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EVALUATION OF TEMPERATURE DISTRIBUTION
AND YIELD OF AN UNDERGROUND NUCLEAR EXPLOSION

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I. INTRODUCTION

In January 1957 Francis B. Porzel, Senior Scientific Advisor at Armour Research Foundation, suggested that the temperature distribution as a function of time in the immediate vicinity surrounding an underground nuclear explosion could be used for two important studies. One is that of calculating the yield of the weapon. Another is a determination of the capacity of the earth to maintain a high temperature heat source which might be employed for the generation of power.

II. THEORETICAL ASPECTS

The hydrodynamic model of the explosion shows that the energy of the bomb is expended in compacting the soil and raising the temperature of the soil in the vicinity of the explosion by the shock waves in the surrounding earth. This is a dissipative effect in which much of the energy appears as waste heat, manifested in the form of a temperature rise in the soil surrounding the bomb cavity. This assumption is conservative because it results in extremely high temperatures concentrated close to the cavity at early times. The actual temperatures produced by the shock process will be lower and already distributed somewhat farther out.

To plan an effective program of temperature measurement, a hypothetical situation approximating the behavior of the weapon and surroundings needs to be postulated. This is for the purpose of evaluating beforehand what temperatures need to be measured, where these measurements should be made, and at what intervals of time. To do this, a simplified situation is assumed in which the surface of a cavity of radius R is suddenly exposed to an instantaneous heat pulse, Q equal to the estimated fraction of the total yield of the bomb which goes into heat. The radius R is chosen to correspond approximately to that which is expected to be formed from the instantaneous vaporization and compaction of the earth immediately surrounding the weapon.

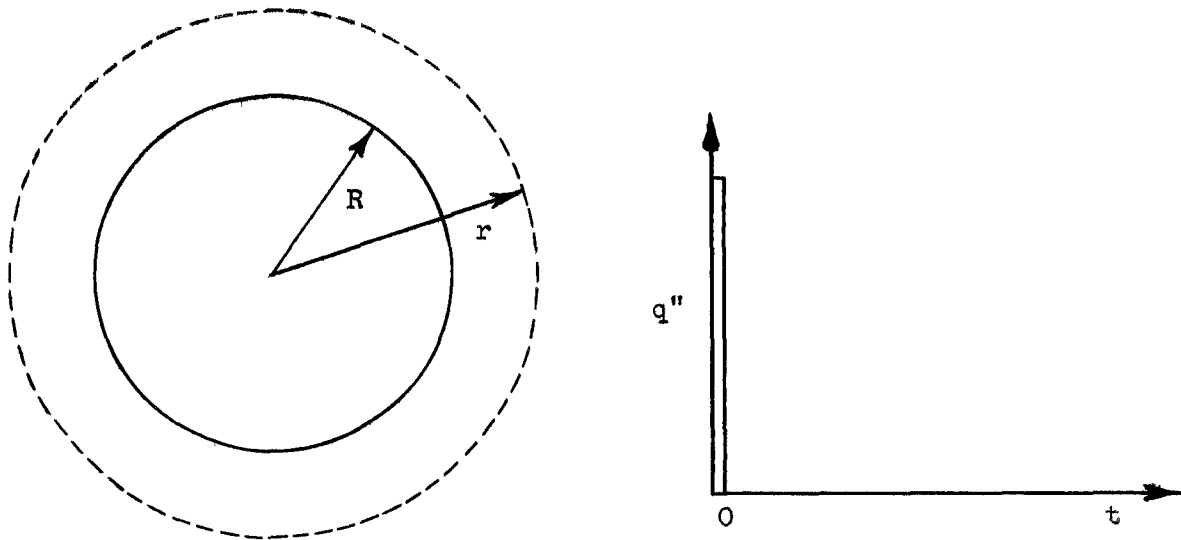


Fig. 1

By means of LaPlace transformation it is possible to obtain the solution for temperature distribution around the cavity as a function of time and distance from its center.

Let Fig. 1a represent the cavity of radius R . The partial differential equation expressing the relationship between the temperature surrounding the cavity of radius r and time, t following the explosion is

$$\frac{\partial^2 u}{\partial r^2} + \frac{2}{r} \frac{\partial u}{\partial r} = \frac{1}{\alpha} \frac{\partial u}{\partial t} \quad (1)$$

where $\alpha = \frac{k}{\rho c}$ = thermal diffusivity of the surrounding soil.

The boundary conditions are:

$$u(r, 0) = 0$$

$$u(r, \infty) = 0$$

$$k \left. \frac{\partial u}{\partial r} \right|_{r=R} = -q'$$

Applying a momentary heat pulse q'' , Fig. 1b, in a delta function the total heat input is

$$Q = 4\pi R^2 \int_0^{\infty} q'' dt \quad (2)$$

Letting $v = ru$

$$\frac{\partial u}{\partial r} = \frac{1}{r} \frac{\partial v}{\partial r} - \frac{v}{r^2}$$

$$\frac{\partial^2 u}{\partial r^2} = \frac{1}{r} \frac{\partial^2 v}{\partial r^2} - \frac{2}{r^2} \frac{\partial v}{\partial r} + \frac{2v}{r^3}$$

$$\frac{\partial^2 u}{\partial r^2} + \frac{2}{r} \frac{\partial u}{\partial r} = \frac{1}{r} \frac{\partial^2 v}{\partial r^2}$$

$$\frac{\partial^2 v}{\partial r^2} = \frac{1}{\alpha} \frac{\partial v}{\partial t}$$

$$\frac{k}{R} \left[\frac{\partial u}{\partial r} - \frac{v}{R} \right]_{r=R} = -q''$$

Applying the LaPlace transformation

$$V(r, s) = \int_0^{\infty} v(r, t) e^{-st} dt$$

$$\frac{d^2 V}{dr^2} = \frac{1}{\alpha} \left[sV - u(r, 0) \right]$$

$$\frac{d^2 V}{dr^2} = \frac{s}{\alpha} V$$

$$V = c e^{-\sqrt{\frac{s}{\alpha}} r}$$

$$\frac{k}{R} \left[\frac{2V}{2r} - \frac{V}{r} \right] = \int_0^{\infty} q'' e^{-st} dt$$

Since q'' has a value only at $t = 0$ where $e^{-st} = 1$

$$\frac{k}{R} \left[\frac{\partial V}{\partial r} - \frac{V}{R} \right] = \frac{-Q}{4\pi R^2}$$

$$\left. \frac{dV}{dr} \right|_{r=R} - \frac{V}{R} = \frac{-Q}{4\pi R k}$$

$$-C \sqrt{\frac{s}{\alpha}} e^{-\sqrt{\frac{s}{\alpha}} R} - \frac{C e^{-\sqrt{\frac{s}{\alpha}} R}}{R} = -\frac{Q}{4\pi R k}$$

or

$$C = \frac{Q}{4\pi R k} \frac{e^{\sqrt{\frac{s}{\alpha}} R}}{\sqrt{\frac{s}{\alpha}} + \frac{1}{R}}$$

so that

$$V = \frac{Q}{4\pi R k} \frac{e^{-\sqrt{\frac{s}{\alpha}} (r-R)}}{\sqrt{\frac{s}{\alpha}} + \frac{1}{R}}$$

Letting $q = \sqrt{\frac{s}{\alpha}}$

$$V = \frac{Q}{4\pi R k} \frac{e^{-q(r-R)}}{q + \frac{1}{R}}$$

$$v(r, t) = \frac{Q}{4\pi R k} \left\{ \sqrt{\frac{\alpha}{\pi t}} e^{-\frac{(r-R)^2}{4\alpha t}} - \frac{\alpha}{R} e^{\left(\frac{r-R}{R} - \frac{\alpha t}{R^2}\right)} \operatorname{erfc}\left(\frac{r-R}{2\sqrt{\alpha t}} + \frac{\sqrt{\alpha t}}{R}\right) \right\}$$

Since

$$u(r, t) = \frac{v(r, t)}{r}$$

$$u(r, t) = \frac{Q}{4\pi r R k} \left\{ \sqrt{\frac{\alpha}{\pi t}} e^{-\frac{(r-R)^2}{4\alpha t}} - \frac{\alpha}{R} e^{\left(\frac{r-R}{R} - \frac{\alpha t}{R^2}\right)} \operatorname{erfc}\left(\frac{r-R}{2\sqrt{\alpha t}} + \frac{\sqrt{\alpha t}}{R}\right) \right\}$$

(3)

where

$u(r, t)$ = temperature at any radius r and time t after the explosion

Q = instantaneous heat pulse equal to the assumed yield of the bomb

$\alpha = \frac{k}{\rho c}$ thermal diffusivity of the surrounding soil

Using this equation, a series of curves of temperature versus radius can be plotted for various times after the explosion giving results of the general shape illustrated in Fig. 2. These curves are taken from actual calculated values assuming a 2 kiloton bomb with 80 per cent of its yield converted into heat. The radius of compaction R is assumed to be 17.5 ft and $\alpha = 0.0229 \text{ ft}^2/\text{hr}$. Q is taken as $6.55 \cdot 10^9$ Btu.

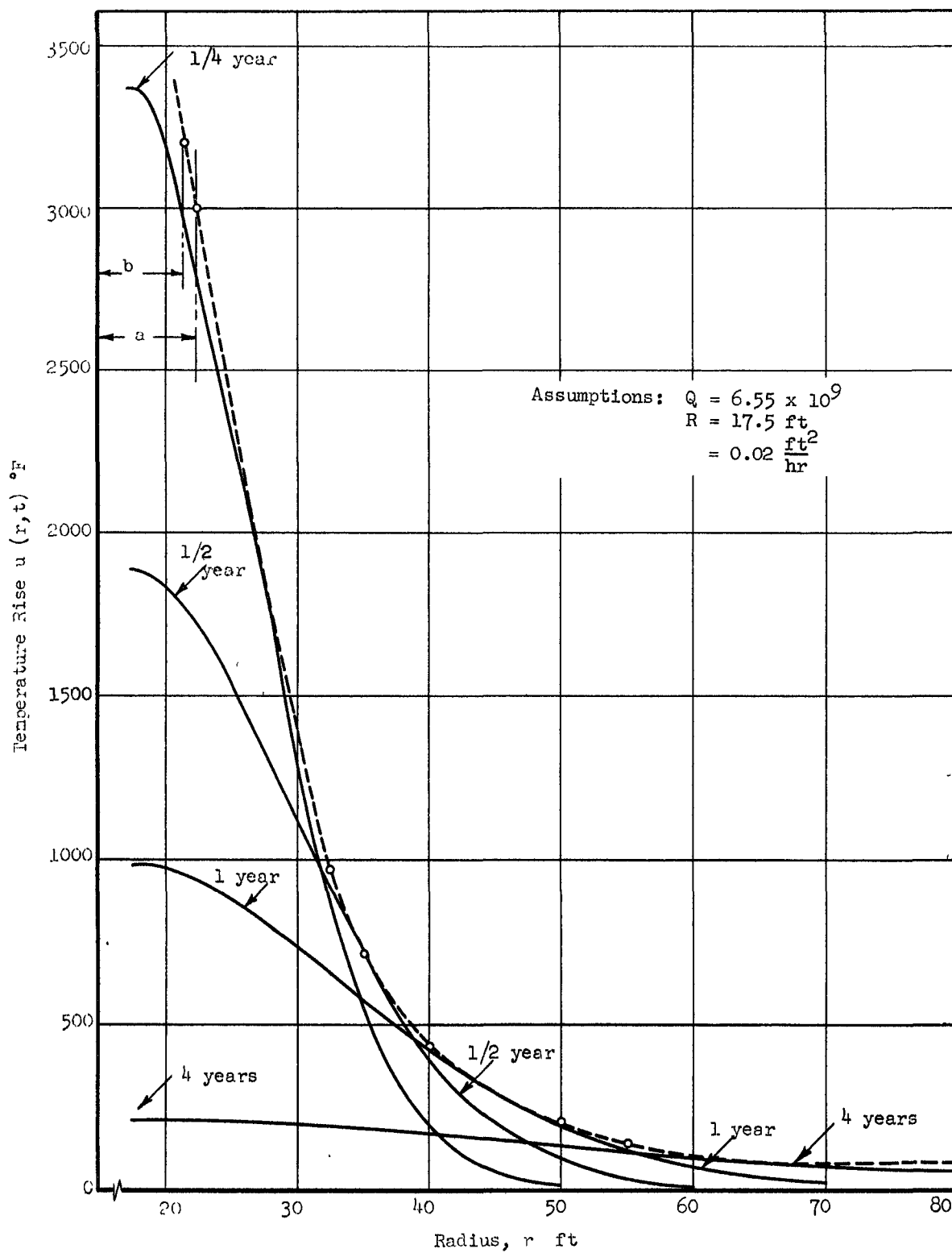


Fig. 2 TEMPERATURE VERSUS RADIUS CURVES FOR CONSTANT TIME

An envelope tangent to the temperature time curves, shown dotted in Fig. 2, may be employed to obtain the maximum temperature which will be realized at any given radius r . From the envelope, one may determine the location nearest the surface of the cavity where temperature measurements may be successfully obtained over a long period of time.

To illustrate, assume that a temperature measuring device capable of withstanding and measuring a maximum temperature rise of 3000°F can be constructed. This device could then be placed at distance "a" from the surface of the cavity and could be used to measure temperature any time after the explosion without being destroyed by excessive temperature. On the other hand, if it were placed closer than the distance "a", for example at distance "b", it would be destroyed as soon as the temperature rise exceeds 3000°F .

From the curves, it can be seen that temperature measurements are possible at radii less than "a" for a short period of time provided the sensors can resist the physical shock that may occur. However, they would not be useful in evaluating the yield of the bomb because at that time most of the energy storage would be within the molten or vaporized material very near the surface of the cavity where the temperature could not be measured.

At the present time, the only way in which bomb yield can be evaluated from temperature measurements is to wait until the cavity and surroundings have been cooled to temperatures that are measurable and then to take measurements of temperature versus distance from the center of the cavity at several different times after the explosion.

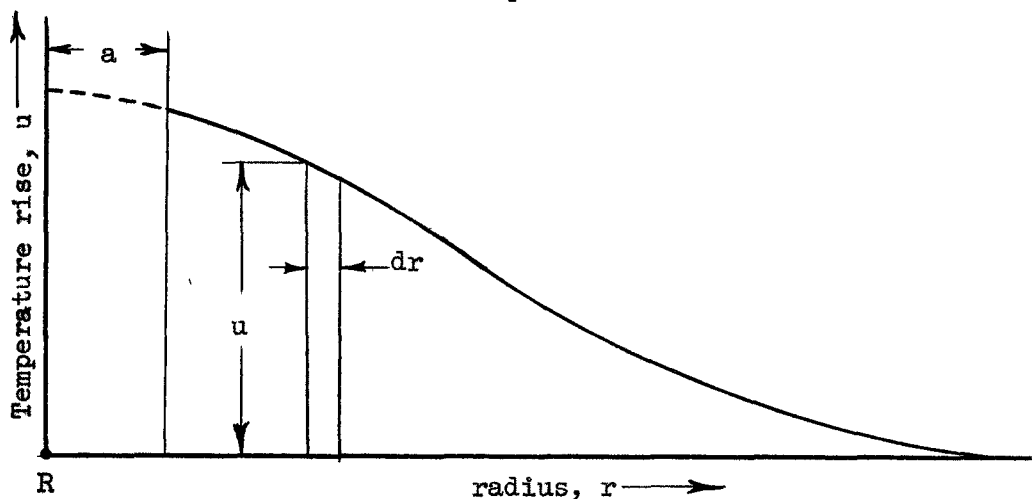


Fig. 3 INTEGRATION UNDER TEMPERATURE VERSUS RADIUS CURVE

By integrating under the temperature versus radius curves as illustrated in Fig. 3, the total energy release neglecting vaporization of water and its subsequent migration through the soil can be evaluated. The equation for accomplishing this is:

$$Q = \int_{r=R}^{r=\infty} 4\pi r^2 \rho c u dr$$

where

- ρ = density of the earth surrounding the cavity, lb/ft³
 c = specific heat of the earth surrounding the cavity, Btu/lb ° F

and where the value of u is obtained from the experimentally measured curve at any given time t after the explosion. The process could be carried out for a number of different curves each measured at a different time.

By taking samples of the soil at different times and distances from the center of the cavity it may be possible to detect the degree of vapor migration, if any, from the heated area surrounding the explosion. In this manner, necessary corrections resulting from this factor could be made. Additional precautions could also be taken to detect moisture migration such as for example, measuring both pressure and temperature at properly selected distances from the center of the cavity. These could be used to determine whether any water in the liquid phase remains present at these points. Three or four of these measurements should be sufficient to establish the influence of this parameter.