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FROM: R. Benumof and M. W. Rosenthal

ABSTRACT

A preliminary study was made of the nuclear stability of reactors in which particles containing U^{233} and Th^{232} are fluidized by liquid water. D_2O moderated reactors were found to be least sensitive to changes in bed height at thorium concentrations in the range 80-120 g/liter, H_2O reactors in the range 1500-2400 g Th/liter, and mixed H_2O-D_2O reactors at thorium concentrations between those for pure moderators. A reactor operated at the thorium concentration at which it is least sensitive to change in bed height was found to be stable in response to a cyclical variation in height.

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A PRELIMINARY STUDY OF THE NUCLEAR STABILITY OF FLUIDIZED BED REACTORS

by

R. Benumof and M. W. Rosenthal

Summary

Analyses were made of some of the problems associated with the nuclear stability of reactors in which particles containing U^{233} and Th^{232} are fluidized by upward-flowing liquid water. D_2O-moderated reactors were found to be least sensitive to changes in bed height if the thorium concentration is in the range 80 - 120 g/liter, and similar H_2O-moderated reactors were least sensitive in the range 1500 - 2400 g Th/liter. With mixed H_2O-D_2O moderators, the thorium concentrations associated with greatest nuclear stability lie between the values for pure moderators. At the optimum concentration for a particular moderator, a 10% change in bed height resulted in approximately 0.15% change in reactivity.

The effects of a sudden onset of sinusoidal variation in bed height on a reactor with the thorium concentration of greatest stability were considered under the assumption of constant core-inlet temperature. For small variations in height, the response of the reactor was found to be stable. All transient power fluctuations disappear with time. The steady-state power response of the reactor consisted of a sinusoidal variation having a frequency twice that of the bed-height oscillation frequency; the average power was slightly less than the initial power. The amplitude of the power fluctuation was a function of the frequency of bed-height oscillation. In one example, for a bed-height oscillation amplitude of 2%, the maximum amplitude of the power oscillation occurred at a frequency of approximately 0.25 cycle/sec and had a magnitude of 1.5% of the average power.

Introduction

A simplified version of a fluidized bed reactor is illustrated in Fig. 1. A moderator, either H_2O, or D_2O, or an H_2O-D_2O mixture is pumped through a bed consisting of uranium-oxide and thorium-oxide particles. The greater the velocity of flow of the moderator, the greater is the height of the bed. The nuclear reaction heats the moderator in the cylindrical core vessel. The moderator gives up this energy in the heat exchanger and then returns to the core.

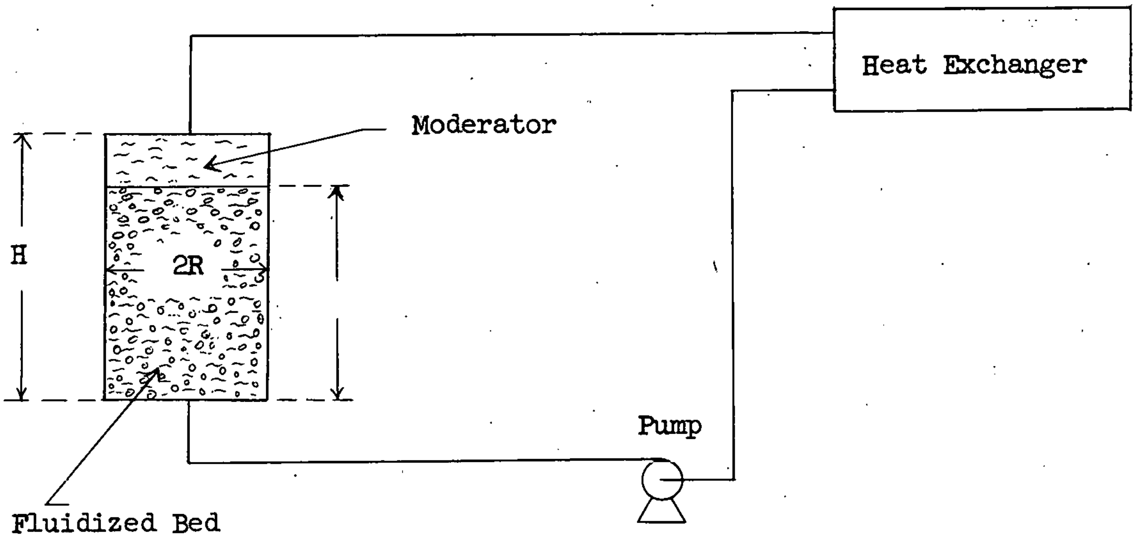

Fig. 1. A Simple Fluidized Bed Reactor

In principle, such a reactor has several advantageous features. First, in the fluidized bed reactor, the pump and the heat exchanger handle moderator only instead of fuel solution or slurry. Secondly, a smaller inventory of fuel and fertile material is needed in the fluidized bed reactor; instead of being dispersed throughout the entire system, the fuel and the fertile material are contained in the core vessel only. Finally, the confinement of the fuel to the core vessel reduces the gamma and delayed-neutron radiation in parts of the system external to the core. These features are obtained without sacrificing the ease of fuel additions and removal characteristic of fluid fuel reactors.

Opposed to these advantages are the possible disadvantages involved in having a fluidized bed. First, inhomogeneities may occur in the bed under various flow conditions and the nuclear stability of the reactor may be affected. Secondly, the very fine particles may be carried out of the core vessel by the moderator. Finally, an increase in temperature results in lower density and lower viscosity of the moderator. If the rise in temperature occurs slowly enough, the bed height tends to decrease. As a consequence, the temperature coefficient of a fluidized bed reactor may be less negative than that of a fluid fuel reactor. (This problem has not been treated.) Of course, there are also the difficulties associated with developing satisfactory particles, achieving good flow distribution, providing for start-up and shutdown, etc.

The Problems Considered in this Paper

Two problems are considered in this paper. First, there is the question of what the thorium concentration should be in order to attain maximum nuclear stability. Secondly, since the bed height would not be absolutely constant, there arises the problem of determining the effects of a cyclical variation in bed height.

Determination of Optimum Concentrations

The number of grams of fuel per liter needed for criticality divided by the number of grams of thorium per liter will be called the critical ratio. If the critical ratio is plotted against various values of thorium concentration (resulting from varying the height of the bed), then a curve similar to the one in Fig. 2 is obtained if the thorium concentration is in the proper range.

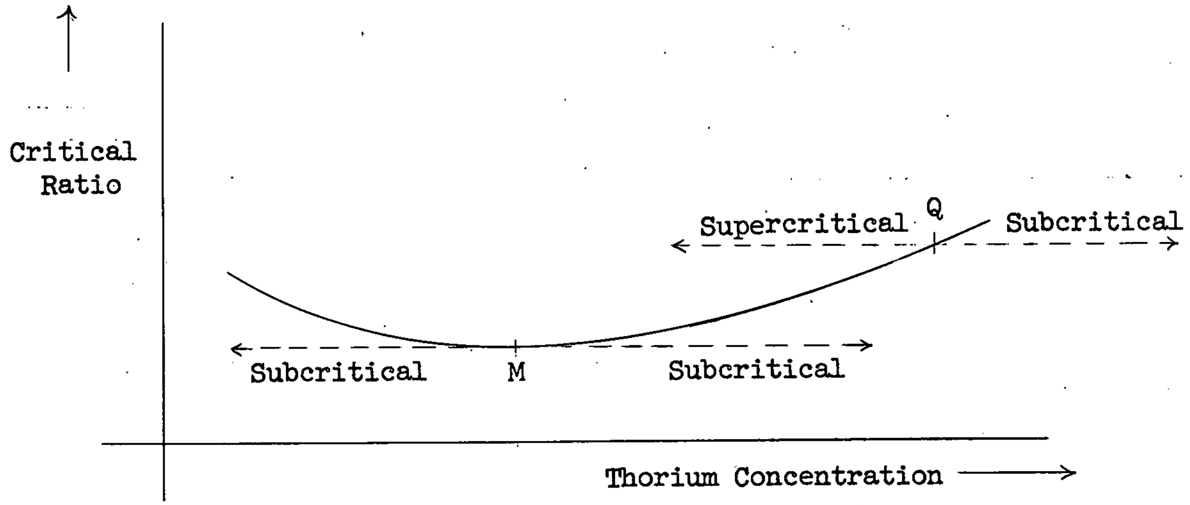

Fig. 2. A Typical Curve of Critical Ratio vs Thorium Concentration

In the actual operation of a fluidized bed reactor, as the height of the bed varies, the ratio of fuel concentration to thorium concentration remains constant, since the particles constituting the bed contain both U^{233} and Th^{232}. Thus, if the minimum point M in Fig. 2 is chosen as the initial operating point, then, as the height of the bed increases or decreases, the reactor tends to become subcritical. On the other hand, if the point Q were chosen as the initial operating point, then, as the height of the bed decreases, the reactor tends to become subcritical, but, as the height of the bed increases, the reactor tends to become supercritical. The reactor is inherently more stable if the initial operating point is M. Thus, the determination of the optimum concentrations (with regard to stability) for a given fluidized bed reactor consists in locating the minimum point M on the curve of critical ratio versus thorium concentration.

That a minimum point actually exists may be easily demonstrated. The critical ratio is proportional to Σ^{23} (crit) divided by Σ^{02} where Σ^{23} is the macroscopic absorption cross-section of U^{233} and Σ^{02} is the macroscopic cross section of Th^{232}. The rate of change of the critical ratio with respect to Σ^{02} is proportional to

$$\frac{\partial\left[\Sigma^{23}(\mathrm{crit})/\Sigma^{02}\right]}{\partial\Sigma^{02}} = \frac{1}{\Sigma^{02}}\left[\frac{\partial\,\Sigma^{23}(\mathrm{crit})}{\partial\,\Sigma^{02}} - \frac{\Sigma^{23}(\mathrm{crit})}{\Sigma^{02}}\right]$$

The minimum occurs when

$$\frac{\partial\,\Sigma^{23}(\mathrm{crit})}{\partial\,\Sigma^{02}} = \frac{\Sigma^{23}(\mathrm{crit})}{\Sigma^{02}} \tag{1}$$

If we assume that $p = e^{-q\,\Sigma^{02}}$ where q is a constant suitable for a given range of values of Σ^{02}, then equation (A6) in Appendix A gives $\Sigma^{23}(\mathrm{crit})$ as a function of Σ^{02}. Combining this with equation (1) above, we obtain the result that the minimum occurs at a value of Σ^{02} given by

$$\Sigma^{02} = \frac{D_1B^2 + \Sigma_1^m}{2}\left\{-1 + \left[1 + \frac{4}{q(D_1B^2 + \Sigma_1^m)}\left(1 - \frac{1.07}{p\eta\, e^{-B^2\tau}}\right)\right]^{1/2}\right\} \tag{2}$$

For H_2O, $q = 7.88$ in the range 1000 - 2000 g Th/liter, and the minimum value of the critical ratio occurs at a thorium concentration of approximately 1500 g Th/liter. For D_2O, the minimum value of the critical ratio occurs at approximately 100 g Th/liter and for this concentration $q = 112$. The significance of these calculations is that it enables us to locate approximately the minimum of the curve of critical ratio vs thorium concentration.

Detailed one-group calculations verify the above results. The one-group calculations for a temperature of 280° are summarized in Table 1. The two-group results for the same temperature are shown in Table 2.

Table 1

Critical Ratio vs. Thorium Concentration

One-Group Results

Moderator	R ft	H ft	h ft	$\frac{\text{g Th}}{\text{liter}}$	Σ^{23}(crit) cm^{-1}	$\frac{\text{g U}^{233}}{\text{g Th}}$
H_2O	2	7	3	2500	8.45×10^{-2}	3.40×10^{-2}
			4	1875	5.43	2.92
			5	1500	4.32	2.90
			6	1250	3.68	2.96
			7	1070	3.16	2.97
H_2O	5	10	6	2100	4.98	2.39
			7	1800	4.17	2.33
			8	1573	3.63	2.32
			10	1260	2.91	2.335
D_2O	2	7	5	100	1.58	16.1
			6	83.3	1.165	14.9
			7	71.5	1.05	15.1
D_2O	5	10	7	100	0.24	2.45
			8	87.5	0.206	2.39
			10	70	0.164	2.40

Table 2

Critical Ratio vs. Thorium Concentration

Two-Group Results

Moderator	R ft	H ft	h ft	g Th / liter	g U^{233} / liter	g U^{233} / g Th
H_2O	2	7	3	3333	83.707	2.511×10^{-2}
			4	2500	58.503	2.340
			5	2000	45.979	2.299
			6	1667	38.555	2.313
			7	1429	33.745	2.362
H_2O	5	10	4	5250	104.564	1.992
			5	4200	80.820	1.924
			7	3000	55.734	1.858
			9	2333	43.164	1.850
			10	2100	39.069	1.860
D_2O	2	7	3.93	125	11.392	9.100
			4.92	100	8.758	8.758
			5.90	83	7.414	8.890
			6.90	71	6.714	9.416
D_2O	5	10	5.90	117	2.674	2.286
			6.90	100	2.241	2.241
			7.86	88	1.955	2.227
			8.85	78	1.748	2.241
			9.90	70	1.592	2.283

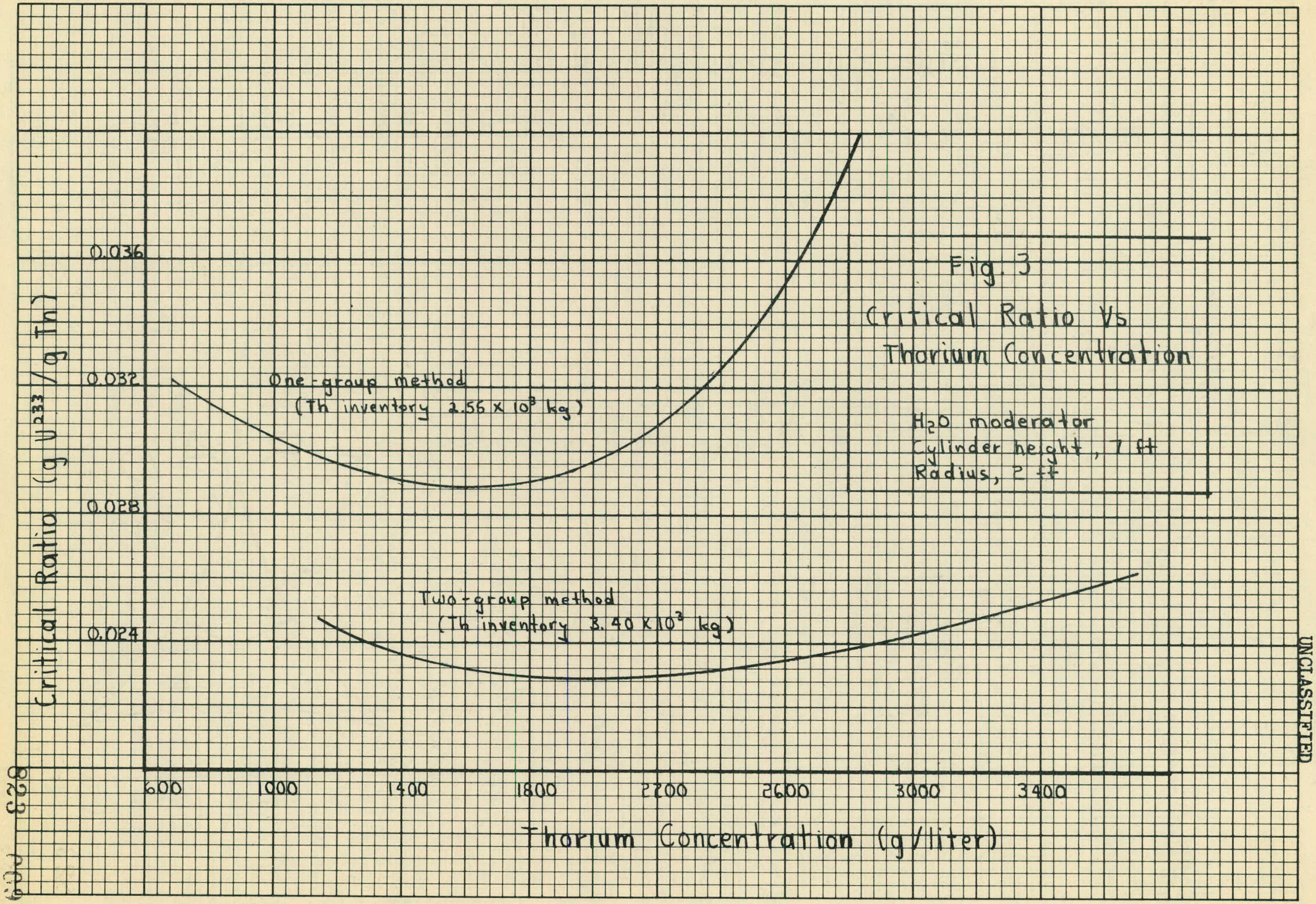
Fig. 3

Critical Ratio Vs Thorium Concentration

H$_2$O moderator

Cylinder height, 7 ft

Radius, 2 ft

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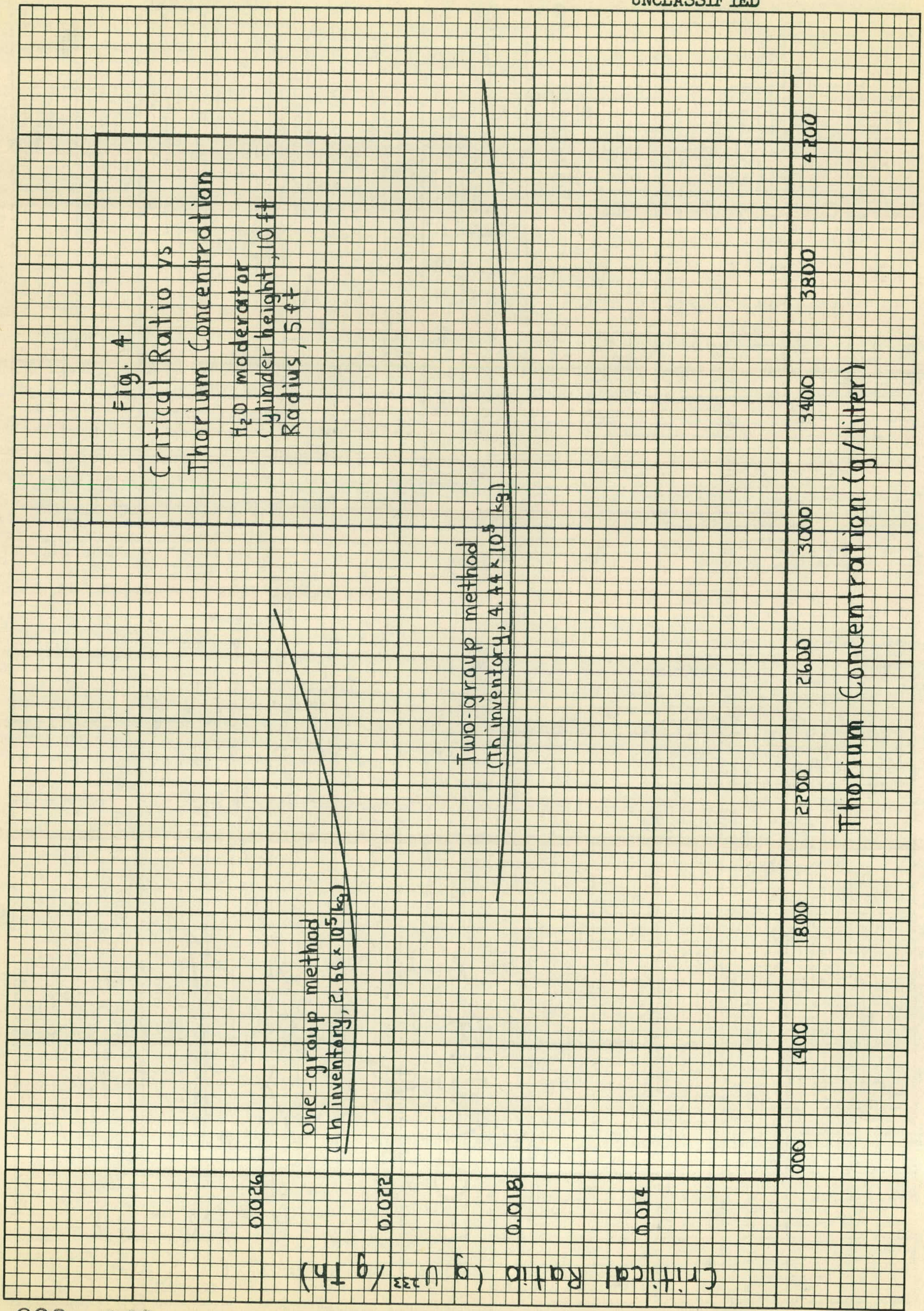

Fig. 4
Critical Ratio vs Thorium Concentration

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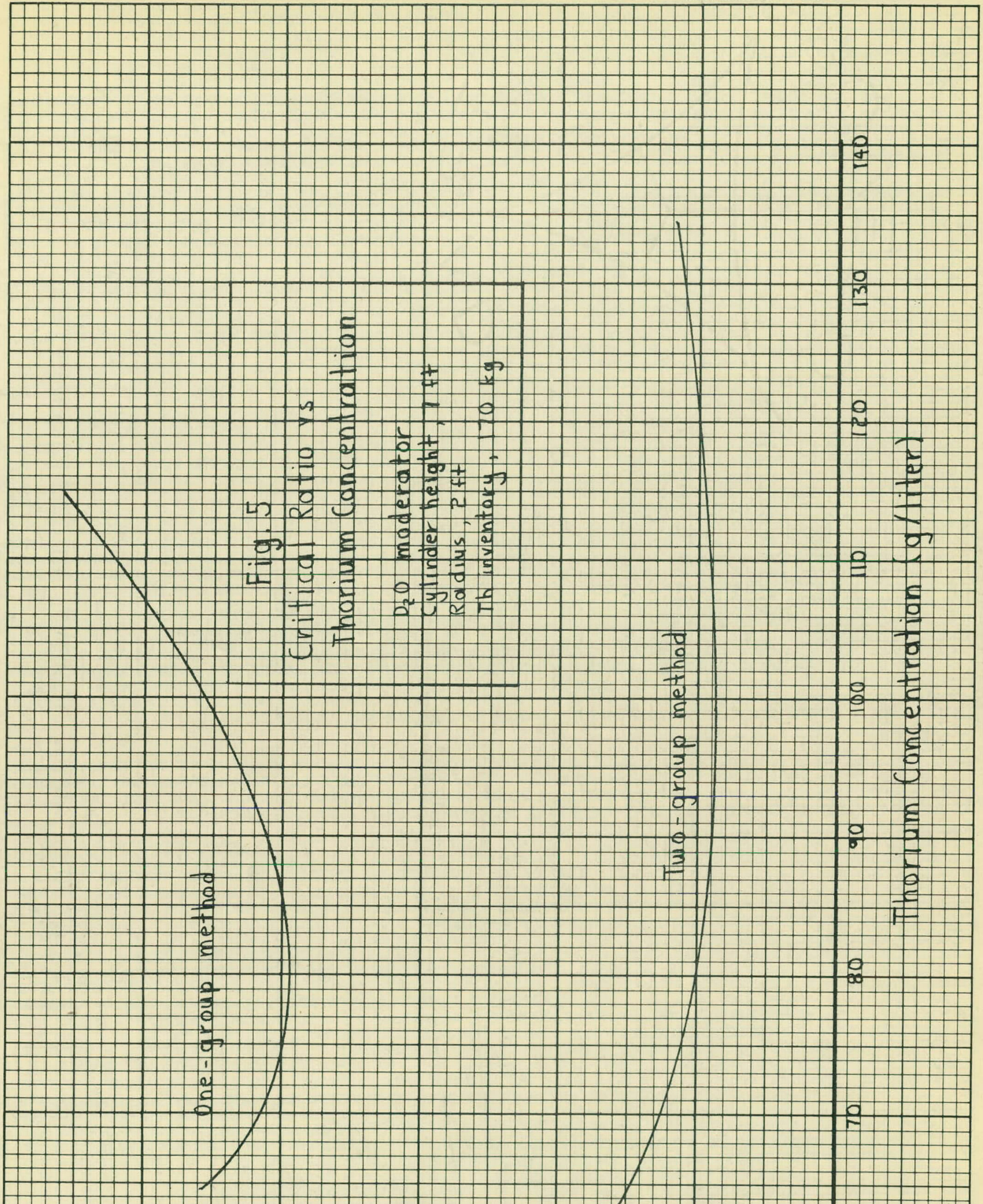

Fig. 5
Critical Ratio vs Thorium Concentration

D_2O moderator
Cylinder height, 7 ft
Radius, 2 ft
Th inventory, 170 kg

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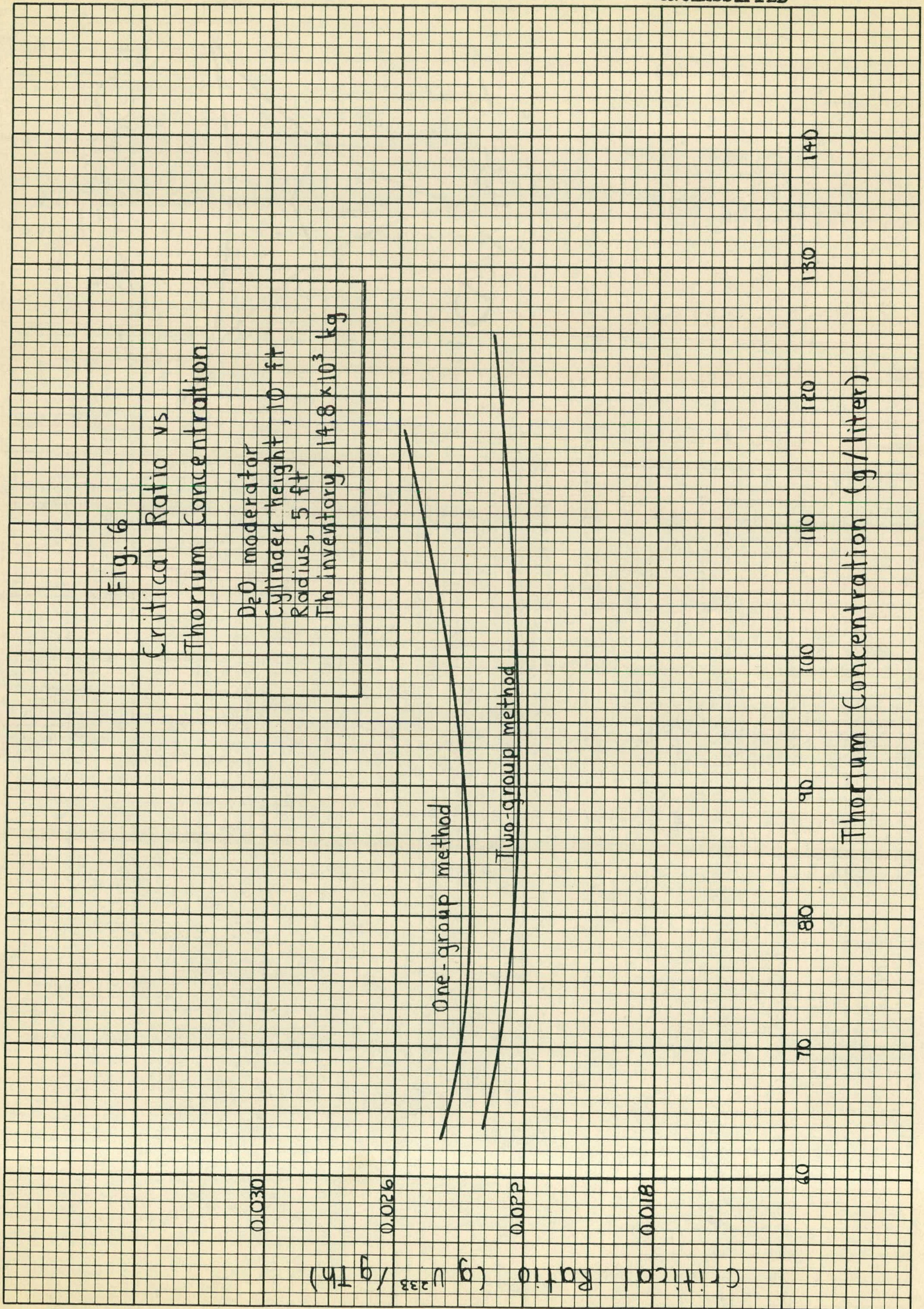

Fig. 6
Critical Ratio vs
Thorium Concentration

D_2O moderator
Cylinder height, 10 ft
Radius, 5 ft
Th inventory, 14.8×10^3 kg

Curves of the results of the one-group and two-group calculations for various reactors are shown in Figs. 3, 4, 5, and 6. The difference between the one-group and the two-group results in Figs. 5 and 6 is to some extent a consequence of the method of taking fast neutron leakage into account. In the one-group calculations, the factor $e^{-B^2\tau}$ is used whereas in the two-group calculations, the comparable factor is $\frac{1}{1 + B^2 \tau}$. Also, in the one-group calculations, the product of the fast fission factor and the resonance escape probability for U^{233} was considered to be unity. These factors were explicitly calculated when the two-group method was used. The optimum operating point for D_2O reactors seems to be at a thorium concentration in the range 80 - 120 g Th/liter and the optimum operating point for H_2O reactors seems to be at a thorium concentration in the range 1500 - 2400 g Th/liter. If a mixture of H_2O and D_2O were used, the optimum thorium concentration would lie between these ranges, the actual value depending on the specific moderator composition.

The approximate change in reactivity with height was determined with the aid of equation (A7) in Appendix A. At the minimum of the critical curve, a 10% change in bed height results in approximately 0.15% change in reactivity. This figure is based upon data obtained by means of two-group calculations.

Stability of an Oscillating Fluidized Bed Reactor

If the height of the fluidized bed is a function of the time, then the power response of the reactor is of interest. The power as a function of time can be expressed as a sum of transient terms associated with the natural frequency of the system plus a constant term representing the average power plus a term representing the forced oscillation. The reactor is considered stable if the transient terms become negligible as time increases and if the amplitude of the forced oscillation does not exceed reasonable values for various frequencies of oscillation.

The behavior of a reactor operated with the thorium concentration at the point of greatest stability has been treated. As shown in Appendix B, if the height of the bed is assumed to be a sinusoidal function of the time, then the transient terms decrease exponentially to zero. The average power subsequent to the onset of a sinusoidal variation in bed height differed only slightly from the power prior to the fluctuation in bed height. For a 2% variation in height, the average power differed from the initial power by 0.15%. The amplitude of the forced oscillation in power depends on the frequency of oscillation of the bed. The maximum value of the power oscillation occurred at a frequency of approximately 0.25 cycle/sec and had a magnitude of approximately 1.5% of the average power.

Discussion of Results

The results obtained above are limited by virtue of the fact that they apply to an admittedly idealized system. To begin with, the core inlet temperature has been assumed constant. A more realistic model would include the effects of the heat exchanger. Mathematically, this would introduce two more equations, the first of which would be a heat balance for the moderator in the heat exchanger and the second of which would be an energy balance for the heat exchanger itself. (Under most conditions, these could be combined into one equation by assuming the heat exchanger temperature to be constant.) We would then have five differential equations instead of the three which were considered above. The final form of the expression for the power would contain five transient terms instead of three. There would still be a term representing the average power and a term describing the forced oscillation. The question of interest in this latter calculation would concern the effect of a cyclical variation of core inlet temperature caused by a cyclical variation in core outlet temperature.

A second limitation on the results obtained above stems from the fact that the equations were simplified by assuming that the changes in power and reactivity were small. The effect of this assumption may be estimated by considering the sum of equations (B3) and (B5), namely,

$$\frac{d(P + C)}{dt} = \frac{k_e - 1}{\ell} P \tag{3}$$

The corresponding sum of the simplified equations, viz., equations (B7) and (B8) is

$$\frac{d(P + C)}{dt} = \frac{(k_e - 1) P_o}{\ell} \tag{4}$$

As long as P does not differ too greatly from P_o, the rate of change of the actual plus the latent power will not be affected very much. For small changes in reactivity, the fluctuations in power are not significant.

Thirdly, it must be realized that the stability of a fluidized bed reactor depends upon the fact that both $\partial^2 k_e/\partial h^2$ and $\partial k_e/\partial T$ are negative. The negative temperature coefficient ($\partial k_e/\partial T$) may not be as great for a fluidized bed reactor as for a homogeneous reactor. The fuel region may not expand with increasing temperature. In fact, because of decreased density and viscosity it may actually tend to decrease. However, because of the change in nuclear characteristics, the temperature coefficient may still be negative. In this paper, the temperature coefficient was taken as -10^{-3} per degree Centigrade, which is in the range of values for reactors in which the ratio of solid to liquid remains constant as the bed density changes. To compensate for a possibly smaller negative temperature coefficient is the fact that, for a reactor operating at the minimum point on the critical curve, any change in bed height tends to reduce the reactivity, viz., ($\partial^2 k_e/\partial h^2$) is negative. At this juncture, there may arise the question of the effect of operating the reactor at a point other than the minimum of the critical curve. Mathematically, this would introduce a term proportional to $\sin \omega t$ in the expression for k_e. The major change in the final expression for the power would be to add a term proportional to $(\partial k_e/\partial h)\ \sin(\omega t + \psi')$ where ψ' is a phase angle. In other words, the forced oscillation would consist of an additional response proceeding with angular frequency ω. The importance of this new term would depend upon the relative magnitude of $(\partial k_e/\partial h)$. What should be investigated is the possibility of resonance effects in connection with this new term. This has not been investigated here.

Finally, the importance of the physical dimensions of the reactor must be considered. The way in which the geometry of the reactor enters into the results may be seen by examining equations (A6) and (A7) in Appendix A. Since B^2 is small, the term $D_1 B^2$ is quite unimportant in comparison to the other terms. The fast neutron nonleakage probability is represented in these same equations by the term $e^{-B^2\tau}$. This factor is of importance only for small D_2O-moderated reactors. Thus, the results of this study depend only slightly upon the particular dimensions of the reactors that were considered.

APPENDIX A

Determination of Critical Fuel Concentration

For the fuel region, the one-group criticality equation was written as

$$D_1 \left(\frac{\partial^2 \phi_1}{\partial r^2} + \frac{1}{r} \frac{\partial \phi_1}{\partial r} + \frac{\partial^2 \phi_1}{\partial z^2} \right) + p\eta e^{-B^2\tau} \Sigma^{23} \phi_1$$

$$- \left(\Sigma^{23} + \Sigma^{02} + \Sigma^{p} + \Sigma_1^{m} \right) \phi_1 = 0 \qquad \text{(A1)}$$

where

D_1 = diffusion constant for fuel region

r = radial distance

z = distance above lower base of cylinder

p = resonance escape probability for thorium

η = average number of fast neutrons emitted per capture of a thermal neutron in fuel

$e^{-B^2\tau}$ = nonleakage probability of fast neutrons

Σ^{23} = macroscopic cross-section of U^{233}

Σ^{02} = macroscopic cross-section of Th^{232}

Σ^{p} = macroscopic cross-section of fission products (poisons)

Σ_1^{m} = macroscopic cross-section of moderator in fuel region

ϕ_1 = thermal flux in fuel region

In equation (A1), we have assumed that the product of the fast fission factor and the resonance escape probability for U^{233} is unity; also, that the fast non-leakage probability for a two-region reactor can be approximated by the fast leakage from the fuel region. We may rewrite equation (A1) as follows:

$$\frac{\partial^2 \phi_1}{\partial r^2} + \frac{1}{r} \frac{\partial \phi_1}{\partial r} + \frac{\partial^2 \phi_1}{\partial z^2} + B^2 \phi_1 = 0 \qquad \text{(A2)}$$

where

$$B^2 = \frac{(e^{-B^2 \tau} p\eta - 1) \Sigma^{23} - \Sigma^{02} - \Sigma^{p} - \Sigma_1^{m}}{D_1} \qquad \text{(A3)}$$

For the pure moderator region (region 2), the diffusion equation is

$$D_2 \left(\frac{\partial^2 \phi_2}{\partial r^2} + \frac{1}{r} \frac{\partial \phi_2}{\partial r} + \frac{\partial^2 \phi_2}{\partial z^2} \right) - \Sigma_2^{m} \phi_2 = 0 \qquad \text{(A4)}$$

where

D_2 = diffusion constant for moderator

Σ_2^{m} = macroscopic absorption cross-section for moderator

ϕ_2 = thermal flux in region 2

The boundary conditions for equations (A2) and (A4) are:

at $r = 0$	ϕ_1 is finite
at $r = R$	$\phi_1 = 0 = \phi_2$
at $z = 0$	$\phi_1 = 0$
at $z = H$	$\phi_2 = 0$
at $z = h$	$\phi_1 = \phi_2$ and $D_1 \frac{\partial \phi_1}{\partial z} = D_2 \frac{\partial \phi_2}{\partial z}$

where

R = radius of cylinder including extrapolation distance

H = height of cylinder including extrapolation distance

h = height of fuel region

By making use of the technique of separation of variables, equations (A2) and (A4) may be solved. By imposing the boundary conditions for $r = 0$ and $z = 0$, we find that

$$\phi_1 = C_1 J_o \left(\frac{2.405}{R} r \right) \sin \omega z$$

$$\phi_2 = \left(C_2 e^{\beta z} + C_2' e^{-\beta z} \right) J_o \left(\frac{2.405}{R} r \right)$$

where

$$\omega^2 = B^2 - \left(\frac{2.405}{R} \right)^2$$

$$\beta^2 = \left(\frac{2.405}{R} \right)^2 + \frac{\Sigma_2^m}{D_2}$$

On imposing the boundary conditions at $z = H$ and $z = h$, we find that the following relation must hold:

$$\tanh \left[\beta(H - h) \right] = - \left(\frac{\beta}{\omega} \right) \left(\frac{D_2}{D_1} \right) \tan \omega h \qquad \text{(A5)}$$

In essence, equation (A5) is the relationship which determines B^2 in terms of the geometrical parameters, R, H, and h

The critical concentration of U^{233} for a given concentration of thorium may be found from equation (A3) Thus

$$\Sigma^{23}(\text{crit}) = \frac{D_1 B^2 + \Sigma^{02} + \Sigma_1^m}{p\eta e^{-B^2\tau} - 1.07} \qquad \text{(A6)}$$

where we have assumed that

$$\Sigma^p = 0.07\ \Sigma^{23}$$

The effective multiplication constant k_e is given by

$$k_e = \frac{p\eta e^{-B^2\tau}\ \Sigma^{23}}{D_1 B^2 + 1.07\ \Sigma^{23} + \Sigma^{02} + \Sigma_1^m} \qquad \text{(A7)}$$

By calculating k_e from equation (A7) for a constant inventory of U^{233} and Th^{232} at different bed heights, the rate of change of reactivity with height can be found.

APPENDIX B

The Kinetic Behavior of the Reactor

In order to analyze the kinetic behavior of the simple reactor illustrated in Fig. 1, we shall make a number of assumptions. First, we shall write

$$h - h_o = A \sin \omega t \tag{B1}$$

where

h_o = initial height of bed

A = amplitude of oscillation

ω = angular frequency of oscillation

Equation (B1) indicates that a sinusoidal variation in bed height is explicitly assumed. Secondly, for small changes in reactivity, the effective multiplication constant k_e may be expanded in a Taylor series as follows:

$$k_e = 1 + \left(\partial k_e/\partial h\right)\left(h - h_o\right) + \frac{1}{2}\left(\partial^2 k_e/\partial h^2\right)\left(h - h_o\right)^2$$

$$+ \left(\partial k_e/\partial T\right)\left(T - T_o\right) + \left(\partial^2 k_e/\partial h \partial T\right)\left(h - h_o\right)\left(T - T_o\right)$$

If the initial operating point is at the minimum of the curve of critical ratio versus thorium concentration, then $\partial k_e/\partial h = 0$; and if $\partial k_e/\partial T =$ constant, then $\partial(\partial k_e/\partial T)/\partial h = 0$. Under these circumstances we have

$$k_e - 1 = \Delta k_e = \frac{1}{2}\left(\partial^2 k_e/\partial h^2\right)\left(h - h_o\right)^2 + \left(\frac{\partial k_e}{\partial T}\right)\left(T - T_o\right) \tag{B2}$$

Thirdly, we shall assume that the delayed neutrons constitute a fraction β of the total number of fission neutrons and that the delayed neutron precursors have a single effective decay constant μ. Under these conditions, the latent power C associated with the delayed neutrons is given by (see reference 1)

$$dC/dt + \mu C = \beta k_e P/\ell \qquad \text{(B3)}$$

where

P = instantaneous reactor power

ℓ = thermal lifetime of neutrons

Fourthly, we shall assume that the temperature of the moderator at the core inlet is constant. In this case, we may write

$$S \frac{dT}{dt} + S' (T - T_o) = P - P_o \qquad \text{(B4)}$$

where

S = heat capacity of core

S' = Wc_p/b, where W is the mass rate of flow, c_p is the specific heat of moderator, and b is the constant which defines the mean core temperature in the equation $T = T_{in} + b (T_{out} - T_{in})$

T_o = initial mean core temperature

P_o = initial power

Finally, the instantaneous power P is related to the other variables by means of the equation (see reference 1)

$$dP/dt = \left[\frac{k_e (1 - \beta) - 1}{\ell} \right] P + \mu C \qquad \text{(B5)}$$

The initial conditions for equations (B3), (B4), and (B5) are:

$$\text{at } t = 0 \qquad P = P_o \qquad k_e = 1$$

$$T = T_o$$

$$\frac{dC}{dt} = 0$$

The initial conditions imply that the initial delayed neutron power C_o is

$$C_o = \beta P_o/\mu \ell \tag{B6}$$

The differential equations, as written above, do not have constant coefficients. We cannot obtain an analytical solution in closed form. We shall therefore assume that the changes in power and reactivity are small. In these circumstances, we may write

$$P = P_o + \Delta P \qquad k_e = 1 + \Delta k_e$$

and we shall then neglect terms of second order. The following mathematical system is the consequence:

$$\frac{dP}{dt} = \alpha_1 - \alpha_1 \cos 2\omega t + \alpha_3 \tau - \gamma P + \mu C \tag{B7}$$

$$\frac{dC}{dt} + \mu C = \gamma P + \alpha_2 - \alpha_2 \cos 2\omega t + \alpha_4 \tau \tag{B8}$$

$$S \frac{d\tau}{dt} + S'\tau = P - P_o \tag{B9}$$

where

$$\alpha_1 = \left(\frac{A^2}{4}\right)\left(\partial^2 k_e/\partial h^2\right)\frac{(1 - \beta)\, P_o}{\ell}$$

$$\alpha_2 = \left(\frac{A^2}{4}\right)\left(\beta\, P_o/\ell\right)\left(\partial^2 k_e/\partial h^2\right)$$

$$\alpha_3 = \left(\partial k_e/\partial T\right)\frac{(1 - \beta)\, P_o}{\ell}$$

$$\alpha_4 = \left(\partial k_e/\partial T\right)\beta\, P_o/\ell$$

$$\gamma = \beta/\ell$$

$$\tau = T - T_o$$

Upon using the Laplace transform and solving for $\overline{P}$, we obtain

$$\overline{P} = \frac{\begin{vmatrix} P_o s\,(s^2 + 4\omega^2) + 4\,\alpha_1\,\omega^2 & -\,\mu & \alpha_3 \\ C_o s\,(s^2 + 4\omega^2) + 4\,\alpha_2\,\omega^2 & s + \mu & \alpha_4 \\ P_o\,(s^2 + 4\omega^2) & 0 & (Ss + S') \end{vmatrix}}{s(s^2 + 4\omega^2)\begin{vmatrix} s + \gamma & -\,\mu & \alpha_3 \\ -\,\gamma & s + \mu & \alpha_4 \\ 1 & 0 & (Ss + S') \end{vmatrix}} \qquad \text{(B10)}$$

where

s = Laplace transformation constant

The determinant in the denominator of the fraction on the right hand side of equation (18) is a third degree polynomial in s. Consequently, the solution for P is of the form

$$P = C_1' e^{C_1 t} + C_2' e^{C_2 t} + C_3' e^{C_3 t} + P_{av} + P'(\omega)\sin(2\omega t + \psi) \qquad \text{(B11)}$$

where the first three terms on the right correspond to the roots of the third degree polynomial, P_{av} results from the factor s in the denominator, and the term $P'(\omega) \sin (2\omega t + \psi)$ results from the factor $(s^2 + 4\omega^2)$. The angle ψ is a phase angle. The reactor is stable if the constants C_1, C_2, C_3, have negative real parts and if the amplitude $P'(\omega)$ is limited to reasonably small values.

We can show that the constants C_1, C_2, C_3 do not have positive real parts. The polynomial of third degree is $Ss^3 + s^2 \left[(\mu + \gamma) S + S' \right]$

$$+ \left[(\mu + \gamma) S' - \alpha_3 \right] s - \mu(\alpha_3 + \alpha_4)$$

The criterion that no root of this polynomial have a positive real part is (see reference 2)

$$\left[\mu + \gamma \frac{S'}{S} \right] \left[(\mu + \gamma) \frac{S'}{S} \quad \frac{\alpha_3}{S} \right] + \mu \left(\frac{\alpha_3}{S} + \frac{\alpha_4}{S} \right) > 0$$

Since this criterion is met for all reasonable values of the reactor parameters, the conclusion is that the reactor is stable insofar as the transient terms are concerned. The orders of magnitude of the reactor parameters are given below.

$$\mu = .1 \text{ sec}^{-1}$$

$$\beta = 2 \ (10^{-3})$$

$$\ell = 2 \ (10^{-4}) \text{ sec}$$

$$\gamma = 10 \text{ sec}^{-1}$$

$$S'/S = 10^{-2} \text{ sec}^{-1}$$

$$\alpha_3 = -2.5 \ (10^{7}) \ \frac{\text{watts}}{\text{deg C-sec}}$$

$$S = 10^{7} \text{ watt-sec/deg C}$$

$$P_o = 5 \ (10^{6}) \text{ watts}$$

$$\alpha_4 = -5 \ (10^{4}) \ \frac{\text{watts}}{\text{deg C-sec}}$$

$$\partial k_e / \partial T = -10^{-3} \text{ per deg C}$$

The constant term P_{av} is given by

$$P_{av} = P_o \left[1 - \frac{S'}{P_o} \, \frac{\frac{A^2}{4}\left(\frac{\partial^2 k_e}{\partial h^2}\right)}{(Dk_e/\partial T)} \right]$$

For a 2% variation in height, $(A^2/4)\ (\partial^2 k_e/\partial h^2) \rightarrow 3(10^{-5})$, so that, if $S' = 1.33\ (10^5)$ watts/deg C and $P_o = S\ (10^6)$ watts, then P_{av} differs from P_o by 0.15%.

Finally, let us consider the forced oscillation term, $P'(\omega)\sin(2\omega t + \psi)$. We have

$$P'(\omega)\sin(2\omega t + \psi) \doteq \frac{A^2}{4}\left(\frac{\partial^2 k_e}{\partial h^2}\right) \frac{P_o}{\ell} \, \frac{\left[16\,\omega^4\gamma^2 + 4\omega^2\left(4\omega^2 + \gamma\mu + \frac{\alpha_3}{S}\right)^2\right]^{1/2}}{4\omega^2\,\gamma^2 + (4\omega^2 + \alpha_3/S)^2} \sin(2\omega t + \psi)$$

where

$$\tan\psi = \frac{4\omega^2\,\gamma}{2\omega\left(4\omega^2 + \gamma\mu + \frac{\alpha_3}{S}\right)}$$

With the values of the reactor parameters as given above, the maximum value of the amplitude of the power oscillation occurs at a frequency of approximately 0.25 cycle/sec and has a magnitude of approximately 1.5% of the average power.

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gcb

List of Symbols

A = amplitude of bed height oscillation

B = buckling of fuel region

C = latent power associated with delayed neutrons

D_1 = thermal diffusion constant for fuel region

D_2 = thermal diffusion constant for pure moderator region

h = height of fluidized bed

H = height of core cylinder

k_e = effective multiplication constant

ℓ = thermal neutron lifetime

P = reactor power

P_o = initial reactor power

p = resonance escape probability

q = coefficient in exponential expression for resonance escape probability

R = radius of core cylinder

r = radial distance

S = total heat capacity of core

S' = mass rate of flow times specific heat of moderator

s = transformation constant

T = average core temperature

T_o = initial average core temperature

t = time

z = vertical distance

List of Symbols - contd.

$\alpha_1 \; = \; \left(\frac{A^2}{4}\right)\left(\partial^2 k_e/\partial h^2\right)\frac{(1-\beta)\,P_o}{\ell}$

$\alpha_2 \; = \; \left(\frac{A^2}{4}\right)\left(\partial^2 k_e/\partial h^2\right)\left(\beta\,P_o/\ell\right)$

$\alpha_3 \; = \; \left(\partial\, k_e/\partial T\right)\frac{(1-\beta)\,P_o}{\ell}$

$\alpha_4 \; = \; \left(\partial\, k_e/\partial T\right)\left(\beta\,P_o/\ell\right)$

β = partial differential equation separation constant or effective fraction of fission neutrons which are delayed

γ = β/ℓ

η = average number of fast neutrons emitted per thermal capture in fuel

μ = disintegration constant of delayed neutron precursors

ϕ = thermal neutron flux

ψ = phase angle

Σ^{23} = macroscopic absorption cross-section of U^{233}

Σ^{02} = macroscopic absorption cross-section of Th^{232}

$\Sigma_1{}^m$ = macroscopic absorption cross-section of moderator in fuel region

$\Sigma_2{}^m$ = macroscopic absorption cross-section of pure moderator

$\Sigma^p = 0.07\,\Sigma^{23}$ = macroscopic absorption cross-section of fission product poisons

τ = Fermi age or $T - T_o$

ω = partial differential equation separation constant or angular frequency of bed height oscillation

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