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CONDITIONS OF SUPERFLUIDITY OF NUCLEAR MATTER

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Summary

The methods developed in the theory of superconductivity are applied to the study of the properties of nuclear matter. The interaction that brings to superfluidity of nuclear matter was considered to be weak. Nucleon-nucleon interaction was taken in the most general form. Asymptotic solutions of equation system were found, which allowed to obtain the conditions of superfluidity of the nuclear matter. The conditions of superfluidity come in general to the requirement of predominance of attraction in nucleon-nucleon potentials at the Fermi surface energy. The data on nucleon-nucleon potentials allow to make conclusion that the conditions of superfluidity of nuclear matter are to be fulfilled.

Introduction

The concept of "nuclear matter" is used when studying the properties of atomic nucleus. It is considered that if the interaction of Coulomb repulsion between the protons doesn't exist then arbitrarily a large number of A nucleons consisting of Z protons and $(A-Z)$ neutrons at $Z \sim \frac{A}{2}$ in their lowest state of energy form a stable configuration which is called nuclear matter. The inner part of medium and heavy nuclei are samples of such nuclear matter. The dynamic properties of nuclear matter can be described in the first approximation by an independent particle model.

The structure of nuclear matter is in many respects similar to the electron structure of metals. Therefore the mathematical methods successfully developed in the theory of superconductivity[1]) are worth to be applied to the study of nuclear matter properties.

N.N. Bogoljubov[2]) drew attention to the fact that nuclear matter can possess the properties of superfluidity. In case of Fermi-gas the system possessing the lowest energy is such, in which the particles successfully occupy all the energy levels up to some energy $E(K_F)$, the energy of Fermi-surface, which depends on the density of gas. Such state of nuclear matter with the energy levels successfully occupied is called normal. Owing to the interaction of fermion pairs possessing equal and opposite momenta, the system of Fermions may acquire the properties of superfluidity. This means that the system of Fermions will have the energy state lower, than the normal one is. Such a state of nuclear matter which is more low than a normal one is called superfluid.

Very weak interactions of electron pairs possessing equal and opposite momenta lead to superfluid state of a solid. Superfluidity is connected with the regulating of the movement of electrons near the Fermi surface. Mathematical methods developed in(1) are considerably connected with the weakness of these interactions.

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1) N.N. Bogoljubov, V.V. Tolmačev, D.V. Shirkov "New Method in the Theory of Superconductivity," Publishing Department of the USSR Academy of Sciences, Moscow, 1958.

2) N.N. Bogoljubov, Docl.Akad.Nauk SSSR, 119, 52 (1958).

It is known that the interaction between free nucleons is a strong one. Nevertheless, the interaction between nucleon pairs possessing equal and opposite momenta bringing to the superfluid state of nuclear matter can be considered a weak one. Indeed, the successful application of the nuclear shell model shows that in the first approximation the interaction forces between the nucleons may be reduced to a certain effective potential which does not affect the appearance of the superfluid state and to the change of the nucleon mass. Besides the interaction between nucleons, which brings to superfluid state, depends on the power of regulation in the movement of the nucleons, which may be not very large.

In the present paper we shan't fortell which of the forces, central or non-central, singlet or triplet, proton-proton or proton-neutron must be responsible for the appearance of superfluid state of nuclear matter. The aim of the present paper is to obtain the conditions which will be satisfied by the most general form of the potential of nucleon-nucleon interaction in order to give superfluid state of nuclear matter possibility to exist.

2. Hamiltonian of the Interaction

Let us consider the part of the Hamiltonian of the interaction which is responsible for the superfluid state of nuclear matter. We consider that as well as in the theory of superconductivity. The interactions of fermion pairs possessing the equal and opposite momenta bring to the superfluid state. The interactions of nucleon pairs possessing the arbitrary momenta bring to some effective potential of nuclear matter which we shall not consider.

Let us present the model Hamiltonian of the interaction as follows:

$$H_{int} = \frac{-1}{V} \sum_{K,K'} \varphi_1^*(K)\, \varphi_2^*(-K)\, V(K,K')\, \varphi_2(-K')\, \varphi_1(K') \quad (1)$$

where V is the volume of the system and $\varphi_1(K)$, $\varphi_2(K)$ are composed of the operators of the production and absorption of protons and neutrons. Let us take the interaction $V(K,K')$ between two nucleons in the most general form:

$$\begin{aligned}
V(K,K') = {} & J_{ss}(K,K')\frac{1}{4}\left(1-\vec{\tau}_1\vec{\tau}_2\right)\frac{1}{4}\left(1-\vec{\sigma}_1\vec{\sigma}_2\right)+ \\
& +J_{st}(K,K')\frac{1}{4}\left(1-\vec{\tau}_1\vec{\tau}_2\right)\frac{1}{4}\left(3+\vec{\sigma}_1\vec{\sigma}_2\right)+ \\
& +J_{ts}(K,K')\frac{1}{4}\left(3+\vec{\tau}_1\vec{\tau}_2\right)\frac{1}{4}\left(1-\vec{\sigma}_1\vec{\sigma}_2\right)+ \\
& +J_{tt}(K,K')\frac{1}{4}\left(3+\vec{\tau}_1\vec{\tau}_2\right)\frac{1}{4}\left(3+\vec{\sigma}_1\vec{\sigma}_2\right)- \qquad (2) \\
& -J_{sT}(K,K')\frac{1}{4}\left(1-\vec{\tau}_1\vec{\tau}_2\right)\left[3(\vec{\sigma}_1\vec{e})(\vec{\sigma}_2\vec{e})-\vec{\sigma}_1\vec{\sigma}_2 e^2\right]- \\
& -J_{tT}(K,K')\frac{1}{4}\left(3+\vec{\tau}_1\vec{\tau}_2\right)\left[3(\vec{\sigma}_1\vec{e})(\vec{\sigma}_2\vec{e})-\vec{\sigma}_1\vec{\sigma}_2 e^2\right]+ \\
& +J_{sl}(K,K')\frac{1}{4}\left(1-\vec{\tau}_1\vec{\tau}_2\right)\left(\vec{\sigma}_1+\vec{\sigma}_2, \vec{l}\right)+ \\
& +J_{tl}(K,K')\frac{1}{4}\left(3+\vec{\tau}_1\vec{\tau}_2\right)\left(\vec{\sigma}_1+\vec{\sigma}_2, l\right).
\end{aligned}$$

Here $\vec{e} = \frac{\vec{K} - \vec{K}'}{2|\vec{K}|}$, $\vec{e} = \frac{\vec{K} \times \vec{K}'}{2|\vec{K}|}$ the first index s or t in the functions $J_{in}(K,K')$ means singlet or triplet states in the isotopic spin space, the second index refers to the usual spin space, s means the singlet interaction, t means a triplet one, T is the interaction of tensor type, and ℓ is the interaction of spin-orbit type.

From the Hermitian conjugation and from the invariance with respect to the operation of reflection of space coordinates we find out that the functions $J_{in}(K,K')$ are real and possess the following properties:

$$J_{in}(K,K') = J_{in}(K',K) \, , \qquad J_{in}(K,K') = J_{in}(-K,-K') \tag{3}$$

Let us present the interaction Hamiltonian (1) in terms of the operators of production and absorption of neutrons and protons, that is:

$$\begin{aligned} H_{int} = \frac{1}{V} \sum_{K,K'} \Big\{ & J_N(K,K') \left[a^+_{K+} a^+_{-K-} a_{-K-} a_{K'+} + b^+_{K+} b^+_{-K-} b_{-K-} b_{K'+} \right] + \\ & + J_+(K,K')\, a^+_{K+} b^+_{-K+} b_{-K'+} a_{K'+} + \\ & + J_-(K,K')\, a^+_{-K-} b^+_{K-} b_{K'-} a_{-K'-} + \\ & + \frac{1}{2} J_y(K,K') \left[a^+_{K+} b^+_{-K-} b_{-K'-} a_{K'+} + a^+_{-K-} b^+_{K+} b_{K'+} a_{-K'-} \right] + \\ & + \frac{1}{2} J_\sigma(K,K') \left[a^+_{K+} b^+_{-K-} b_{K'+} a_{-K'-} + a^+_{-K-} b^+_{K+} b_{-K'-} a_{K'+} \right] + \end{aligned} \tag{4}$$

$$+J_{I}^{(1)}(K,K')a^{+}_{K+}b^{+}_{-K+}b_{-K'-}a_{K'+}+J_{I}^{(1)*}(K,K')a^{+}_{K+}b^{+}_{-K-}b_{-K'+}a_{K'+}+$$

$$+J_{I}^{(2)}(K,K')a^{+}_{K+}b^{+}_{-K+}b_{K'+}a_{-K'-}+J_{I}^{(2)*}(K,K')a^{+}_{-K-}b^{+}_{K+}b_{-K'+}a_{K'+}+ \tag{4}$$

$$+J_{I}^{(3)}(K,K')a^{+}_{K+}b^{+}_{-K-}b_{K'-}a_{-K'-}+J_{I}^{(3)*}(K,K')a^{+}_{-K-}b^{+}_{K-}b_{-K'-}a_{K'+}+$$

$$+J_{I}^{(4)}(K,K')a^{+}_{-K-}b^{+}_{K+}b_{K'-}a_{-K'-}+J_{I}^{(4)*}(K,K')a^{+}_{-K-}b^{+}_{-K-}b_{K'+}a_{-K'-}+$$

$$+J_{II}(K,K')a^{+}_{K+}b^{+}_{-K+}b_{K'-}a_{-K'-}+J_{II}^{*}(K,K')a^{+}_{-K-}b^{+}_{K-}b_{-K'+}a_{K'+}\Big\}$$

Here $a^{+}_{K\pm}$, $b^{+}_{K\pm}$ are the operators of proton and neutron production $a_{K\pm}$, $b_{K\pm}$ are their absorption operators, and $\pm$ characterizes the spin direction. The functions $J_{N}(K,K')$, $J_{I}(K,K')$ and the other ones are presented in terms of the functions $J_{in}(K,K')$.

Thus we shall present the model Hamiltonian as follows:

$$H=\sum_{K,S}\left\{E(K)-E_{F}\right\}C^{+}_{KS}C_{KS}+ \tag{5}$$

$$+\frac{1}{V}\sum_{K,K'}I\left(K,K'/S_{1},S_{2};S'_{2},S'_{1}\right)C^{+}_{KS_{1}}C^{+}_{KS_{2}}C_{K'S'_{2}}C_{K'S'_{1}},$$

where E_{F} - is the parameter which plays the role of the chemical potential; in normal state it is equal to the energy of Fermi surface; S - is the discrete index which characterizes the spin, isotopic spin and the sign momentum K.

3. The System of Fundamental Equations

Below we follow the considerations of Bogoljubov's paper[2]. Let us perform the canonical transformation

$$a_{KS} = \sum_{s'} \left\{ u(K,s,s')\alpha_{Ks'} + v(K,s,s')\alpha^{+}_{Ks'} \right\} \tag{6}$$

Let us determine the new vacuum state $\alpha_{KS} C_0 = 0$ and find out the average value $\overline{H} = \langle C_0^{*} H C_0 \rangle$. In $\overline{H}$ let us neglect the terms which remain finite at $V \to \infty$. Let us find out the functions $u(K,s,s')$, $v(K,s,s')$ out of the requirements of extremum $\overline{H}$ while fulfilling the additional conditions. The equation obtained yields the solution that corresponds to normal state $C^{(n)}$.

In order to obtain the conditions at which the energy of normal state $C^{(n)}$ won't be minimal and therefore the superfluid state $C^{(s)}$ will appear, let us find out the second variation of $\overline{H}$. If the second variation of $\overline{H}$ over $u(K,s,s')$ and $v(K,s,s')$ in the presence of additional conditions is less than zero for the solution corresponding to normal state $C^{(n)}$, that means that the superfluid state $C^{(s)}$ appears.

Thus we obtain that superfluid state of nuclear matter exists only then when there are the solutions with negative eigenvalues E of the following set of equations:

$$2/E(K) - E_F/\varphi(K) + \frac{1}{V}\sum_{K'} J_N(K,K')\varphi(K') = E\varphi(K), \tag{7}$$

$$2/E(K) - E_F/\varphi_+(K) + \frac{1}{V}\sum_{K'} \left\{ J_+(K,K')\varphi_+(K') + J_I^{(1)}(K,K')\varphi_1(K') + \right. \tag{8}$$

$$\left. + J_I^{(2)}(K,K')\varphi_2(K') + J_{II}(K,K')\varphi_-(K') \right\} = E\varphi_+(K),$$

$$2/E(K)-E_F/\varphi_-(K)+\frac{1}{V}\sum_{K'}\Big\{J_-(K,K')\varphi_-(K')+J_I^{(3)*}(K,K')\varphi_1(K')+ \tag{9}$$

$$+J_I^{(4)*}(K,K')\varphi_2(K')+J_{II}^{*}(K,K')\varphi_+(K')\Big\}=E\varphi_-(K),$$

$$2/E(K)-E_F/\varphi_1(K)+\frac{1}{V}\sum_{K'}\Big\{\frac{1}{2}J_\nu(K,K')\varphi_1(K')+\frac{1}{2}J_\sigma(K,K')\varphi_2(K')+ \tag{10}$$

$$+J_I^{(1)*}(K,K')\varphi_+(K')+J_I^{(3)}(K,K')\varphi_-(K')\Big\}=E\varphi_1(K),$$

$$2/E(K)-E_F/\varphi_2(K)+\frac{1}{V}\sum_{K'}\Big\{\frac{1}{2}J_\nu(K,K')\varphi_2(K')+\frac{1}{2}J_\sigma(K,K')\varphi_1(K')+ \tag{11}$$

$$+J_I^{(2)*}(K,K')\varphi_+(K')+J_I^{(4)}(K,K')\varphi_-(K')\Big\}=E\varphi_2(K).$$

4. The Conditions of Superfluidity in Case of p-p and n -n Interactions

Before proceeding to investigations of the conditions of superfluidity of nuclear matter in the most general form, we shall consider two particular cases. The first one, when proton-proton and neutron-neutron interactions are considered and proton-neutron interactions are neglected. Apparently this case is the most interesting one, when passing from nuclear matter to the atomic nucleus.

Let us find out at which $J_N(K,K')$ it is possible to solve the equation

$$2/E(K)-E_F/\varphi(K)+\frac{1}{V}\sum_{K'}J_N(K,K')\varphi(K')=E\varphi(K) \tag{7}$$

with negative eigenvalues $E=-2\delta$, $\delta>0$. With this aim we investigate the asymptotic form of solutions (7) when $J_N(K,K')$ tends to zero as well as E which remains negative.

Let

$$\varphi(K)=\frac{\chi(K,\Omega)}{|E(K)-E_F|+\delta},$$

We proceed from the sum to the integral and get

$$2\chi(K,\Omega)+\frac{1}{(2\pi)^3}\int d\Omega'\int_0^\infty K'^2 dK'\,\frac{J_N(|K|,|K'|,\cos\alpha)}{|E(K')-E_F|+\delta}\chi(K',\Omega')=0, \qquad (12)$$

where $\cos\alpha=\cos\vartheta\cos\vartheta'+\sin\vartheta\sin\vartheta'\cos(\varphi-\varphi_0')$

Taking into account that at $\delta\to 0$ the integral in (12) becomes logarithmically divergent near the Fermi surface let us consider the approximated equation

$$\chi(K,\Omega)+\ln\left\{\frac{mE'(K_F)}{\delta}\right\}\frac{K_F^2}{E'(K_F)}\frac{1}{(2\pi)^3}\int d\Omega' J_N(|K|,K_F,\cos\alpha)\chi(K_F,\Omega')-$$
$$-\frac{1}{2(2\pi)^3}\int d\Omega'\int_0^\infty dK'\ln|K'-K_F|\,\frac{d}{dK'}\left[J_N(|K|,|K'|,\cos\alpha)\frac{K'^2}{E(K')}\chi(K',\Omega')\right]=0, \qquad (13)$$

which at small $J_N(K,K')$ asymptotically coincides with (12). Let us introduce a certain function

$$f(K',\Omega')=\frac{\chi(K',\Omega')}{\chi(K_F',\Omega')}\,\frac{1}{\ln\frac{mE'(K_F)}{\delta}}$$

and

$$f(K_F)=\frac{1}{\ln\frac{mE'(K_F)}{\delta}}>0$$

Then the equation for $f(K,\Omega)$ is

$$f(K,\Omega)+\frac{K_F^2}{E'(K_F)}\frac{1}{(2\pi)^3}\int d\Omega' J_N(|K|,K_F,\cos\alpha)\frac{\chi(K_F,\Omega')}{\chi(K_F,\Omega)}- \tag{14}$$

$$-\frac{1}{2}\frac{1}{(2\pi)^3}\int d\Omega'\int_0^\infty dK' \ln|K'-K_F|\frac{d}{dK'}\left[J_N(|K|,|K'|,\cos\alpha)\frac{K'^2}{E'(K')}f(K',\Omega')\frac{\chi(K_F,\Omega')}{\chi(K_F,\Omega)}\right]=0$$

The solution of the equation (14) at $K=K_F$ since $J_N(K_F,K_F,\cos\alpha)$ tends to zero exists only when

$$\int_0 d\Omega'\frac{\chi(K_F,\Omega')}{\chi(K_F,\Omega)}J_N(K_F,K_F,\cos\alpha)<0\,. \tag{15}$$

Actually $f(K_F)>0$ and the last term in (14) is the value of higher order of smallness, if for $J_N(K_F,|K'|\cos\alpha)$ there doesn't exist any region in K' - space where it rapidly changes.

Let us expand in Legendre polinomials

$$J_{in}(K_F,K_F,\pm\cos\vartheta)=\sum_{\ell=0}^{\infty}J_{in}^{\ell}(K_F)P_\ell(\pm\cos\vartheta),$$

$$\chi(K_F,\cos\vartheta)=\sum_{\ell=0}^{\infty}d^{\ell}(K_F)P_\ell(\cos\vartheta).$$

and perform corresponding integration. Then we obtain the conditions of superfluidity of nuclear matter

$$J_{ts}^{\ell=0}(K_F)<0\,,\qquad J_{ts}^{\ell=2}(K_F)<0 \tag{16}$$

$$J_{tt}^{\ell=1}(K_F)+0{,}8\,J_{tT}^{\ell=1}(K_F)<0 \tag{17}$$

if each state with $\ell \geq 2$ contributes relatively little to the p-p interaction.

5. The conditions of superfluidity in case of central forces

It is of interest to consider the case with central forces. We come to it if we suppose that tensor and spin-orbit forces tend to zero more that central forces.

In case of central forces a set of equations (7) - (11) is reduced to three independent equations:

$$2\left|E(K)-E_F\right|\varphi(K)+\frac{1}{V}\sum_{K'}J_N^{\circ}(K,K')\varphi(K')=E\varphi(K) \tag{18}$$

$$2\left|E(K)-E_F\right|\varphi_+(K)+\frac{1}{V}\sum_{K'}J_+^{\circ}(K,K')\varphi_+(K')=E\varphi_+(K) \tag{19}$$

$$2\left|E(K)-E_F\right|\varphi_1(K)+\frac{1}{2V}\sum_{K'}\left\{J_\nu^{\circ}(K,K')+J_\alpha^{\circ}(K,K')\right\}\varphi_1(K')=E\varphi_1(K) \tag{20}$$

Following the considerations of the previous section we get the conditions of superfluidity of nuclear matter

$$J_{ts}^{\ell=0}(K_F)<0,\quad J_{ts}^{\ell=2}(K_F)<0,\quad J_{ss}^{\ell=1}(K_F)<0 \tag{21}$$

$$J_{tt}^{\ell=1}(K_F)<0$$

if the contribution of the states with $\ell > 2$ is not large.

6. The conditions of superfluidity in general case

Now let us investigate the restrictions on the functions $J_{in}(\kappa,\kappa')$ in order that the set of equations (7) - (11) has solutions with negative eigenvalues E. With this aim we suppose

$$\varphi_{\pm}(\kappa)=\frac{\theta_{\pm}(\kappa,\Omega)}{|E(\kappa)-E_F|+\delta},\quad \varphi_{1,2}(\kappa)=\frac{\theta_{1,2}(\kappa,\Omega)}{|E(\kappa)-E_F|+\delta}.$$

As in the section 4 we proceed to the approximated set of equations which at small J_{in} asymptotically coincides with the set of equations (7) - (11). Then we introduce some unknown functions $f_i(\kappa,\Omega)$ coinciding between themselves at $\kappa=\kappa_F$. Suppose that the functions $J_{in}(\kappa_F,|\kappa'|,\Omega')$ have no regions in κ'-space, where they change very rapidly. It is convenient to introduce the notation

$$J_{in}^{(\pm)}=\frac{1}{2}\left\{J_{in}(\vec{\kappa}_F,\vec{\kappa}_F')\pm J_{in}(\vec{\kappa}_F,-\vec{\kappa}_F')\right\}.$$

Then the conditions of superfluidity of nuclear matter for the most general case of nucleon-nucleon interaction are

$$\int d\Omega'\,\frac{\chi(\kappa_F,\Omega')}{\chi(\kappa_F,\Omega)}\left[J_{ts}^{(+)}+J_{tt}^{(-)}+2(3e_z^2-e^2)J_{tT}^{(-)}\right]<0, \qquad (22)$$

$$Re\int d\Omega'\Big\{\frac{\theta_+(K_F,\Omega')}{\theta_+(K_F,\Omega)}\Big[J_{st}^{(+)}+J_{tt}^{(-)}-(3e_z^2-e^2)(J_{sT}^{(+)}+J_{tT}^{(-)})+2\ell_z(J_{s\ell}^{(+)}+J_{t\ell}^{(-)})\Big]+$$

$$+\frac{\theta_1(K_F,\Omega')}{\theta_+(K_F,\Omega)}\Big[(\ell_x-i\ell_y)(J_{s\ell}^{(+)}+J_{t\ell}^{(-)})-3e_z(e_x-ie_y)(J_{sT}^{(+)}+J_{tT}^{(-)})\Big]+$$

$$+\frac{\theta_2(K_F,\Omega')}{\theta_+(K_F,\Omega)}\Big[(\ell_x-i\ell_y)(J_{s\ell}^{(+)}-J_{t\ell}^{(-)})-3e_z(e_x-ie_y)(J_{sT}^{(+)}-J_{tT}^{(-)})\Big]- \qquad (23)$$

$$-\frac{\theta_-(K_F,\Omega')}{\theta_+(K_F,\Omega)}(3e_x^2-3e_y^2-6ie_xe_y)\Big[J_{sT}^{(+)}-J_{tT}^{(-)}\Big]\Big\}<0\;.$$

$$Re\int d\Omega'\Big\{\frac{\theta_-(K_F,\Omega')}{\theta_-(K_F,\Omega)}\Big[J_{st}^{(+)}+J_{tt}^{(-)}-(3e_z^2-e^2)(J_{sT}^{(+)}+J_{tT}^{(-)})-2\ell_z(J_{s\ell}^{(+)}+J_{t\ell}^{(-)})\Big]+$$

$$+\frac{\theta_1(K_F,\Omega')}{\theta_-(K_F,\Omega)}\Big[(\ell_x+i\ell_y)(J_{s\ell}^{(+)}-J_{t\ell}^{(-)})+3e_z(e_x+ie_y)(J_{sT}^{(+)}-J_{tT}^{(-)})\Big]+ \qquad (24)$$

$$+\frac{\theta_2(K_F,\Omega')}{\theta_-(K_F,\Omega)}\Big[(\ell_x+i\ell_y)(J_{s\ell}^{(+)}+J_{t\ell}^{(-)})+3e_z(e_x+ie_y)(J_{sT}^{(+)}+J_{tT}^{(-)})\Big]-$$

$$-\frac{\theta_+(K_F,\Omega')}{\theta_-(K_F,\Omega)}(3e_x^2-3e_y^2+6ie_xe_y)\Big[J_{sT}^{(+)}-J_{tT}^{(-)}\Big]\Big\}<0\;,$$

$$
\begin{aligned}
Re\int d\Omega'\Big\{ & \frac{1}{2}\,\frac{\Theta_1(K_F,\Omega')}{\Theta_1(K_F,\Omega)}\Big[\overset{(-)}{J}_{ss}+\overset{(+)}{J}_{st}+\overset{(+)}{J}_{ts}+\overset{(-)}{J}_{tt}+(3e_z^2-e^2)\big(\overset{(+)}{J}_{sT}+\overset{(-)}{J}_{tT}\big)\Big]+\\
&+\frac{1}{2}\,\frac{\theta_2(K_F,\Omega')}{\Theta_1(K_F,\Omega)}\Big[\overset{(-)}{J}_{ss}+\overset{(+)}{J}_{st}-\overset{(+)}{J}_{ts}-\overset{(-)}{J}_{tt}+(3e_z^2-e^2)\big(\overset{(+)}{J}_{sT}-\overset{(-)}{J}_{tT}\big)\Big]+\\
&+\frac{\Theta_+(K_F,\Omega')}{\Theta_1(K_F,\Omega)}\Big[(\ell_x+i\ell_y)\big(\overset{(+)}{J}_{s\ell}+\overset{(-)}{J}_{t\ell}\big)-3e_z(\ell_x+i\ell_y)\big(\overset{(+)}{J}_{sT}+\overset{(-)}{J}_{tT}\big)\Big]+\\
&+\frac{\Theta_-(K_F,\Omega')}{\Theta_1(K_F,\Omega)}\Big[(\ell_x-i\ell_y)\big(\overset{(+)}{J}_{s\ell}-\overset{(-)}{J}_{t\ell}\big)+3e_z(e_x-i\ell_y)\big(\overset{(+)}{J}_{sT}-\overset{(-)}{J}_{tT}\big)\Big]\Big\}<0\,,
\end{aligned}
\tag{25}
$$

$$
\begin{aligned}
Re\int d\Omega'\Big\{ & \frac{1}{2}\,\frac{\Theta_2(K_F,\Omega')}{\Theta_2(K_F,\Omega)}\Big[\overset{(-)}{J}_{ss}+\overset{(+)}{J}_{st}+\overset{(+)}{J}_{ts}+\overset{(-)}{J}_{tt}+(3e_z^2-e^2)\big(\overset{(+)}{J}_{sT}+\overset{(-)}{J}_{tT}\big)\Big]+\\
&+\frac{1}{2}\,\frac{\Theta_1(K_F,\Omega')}{\Theta_2(K_F,\Omega)}\Big[\overset{(-)}{J}_{ss}+\overset{(+)}{J}_{st}-\overset{(+)}{J}_{ts}-\overset{(-)}{J}_{tt}+(3e_z^2-e^2)\big(\overset{(+)}{J}_{sT}-\overset{(-)}{J}_{tT}\big)\Big]+\\
&+\frac{\Theta_+(K_F,\Omega')}{\Theta_2(K_F,\Omega)}\Big[(\ell_x+i\ell_y)\big(\overset{(+)}{J}_{s\ell}-\overset{(-)}{J}_{t\ell}\big)-3e_z(e_x+i\ell_y)\big(\overset{(+)}{J}_{sT}-\overset{(-)}{J}_{tT}\big)\Big]+\\
&+\frac{\Theta_-(K_F,\Omega')}{\Theta_2(K_F,\Omega)}\Big[(\ell_x-i\ell_y)\big(\overset{(+)}{J}_{s\ell}+\overset{(-)}{J}_{t\ell}\big)+3e_z(e_x-i\ell_y)\big(\overset{(+)}{J}_{sT}+\overset{(-)}{J}_{tT}\big)\Big]\Big\}<0
\end{aligned}
\tag{26}
$$

and the imaginary parts of the expressions (23) - (26) must be of the same order of smallness as the terms neglected.

The conditions of superfluidity of nuclear matter (22) - (26) and the restrictions for the imaginary parts are very complicated. Nevertheless as it was shown in the previous sections, they give us possibility to obtain simple conditions for the particular cases of interactions.

7. Conclusion

As is shown by the theory of superconductivity(I) the Fermi systems with predominant attractive forces possess the property of superfluidity.

The general tendency of predominant attractive forces reveals itself in the conditions of superfluidity of nuclear matter although the conditions of superfluidity for the most general form of nucleon-nucleon interaction are rather complicated and the dependence on the potential is rather complicated too. The existence of superfluid state of nuclear matter is connected with the behaviour of the even states

$J_{st}(\kappa_F)$, $J_{ts}(\kappa_F)$, $J_{sT}(\kappa_F)$, $J_{s\ell}(\kappa_F)$

and odd states $J_{ss}(\kappa_F)$, $J_{st}(\kappa_F)$, $J_{tt}(\kappa_F)$, $J_{tT}(\kappa_F)$, $J_{t\ell}(\kappa_F)$ on the Fermi surface. In particular cases when only p-p interactions or central interactions are considered, the conditions of superfluidity are reduced to the requirement that the potentials

$$J_{ts}^{\ell=0}(K_F),\ J_{ts}^{\ell=2}(K_F),\ J_{tt}^{\ell=1}(K_F)+0{,}3\ J_{tr}^{\ell=1}(K_F),\ J_{ss}^{\ell=1}(K_F),\ J_{tt}^{\ell=1}(K_F)$$

are the attractive potentials. Since the states with $\ell > 2$ make a considerably small contribution into nucleon-nucleon interactions of the corresponding energy region no restrictions exist for them.

Let us compare the conditions of superfluidity of nuclear matter with the potentials, obtained from the analysis of the experimental data of nucleon-nucleon scattering. It is known that the particle density of nuclear matter is approximately $2{,}21\cdot 10^{38}\ \frac{\text{particles}}{\text{cm}^3}$. On this condition the wave length that corresponds to the Fermi energy is $0{,}675\cdot 10^{-13}$ cm. On the basis of a number of papers(3) the conclusion may be drawn that the potentials have a repulsive character when the wave length is of the order of $(0{,}2 - 0{,}5)\cdot 10^{-13}$ cm. and less; at larger wave lengths they have an attractive character. Thus near the Fermi surface the nucleon-nucleon potentials are in general attractive ones.

According to the data on nucleon-nucleon potentials we may come to the conclusion that the conditions of superfluidity of nuclear matter are in general fulfilled if the interactions bringing to superfluidity are weak.

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