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SUBJECT: FLUX-TRAP REACTOR WITH ABSORBER IN THE CENTER

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FLUX-TRAP REACTOR WITH ABSORBER IN THE CENTER

ABSTRACT

The idealized flux-trap reactor treated in Refs. 1 and 2 is modified by the insertion of an absorber. It is shown that, for appreciable absorption, a flux depression results, and the remaining flux is proportional to the diffusion constant D times the center flux in the nonabsorption case. This factor D just cancels the factor 1/D in the expression for this center flux (Ref. 1) so that the flux in the case with absorber is independent of D. In the case with absorber the advantage of Be and BeO largely disappears.

In two previous memoranda[1,2] we considered a flux-trap reactor idealized in the following manner. The fuel is concentrated in a spherical shell, embedded in an infinite moderator, that is, the moderator occupies the space inside as well as outside the shell. The shell is "infinitely thin," but "black" for thermal neutrons, that is, the thickness of the shell is small compared with its radius, but large compared with the diffusion length in the fuel. The shell emits ηf fission neutrons for each thermal neutron absorbed. Absorptions at epithermal energies, both in the fuel and the moderator, are neglected. In this model the fission neutrons emitted from the shell slow down according to the age kernel and then diffuse according to diffusion theory, with the boundary conditions created by the "black" shell.

The flux trap is formed by the moderator enclosed by the fuel shell. The thermal flux at the center of this flux trap, normalized to the emission of one fission neutron by the shell, was computed by means of the last equation in Ref. 1. This equation assumed that the only absorption in the flux trap was the absorption by the moderator itself. In other words, no sample of appreciable absorption had been introduced.

As soon as absorbing material is inserted at the center of the internal thermal column, the flux at this location will be depressed as compared to the flux obtained in the previous memoranda for the idealized case without such absorber. In order to compute the effect of this depression, we idealized the absorber as a sphere at the surface of which the flux has to vanish. The radius of the sphere is called the effective radius r_1. The total current into the sphere is a measure of the depressed flux.

Equation (3) of Ref. 1, which gives the thermal flux caused by a shell source at $r = r'$, has to be modified to fit the new boundary condition at

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1. W. K. Ergen, Flux Distribution in a Reactor Consisting of a Spherical Shell of Fuel in an Infinite Moderator, ORNL CF-57-12-100 (Dec. 24, 1957).
2. W. K. Ergen, Fluxes Obtainable in a Flux-Trap Reactor, ORNL CF-58-1-4 (Jan. 15, 1958).

These memoranda give the notation used here.

$r = r_1$. The following expression vanishes at $r = r_1$ and $r = r''$, has the correct discontinuity of its derivative at $r = r'$, and satisfies the differential equation $\nabla^2\phi - \mathcal{K}^2\phi = 0$ for all $r_1 \leqq r \leqq r''$, $r \neq r'$.

$$d\phi_1 = \frac{S(r')\,dr'}{8\pi\, r\, r' \mathcal{K} D}\Bigg\{ e^{-\mathcal{K}\left|(r-r_1)-(r'-r_1)\right|} - e^{-\mathcal{K}\left[(r-r_1)+(r'-r_1)\right]} \tag{1}$$

$$- \frac{e^{-\mathcal{K}(r''-r_1)}}{e^{\mathcal{K}(r''-r_1)} - e^{-\mathcal{K}(r''-r_1)}}\left[e^{\mathcal{K}(r'-r_1)} - e^{-\mathcal{K}(r'-r_1)}\right]\left[e^{\mathcal{K}(r-r_1)} - e^{-\mathcal{K}(r-r_1)}\right]\Bigg\}$$

The current into the sphere of radius r_1 is given by:

$$j = -4\pi\, r_1^2\, D \left|\frac{\partial}{\partial(-r)}\, d\phi_1\right|_{r=r_1} = \frac{r_1}{r'}\, S(r')\, dr'\, \frac{e^{\mathcal{K}(r''-r')} - e^{-\mathcal{K}(r''-r')}}{e^{\mathcal{K}(r''-r_1)} - e^{-\mathcal{K}(r''-r_1)}} \quad \ldots \ (2)$$

We substitute for $S(r')\,dr'$ from Eqs. (1) and (2) of Ref. 1 and integrate over r' from r_1 to r''. Using Eq. (1) of Ref. 1 implies that we neglect the influence of the sample on the slowing-down process.

$$j = \int_{r_1}^{r''} dr'\, \frac{r_1}{r'}\, \frac{e^{+\mathcal{K}(r''-r')} - e^{-\mathcal{K}(r''-r')}}{e^{\mathcal{K}(r''-r_1)} - e^{-\mathcal{K}(r''-r_1)}}\, \frac{4\pi r'^2}{8\pi^{3/2}\,\tau^{1/2}\, r' r''}\left[e^{-(r'-r'')^2/4\tau} - e^{-(r'+r'')^2/4\tau}\right]$$

$$= \frac{r_1}{2r''}\, \frac{e^{\mathcal{K}^2\tau}}{e^{\mathcal{K}(r''-r_1)} - e^{-\mathcal{K}(r''-r_1)}}\Bigg\{ 2\, \mathrm{erf}\,\mathcal{K}\sqrt{\tau} + \mathrm{erf}\left(\frac{r''-r_1}{2\sqrt{\tau}} - \mathcal{K}\sqrt{\tau}\right) - \mathrm{erf}\left(\frac{r''-r_1}{2\sqrt{\tau}} + \mathcal{K}\sqrt{\tau}\right)$$

$$- e^{2\mathcal{K}r''}\left[\mathrm{erf}\left(\frac{r''}{\sqrt{\tau}} + \mathcal{K}\sqrt{\tau}\right) - \mathrm{erf}\left(\frac{r_1+r''}{2\sqrt{\tau}} + \mathcal{K}\sqrt{\tau}\right)\right] + e^{-2\mathcal{K}r''}\left[\mathrm{erf}\left(\frac{r''}{\sqrt{\tau}} - \mathcal{K}\sqrt{\tau}\right) - \mathrm{erf}\left(\frac{r_1+r''}{2\sqrt{\tau}} - \mathcal{K}\sqrt{\tau}\right)\right]\Bigg\}$$

For $r_1 \ll r''$

$$j = \frac{r_1}{2r''} \frac{e^{K^2}}{e^{K\rho} - e^{-K\rho}} \Big\{ 2 \text{ erf } K - e^{2K\rho} \text{ erf}(K+\rho) + (e^{2K\rho} - 1) \text{ erf}(K+\tfrac{\rho}{2})$$

$$+ (e^{-2K\rho} - 1) \text{ erf}(K-\tfrac{\rho}{2}) - e^{-2K\rho} \text{ erf}(K-\rho) \Big\} = 4\pi D r_1 \phi_c,$$

where ϕ_c is the center flux of reactor without absorber as given by the last equation of Ref. 1.

If there were no flux depression the sphere would absorb $4\pi r_1^2 \phi_c$ neutrons. The neutron absorption is thus reduced by the flux depression in the ratio D/r_1. Since our use of diffusion theory presupposes that r_1 is larger than a transport mean free path λ_{tr}, and since $\lambda_{tr} = 3D$, we really have a reduction in flux. The transport mean free path is, for the moderators considered, about 1 1/2 to 2 1/2 cm, and the radii of the flux-trap reactor are about 20 to 30 cm (see Table 2 in Ref. 2). Hence the assumption that $r_1 > \lambda_{tr}$ is not in contradiction to the assumption that r_1 should be considerably smaller than the radius of reactor.

For instance, for a beryllium moderator, D is 0.48, and for an effective radius of r_1 4.8 cm, the flux would be depressed to only 0.1 of the flux computed for the case without absorber. In D_2O, the same effective radius would result only in a flux depression of 0.883/4.8 = 0.184. Generally speaking, substances with large D give low fluxes in a flux-trap reactor without absorption; but when the sample has appreciable absorption, the factor D in the flux depression just cancels the factor 1/D in the expression for the center flux (last equation of Ref. 1), and the depressed flux is independent of the diffusion constant. In Table 1, the maximum center fluxes, taken from Table 2 of Ref. 2, are multiplied by D to make them comparable for the case of appreciable absorption. It may be seen that the advantages of Be and BeO largely disappear for the absorption case. For small effective radius r_1

the transport theory would have to be used in the computation. As r_1 becomes smaller than λ_{tr}, the flux depression disappears as a neutron absorbed by the sample would not have returned to the sample even if it had survived.

TABLE 1. RELATIVE VALUES $\phi_c D$ OF CENTRAL FLUX IN FLUX TRAP-REACTOR WITH ABSORBER

Moderator	Density (g/cm^3)	$\Phi_c D$ (n/cm/fission n)
D_2O	1.1	0.000441
Be	1.85	0.000455
BeO	3.0	0.0004507
C	2.0	0.0003068