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MEASUREMENT OF THE DOPPLER TEMPERATURE EFFECT IN AN EBR-I TYPE ASSEMBLY

by

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ABSTRACT

A measurement of the Doppler temperature effect has been made in the fast spectrum of a mock-up of the EBR-I reactor in ZPR-III. The effect was measured by thermally cycling samples of enriched uranium, natural uranium and plutonium, and detecting the small changes in reactivity. The pile oscillation technique using a resonant detector was employed to measure the small oscillating component of the neutron flux. An upper limit, 0.5×10^{-8} $\Delta k/\Delta T$, was obtained for a 506-gram sample of U^{235} in the spectrum of an EBR-I mock-up, and for a 235-gram sample of Pu^{239} in a plutonium-fueled assembly of the same configuration. The results for natural uranium were inconclusive.

I. INTRODUCTION

Knowledge of the prompt temperature coefficient of reactivity for fast reactors is important because of its role in the safety of these systems. The coefficient determines the size of the reactivity change which would result from a temperature excursion. It is, therefore, an important factor in determining the limit of any power excursion which the reactor may incur. One source of the prompt temperature coefficient is the Doppler temperature effect, which is of particular interest because it can be either positive or negative.

The Doppler effect arises from the temperature variation of the distribution of relative velocities between neutrons and nuclei. The distribution of velocities results from thermal motion. The effect essentially increases the width and decreases the height of resonance levels. If the temperature rises, the effect increases. As a result, the neutron cross section is a function of temperature.

Changes in the cross sections of atoms in a reactor cause changes in reactivity only if they change the neutron population. The rate at which neutrons are absorbed is proportional to the cross section multiplied by the flux. The over-all change of neutron population in the reactor is proportional to the integral of this product over the neutron energy spectrum.

Doppler broadening changes the cross section but not the integral, unless the flux also varies with energy. A suitable variation of flux results, for example, if a resonance is strong enough to result in self-shielding. Self-shielding occurs when a resonance depletes the flux to such an extent that the effective volume cross section is considerably less than the total volume cross section. Doppler effect causes material to absorb neutrons not only from the energy region of the natural width of the resonance, but also from the additional width due to the broadening. This results in an increase in the neutron absorption rate and, thus, a change of reactivity.

In the thermal and resonance energy regions, the magnitude of the Doppler effect may be determined theoretically, since resonance level parameters are well known for U^{235} and U^{238}. In the energy spectrum of fast reactors, however, level distributions may only be estimated since the resolution of present-day neutron spectrometers is inadequate to obtain the parameters. At energies around 200 kev, the resonance levels are broadened and so closely spaced they tend to overlap; this decreases the change in the cross section due to Doppler broadening. There is, however, the possibility that the distribution of level strengths and spacing is such that, in spite of the tendency of the levels to overlap, there is a change in the absorption characteristics of the atom in question on heating.

Feshbach, Goertzel and Yamauchi at NDA,[1] Lane, Lynn and Story of AERE,[2] and Bethe[3] have investigated theoretically the Doppler effect for U^{235} and U^{238} for fast reactors. Their results indicate a strong dependence on the resonance parameters and level distribution in the hundreds-of-kilovolts energy region. In view of the importance of the Doppler effect and the uncertainty in the parameters in this energy region, it was thought desirable to make an integral measurement which would determine the sign and magnitude of the effect for U^{235}, U^{238}, and plutonium for a particular fast assembly.

The measurements were made on a mock-up of the EBR-I core in the Argonne Fast Critical Assembly (ZPR-III)[4] shown in Fig. 1.

Since it would be difficult to heat up an entire core and separate the Doppler effect from the effects due to thermal expansion, samples representing a small fraction of the total fuel in the core were heated. The change in reactivity due to the Doppler effect in a sample of average worth is proportional to the ratio of the sample mass to the critical mass of the assembly.

The size of the samples determined the measurement technique. The method employed was a modification of the Langsdorf technique.[5] The small changes in reactivity anticipated ($\sim 5 \times 10^{-7} \Delta k/k$, or 2.0 inhours) precluded their measurement by the period technique. Changes of ambient temperature and pressure cause continual changes of reactivity sufficient to completely drown out the changes to be measured. In addition, the heat

from the sample would gradually heat the surrounding assembly and cause greater changes in reactivity. The effects of slow drifts of reactivity on the measurement were eliminated by using an oscillation technique.

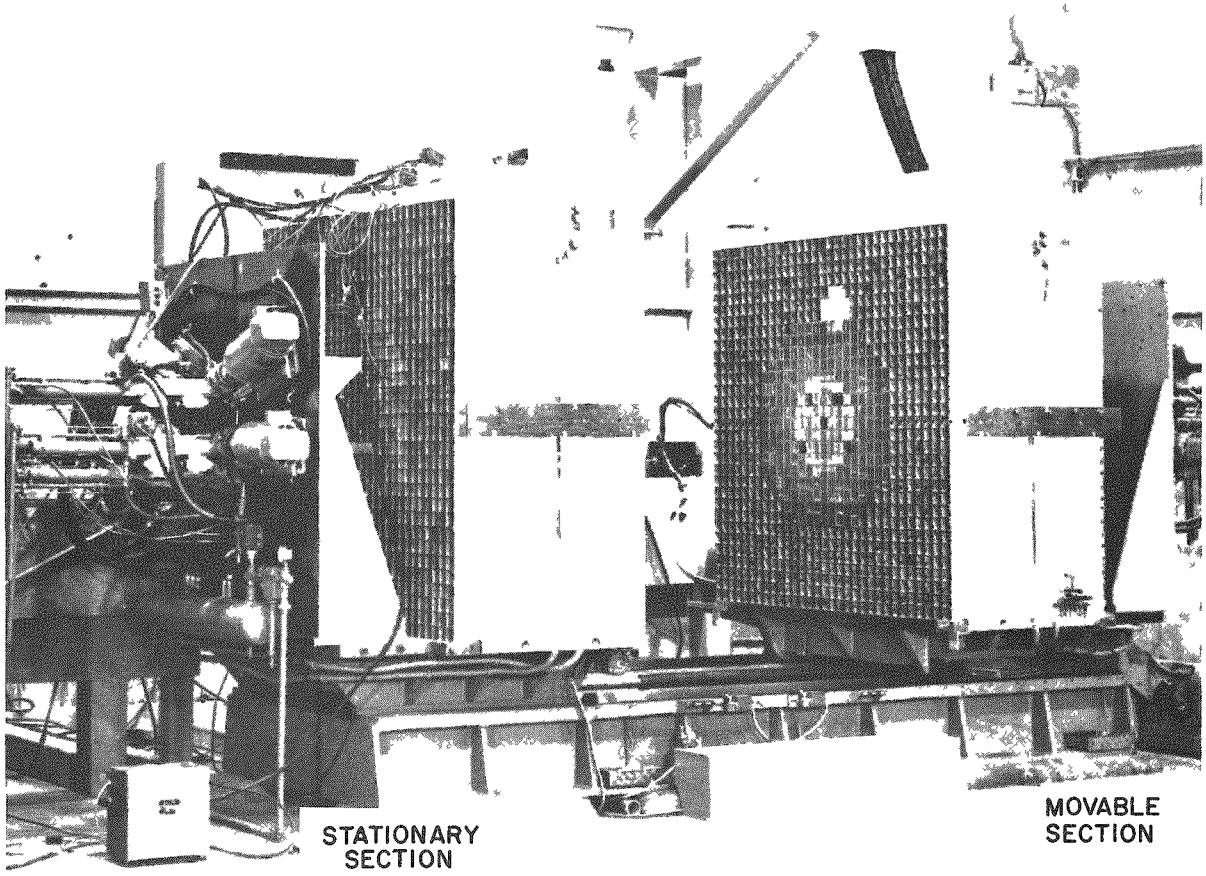

FIG. I
ZPR-III CRITICAL ASSEMBLY

II. OSCILLATION THEORY

When small sinusoidal oscillations are introduced in a reactor which is just critical, the population of reactor neutrons oscillates sinusoidally with an amplitude proportional to the reactivity change Δk;

$$\Delta N/N = A(\omega)\ \Delta k/k \quad ,$$

where ω is the oscillation frequency. The change (ΔN) will be shifted in phase from the reactivity change by $\phi(\omega)$.

The functions A and ϕ depend on the neutrons of the delayed characteristics of the particular assembly. They may be obtained by computation, knowing the characteristics of the delayed neutrons, or by direct measurement. For nonsinusoidal oscillations the neutron population will vary according to the Fourier components of the driving function. However, each harmonic will be amplified and shifted in phase by different amounts; consequently, the neutron population will have a different shape as a function of time than the driving function. For this reason, in an oscillation experiment it is simplest to consider only the fundamental components of the driving function and population.

The sensitivity of the oscillation technique is limited by the noise in the output of system used for detecting reactivity changes. There are two principal sources of this noise. The first source consists of fluctuations of current in the ion chambers used to detect the oscillations. These fluctuations result from statistical fluctuations of the number of ionizations which occur in the chamber per unit time. The fluctuations can be minimized by using chambers which detect the greatest number of neutrons. The amplitude of the noise contributions can be estimated, using the theoretical results of Brownrigg and Littler,[6] Frisch and Littler,[7] and Feiner, Frost, and Hurwitz.[8]

The chamber noise can be estimated if the chamber current is known. Assuming the average charge collected per neutron detected is one-fifth that which would have been produced in a BF_3 chamber (or 2×10^{-15} coulomb per event), the number of events detected per unit chamber current was about 5×10^8 events per second at 10 μamp.

Using the formula of Feiner, et al.,[8] the expected root-mean-square noise amplitude detected due to chamber noise (N_2) may be obtained from

$$N_2^2 = 2CI_0 \, D_1^2(\omega_0) \frac{1}{T} \quad ,$$

where

I_0 = chamber current
C = charge collected per fission detected
$D_1(\omega)$ = detector response per unit change of chamber current
T = total time of a run.

For a run of 53 min at 10 μamp, the value of N_2 is 0.013 scale division.

The second and most important source of noise is the assembly proper. The prompt fissions tend to follow each other in chains: prompt neutrons cause prompt fissions which, in turn, cause prompt neutrons.

These chains appear as bursts of neutrons at the chambers and cause fluctuations of the chamber current in addition to those caused by statistical fluctuations of the ionizing events. The theory of Feiner affords an estimate of the root-mean-square amplitude of the "reactor" noise (N_1), provided the fission rate is known:

$$N_1^2 = 2X_2 \left(\frac{A^2(\omega_0)}{F} \right) D^2(\omega_0) \frac{1}{T} \quad ,$$

where

F = $\frac{\text{fission rate}}{}$
$X_2 = \overline{\nu(\nu-1)}$, where ν = number of neutrons per fission
$D(\omega)$ = detector response function per unit fractional change of assembly power.
T = total time of run
$A(\omega)$ = pile amplification.

The fission rate in the ZPR-III was estimated by comparing the count rate of a particular counter in the present core with previous loadings for which the total fission rate had been computed. For the same run mentioned above, the value of N_1 was 0.83 scale division.

The theory of Frisch,[7] which employs a different derivation to obtain a similar formula, predicts this noise should be a factor of $\overline{\nu}$ ($\overline{\nu} \cong 2.5$) less.

There are two ways to improve the measurement of a signal in the presence of noise: (1) increase the reactor power; and (2) make observations over a greater number of cycles. The signal ΔN detected is proportional to the total neutron population N:

$$\Delta N = A(\omega)\, N \Delta k \quad .$$

The noise, on the other hand, is proportional to the square root of the neutron population, so the signal-to-noise ratio is proportional to the square root of the neutron population or the power. In this experiment the power level was set at about 5 watts to avoid making the fuel too active to be handled and, at the same time, to give an adequate signal-to-noise ratio.

In measuring a small sinusoidal signal in the presence of noise, it is possible to measure the output of the experiment at only those instants where the signal is expected to be maximum or minimum. However, the output of the experiment also contains information during intra-peak periods when the signal is rising or falling. In fact, the greatest signal-to-noise ratio is obtained by taking an appropriate average of the signals weighted according to the expected time dependence.

If the output of the ion chambers is of the form

$$B \sin \omega t + N(t) \quad ,$$

where B sin ωt represents the signal, and N(t) the noise component, the average is taken by multiplying by sin ωt as the weighting function and integrating from t_0 to

$$t = t_0 + nT.$$

Thus

$$\overline{B} = \frac{1}{n\pi} \int_{t_0}^{t_0 + nT} \left[A \sin \omega t + N(t) \right] \sin \omega t \, dt \quad ,$$

where the average is to be taken over n complete cycles of period T. This average $\overline{B}$ is the first Fourier coefficient of the chamber current and is also zero delay cross correlation coefficient between the current and sin ωt. It can also be shown that $\overline{B}$ is the best least squares estimate of B. Brownrigg and Littler[6] have shown experimentally that, in the absence of signal, the noise contribution closely fits a Gaussian distribution for repeated runs.

The integration can be performed in several ways,[5-7] the simplest being that of Langsdorf as perfected by Frost.[10] They used galvanometers as resonant detectors and integrators. The use of a resonant detector has the advantage of displaying, at any time during an experimental run, a deflection representing an average over a number of oscillation cycles. For the experiment reported here, the galvanometer was replaced by an electronic amplifier having similar characteristics. It was less sensitive to vibration and overload than the galvanometer and could be attached directly to a strip chart recorder.

The response R(t) of a resonant detector to a signal S(t) + N(T) can be written as an integral:

$$R(t) = \int_{t_0}^{t} \left[S(\tau) + N(\tau) \right] \cos \omega(t - \tau) \exp \left[-(\omega/2Q)(t - \tau) \right] d\tau \quad .$$

The arguments of the cosine and exponential represent the fact that the integration, as far as the detector is concerned, is from the time t back to t_0. The exponential results from damping in the circuit with the constant Q chosen by exact analogy to the Q used in radio circuit analysis.

The tuned amplifier would be a true integrator if it were not for the exponential term in the integral. Because of the exponential term it can be thought of as integrating approximately over the preceding Q/π cycles. In essence, it exponentially forgets the cycles before that. The principal advantage is that at the instants corresponding to peaks of the ion chamber signal component, the resonant detector output contains a signal-to-noise ratio of about $\sqrt{Q/\pi}$ greater than that shown by an untuned detector. This makes it easier to see a small signal. In addition, the tuned amplifier can be used to help perform an integration over a number of cycles. This is done by reading and averaging the amplitudes of its output at times t_i corresponding to successive maxima of the signal. To see this, consider the integral as written in two parts:

$$R(t) = \int_0^{t_i - \frac{2\pi}{\omega}} \left[S(\tau) + N(\tau)\right] \cos \omega (t-\tau) \exp \left[-(\omega/2Q)(t-\tau)\right] d\tau$$

$$+ \int_{t_i - \frac{2\pi}{\omega}}^{t_i} \left[S(\tau) + N(\tau)\right] \cos \omega(t-\tau) \exp \left[-(\omega/2Q)(t-\tau)\right] d\tau \quad .$$

The second integral, except for the small change due to the exponential, is the desired integral. For a sinusoidal signal and no noise the second integral is approximately $(1 - \pi/2Q)$ times the desired integral. For a Q of 25 the difference is only 6%. The first integral, however, contains contributions from all of the preceding cycles of the run. Thus, for an average taken over a run of n cycles, only $n\ \pi/Q$ of the readings can be considered independent. The signal-to-noise ratio of the entire run is therefore improved over that for a single cycle by about

$$\sqrt{Q/\pi}\ \sqrt{n\ \pi/Q} = \sqrt{n} \quad ,$$

as predicted by the Frisch[7] theory.

III. DESCRIPTION OF EQUIPMENT

In the Spring of 1956 attempts were made to measure the Doppler effect in an assembly with a large critical mass: an EBR-II mock-up in ZPR-III with an 89.1-gram sample of U^{235} as oxide. The results indicated the reactivity change was smaller than could be observed. To increase the signal it was decided to use metallic uranium, which would increase the sample density by a factor of about four.

A major problem in trying to measure the Doppler effect is that any motion of the thermally cycled element upon expansion and contraction can cause a change in the reactivity which is of the same order of magnitude as expected from the Doppler effect. The magnitude of the effect due to motion depends on the gradient of the uranium importance distribution and the coefficient of thermal expansion. To minimize the effect of motion, the elements to be thermally cycled were fabricated in the form of a helix or spring.(11,12)

A. Helical Elements

The uranium elements consisted of natural uranium, or enriched uranium rods (1/8 in. dia.), jacketed in nickel tubes (wall thickness of 5 mils), and wound into helices (1 in. ID). The tubes were sealed with stainless steel plugs. The natural uranium helix had 29 turns, an over-all length of $6\frac{1}{2}$ in., and contained 387.1 gm of natural uranium. The enriched helix (Fig. 2) had 41 turns, an over-all length of about $7\frac{1}{2}$ in., and contained 506 gm of U^{235}.

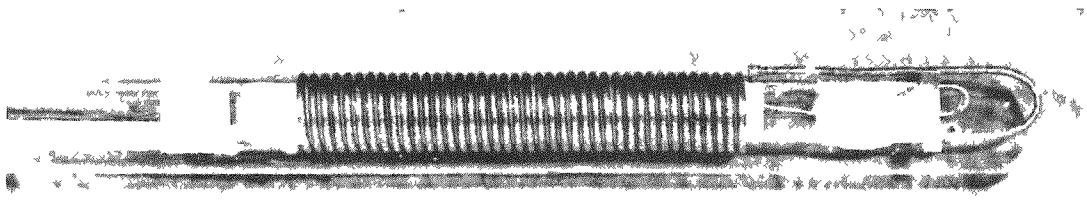

Fig. 2
ENRICHED URANIUM HELIX

Each helix was mounted on a porous frit filter (1 in. dia.) which was fused to a Pyrex tube. Electrical connections (copper rods) were soldered to the stainless steel plugs. Thermocouples were spot welded about 3/4 of a turn from either end of the helix. The entire assembly was mounted in a square aluminum tube which fit inside one of the matrix elements of the critical facility.

The plutonium elements consisted of three extruded plutonium rods (0.103 in. OD) screwed together and jacketed in a titanium tube (~8 ft long, 11-mil wall thickness). The tubing was swaged tightly over the plutonium rod, and both ends were sealed with welded titanium plugs, 1/2 in. long. The jacketed rod (0.125 in. OD) was wound into a helix with a 1-in. ID, 32 turns, and an over-all length of $6\frac{1}{4}$ in. Copper tube electrical connections were swaged over the ends of the coil.

All helices were thermally cycled by alternately passing a current (~130 amp) through the coils for 10 sec, and then flowing chilled helium across the turns for the cooling cycle. The helium was cooled through a coil of copper tubing immersed in a bath of dry ice and acetone. Figure 3 shows the electrical circuit used to control the heating cycle, and the flow of helium coolant.

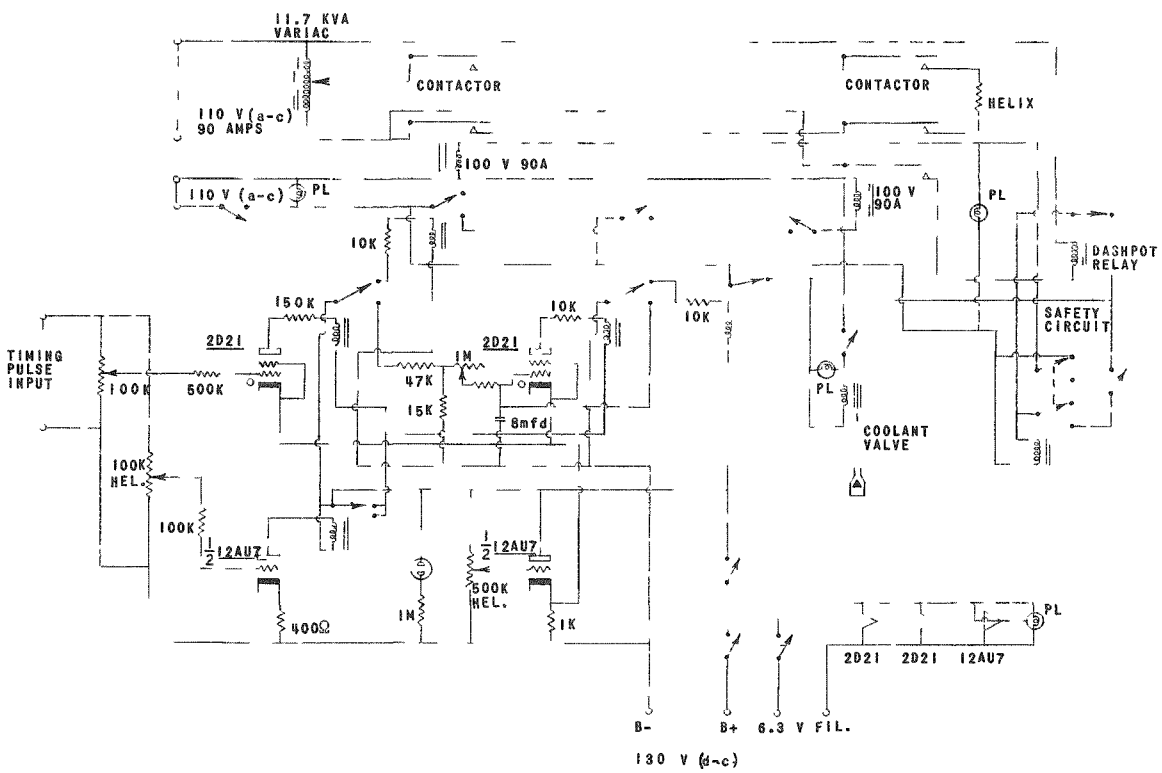

FIG. 3
HELIX HEATING AND COOLING CIRCUIT

B. Neutron Detectors

Special boron-10 ion chambers to detect fast neutrons were constructed. Since the absorption cross section of boron-10 is quite low at neutron energies of ~200 kev, each chamber (2-in. dia. cylinders) contained 100 parallel circular plates heavily coated with enriched boron-10. To increase their sensitivity, the chambers were filled with $B^{10}F_3$ gas at about 3 atmospheres. In addition, polyethylene (1/2 in. thick) was placed around three sides of the chambers in the reactor to lower the average energy of the neutron spectrum seen by the chambers.

The neutron-detecting circuit diagram is shown in Fig. 4. The highpass filter was a conventional RC filter of three sections which prevented long-term variations of the reactor power level from affecting the detector. The d-c amplifier was a conventional feedback amplifier whose circuit is shown in Fig. 5. The electronic equivalent to the galvanometer consisted of a stable d-c amplifier with a "twin T" negative feedback loop tuned to the oscillation frequency. The "twin T" amplifier had a gain of essentially unity far above or below resonance, and a gain of about 70 at resonance. The circuit of the "twin T" amplifier is shown in Fig. 6.

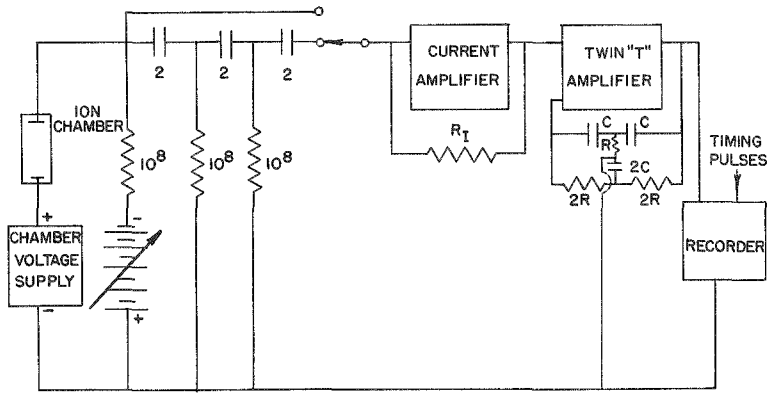

FIG 4
SCHEMATIC DIAGRAM OF FLUX MEASURING SYSTEM

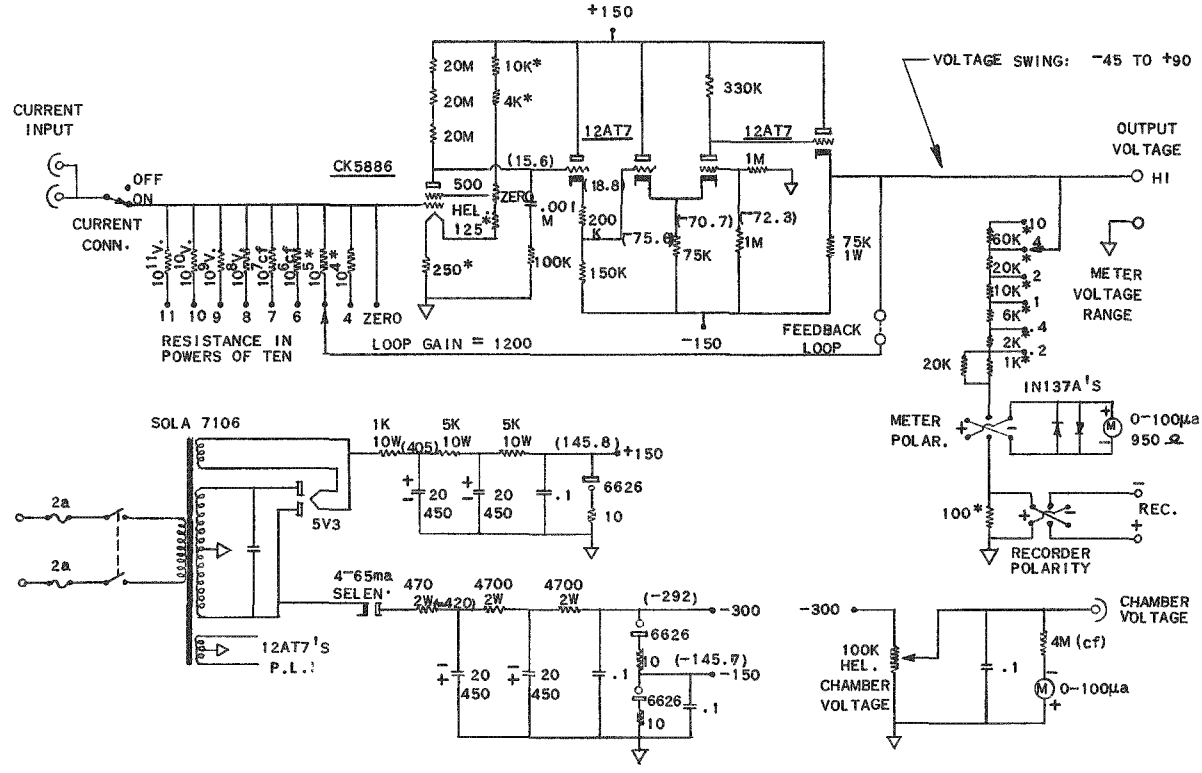

FIG. 5
GENERAL PURPOSE ELECTROMETER AMPLIFIER

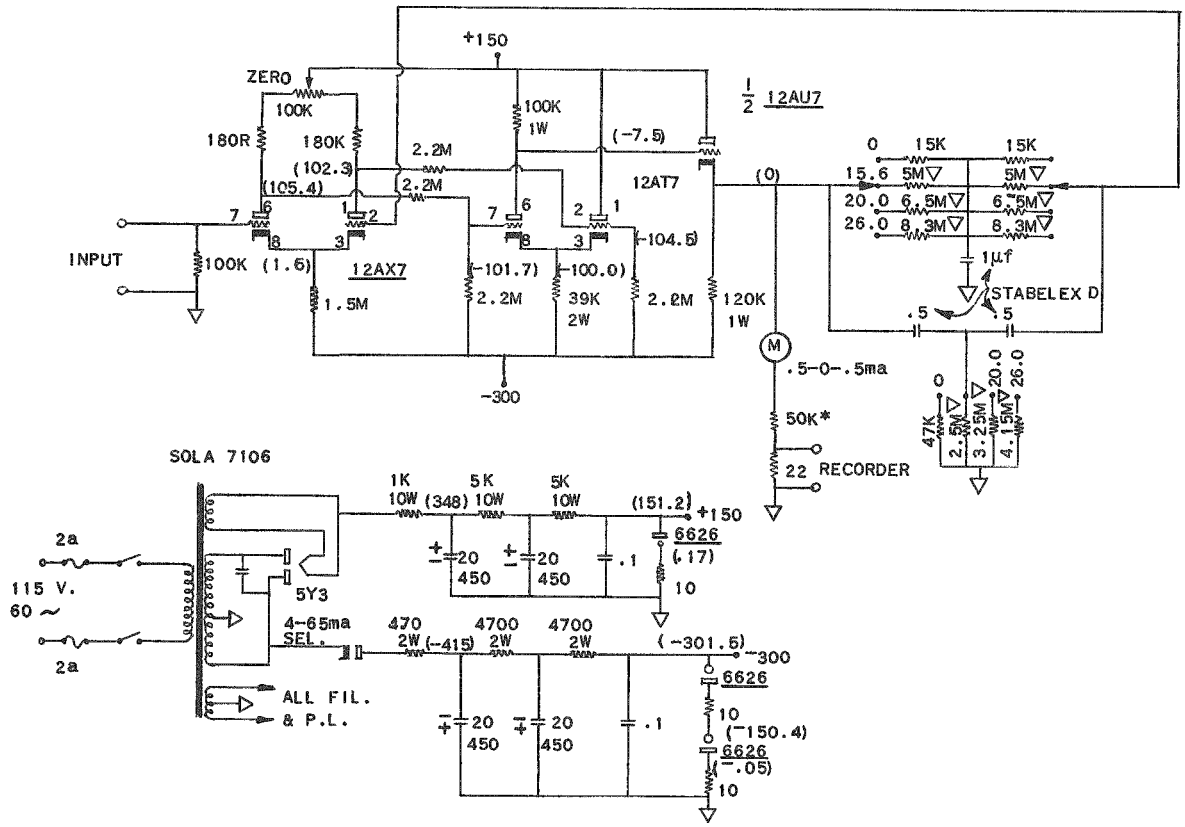

FIG. 6
TWIN T AMPLIFIER

C. Mechanical Oscillator

A mechanical oscillator was used to check the gain of the detecting system. The oscillator consisted of a horizontally mounted steel rod which could be oscillated in and out around a fixed point with varying amplitudes up to 3 in. A small sample of enriched uranium (9 gm) was fixed to the rod. The movement of the rod with the enriched uranium caused a sinusoidal reactivity change which could be observed with the resonant circuit detecting system. The rod was driven by a scotch link geared to a synchronous Bodine motor. By varying the frequency of the current input, the speed of the motor and, hence, the frequency of the mechanical oscillator drive, could be varied. The current was supplied by a power amplifier (Fig. 7) which, in turn, was driven by a standard Hewlett Packard audio oscillator. After warmup, the frequency of the latter was sufficiently constant to remain in resonance with the tuned amplifier. In order to adjust the frequency of the audio oscillator over small increments, a vernier in the form of a padding condenser was added across one of the main tuning

condensers of the oscillator. The power amplifier could also be switched to power a second Bodine synchronous motor which provided timing signals through the use of a rotating cam and microswitches. The timing signals were used to control the temperature cycle frequency through the temperature controller (Fig. 3) and to provide timing pips on the Leeds and Northrup output recorder. Thus the same frequency was used for mechanical oscillation and for the thermal cycling.

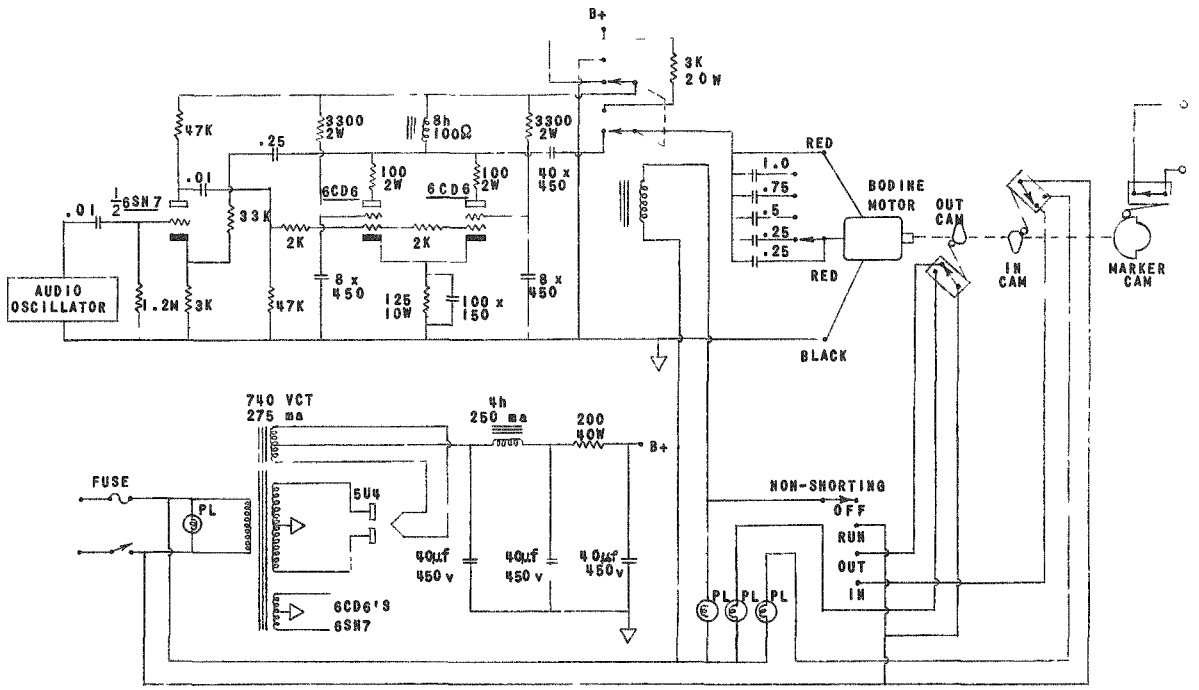

FIG. 7
TIMER OR MECHANICAL OSCILLATOR DRIVE CURRENT AMPLIFIER

D. Temperature Measurement

The temperature of the fuel elements indicated by the thermocouples was read with both a Brown "Electronik" potentiometer* and a hand-operated potentiometer. The temperature distribution along the coils and the calibration of the thermocouples were checked with a material of known melting point (Tempilstiks**). Prior to installation in the assembly, the elements were cycled and the voltages of the thermocouples read when

*Brown Instrument Division, Minneapolis-Honeywell Regulator Co., Philadelphia, Pa.

**Tempil Corporation, New York, N. Y.

the Tempilstik marks melted. The Tempilstiks and thermocouples agreed to within 25°C. However, the measurements indicated some irregularities in the performance of both; therefore it was assumed there was an uncertainty of 20% in the temperature measurements.

E. Assembly Loading

Earlier attempts to measure the Doppler effect in an assembly whose critical mass was 180 kg of U^{235} indicated insufficient signal for detection. Since the change in reactivity for a thermally cycled element is proportional to the ratio of the mass of the sample to the mass of the corresponding material in the core of the reactor, it was decided to try the experiment in an EBR-I mock-up which would have a critical mass of about 1/3 that of the assembly previously used. The ZPR-III assembly contained 62.8 kg of U^{235} in the uranium-fueled mock-up. The plutonium-fueled assembly contained approximately 34.9 kg of Pu and 2.0 kg of U^{235}.

The mock-up of the EBR-I core in ZPR-III consisted of a three-drawer by three-drawer array. However, in order to accommodate the helical element and the mechanical oscillator, the loading was modified as shown in Fig. 8. The helical element was positioned in drawer P-15 such that one-half of the helix was in the stationary section and the other half in the movable section, and surrounded by fuel drawers. The mechanical oscillator was located in 1-P-17. (The prefix 1 denotes the stationary section of the critical assembly.)

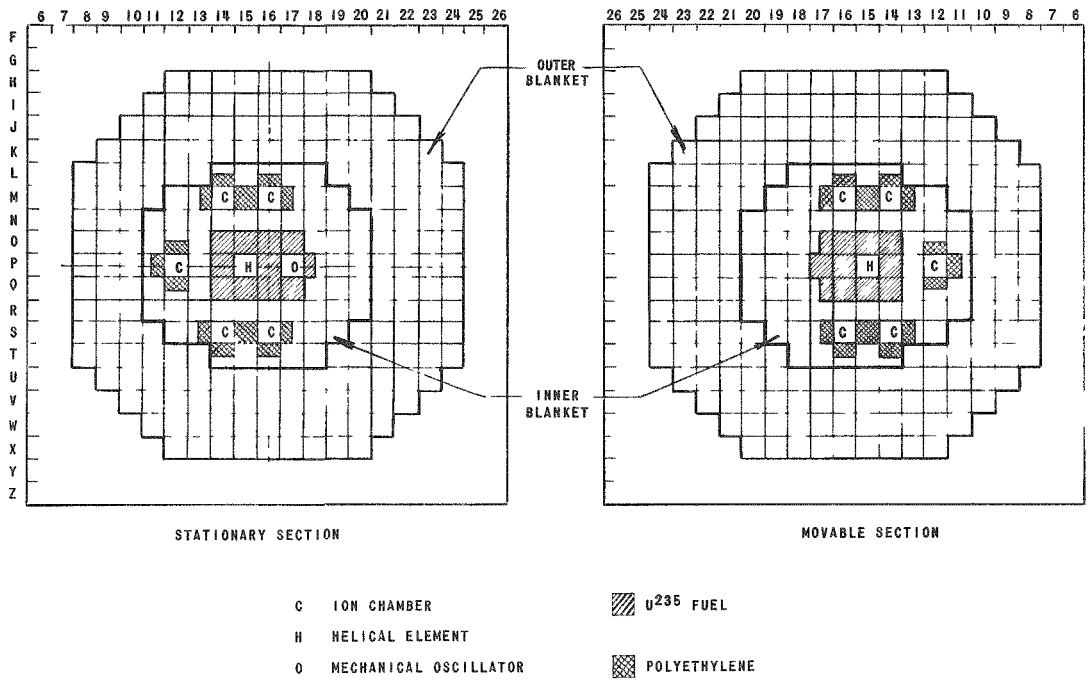

FIG. 8
EBR-I MOCK-UP IN ZPR-III FAST CRITICAL ASSEMBLY

The loading diagram shows five ion chambers inserted in the ZPR-III assembly. Four of the chambers were connected in parallel to the resonant circuit detecting system and surrounded on three sides by a polyethylene moderator to obtain sufficiently large currents for the experiment. The fifth chamber, also surrounded on three sides by polyethylene, was used to monitor the power level of the reactor.

IV. EXPERIMENTAL PROCEDURE

The experiment was conducted in two parts. First, the enriched and natural uranium and plutonium helices were thermally cycled in the EBR-I mock-up using U^{235} as fuel. In the second part, the assembly was reloaded with plutonium fuel, maintaining approximately the same geometry, and the three helices were again thermally cycled.

The runs in the plutonium-fueled assembly served two purposes. The first was to obtain a measurement of reactivity change caused by the thermal expansion of the elements. By making the assemblies similar, geometrically, it can be assumed that the spatial variation of the fuel worth was the same for both assemblies. The reactivity changes caused by thermal expansion, therefore, should have been the same. Since the enriched and natural uranium helices in the plutonium assembly should give rise to little or no Doppler effect, cycling in two assemblies gave an estimate of the reactivity worth of the motion of the helix due to expansion. The second purpose for the plutonium assembly was to cycle a plutonium element in both assemblies in order to obtain an estimate of the Doppler effect for plutonium.

In order to make an estimate of the reactor noise and to see whether the flow of helium during the cooling cycle had any effect on the reactivity, a run was also made during which the element was not heated, but the helium was periodically passed through the helix.

The cycling of the elements in the uranium-fueled assembly was carried out at a power level of about 5 watts, which produced a current of about 10 μamp total in the four ion chambers connected in parallel. Since the detecting circuit was fairly sensitive to slight changes in power level, the procedure adopted was to adjust the control rods so that the assembly was just critical or on a slightly positive period. In general, the assembly tended to go into a negative period due to the heating of the fuel; a slight positive period meant that the power level would rise slowly at first, then level off, and, finally, after about 30 min, when the increase of fuel temperature overcame the positive period the level would start to decrease. In

this manner it was possible to make runs of the order of 40 to 60 min during which the power level varied less than 10%, without necessitating any change in position of the control rod. The oscillation period for all runs was adjusted to 19.8 sec to match the resonant frequency of the tuned amplifier. It was, therefore, possible to obtain between 120 and 180 complete cycles of the temperature for each run.

To assist in estimating the effect of motion, the axial variation of importance of U^{235} was measured by moving a piece of U^{235} along the center of a mounted U^{235} helix and measuring the change of period caused by the motion. Figure 9 shows a plot of the variation relative to the importance at the reactor midplane. The difference between the measurements for the two sections is probably due to either an error in measuring the location of the slug, which was known only to within 1/16 in., or to the asymmetry between the loading of the halves.

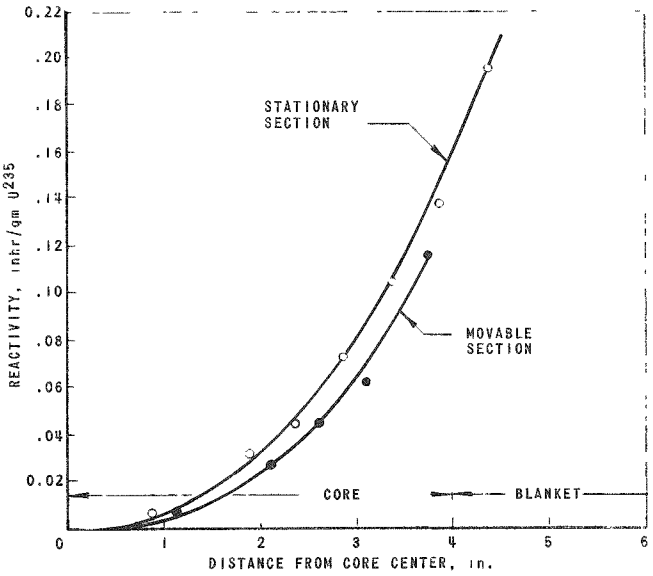

FIG. 9
AXIAL REACTIVITY LOSS OF U^{235}

A measurement of the radial variation of importance was also made. The shape of this curve in the void in which the element was located was checked in the plutonium assembly by mounting pieces of uranium (1/2 x 1/8 x 2 in.) off center on a rod. The rod was rotated 180 degrees, thereby moving the pieces radially 1/2 in. The results of these measurements are plotted in Fig. 10. Both depleted and enriched uranium were used to measure the radial variation of importance for both isotopes. The measurements of the radial variation of importance at the location of the helix were made in the plutonium assembly where the spontaneous fission neutron background due to Pu^{240} prevented the measurement of small reactivity change by the period technique. The reactivity changes, therefore, were measured by measuring the motion of a control rod necessary to restore the reactor to critical. The rods in turn were calibrated by making period measurements in the uranium assembly. No intercalibration of the control rod was attempted between the two assemblies because of the difficulty of making good period measurements in the plutonium assembly.

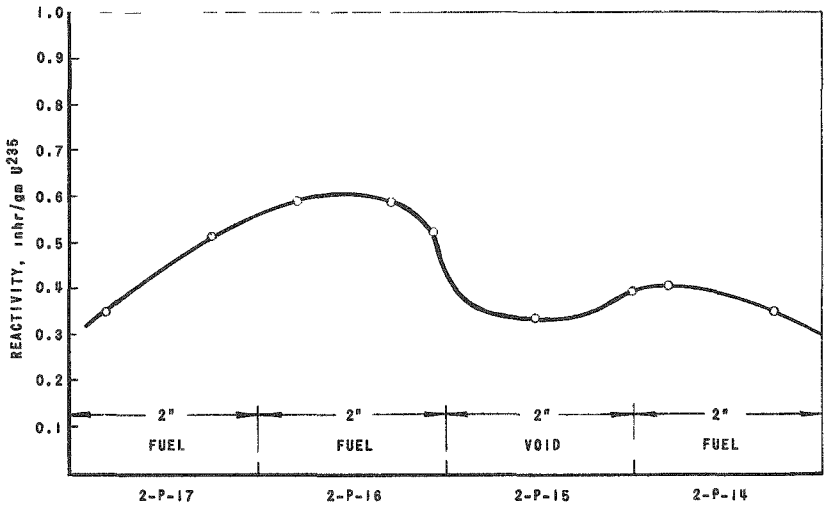

FIG. 10
RADIAL WORTH OF U^{235}

Normalization between reactivity measurements made in the uranium-fueled and in the plutonium-fueled assemblies was accomplished by making danger coefficient measurements with enriched uranium and plutonium elements in drawers 2-P-14 and 2-O-16 at midplane of the reactor. The ratios of the reactivity worth of enriched uranium and plutonium in the uranium-fueled to the plutonium-fueled assemblies gave the required normalizations, which are tabulated below.

	Reactivity, inhours/gm		
	U-Fueled Assembly	Pu-Fueled Assembly	Pu/U
Pu in P-14	0.453	1.262	2.79
U in P-14	0.283	0.782	2.78

While the runs in the plutonium assembly gave a measurement of the effects of thermal expansion of the elements, a second check was made by actually measuring the expansion. This was done with a microscope equipped with a micrometric reticle in the eyepiece, calibrated with a ruled slide. The expansion of every third or fourth turn along the coil was measured. Then, by using the radial importance measurements, the reactivity change due to this motion was estimated. As might be expected, the motion of the coils was somewhat erratic, the entire turns moving laterally in some instances as much as they expanded. Therefore, the estimate of reactivity change based on the measurements can only be considered approximate.

The expansions of the plutonium and enriched uranium samples were also checked by taking photographs of them, both hot and cold. The photographs showed only that the diametric expansion and laterial motion of these samples were less than 0.010 in.

V. DATA ANALYSIS

The actual technique used to perform the integration was to read the recorded detector output at eight different times during the temperature cycle of the sample element. It would have been sufficient to read the record at the time corresponding to the peaks of the output signal if the phase of the output had been known for certain. However, the signal due to expansion of the thermally cycled elements apparently caused a shift of phase. By taking eight readings each cycle, the phase, as well as the amplitude, was measured. The differences were computed between the first and fifth, second and sixth, third and seventh, and fourth and eighth readings corresponding to phases of 0, 45, 90, and 135 degrees with respect to the coolest part of the heating cycles.

Histograms of these differences are shown for one run in Fig. 11. The average of the differences gives the signal at that phase. The root-mean-square deviation, σ, of the differences at one phase provides an estimate of the error of each average. Quantitatively, $\sigma/\sqrt{n\pi/Q}$ is the root-mean-square error of the average and therefore is the root-mean-square error for the run due to noise. By plotting the differences as a function of angle, it is possible to estimate the amplitude of the signal. The maximum amplitude may be correlated with the effective peak temperatures obtained during the thermal cycling. Using the amplitude with the calibrated gain of the system, it was possible to obtain $\Delta k/k$ for the maximum temperature difference.

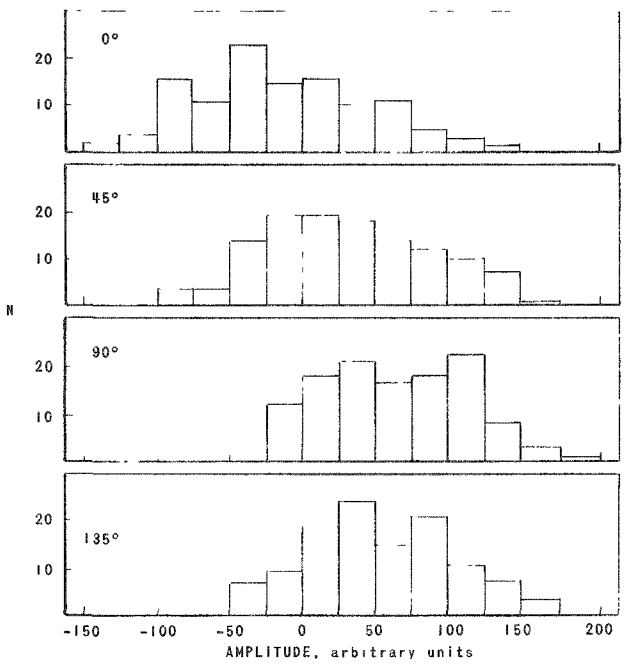

FIG. 11
HISTOGRAM OF AMPLITUDES FOR FOUR PHASE ANGLES

Since the temperature variation was not sinusoidal, but more nearly a saw-tooth wave (Fig. 12), it was necessary to make a Fourier

analysis of the temperature function to obtain the effective temperatures. The Fourier analysis for the temperature cycle of the enriched uranium helix is given below, in millivolts from a chromel P-alumel thermocouple:

$$f(\omega t) = 43.2 + 9.1 \sin \omega t - 22.7 \cos \omega t - 2.1 \sin 2\omega t + 3.6 \cos 2\omega t + 0.6 \sin \omega t - 4.4 \cos 3\omega t - 0.15 \sin 4\omega t + 1.4 \cos 4\omega t$$

$$f(\omega t) = 43.2 - 24.4 \cos (\omega t + 22) + \ldots$$

where

$$10 \text{ mv} \cong 25\text{C} \ ,$$

The observed peak to peak temperature was $\Delta T = 58.5$ mv = 143C. The effective peak to peak temperature, however, was twice the amplitude of the first Fourier component which, from the Fourier analysis, becomes $\Delta T = 48.8$ mv $\cong$ 120C, with the average temperature equal to about 106C.

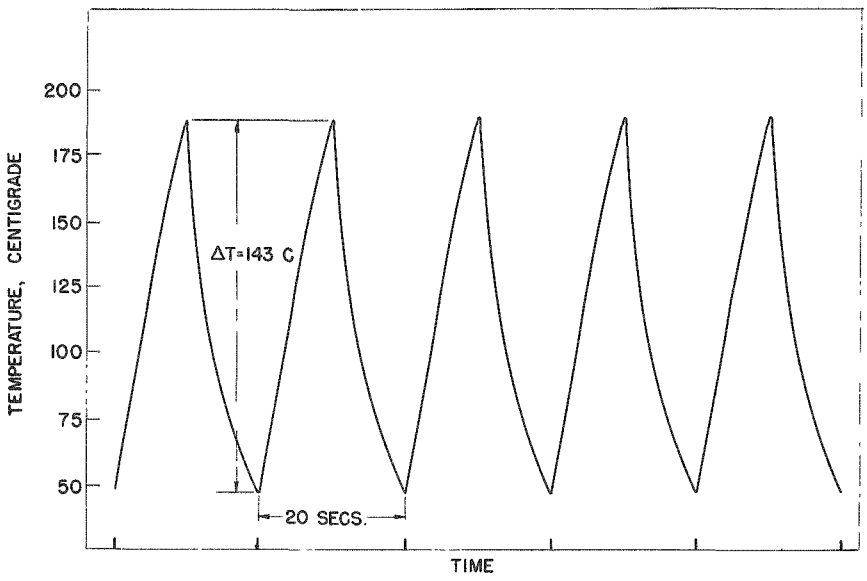

FIG. 12
TEMPERATURE CYCLE OF URANIUM HELIX

VI. CALIBRATION

The average amplitude of oscillation, $\overline{C}$, of the output of the resonant amplifier as recorded on the Leeds and Northrup Speedomax recorder was proportional to the change in neutron flux as seen by the ion chamber:

$$\overline{C} \sim \Delta N$$

or

$$g_0 \bar{C} = \Delta N \quad ,$$

where g_0 is a constant involving the charge produced by the (n,α) reaction in the chamber, the gain of the d-c amplifier, filter, resonant amplifier and L & N recorder.

Since

$$\Delta N = N A(\omega) \Delta k \quad ,$$

where $A(\omega)$ is the pile amplification factor, N the neutron flux, and $\Delta k/k$ the change in reactivity,

$$g_0 \bar{C} = N A(\omega) \Delta k \quad .$$

The neutron flux N is proportional to the d-c value of the ion chamber current I_0; or

$$I_0 \sim N \quad ,$$

or

$$N = g_1 I_0 \quad ,$$

where g_1 is a constant involving the charge produced by the (n,α) reaction in the chamber, and the gain of the d-c amplifier.

Hence:

$$g_0 \bar{C} = g_1 I_0 A(\omega) \Delta k \quad .$$

If g_0, g_1 and $A(\omega)$ were known, then the change in reactivity Δk could be obtained with a measurement of $\bar{C}$ and I_0. Using the mechanical oscillator for calibration effectively lumps the three constants into one:

$$G = g_1 \frac{A(\omega)}{g_0} \quad .$$

Hence:

$$\frac{\bar{C}}{I_0} = G \Delta k \quad .$$

Another way to obtain the relationship between the experimental measurements $\bar{C}$ and the reactivity change is to measure the ratio g_1/g_0, which was just the combined gain of the resonant amplifier, L & N recorder and filter, and to use the theoretical value for $A(\omega)$.

Both methods were used since the first gave an experimental check of the theoretical $A(\omega)$. The reactivity worth of the mechanical oscillator was obtained by a simple period measurement when the oscillator sample

was moved from one extreme position to the other. The worth of the oscillator was found to be 2.0 ± 0.1 inhours or $5.2 \times 10^{-5} \Delta k$ for a 2.5 in. movement of the sample. This reactivity measurement was based on the period-to-reactivity relationship computed from the Hughes, et al.(13) values for the delayed fractions for U^{235}. This does not take into account the possible contribution from fissions of U^{238} which could change the effective delay fractions. Mechanically oscillating the same sample with a frequency of 1/20 cps in run No. 5 gave an average amplitude equivalent to 42.5 divisions on the L & N recorder at an ion chamber current of 1.75 μamp. Since the amplitudes obtained from thermal cycling of the helices were corrected to 11.7 μamp, the resulting average amplitude for the mechanical oscillator, when corrected, was 284 divisions. Inserting these values for C, I_0, and Δk, respectively, are $G = 4.7 \times 10^5$. The values of Δk were obtained by dividing the average amplitudes by the product of G and the current in μamp. In this manner the entire detection system was calibrated directly without the necessity of knowing the exact properties of each component. As a check, the gain of the d-c amplifier, resonant amplifier, filter and L & N recorder was obtained by putting a known oscillating current into the input of the high-pass filter. The gain for the system was found to be $g_1/g_0 = 26.4 \times 10^8$. From this the pile amplification $A(\omega)$ was calculated to be 177. This may be compared with the value of 200 as obtained from theoretical pile amplification curve for EBR-I (Fig. 13) as reported by Brittan.(14)

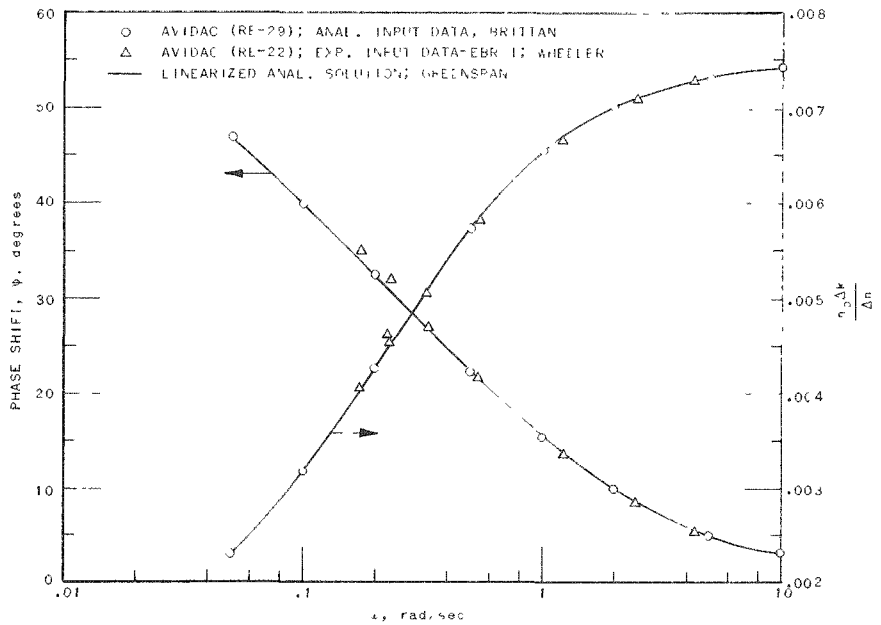

FIG. 13
RESPONSE TO AN IMPRESSED PERIODIC VARIATION IN REACTIVITY ON A REACTOR AT ZERO POWER HAVING AN EFFECTIVE LIFETIME OF 4×10^{-8} seconds

VII. RESULTS

The average amplitudes at 0, 45, 90 and 135 degrees are given in Table I. Table II shows these amplitudes after the effects of the helium cycling measured in run No. 17 have been subtracted. Figure 14 is a composite plot of the amplitudes as a function of angle. The values for angles greater than 135 degrees were obtained by reversing the signs of the values from 0 to 135 degrees.

Table I

MEASURED AMPLITUDES

Run No.		Phase angle, degrees 0	45	90	135	No. Cycles
	U-Fueled Assembly					
4-7	Enriched U	- 1.35	+3.09	+ 5.33	+ 4.81	137
	rms, deviation, σ	± 4.45	±3.42	± 3.20	± 4.27	
10	Natural U	- 0.91	-2.79	- 3.36	- 1.73	132
	rms, deviation, σ	± 2.46	±1.12	± 2.19	± 2.48	
8	Pu	- 2.44	+0.58	+ 3.27	+ 4.00	160
	rms, deviation, σ	± 2.56	±2.59	± 2.34	± 2.33	
5	Mechanical Oscillator (2.0 inhr)	-271	-111	+116	+270	17
	rms, deviation, σ	± 7	± 10	± 6	± 6	
	Pu-Fueled Assembly					
13	Enriched, U	- 1.70	+2.90	+ 6.30	+ 5.40	121
	rms deviation, σ	± 6.0	±6.0	± 5.0	± 5.5	
12	Natural U	-12.4	-1.6	+10.9	+16.4	141
	rms deviation, σ	± 4.9	±5.0	± 6.3	± 6.5	
15	Pu	- 6.4	-2.8	+ 2.3	+ 6.1	134
	rms deviation, σ	± 6.7	±6.5	± 5.9	± 6.2	
17	HeCycling(Noise)	- 2.9	-2.4	- 0.53	+ 1.6	109

Table II

MEASURED AMPLITUDES LESS HELIUM CYCLING EFFECTS

Run No.	Helix	Phase angle, degrees 0	45	90	135
	U-Fueled Assembly				
4-7	Enriched Uranium	-0.31	+3.94	+5.52	+4.24
10	Natural Uranium	+0.13	-1.94	-3.17	-2.30
8	Plutonium	-1.40	+1.43	+3.46	+3.43
	Pu-Fueled Assembly				
13	Enriched Uranium	0.43	+1.89	+2.4	+1.36
12	Natural Uranium	-3.34	+0.36	+4.07	+5.29
15	Plutonium	-1.25	-0.14	+1.00	+1.61

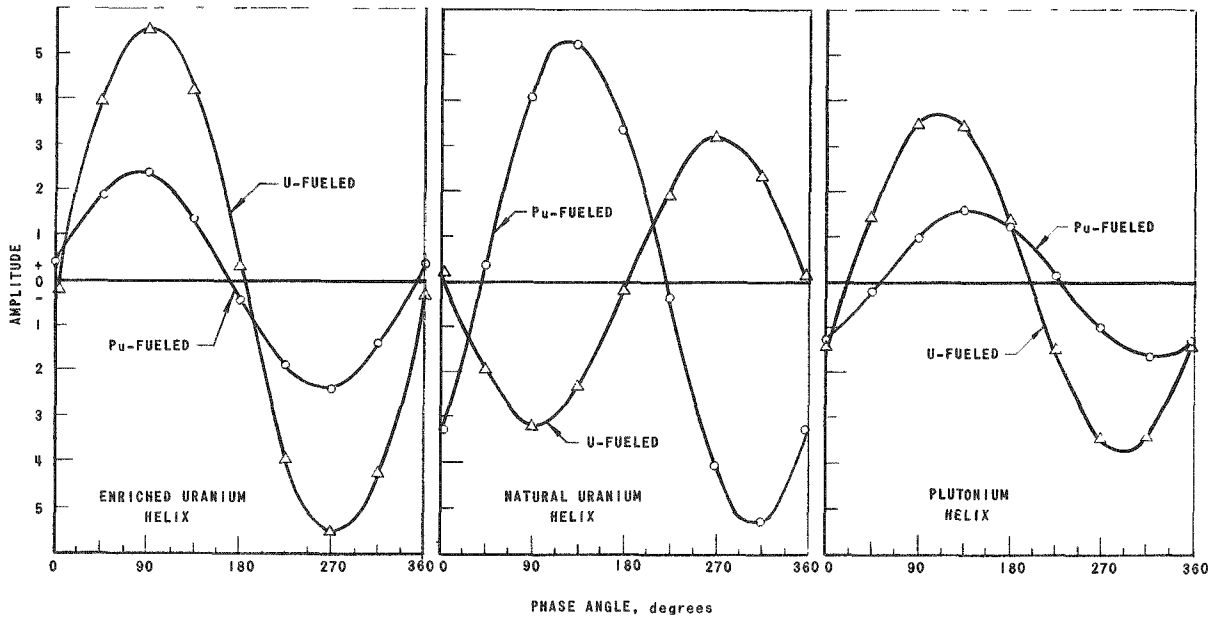

FIG. 14
AMPLITUDE VS PHASE ANGLE AS A FUNCTION OF SAMPLE AND ASSEMBLY FUELING

The data in Table II were used to calculate the changes in reactivity listed in Table III.

Table III

EXPERIMENTAL VALUES OF $\Delta k/k$ and $\Delta k/\Delta T$

	U^{235} - Fueled Assembly				Pu^{239} - Fueled Assembly			
Helix	Amplitude (Max.)	$\Delta k/k$ ($\times 10^{+6}$)	ΔT, C	$\Delta k/\Delta T$ ($\times 10^{+8}$)	Amplitude (Max.)	$\Delta k/k$ ($\times 10^{+6}$)	ΔT, C	$\Delta k/\Delta T$ ($\times 10^{+8}$)
Enriched U (542.8 gm)	+5.5	+1.0	120	+0.83	+2.4	+0.44	140	+0.31
Nat. U (387.1 gm)	-3.2	-0.58	140	-0.41	+5.3	+0.97	140	+0.69
Pu (234.7 gm)	+3.7	+0.68	120	+0.57	+1.6	+0.29	120	+0.24

The magnitude of the effect of motion was estimated from a knowledge of the radial worth distributions for enriched and natural uranium The measurements of the radial worth distributions were made in the plutonium-fueled assembly with the assumption that the distribution should be the same as in the uranium-fueled assembly since the geometry remained fixed. The results and the estimated $\Delta k/k$ for the helices due to the motion are shown in Table IV.

Table IV

COMPARISON OF THEORETICAL AND EXPERIMENTAL CORRECTIONS FOR THERMAL EXPANSION

	Enriched Uranium (542.8 gm)	Natural Uranium (387.1 gm)
Radial Worth, $(\Delta k/k)$/in.	38×10^{-5}	16×10^{-5}
Radial Expansion for ΔT = 120C		
Measured (avg.), in.	1.8×10^{-3}	1.9×10^{-3}
Theoretical	1.36×10^{-3}	1.36×10^{-3}
$\Delta k/\Delta T$		
Measured Expansion	0.57×10^{-8}	0.25×10^{-8}
Theoretical Expansion	0.43×10^{-8}	0.18×10^{-8}

The estimate for the expansion of the natural uranium helix applies only to the run in the plutonium-fueled assembly. After the run in the uranium-fueled assembly, one end of the helix was found loosely taped to the mounting tube. More tape was added before making either the runs in the plutonium-fueled assembly, or the microscopic measurements of expansion. The retaping may have reduced the expansion by as much as a factor of three from what it was during the runs in the uranium-fueled assembly.

VIII. CONCLUSION

The noise amplitudes were found to be $2.46/\sqrt{20} = 0.55$ scale division, assuming 20 uncorrelated measurements for a run of 160 cycles. Using the theoretical formulas of Feiner, Frost, and Hurwitz,(8) the estimates of noise were: 0.013 scale division for chamber noise, and 0.83 scale division for reactor noise. Thus, the observed noise amplitude falls between that predicted by the Feiner, Frost, and Hurwitz theory, and the Frisch theory.(7)

It is interesting to note that the reactor noise amplitude is proportional to the amplification. In the plutonium assembly, therefore, the noise should be correspondingly greater than in the uranium assembly. Actually, it was found to be a factor of 2.1 ± 0.4 higher. This value is less than the factor of 2.8 measured by the period technique. The difference is within the limits of experimental accuracy.

The sensitivity of this experiment for measuring Doppler effect was determined by the accuracy with which temperature changes, reactor noise fluctuation, and expansion effects were measured. The various temperature measurements agreed to within 20%. The reactor and detector noise is known to fit a gaussian distribution,(6) which in this experiment

had a standard deviation, σ_N, of 0.55 scale division, which is equivalent to 0.08×10^{-8} $\Delta k/\Delta T$ for 160 cycles. The measurement of the helium-cycling correction, made over only 109 cycles, had a standard deviation equivalent to 0.1×10^{-8} $\Delta k/\Delta T$.

The remaining and most important source of experimental inaccuracy was the expansion effect. The estimates of the effect for the enriched samples were 0.57×10^{-8} and 0.43×10^{-8} $\Delta k/\Delta T$, as seen in Table IV. The reactivity change measured from cycling of this helix in the plutonium assembly, which is presumed entirely due to expansion, was 0.31×10^{-8} $\Delta k/\Delta T$.

The effect of a lateral motion of the helix must also be considered because the fit of the helix on the glass frit filter was loose; 0.05 mil lateral motion of the whole sample would produce 0.8×10^{-8} $\Delta k/\Delta T$. A change of expansion pattern due to remounting might therefore cause a change as large as 0.8×10^{-8} $\Delta k/\Delta T$, but on the average a much smaller change can be expected.

With reference to Table III, a $\Delta k/\Delta T$ of $+0.83 \times 10^{-8}$ was obtained for the enriched uranium helix thermally cycled in the U-fueled assembly between 50C and 170C at an average temperature of about 106C. The same helix thermally cycled in the plutonium-fueled assembly gave a $\Delta k/\Delta T$ of $+0.31 \times 10^{-8}$. Since there is little or no Doppler effect expected for the enriched uranium helix thermally cycled in a plutonium-fueled assembly, it is concluded the observed change in reactivity may be due entirely to motion of the helix. Since there is considerable uncertainty in the magnitude of the effect of motion, it is possible only to estimate an upper limit to the Doppler effect for an EBR-I type assembly. If the effect of motion is taken to be at least 0.3×10^{-8} $\Delta k/\Delta T$ for the entire helix, the upper limit for the Doppler effect for this sample in an EBR-I assembly becomes 0.5×10^{-8} $\Delta k/\Delta T$.

Goertzel and Klahr[15] at NDA have formulated an expression to relate the reactivity change obtained by thermally cycling a sample to the reactivity change obtained if the entire core were heated.

$$\Delta k \text{ heated reactor} = \frac{\Delta k \text{ (experiment)}}{\frac{\Delta m}{m} C \left[\left(\sqrt{\frac{2T_0}{T_0 + T_a}} - \sqrt{\frac{2T_0}{T_0 + T_b}}\right) \Big/ \left(\sqrt{\frac{T_0}{T_a}} - \sqrt{\frac{T_0}{T_b}}\right)\right]},$$

where

Δm = mass of U^{235} in sample
m = mass of U^{235} in core
T_0 = temperature of core
T_a = minimum temperature of thermal cycling
T_b = maximum temperature of thermal cycling
C = constant involving flux and importance average.

Although the reactivity worth of the 506-gm enriched uranium helix was found to be approximately 293 inhours, it is difficult to assign a value to the constant C because of the unusual shape of the mock-up used. It is necessary to perform a careful measurement of the reactivity worth of fuel throughout the mock-up in order to obtain an average reactivity worth of fuel. Thus it is necessary to assume that the helix was located in a position of average importance and that C = 1.

Substituting:

Δm = 506 gm U^{235}
m = 62.8 kg U^{235}
T_0 = 293C
T_a = 323C
T_b = 443C

Δk (heated reactor) = 2.1×10^2 Δk (experiment) at an average temperature of 106C

or

$$\left(\frac{\Delta k}{\Delta T}\right) \text{ at } 20C = 2.1 \times 10^2 \left[1.49 \frac{\Delta k}{\Delta T} \text{ (experiment)}\right]$$

$\left(\frac{\Delta k}{\Delta T}\right)$ at 20C $\leq 1.6 \times 10^{-6}/C$ for U^{235} in the EBR-I type assembly in ZPR-III.

Klahr[15] also has made a formulation for the case of the entire core being heated to a very high temperature. He obtains:

$$\frac{\Delta k_{\text{heated}\ (T_a \rightarrow T_b)}}{\Delta k_{\text{heated}\ (T_a \rightarrow \infty)}} = 1 - \frac{T_a}{T_b} \quad .$$

Thus:

$$\Delta k_{\text{heated}\ (T_a \rightarrow \infty)} = 0.56 \times \frac{10^{-6} \times 2.1 \times 10^2}{0.146} = 0.80 \times 10^{-3}$$

$$= \sim 10 \text{ cents}$$

If the entire core were heated from about 110C to a very high temperature, the Doppler effect would contribute a maximum of about 10 cents worth of reactivity.

Upon examining the experimental results of Table III it is found that a large negative effect is obtained by thermally cycling the natural uranium helix in the uranium-fueled assembly, and a positive effect in the plutonium-fueled assembly. This is difficult to explain in terms of a

Doppler effect, since there is very little U^{238} in the core of the assembly. Unfortunately, there was not sufficient time available to make measurements of the danger coefficient and axial variations of worth of natural uranium in the position of the helices. As a result, it is impossible to compute the worth of the natural uranium helix and, therefore, to obtain a value for the Doppler effect of U^{238}.

The meaning of the results obtained with the plutonium sample is uncertain. It was anticipated that thermal cycling of the plutonium helix in the plutonium-fueled assembly would produce a larger effect than in the uranium-fueled assembly. This was not so. Since the geometry of the core was the same in both assemblies and, hence, the worth distribution of plutonium was similar, the effect due to the motion of the helix should also have been the same. One explanation is that the motion caused by thermal expansion did not reproduce when the elements were removed and reinstalled in the assemblies. The thermal expansion of the plutonium sample was not measured directly, but the magnitude of the reactivity changes observed was similar to the changes observed with the enriched uranium. It is very unlikely the Doppler effect is negative; therefore the same conclusion holds that the reactivity change produced by Doppler effect in the plutonium sample must also be less than $0.5 \times 10^{-8} \Delta k/\Delta T$.

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