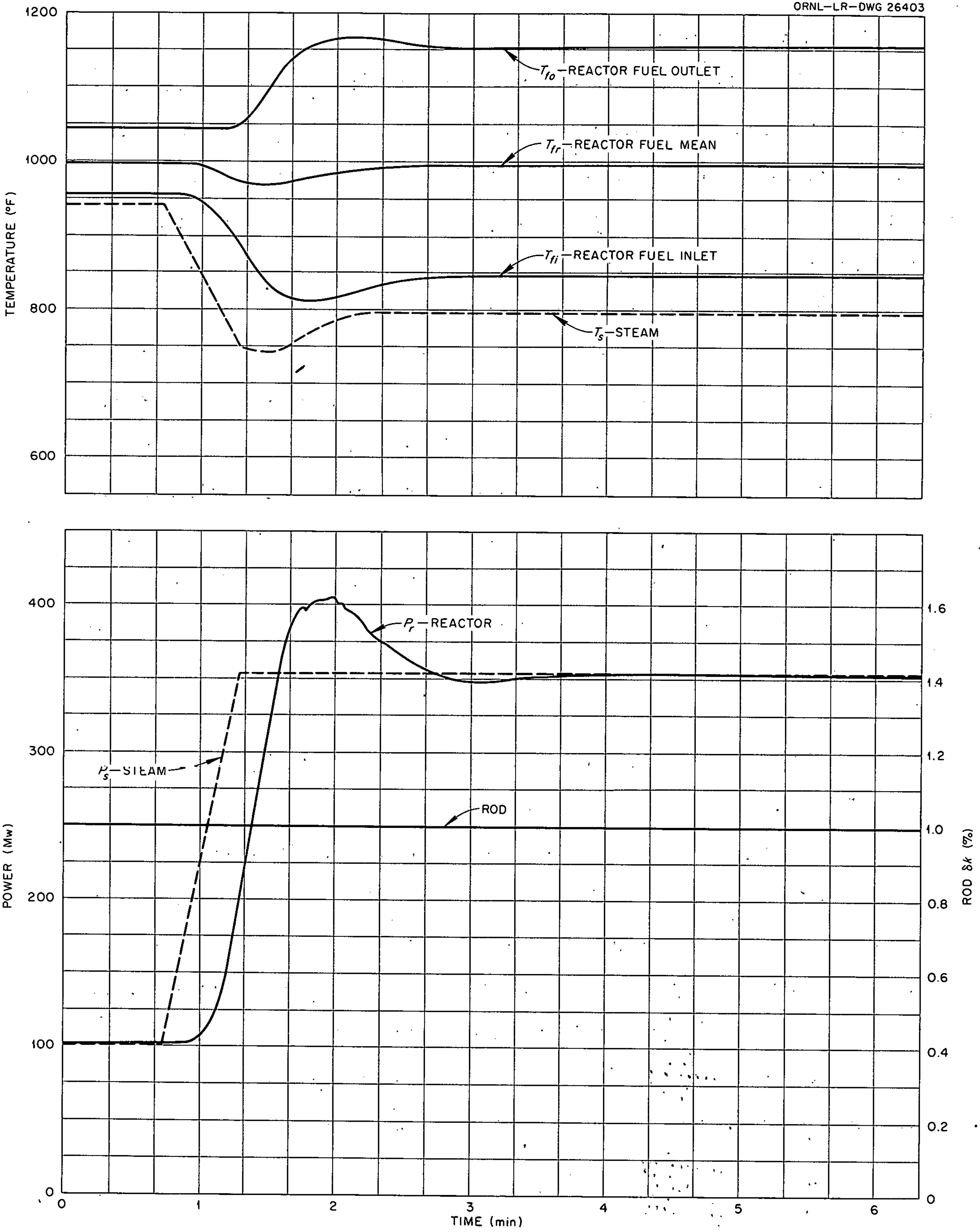
ORNL-LR-DWG 26403



Load Increase; Simulation of ORSORT Buttermilk Reactor.

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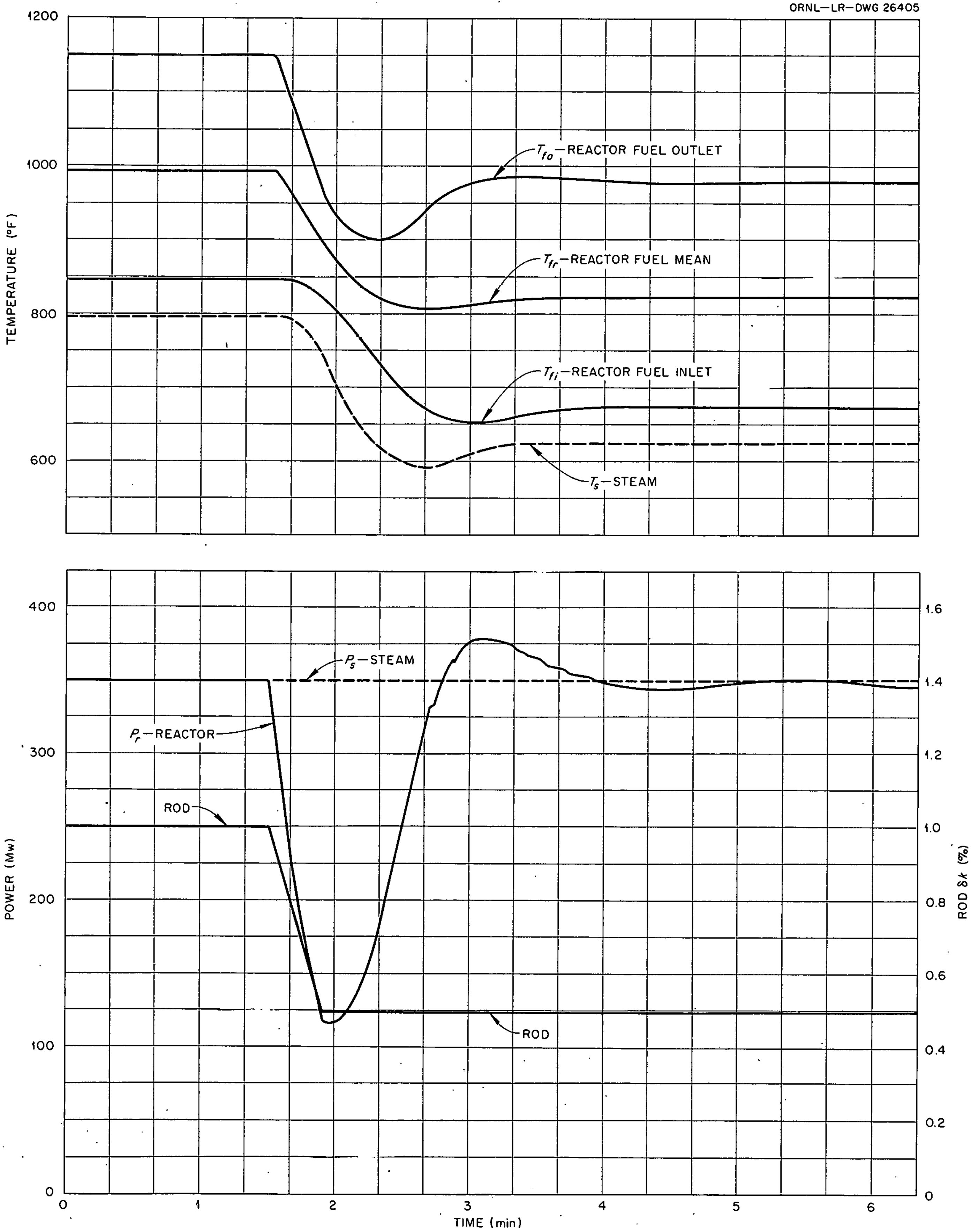
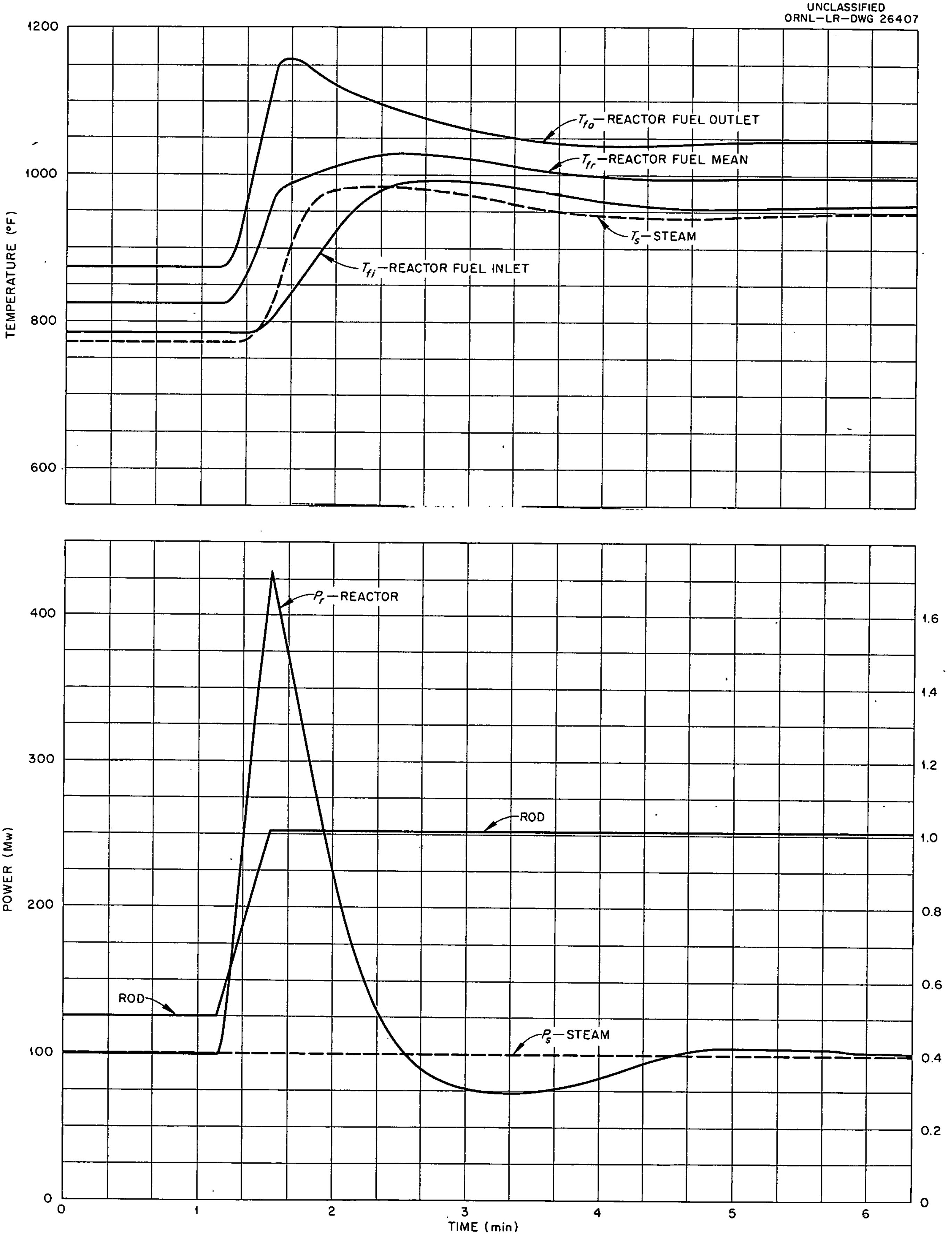
ORNL—LR—DWG 26405



Rod Insertion; Simulation of ORSORT Buttermilk Reactor.

143 041



Rod Withdrawal; Simulation of ORSORT Buttermilk Reactor.

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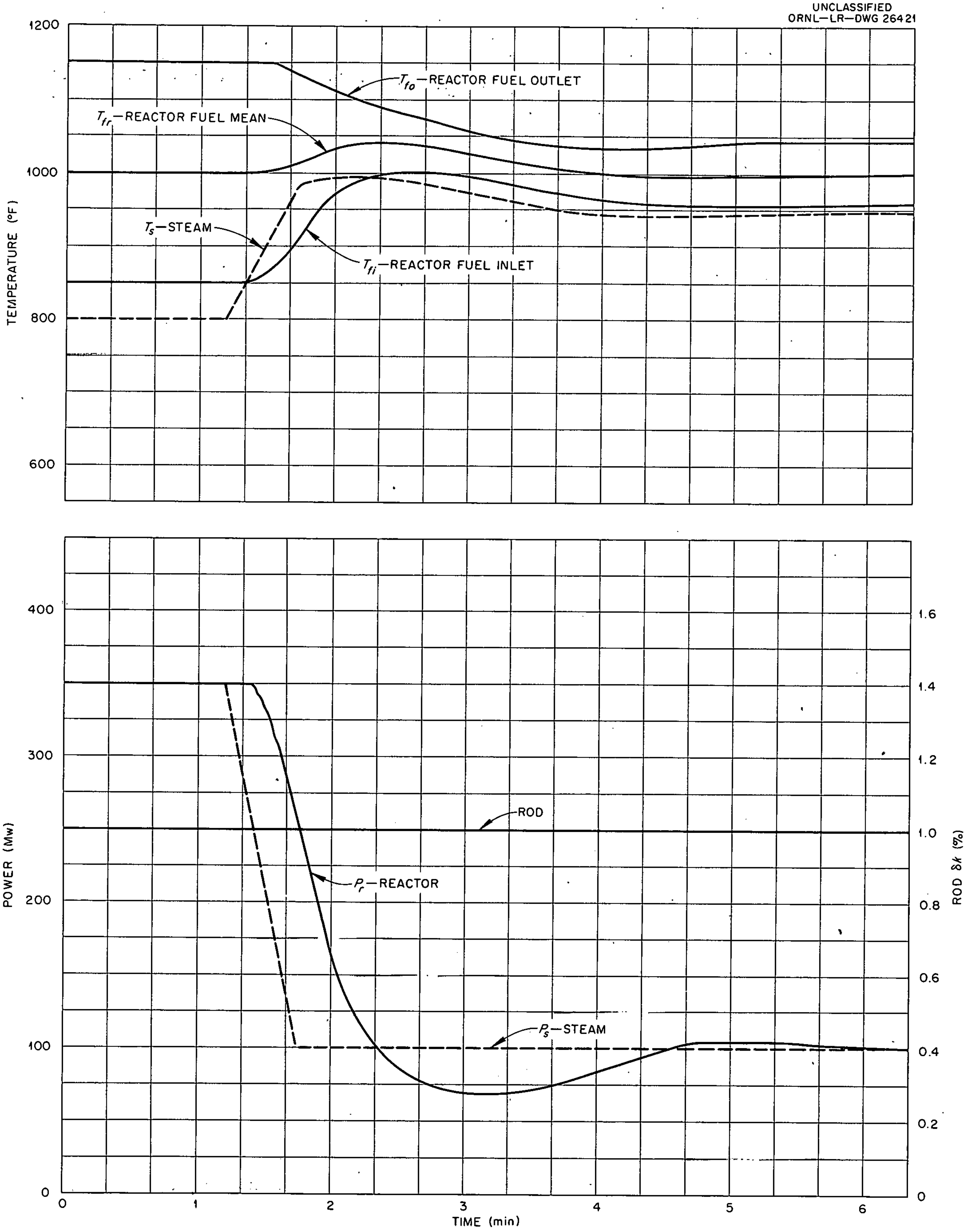
ORNL—LR—DWG 26421



Load Decrease; Simulation of ORSORT Buttermilk Reactor.

143 043

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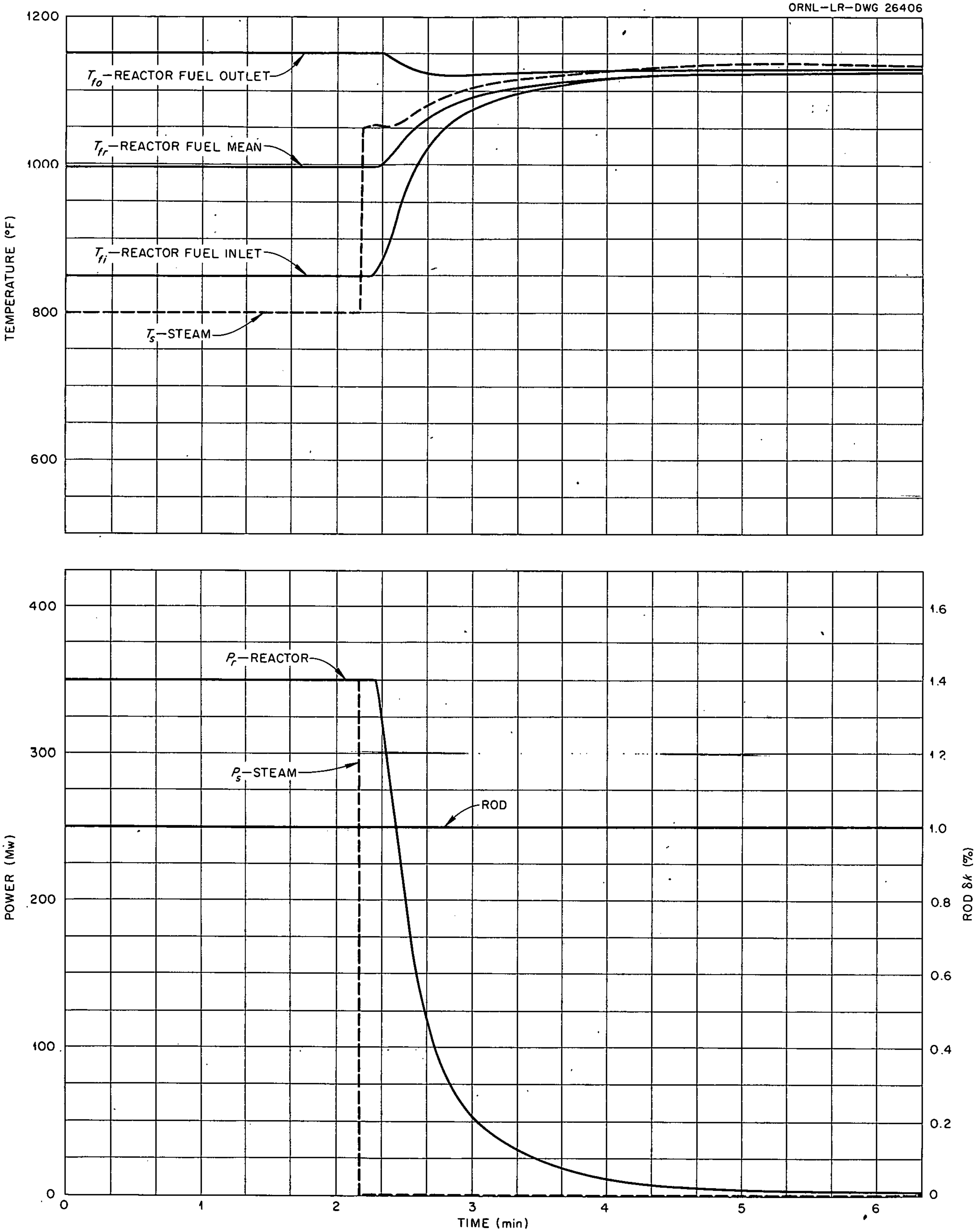
ORNL-LR-DWG 26406



Load Drop; Simulation of ORSORT Buttermilk Reactor.

143 044

H. Period and Log N

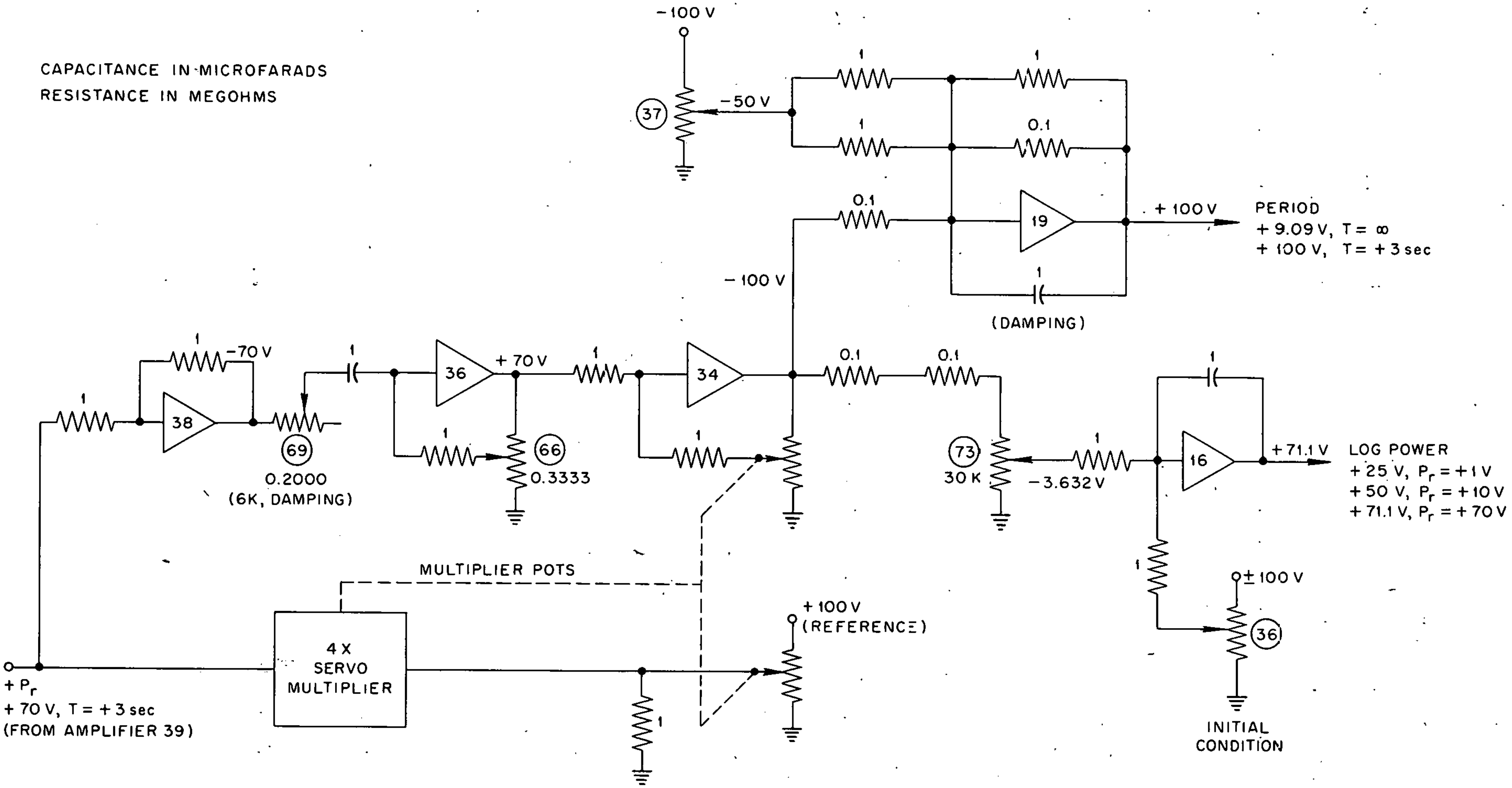
The computation of the reactor period and the logarithm of the power were not important to the system studied. This portion of the simulation is included here as an example of one of the types of calculations that an analog computer can perform.

The principle employed was that of dividing $\frac{dn}{dt}$ by n to obtain a voltage proportional to the reciprocal period. Refer to page 25 of CF 57-1-1. The voltage proportional to $\frac{\frac{dn}{dt}}{n}$ was then integrated with respect to time to produce a voltage proportional to the logarithm of n.

The explicit circuit is drawn in ORNL-LR-DWG 26400. The time derivative of P_r, or n, was taken by amplifier 36. Amplifier 38 merely reversed the sign of the voltage P_r. Pot 69 was connected to provide a certain amount of smoothing of the differentiated voltage. Without this 6000 ohms of damping resistance, the output of amplifier 36 was quite noisy. Amplifier 34 together with servo multiplier 4 computed the quotient of $\frac{dn}{dt}$ divided by n. When the period was 3 sec., the output of amplifier 34 was -100 volts. When the period was infinite, this output was zero. The servo multiplier positioned the slider of the pot having the 100 volt reference to obtain a voltage from this slider equal to the input voltage P_r. Since the slider of the multiplier pot in the output of amplifier 34 was also being positioned, the coefficient of the output voltage of amplifier 34 was continuously varied by the servo multiplier. The output of amplifier 34 may be stated as follows:

UNCLASSIFIED
ORNL-LR-DWG 26400

CAPACITANCE IN MICROFARADS
RESISTANCE IN MEGOHMS



Explicit Circuit for Period and Log N for Simulation of ORSORT Buttermilk Reactor.

$$\left[\begin{array}{c}\text{Output of}\\ \text{amplifier 34}\end{array}\right] = -\frac{\left[\begin{array}{c}\text{Output of}\\ \text{amplifier 36}\end{array}\right]}{\left[\begin{array}{c}\text{Voltage ratio of}\\ \text{multiplier pots}\end{array}\right]}$$

The multiplier pot voltage ratio was 0.700 at the condition taken in the circuit diagram, causing the output of amplifier 34 to be -100 volts.

Amplifier 16 computed the logarithm of the power by integrating the output of amplifier 34. Pot 73 was set to have an actual output of -3.623 volts when the output of amplifier 34 was -100 volts. This value of -3.623 volts would increase the output of amplifier 16 by 25 volts after a time lapse of 6.9078 sec. The log power recorder required 25 volts to cover one decade of power change. A 3 sec. period will produce one decade of power change in 6.9078 sec. In order that the scale of the log power recorder was properly indexed, the output of pot 36 was applied to amplifier 16 to set the recorder at the desired value corresponding to the reactor power. The voltage from pot 36 was zero during the recording of a transient.

The output of amplifier 34 was supplied to amplifier 19 in order that the standard period chart paper could be used on the period recorder. The voltage from pot 37 properly indexed the recorder to indicate an infinite period when the output of amplifier 34 was zero. The circuit resistances provided that the recorder indicated full scale when the period was 3 sec. A capacitor to smooth out the rapid variations of the output of amplifier 34 was necessary, especially at low values of reactor power.

The results of a step increase in nuclear multiplication are described by the curves in ORNL-LR-DWG 26404. The reactor temperature coefficient was eliminated, and the delayed neutron precursors were set up for stationary uranium -235. Note the noisy indication of reactor period at low power levels. Also, note that the log power curve is asymptatically approaching a straight line as the period becomes longer and approaches the asymptatic value of 22.7 sec.

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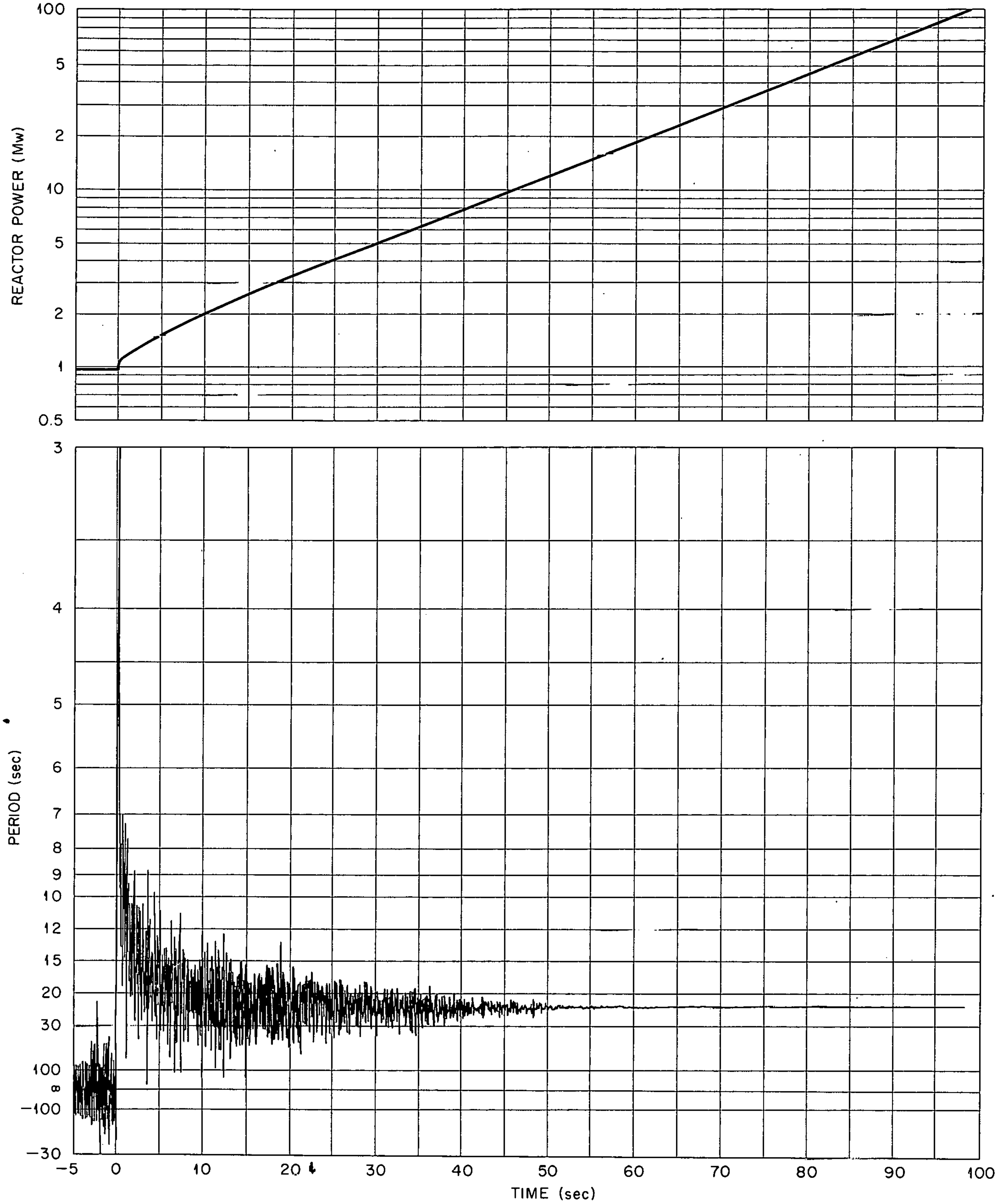

Step Insertion of 0.198 % δk, Stationary Fuel, No Temperature Coefficient; Simulation of ORSORT Buttermilk Reactor.