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SUBJECT: Survey of the Static Nuclear Characteristics of Small One-Region Slurry Reactors: Part III

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FROM: B. E. Prince and M. P. Lietzke

ABSTRACT

A summary is given of computed criticality parameters for U^{235} fueled, spherical ThO_2-H_2O-D_2O reactors smaller than 3 ft in diameter and moderated with H_2O-D_2O mixtures above 20% in H_2O. Thorium concentrations were varied between 200 and 1000 g/liter, and the temperature range was 20°C to 200°C. The parameters calculated were critical uranium-to-thorium ratios and the reactivity coefficients of temperature, void, and slurry concentration. As a typical example, a 2.5-ft reactor, moderated with an 80% D_2O-20% H_2O mixture, has a minimum critical mass ratio and a maximum critical temperature at about 500 g Th/liter. These values were about 0.08 g U^{235}/g Th and 200°C, respectively.

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INTRODUCTION

Studies have previously been reported on the dependence of the criticality parameters of single-region slurry reactors upon various design variables.[1] These variables included operating temperature, size, slurry concentration, moderator, and fuel. The present memorandum is an extension of these results to U^{235}-fueled, thorium-oxide slurry reactors smaller than 3 ft in diameter with moderator mixtures of light and heavy water. An appendix is included which describes the IBM-704 routine used in the computations.

SUMMARY

The present memorandum presents results for U^{235}-fueled, spherical ThO_2-H_2O-D_2O reactors with reactor vessel diameters of 2.5, 2.0, and 1.5 ft and moderator mixtures of 100, 50, and 20 mol-percent H_2O. The thorium concentration was varied between 200 and 1000 g/liter, and the temperature between 20°C and 200°C. Characteristics of the reactors studied were:

(a) For moderator mixtures of up to 50 mol-percent D_2O in H_2O, the minimum critical mass ratio occurs in the vicinity of 1000 g Th/liter or greater. At larger percentages of D_2O, the minimum critical mass ratio increases and occurs at smaller thorium concentrations, e.g., 400 to 600 g/liter at 80% D_2O.

(b) For reactors moderated with 80% D_2O - 20% H_2O, maximum critical temperatures occur between 400 and 600 g/liter, for a given uranium-to-thorium ratio.

RESULTS

The Oracle one-region reactor survey program used in previous studies was revised for the IBM-704 and extended to compute reactivity coefficients. Reactivity changes for the following conditions were considered:

(a) Temperature increases in which the moderator expands uniformly and the mass ratios of uranium and thorium to moderator remain constant throughout the reactor.

(b) Moderator density is changed due to formation of voids; e.g., from radiolytic gas or bubbles due to boiling. Here also, the density change was assumed uniform throughout the reactor.

(c) Uniform small changes in thorium concentration, made at constant temperature and constant ratio of uranium to thorium.

In the present study, reactor vessel diameters of 2.5, 2.0, and 1.5 ft and moderator mixtures of 0, 50, and 80 mol per cent D_2O were considered. The reactor temperature was varied between 20 and 200°C, and the thorium concentration between 200 and 1000 g per liter.

Figures 1a, b, c give the critical mass ratios in the above reactors as functions of thorium concentration. In these calculations, a diameter increment of 4 inches (reflector savings of 2 in.) was assumed to account for the presence of the pressure vessel. For example, Fig. 1a shows the characteristics of a 34-in. diameter bare sphere, or approximately a 2.5-ft ID vessel-enclosed reactor.

Figures 2a, b, c are plots of the critical temperatures in these systems as functions of thorium concentration. Each temperature curve is for a fixed ratio of U^{235} to thorium.

Tables 1a, b, c, list the values obtained for the temperature and void coefficients of reactivity in the above reactors; the values given are for the extremes of the thorium concentration range studied. For thorium concentrations between those listed, the coefficients decreased nearly linearly with increasing concentration.

The slurry concentration coefficients of reactivity are plotted in Figs. 3a, b, c. This coefficient is the change in reactivity following a unit change in thorium concentration from the just-critical condition.

DISCUSSION

As an example of the use of the reactivity coefficients in evaluating a particular reactor system, Fig. 1a shows that a 2.5-ft reactor, moderated with a mixture of 80% D_2O and 20% H_2O, is just critical at 200°C, 400 g Th/liter and about 0.08 g U^{235}/g Th. If a change in concentration is made which is equivalent to a uniform increase of 20 g Th/liter throughout the reactor, Fig. 3a shows that the reactivity added is:

$$\Delta k_e \simeq + (0.14 \times 10^{-3}) (20) \simeq + 0.3\%$$

From Table 1a, the temperature coefficient for the above reactor is about -1.1×10^{-3}/°C. Then, if the addition of 0.3% Δk_e were made slowly, in equilibrium with the temperature rise, the approximate rise would be about 3°C. Also, from Table 1a, the uniform void coefficient for the reactor is about $- 0.75\ \Delta k_e$/void vol/core vol. The amount of voids which would compensate for 0.3% Δk_e would therefore be about 0.4% of the core volume.

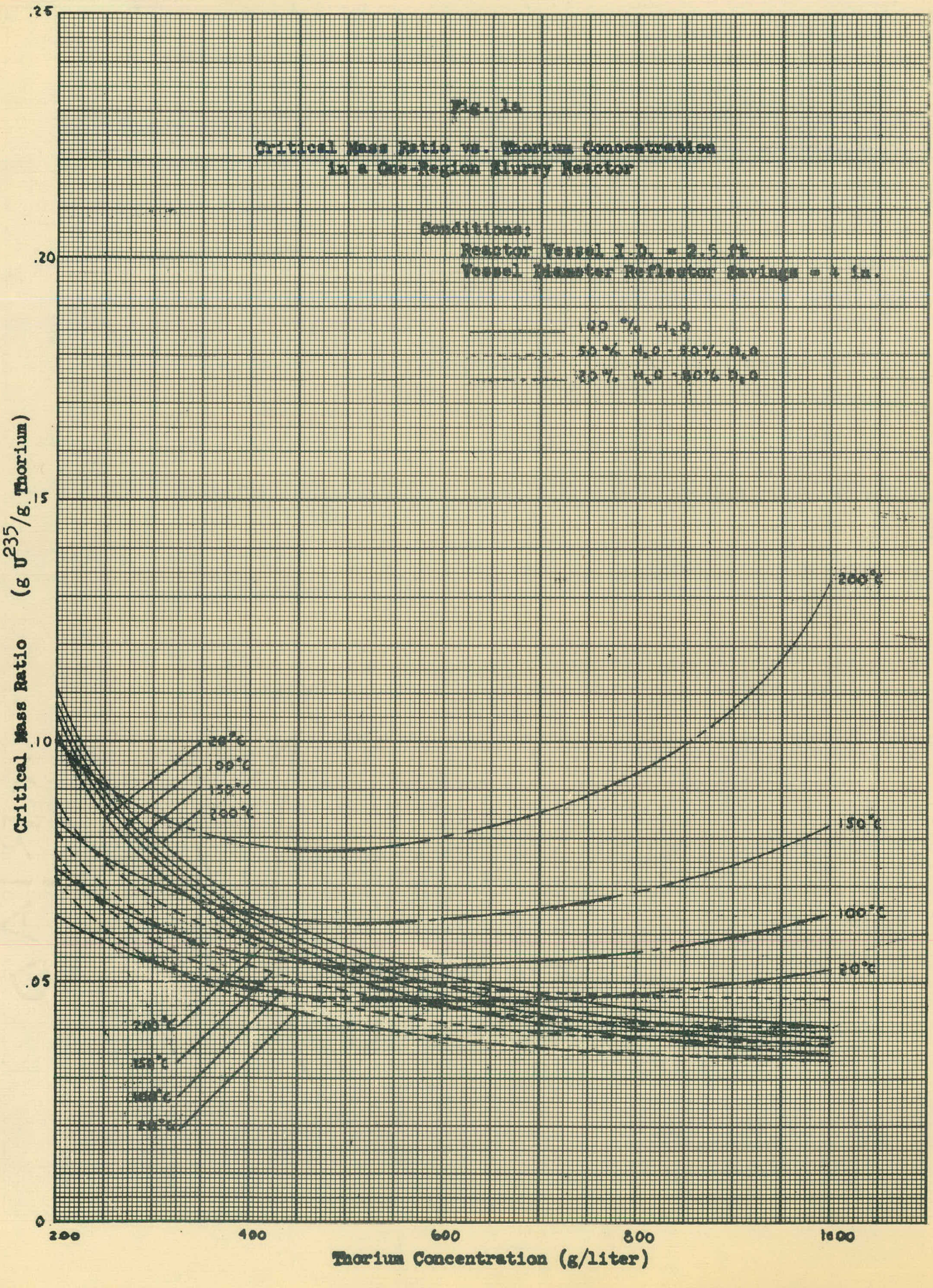
Fig. 1a
Critical Mass Ratio vs. Thorium Concentration
in a One-Region Slurry Reactor
Conditions:
Reactor Vessel I.D. = 2.5 ft
Vessel Diameter Reflector Savings = 4 in.
100 % H_2O
50 % H_2O - 50 % D_2O
20 % H_2O - 80 % D_2O
Critical Mass Ratio (g U^{235}/g Thorium)
Thorium Concentration (g/liter)
.25
.20
.15
.10
.05
0
200
400
600
800
1000
200°C
150°C
100°C
20°C
20°C
100°C
150°C
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150°C
100°C
20°C

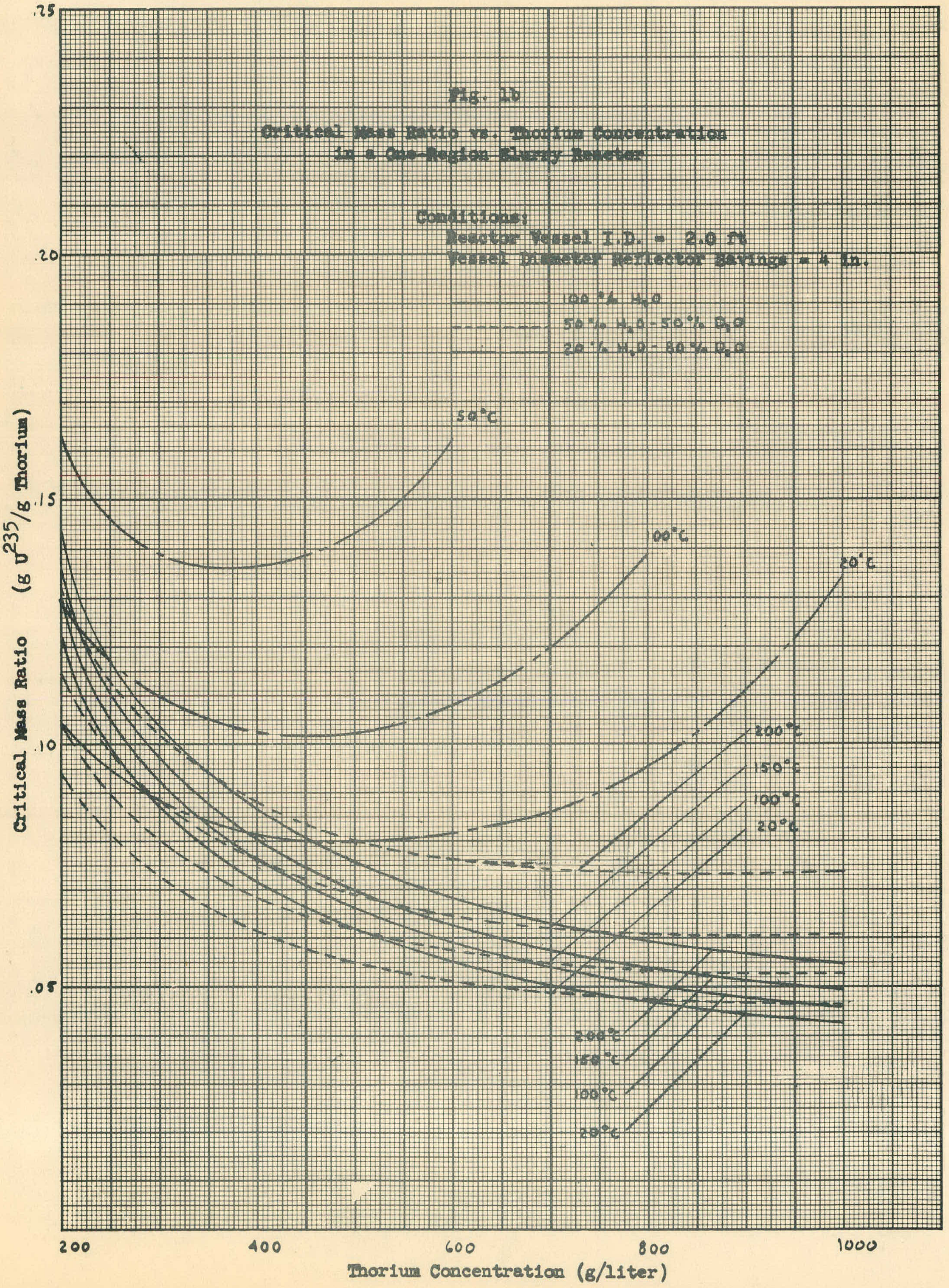
Fig. 1b
Critical Mass Ratio vs. Thorium Concentration
in a One-Region Slurry Reactor
Conditions:
Reactor Vessel I.D. = 2.0 ft
Vessel Diameter Reflector Savings = 4 in.
100 % H2O
50 % H2O - 50 % D2O
20 % H2O - 80 % D2O
50°C
100°C
20°C
200°C
150°C
100°C
20°C
200°C
150°C
100°C
20°C
.25
.20
.15
.10
.05
Critical Mass Ratio (g U235/g Thorium)
200
400
600
800
1000
Thorium Concentration (g/liter)

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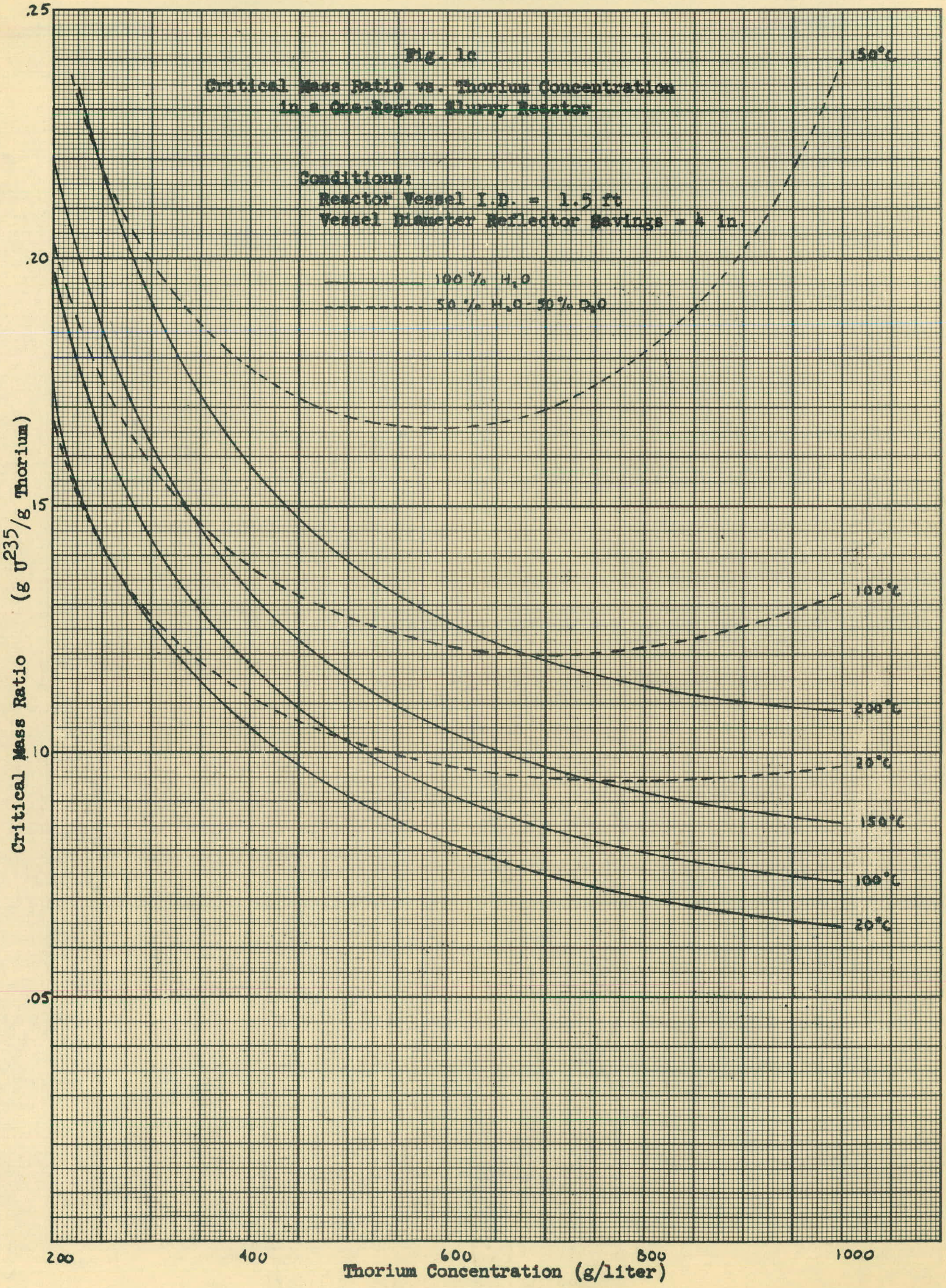

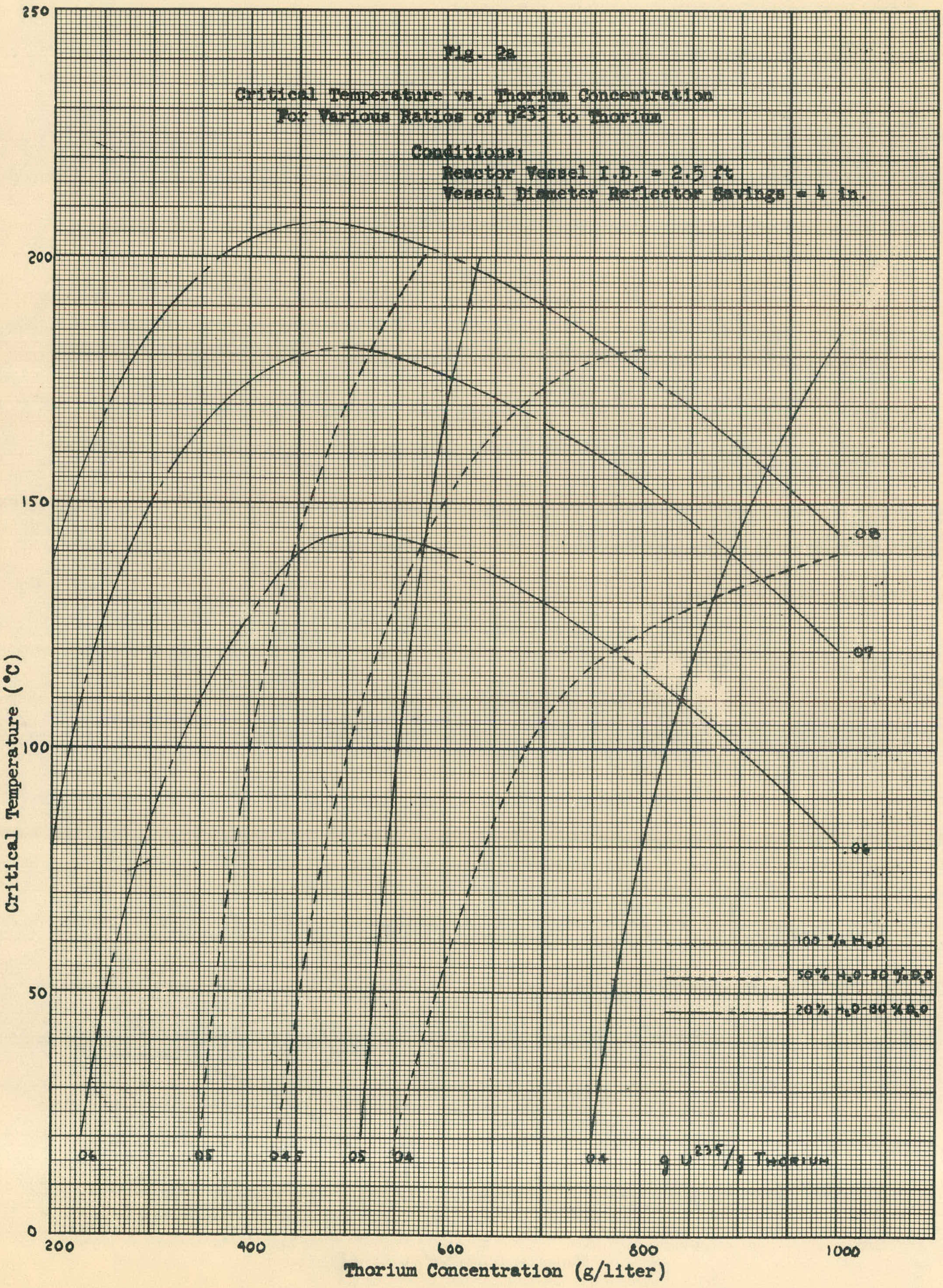
Fig. 2a
Critical Temperature vs. Thorium Concentration
For Various Ratios of U235 to Thorium
Conditions:
Reactor Vessel I.D. = 2.5 ft
Vessel Diameter Reflector Savings = 4 in.
250
200
150
100
50
0
200
400
600
800
1000
Critical Temperature (°C)
Thorium Concentration (g/liter)
.08
.07
.06
.06
.05
.045
.05
.04
.04
g U235/g Thorium
100 % H2O
50 % H2O-50 % D2O
20 % H2O-80 % D2O

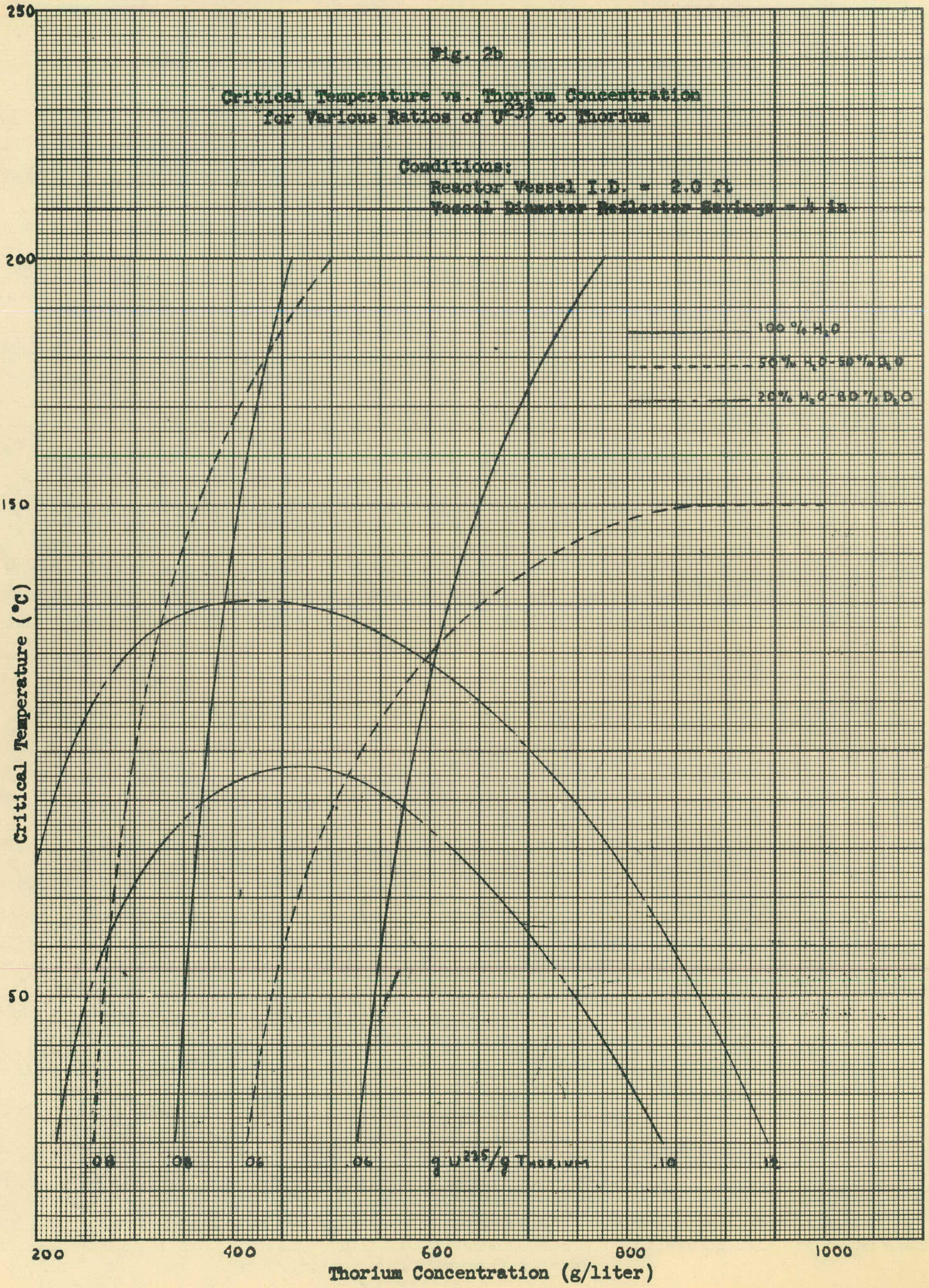
Fig. 2b
Critical Temperature vs. Thorium Concentration
for Various Ratios of U235 to Thorium
Conditions:
Reactor Vessel I.D. = 2.0 ft
Vessel Diameter Reflector Savings = 4 in.
100 % H2O
50 % H2O-50 % D2O
20% H2O-80 % D2O
250
200
150
50
Critical Temperature (°C)
.08
.08
.06
.06
g U235/g Thorium
.10
.12
200
400
600
800
1000
Thorium Concentration (g/liter)

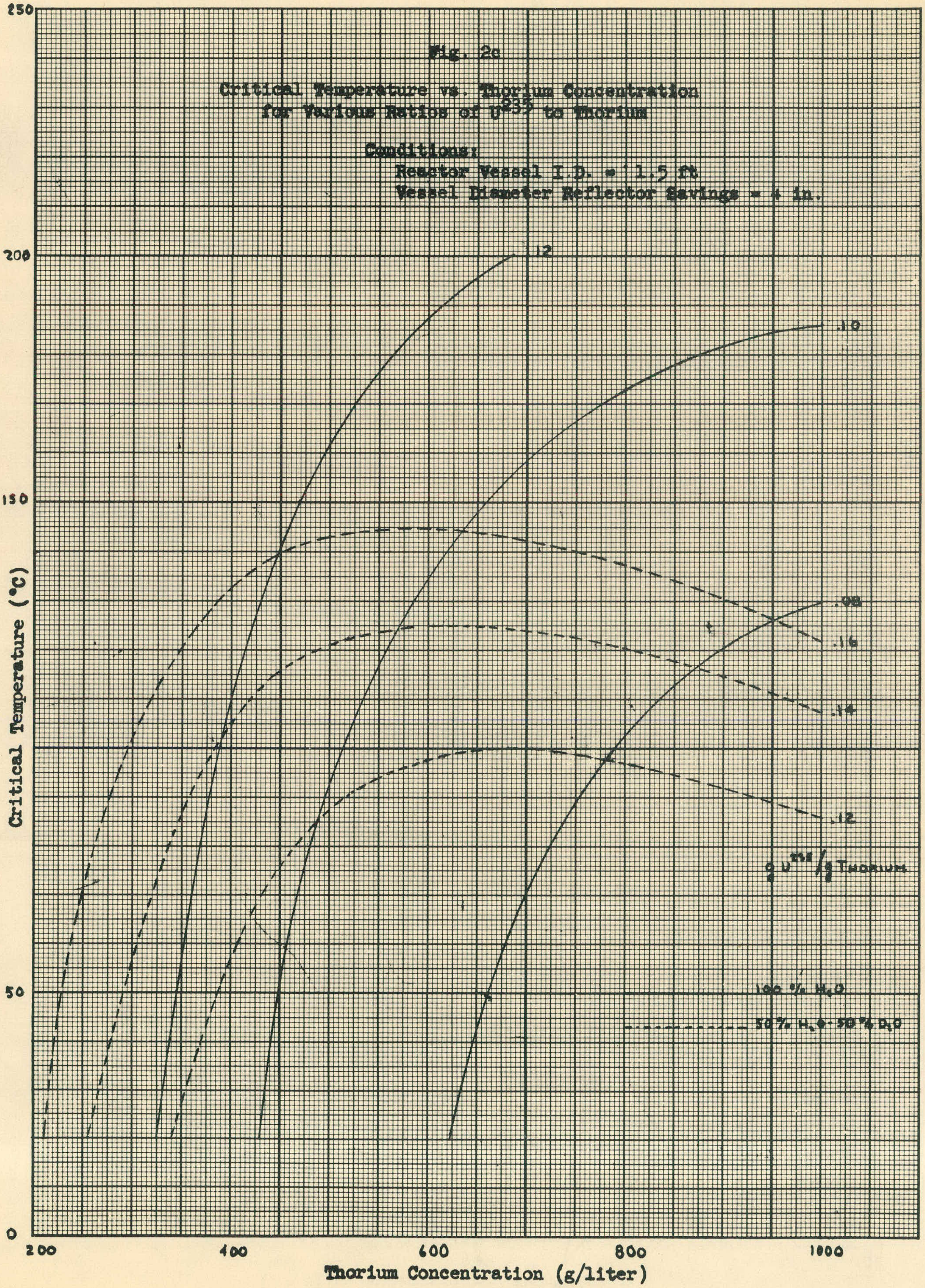
Fig. 2c
Critical Temperature vs. Thorium Concentration
for Various Ratios of U235 to Thorium
Conditions:
Reactor Vessel I.D. = 1.5 ft
Vessel Diameter Reflector Savings = 4 in.
Critical Temperature (°C)
Thorium Concentration (g/liter)
250
200
150
50
0
200
400
600
800
1000
.12
.10
.08
.16
.14
.12
g U235/g THORIUM
100 % H2O
50 % H2O - 50 % D2O

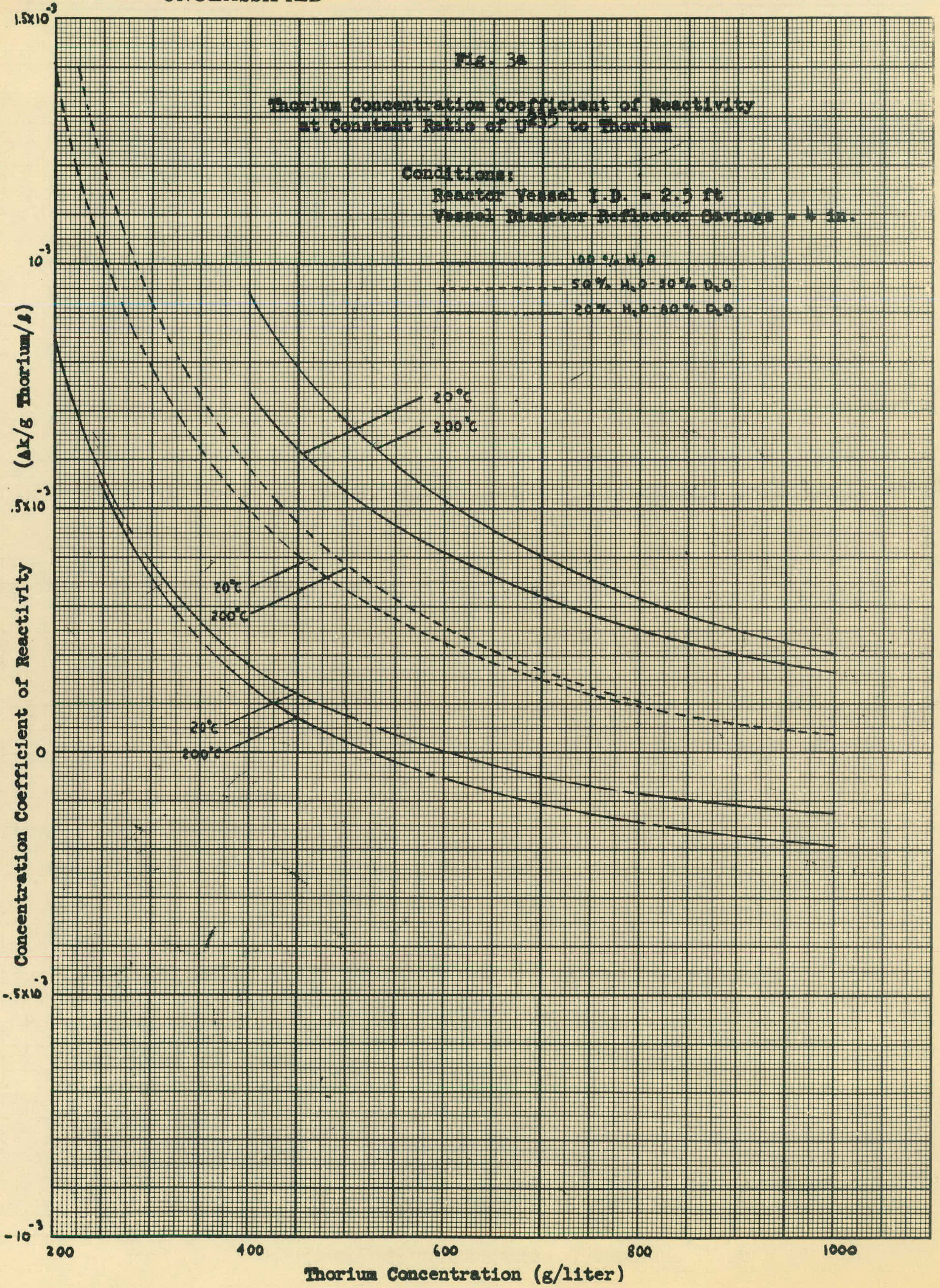
Fig. 3a
Thorium Concentration Coefficient of Reactivity at Constant Ratio of U235 to Thorium
Conditions:
Reactor Vessel I.D. = 2.5 ft
Vessel Diameter Reflector Savings = 4 in.
100 % H2O
50% H2O-50% D2O
20% H2O-80% D2O
20°C
200°C
20°C
200°C
20°C
200°C
1.5x10-3
10-3
.5x10-3
0
-.5x10-3
-10-3
Concentration Coefficient of Reactivity (Δk/g Thorium/l)
200
400
600
800
1000
Thorium Concentration (g/liter)

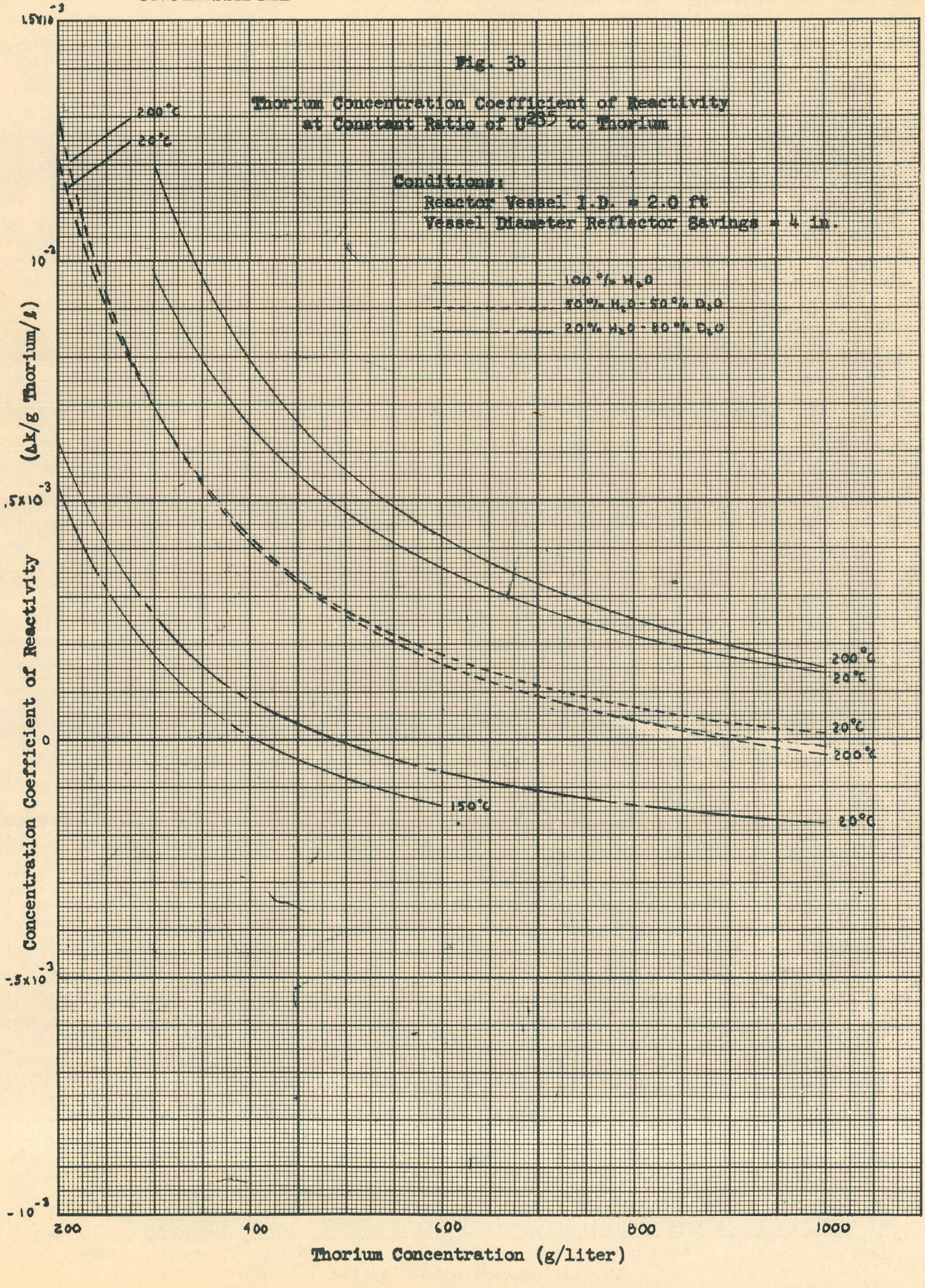
Fig. 3b
Thorium Concentration Coefficient of Reactivity
at Constant Ratio of U235 to Thorium
Conditions:
Reactor Vessel I.D. = 2.0 ft
Vessel Diameter Reflector Savings = 4 in.
100 % H2O
50 % H2O - 50 % D2O
20 % H2O - 80 % D2O
200 °C
20 °C
150 °C
1.5x10-3
10-3
.5x10-3
0
-.5x10-3
-10-3
200
400
600
800
1000
Concentration Coefficient of Reactivity (Δk/g Thorium/l)
Thorium Concentration (g/liter)

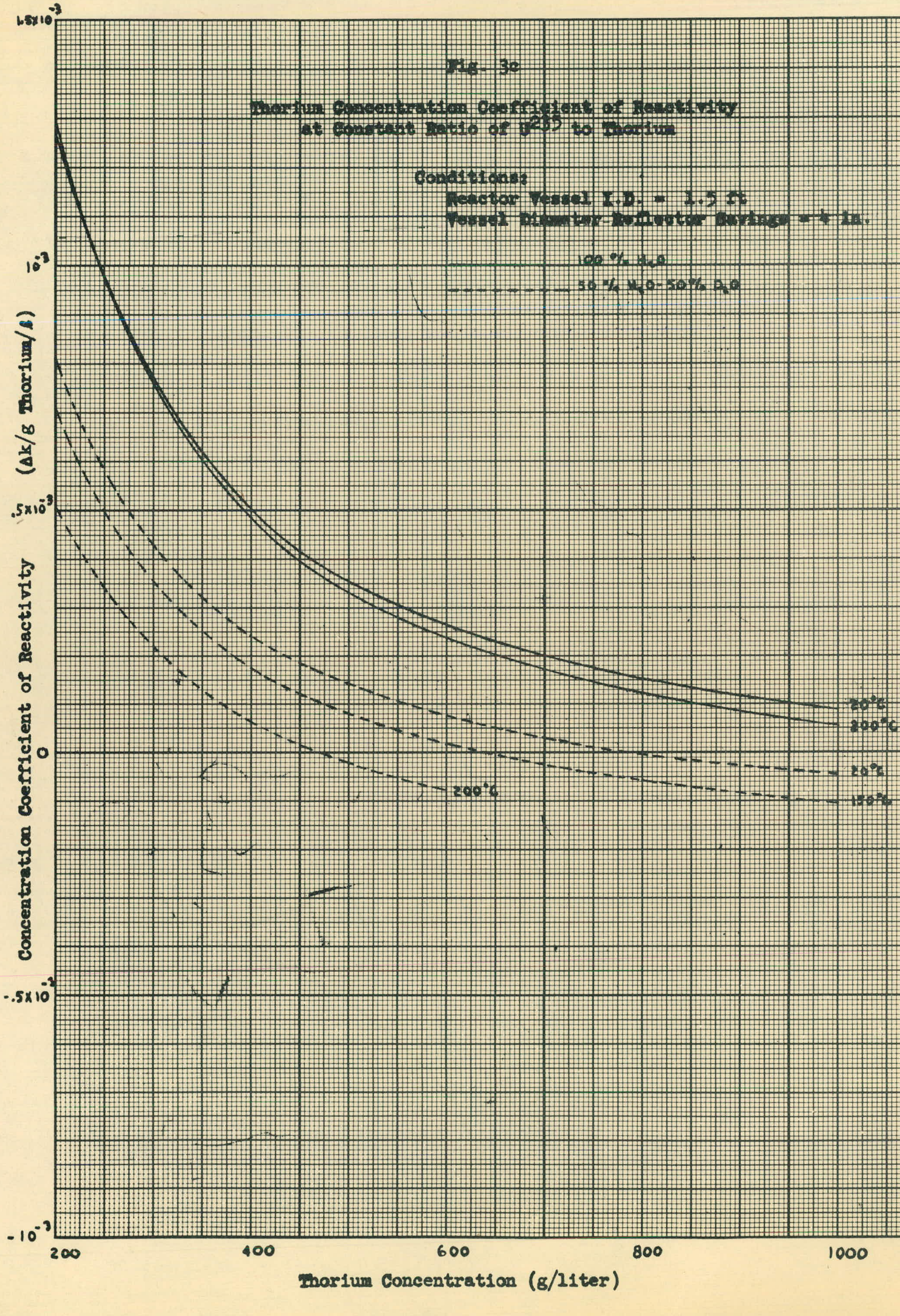
Fig. 3c
Thorium Concentration Coefficient of Reactivity at Constant Ratio of U235 to Thorium
Conditions:
Reactor Vessel I.D. = 1.5 ft
Vessel Diameter-Reflector Savings = 4 in.
100 % H2O
50 % H2O-50% D2O
1.5x10-3
10-3
.5x10-3
0
-.5x10-3
-10-3
Concentration Coefficient of Reactivity (Δk/g Thorium/l)
20°C
200°C
20°C
150°C
200°C
200
400
600
800
1000
Thorium Concentration (g/liter)

Some of the more important characteristics of the reactors studied may be summarized as follows:

(a) For moderator mixtures of up to 50 mol percent D_2O in H_2O, the minimum critical mass ratio occurs in the vicinity of 1000 g Th/liter or greater. At larger percents of D_2O, the minimum mass ratio increases and occurs at smaller thorium concentrations, e.g., 400 to 600 g/liter at 80% D_2O.

(b) In the range of reactor variables studied, approximate lower limits of thorium concentration for criticality at uranium-to-thorium ratios less than 0.10 are given in Table 2. The values given are summarized from Figs. 1a, b.

Table 1a. Temperature and Uniform Void Coefficients of Reactivity in a 2.5-ft Diameter Reactor

Temperature (°C)	Mol % D_2O	Temperature Coefficient (Δk/°C) x 10^{+3}		Uniform Void Coefficient (Δk/void vol./core vol.)	
		200 g Th/ℓ	1000 g Th/ℓ	200 g Th/ℓ	1000 g Th/ℓ
20	0	- 0.15	- 0.13	- 0.33	- 0.31
	50	- 0.26	- 0.18	- 0.44	- 0.40
	80	- 0.43	- 0.23	- 0.64	- 0.53
100	0	- 0.32	- 0.29	- 0.36	- 0.34
	50	- 0.48	- 0.38	- 0.48	- 0.43
	80	- 0.73	- 0.47	- 0.69	- 0.56
150	0	- 0.47	- 0.43	- 0.39	- 0.37
	50	- 0.68	- 0.55	- 0.53	- 0.46
	80	- 0.97	- 0.67	- 0.74	- 0.59
200	0	- 0.65	- 0.59	- 0.43	- 0.41
	50	- 0.91	- 0.77	- 0.58	- 0.52
	80	- 1.25	- 0.88	- 0.79	- 0.63

Table 1b. Temperature and Uniform Void Coefficients of Reactivity in a 2.0-ft Diameter Reactor

Temperature (°C)	Mol % D_2O	Temperature Coefficient ($\Delta k/°C$) x 10^{+3}		Uniform Void Coefficient (Δk/void vol./core vol.)	
		200 g Th/ℓ	1000 g Th/ℓ	200 g Th/ℓ	1000 g Th/ℓ
20	0	- 0.21	- 0.18	- 0.45	- 0.43
	50	- 0.33	- 0.24	- 0.58	- 0.53
	80	- 0.47	- 0.25	- 0.80	- 0.67
100	0	- 0.44	- 0.40	- 0.49	- 0.46
	50	- 0.62	- 0.49	- 0.63	- 0.57
	80	- 0.82	- 0.55	- 0.84	- 0.70
150	0	- 0.63	- 0.58	- 0.53	- 0.50
	50	- 0.86	- 0.70	- 0.68	- 0.61
	80	- 1.08	-	- 0.88	-
200	0	- 0.86	- 0.79	- 0.58	- 0.55
	50	- 1.14	- 0.97	- 0.74	- 0.67
	80	- 1.36	-	- 0.91	-

Table 1c. Temperature and Uniform Void Coefficients of Reactivity in a 1.5-ft Diameter Reactor

Temperature (°C)	Mol % D_2O	Temperature Coefficient ($\Delta k/°C$) x 10^{+3}		Uniform Void Coefficient (Δk/void vol./core vol.)	
		200 g Th/ℓ	1000 g Th/ℓ	200 g Th/ℓ	1000 g Th/ℓ
20	0	- 0.29	- 0.25	- 0.64	- 0.61
	50	- 0.40	- 0.29	- 0.79	- 0.73
100	0	- 0.60	- 0.55	- 0.68	- 0.65
	50	- 0.77	- 0.62	- 0.83	- 0.76
150	0	- 0.85	- 0.79	- 0.73	- 0.70
	50	- 1.04	- 0.87	- 0.88	- 0.80
200	0	- 1.14	- 1.06	- 0.78	- 0.75
	50	- 1.34	-	- 0.92	-

Table 2. Limiting Thorium Concentrations for Critical Mass Ratios Less than 0.10*

Reactor Diameter (ft)	2.5			2.0		
Moderator % D_2O	0%	50%	80%	0%	50%	80%
Minimum Thorium Concentration for Criticality at 200°C (g/liter)	225	< 200	200	320	310	No Critical Reactor
Minimum Thorium Concentration for Criticality at 150°C (g/liter)	215	< 200	< 200	290	250	No Critical Reactor
Minimum Thorium Concentration for Criticality at 100°C (g/liter)	210	< 200	< 200	270	210	No Critical Reactor

* The 1.5-ft diameter systems are not included since most of the reactors are subcritical in the tabulated regions of interest.

APPENDIX

A. Criticality Parameters.

The criticality model used in the reactor survey program has been described in reference 1. A summary of the program equations is given below. Although reference is made to the uranium-thorium oxide system, the calculation routine can be applied if the fertile material is U^{238}.

The basic equation for the effective multiplication constant is:*

$$k_e = \eta_T \epsilon \frac{p_o \, p_1 \, f \, g}{1 + L^2 B^2} \qquad (1)$$

where

$$f = \frac{\Sigma_1}{\Sigma_1 (1 + x) + \Sigma_o + \Sigma_m} \qquad (1a)$$

$$\epsilon = 1 + \left(\frac{\bar{\eta}_R}{\eta_T} \right) \frac{(1 - p_1)}{p_1} \frac{(1 + L^2 B^2)}{f} \qquad (1b)$$

The definitions of the symbols are:

k_e = Reactor effective multiplication constant

η_T = Neutrons produced per neutron absorbed at thermal energies in fuel

$\left(\frac{\bar{\eta}_R}{\eta_T} \right)$ = Average ratio of η for neutron absorption at resonance energies in fuel to thermal η

f = Thermal utilization

p = Resonance escape probability; subscripts 0 = fertile material, 1 = fuel

g = Nonleakage probability for neutrons slowing down to thermal energies

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* Equation 1 differs slightly from that given in reference 1. In the latter, an average value for p_o was used in the calculation of the resonance effect (ϵ).

P = Thermal nonleakage probability = $\frac{1}{1 + L^2 B^2}$

L^2 = Thermal diffusion length in slurry (cm^2)

B^2 = Effective reactor geometric buckling (cm^{-2})

ϵ = Resonance effect (neutrons produced from absorption in fuel at all energies, per neutron produced from thermal absorption in fuel)

Σ = Macroscopic absorption cross section in slurry (cm^{-1}); subscripts 0 = fertile material, 1 = fuel, m = moderator

x = Equilibrium poison fraction (fraction of thermal absolute cross section of fuel which is thermal neutron poisons)

The critical mass ratio, which is proportional to Σ_1/Σ_o, is obtained by iterative solution of equation 1, with k_e equal to unity.

In equation 1, resonance escape probabilities (p) were computed from the formulas given below. For fertile material an empirical formula was used for the effective resonance integral (R):

$$p_o = \exp\left(- \frac{N_o R_o}{\xi \Sigma_s}\right) \qquad (1c)$$

$$R_o = c_1 \left(\frac{\Sigma_s}{N_o}\right)^{c_2} \qquad 0 \leq \frac{\Sigma_s}{N_o} \leq c_3$$

$$R_o = R_o(\infty) \qquad \frac{\Sigma_s}{N_o} > c_3$$

In the present study, the values used for thorium were:[2]

$c_1 = 8.33$ $\qquad c_2 = 0.253$ $\qquad c_3 = 4000$ barns $\qquad R_o(\infty) = 69.8$ barns

For fuel, the following theoretical relation for R_1 was used:*

$$p_1 = e^{\frac{-N_1}{\xi \Sigma_s} R_1} \qquad \text{(1d)}$$

$$R_1 = 2R_1(\infty)\left[\sqrt{(1+\beta)\beta} - 1\right] \qquad 0 \leq \frac{\Sigma_s}{N_o} \leq \infty$$

$$\beta = \frac{0.1\pi}{R_1(\infty)} \frac{\Sigma_s}{N_1}$$

In the above formulas:

N = Atoms per barn-cm in slurry; subscripts o = fertile material, 1 = fuel

$\xi \Sigma_s$ = Slurry macroscopic slowing down power in resonance energy region (cm^{-1})

Σ_s = Slurry macroscopic scattering cross section in resonance energy region (cm^{-1})

The effective geometric buckling (B^2) for a spherical vessel-enclosed reactor was approximated by:

$$B^2 = \left(\frac{\pi}{R + d}\right)^2 \qquad \text{(1e)}$$

where R is the inside radius of the vessel and d is an effective radial reflector savings for the vessel.

The functional relation between the fast nonleakage probability (g) and the neutron age (τ) was approximated by:

$$g = \frac{\exp(-B^2 \gamma_1 \tau)}{(1 + B^2\gamma_2\tau)(1 + B^2\gamma_3\tau)(1 + B^2\gamma_4\tau)} \qquad \text{(1f)}$$

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* Due to L. Dresner (unpublished)

where $\gamma_{1,..4}$ were assumed dependent only on the moderator D_2O-H_2O ratio. Based on the experiments of Friedman and Wattenberg, the following γ values can be used for 99.8% D_2O:*

$$\gamma_1 = 0.6 \qquad \gamma_2 = 0.4 \qquad \gamma_3 = \gamma_4 = 0$$

In the present study, the fast nonleakage was approximated by a single "Yukawa" kernel (corresponding to the ordinary two-group approximation):

$$\gamma_1 = \gamma_2 = \gamma_3 = 0 \; ; \quad \gamma_4 = 1.0$$

For application to mixtures of H_2O and D_2O, Tobias and Fowler[3] have recently correlated the experimental data of Wade[4] by an expression of the above form with $\gamma_3 = \gamma_4 = 0$. Comparison of their results with the single "Yukawa" approximation was found to be fair, except for moderator mixtures greater than 80% D_2O, and spheres smaller than about 2 ft in diameter.

B. Reactivity Coefficients

The reactivity coefficients are the logarithmetic partial derivatives of k_e with respect to the quantity producing the reactivity change. Since the differentiation of equation 1 is mathematically straightforward, only the resulting formulas for the coefficients are given below:

Temperature Coefficient

$$\frac{1}{k_e}\frac{\partial k_e}{\partial T} \simeq \frac{1}{g}\frac{\partial g}{\partial T} + \frac{1}{P}\frac{\partial P}{\partial T} + \frac{1}{\epsilon}\frac{\partial \epsilon}{\partial T} \qquad (2)$$

Fast leakage:

$$\frac{1}{g}\frac{\partial g}{\partial T} = + 2\,\delta_1 \left\{ B^2 \gamma_1 \tau + \frac{B^2 \gamma_2 \tau}{1 + B^2 \gamma_2 \tau} + \frac{B^2 \gamma_3 \tau}{1 + B^2 \gamma_3 \tau} + \frac{B^2 \gamma_4 \tau}{1 + B^2 \gamma_4 \tau} \right\} \qquad (2a)$$

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* See reference 1 for discussion.

where:

$$\delta_1(T) = \left(\frac{1}{\rho} \frac{\partial \rho}{\partial T} \right)_{\text{moderator}}$$

Thermal leakage:

$$\frac{1}{P} \frac{\partial P}{\partial T} = - \frac{L^2 B^2}{1 + L^2 B^2} \left(\frac{1}{L^2} \frac{\partial L^2}{\partial T} \right)$$

$$\frac{1}{L^2} \frac{\partial L^2}{\partial T} = \frac{\left(y/D_{D_2} \right) \delta_2 + \left((1 - y)/D_{H_2O} \right) \delta_3}{y/D_{D_2O} + (1-y)/D_{H_2O}} + \frac{1}{2T} - v_m \delta_1$$

where:

y = Mol fraction D_2O in moderator

T = °C + 273

v_m = volume moderator/volume slurry

$$\delta_2(T) = \left(\frac{1}{D} \frac{dD}{dT} \right)_{D_2O}$$

$$\delta_3(T) = \left(\frac{1}{D} \frac{dD}{dT} \right)_{H_2O}$$

Resonance Effect:

$$\frac{1}{\epsilon} \frac{\partial \epsilon}{\partial T} \simeq - \frac{\epsilon - 1}{\epsilon} \left(\frac{1}{P} \frac{\partial P}{\partial T} \right) \quad (2c)$$

Assumptions made in obtaining equation (2) are:

(a) The temperature coefficients of the thermal and resonance "etas" for the fuel and the coefficients of the effective resonance integrals for fertile and fuel material have been assumed negligible with respect to the leakage coefficients.

For most conditions of thorium concentration and temperatures of interest in slurry reactor design, this approximation will be adequate, due to the large density changes of the moderator with temperature.

(b) During a temperature increase, the moderator expands uniformly throughout the reactor; also, the mass ratios of uranium and thorium to moderator remain constant. Thermal expansion of the thorium oxide particles was neglected.

(c) The reflector savings (d) is assumed independent of temperature. This approximation is warranted only if d is small compared to the vessel diameter.

(d) Absorption cross sections vary inversely as the neutron velocity.

Uniform Void Coefficient

If voids are assumed to be formed homogeneously throughout the reactor moderator volume, by setting $\rho = \rho_o(1-v)$ and differentiating equation 1;

$$\frac{1}{k_e}\frac{\partial k_e}{\partial v} = \frac{1}{g}\frac{\partial g}{\partial v} + \frac{1}{P}\frac{\partial P}{\partial v} + \frac{1}{\epsilon}\frac{\partial \epsilon}{\partial v} \qquad (3)$$

where v = void volume/core volume and where each term on the right hand side of equation 3 is evaluated at v = 0.

Fast Leakage:

$$\frac{1}{g}\frac{\partial g}{\partial v} = -2\left\{+B^2\gamma_1\tau + \frac{B^2\gamma_2\tau}{1+B^2\gamma_2\tau} + \frac{B^2\gamma_3\tau}{1+B^2\gamma_3\tau} + \frac{B^2\gamma_4\tau}{1+B^2\gamma_4\tau}\right\} \qquad (3a)$$

Thermal Leakage:

$$\frac{1}{P}\frac{\partial P}{\partial v} = -\frac{L^2B^2}{1+L^2B^2}(1+v_m) \qquad (3b)$$

Resonance Effect:

$$\frac{1}{\epsilon}\frac{\partial \epsilon}{\partial v} = -\frac{\epsilon - 1}{\epsilon}\left(\frac{1}{P}\frac{\partial P}{\partial v}\right) \tag{3c}$$

Slurry Concentration Coefficient

The reactivity coefficient of slurry concentration is obtained by differentiating equation 1 with respect to the fertile material concentration (G_o), after imposing the conditions:

(a) $\frac{G_1}{G_o}$ = g fuel/g fertile = constant

(b) Temperature = constant

Thus:

$$\frac{1}{k_e}\frac{\partial k_e}{\partial G_o} = \frac{1}{f}\frac{\partial f}{\partial G_o} + \frac{1}{P}\frac{\partial P}{\partial G_o} + \frac{1}{p_o}\frac{\partial p_o}{\partial G_o} + \frac{1}{p_1}\frac{\partial p_1}{\partial G_o} + \frac{1}{\epsilon}\frac{\partial \epsilon}{\partial G_o} \tag{4}$$

Thermal Utilization:

$$\frac{1}{f}\frac{\partial f}{\partial G_o} = \frac{f}{G_o}\frac{y(\Sigma_a)_{D_2O} + (1-y)\ (\Sigma_a)_{H_2O}}{\Sigma_1} \tag{4a}$$

Thermal Leakage:

$$\frac{1}{P}\frac{\partial P}{\partial G_o} = -\frac{L^2 B^2}{1 + L^2 B^2}\left(\frac{1}{f}\frac{\partial f}{\partial G_o} - \frac{1}{G_o}\right) \tag{4b}$$

Resonance Escape Probability (fertile):

$$\frac{1}{p_o}\frac{\partial p_o}{\partial G_o} = -\frac{N_o R_o}{\xi \Sigma_s}\left\{\frac{1}{R_o}\frac{\partial R_o}{\partial G_o} + \frac{\xi \Sigma_s}{N_o}\frac{\partial}{\partial G_o}\left(\frac{N_o}{\xi \Sigma_s}\right)\right\} \tag{4c}$$

$$\frac{1}{R_o}\frac{\partial R_o}{\partial G_o} = -\frac{c_2}{G_o}\left\{\frac{y(\Sigma_s)_{D_2O} + (1-y)(\Sigma_s)_{H_2O}}{\Sigma_s}\right\} \qquad \left[0 \leq \frac{\Sigma_s}{N_o} \leq c_3\right]$$

$$= 0 \qquad \frac{\Sigma_s}{N_o} > c_3$$

$$\frac{\xi\Sigma_s}{N_o} \frac{\partial}{\partial G_o} \left(\frac{N_o}{\xi\Sigma_s} \right) = \frac{1}{G_o} \left\{ \frac{y(\xi\Sigma_s)_{D_2O} + (1-y)(\xi\Sigma_s)_{H_2O}}{\xi\Sigma_s} \right\}$$

Resonance Escape Probability (fuel):

$$\frac{1}{p_1} \frac{\partial p_1}{\partial G_o} = - \frac{N_1 R_1}{\xi\Sigma_s} \left\{ \frac{1}{R_1} \frac{\partial R_1}{\partial G_o} + \frac{\xi\Sigma_s}{N_1} \frac{\partial}{\partial G_o} \left(\frac{N_1}{\xi\Sigma_s} \right) \right\}$$

$$\frac{\xi\Sigma_s}{N_1} \frac{\partial}{\partial G_o} \left(\frac{N_1}{\xi\Sigma_s} \right) = \frac{\xi\Sigma_s}{N_o} \frac{\partial}{\partial G_o} \left(\frac{N_o}{\xi\Sigma_s} \right)$$

$$\frac{1}{R_1} \frac{\partial R_1}{\partial G_o} = - \frac{0.2\pi}{R_1 G_o^-} \left(\frac{1 + 2\beta}{2\sqrt{(1+\beta)\beta}} - 1 \right) \left(\frac{y(\Sigma_s)_{D_2O} + (1-y)(\Sigma_s)_{H_2O}}{N_1} \right) \tag{4d}$$

Resonance Effect:

$$\frac{1}{\epsilon} \frac{\partial \epsilon}{\partial G_o} = - \frac{\epsilon - 1}{\epsilon} \left\{ \frac{1}{1-p_1} \left(\frac{1}{p_1} \frac{\partial p_1}{\partial G_o} \right) + \frac{1}{f} \cdot \frac{\partial f}{\partial G_o} + \frac{1}{P} \frac{\partial P}{\partial G_o} \right\} \tag{4e}$$

Assumptions made in obtaining equation 4 are:

(a) The change in slurry concentration is uniform throughout the reactor.

(b) For small changes in concentration, changes in the fast leakage are small and may be neglected. This assumption is justified inasmuch as the neutron age (τ) is a slowly varying function of thorium concentration, in the concentration region of interest in slurry reactor design.

C. Nuclear Constants:

Table 3. Moderator Densities and Corresponding Temperature Coefficients at Various Reactor Temperatures[5]

Temperature (°C)	ρ (g/cm^3)		$\frac{1}{\rho}\frac{\partial\rho}{\partial T}$ (°C)$^{-1}$ x 10^{+3}
	H_2O	D_2O	(H_2O or D_2O)
20	1.000	1.105	- 0.330
100	0.962	1.062	- 0.769
150	0.916	1.015	- 1.070
200	0.864	0.958	- 1.366
250	0.798	0.880	- 1.905
280	0.749	0.828	- 2.296

Table 4. Thermal Diffusion Coefficients and Corresponding Temperature Coefficients for H_2O and D_2O

Temperature (°C)	D(cm)		$\frac{1}{D}\frac{\partial D}{\partial T}$ (°C)$^{-1}$ x 10^{+3}	
	H_2O (6)	D_2O (2)	H_2O	D_2O
20	0.153	0.828	1.827	1.087
100	0.178	0.905	2.019	1.282
150	0.198	0.971	2.150	1.473
200	0.221	1.048	2.446	1.803
250	0.252	1.160	3.079	2.345
280	0.276	1.245	3.379	2.570

In the present study, the demsity and thermal transport cross sections of thorium oxide (ThO_2) were assumed independent of temperature. The values used were:

$$\rho_{ox} = 9.7 \text{ g/cm}^3$$

$$(\sigma_{tr})_{ox} = 26.0 \text{ barns}$$

Table 5. Maxwell-Boltzman Averaged Thermal Absorption Cross Sections[2]

Temperature (°C)	σ_a(barns)	σ_1(barns)	$\Sigma_a(cm^{-1})$	
	Thorium	U^{235} ($\eta_T = 2.08$; $\nu = 2.46$)	H_2O	D_2O
20	6.60	595	0.0196	8.02×10^{-5}
100	5.85	515	0.0167	6.85×10^{-5}
150	5.50	479	0.0149	6.15×10^{-5}
200	5.20	449	0.0133	5.49×10^{-5}
250	4.94	430	0.0117	4.86×10^{-5}
280	4.81	411	0.0107	4.44×10^{-5}

Table 6. Microscopic Scattering Cross Section (σ_s) and Slowing Down Power ($\xi\sigma_s$) in Resonance Region for H_2O, D_2O, and ThO_2

Moderator	σ_s(barns)	$\xi\sigma_s$(barns
H_2O	43.8	40.5
D_2O	10.5	5.33
ThO_2	20.1	1.02

Table 7. Resonance Absorption Integrals for U^{235} at Infinite Dilution*

Temperature (°C)	$R_1(\infty)$ barns
20	696.5
100	627.5
150	594.6
200	570.3
250	542.1
280	528.9

Age of fission neutrons in mixtures of H_2O and D_2O were obtained from the experiments of Wade.[4] These values are summarized in Fig. 4. For slurry concentrations below 1000 g thorium per liter, the increment in age due to replacement of the moderator by ThO_2 is small compared with the total age.** In the present study, this increment was neglected and the age was assumed to be that in the pure moderator.

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* These values are obtained by numerical integration of the fission cross section for U^{235} given in BNL-325. The lower limit of the integral was assumed to be 5 kT. The absorption integral may be obtained from the average resonance-to-thermal "eta" ratio:

$$\frac{R_{f1}(\infty)}{R_{a1}(\infty)} = \left(\frac{\bar{\eta}_R}{\eta_T}\right)\frac{\eta_T}{\nu}$$

The values in Table 7 are based on an "eta" ratio of 0.9.

** Based on reference 2, and additional private communications with M. Tobias.

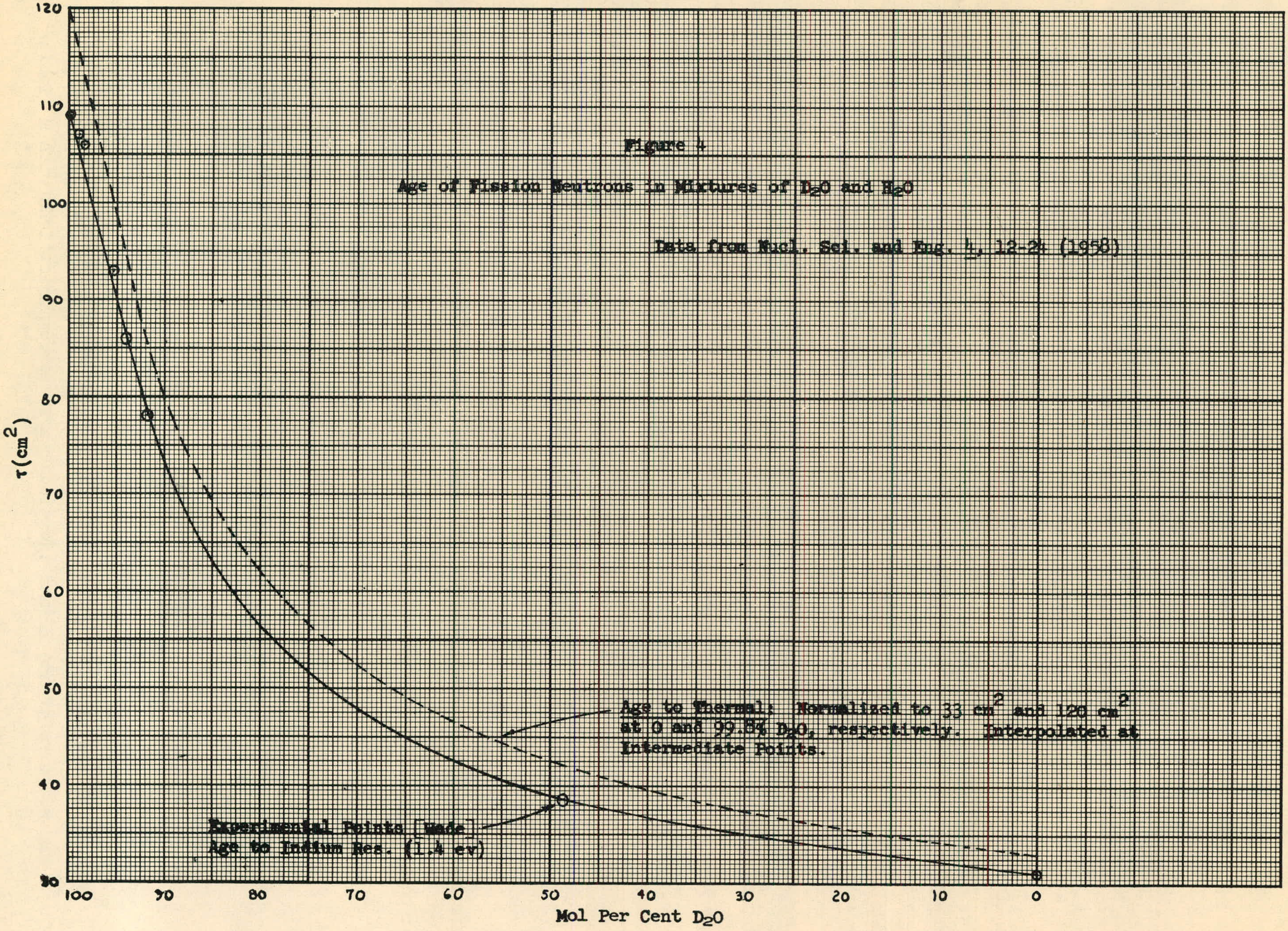

Figure 4

Age of Fission Neutrons in Mixtures of D_2O and H_2O

Data from Nucl. Sci. and Eng. 4, 12-24 (1958)

D. List of Machine Routine Input Parameters

	Symbol
Reactor Variables:	
Concentration of fertile material (g/liter)	G_o
Inside diameter of reactor vessel (ft)	ID
Radial reflector savings of reactor vessel (ft)	d
Temperature (^oC)	T
Equilibrium poison fraction	x
Mol fraction D_2O in moderator	y
Parameters Independent of Temperature (T)	
Atomic weight of fertile and fuel materials	A_o, A_1
Ratio of molecular weight of oxide of fertile material to atomic weight (A_o)	m
Microscopic slowing down power in resonance region (barns);	$(\xi\sigma_s)_{H_2O}$
subscripts H_2O, D_2O, and ox refer to H_2O, 99.8% D_2O, and	$(\xi\sigma_s)_{D_2O}$
oxide of fertile material (per fertile atom), respectively.	$(\xi\sigma_s)_{ox}$
Scattering cross section in resonance region (barns)	$(\sigma_s)_{H_2O}$
	$(\sigma_s)_{D_2O}$
	$(\sigma_s)_{ox}$
Parameters Dependent on Temperature (T)	
Thermal neutron diffusion coefficients for H_2O and 99.8% D_2O (cm)	D_{H_2O}, D_{D_2O}
Temperature coefficients of D for H_2O and 99.8% D_2O, respectively $(^\circ C)^{-1}$	δ_3, δ_2
Densities of H_2O, D_2O, and oxide of fertile material (g/cm^3)	ρ_{H_2O}, ρ_{D_2O}, ρ_{ox}
Temperature coefficient of density for H_2O or D_2O $(^\circ C)^{-1}$	δ_1
Average "eta" for thermal neutron absorption in fuel	η_T

	Symbol
Parameters Dependent on Temperature (T) - contd.	
Average ratio for "eta" for neutron absorption above thermal energies in fuel, to thermal "eta"	$\left(\overline{\eta_R/\eta_T} \right)$
Resonance absorption integrals at infinite dilution for fertile and fuel materials (barns)	$R_o(\infty)$, $R_1(\infty)$
Constants in emperical formula for fertile material resonance integral	c_1, c_2, c_3
Macroscopic absorption cross section for H_2O and 99.8% D_2O, respectively (cm^{-1})	$(\Sigma_a)_{H_2O}$, $(\Sigma_a)_{D_2O}$
Microscopic absorption cross sections for H_2O and 99.8% D_2O, respectively (barns)	σ_{ao}, σ_{a1}
Thermal transport cross section of oxide of fertile material, per fertile atom (barns)	$(\sigma_{tr})_{ox}$
Parameters dependent on mol-fraction D_2O in moderator (y):	
Normalized "Gaussian" (subscript 1) and "Yukawa" (subscripts 2, 3, 4) neutron ages	γ
Parameters dependent on T, y, and G_o:	
Neutron age to thermal energies (cm^2)	τ

References

1. B. E. Prince and M. W. Rosenthal, Survey of the Static Nuclear Characteristics of Small, One-Region Slurry Reactors (Part I), ORNL CF-58-7-76, July 28, 1958; (Part II), ORNL CF-58-11-98 (to be issued).

2. T. B. Fowler and M. Tobias, Two-Group Constants for Aqueous Homogeneous Reactor Calculations, ORNL CF-58-1-79, January 22, 1958.

3. M. Tobias, personal communication, October 25, 1958.

4. James W. Wade, "Neutron Age in Mixtures of D_2O and H_2O," Nuclear Science and Engineering 4, 12-24 (1958).

5. M. Tobias, Certain Physical Properties of Aqueous Homogeneous Reactor Materials, ORNL CF-56-11-135.

6. C. D. Petrie, et al, "Calculation of Thermal Group Constants for Mixtures Containing Hydrogen," Nuclear Science and Engineering 2, 728-744 (1957).

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