ANL-5873
Reactors - General
(TID-4500, 14th Ed.)
AEC Research and
Development Report

ARGONNE NATIONAL LABORATORY
P. O. Box 299
Lemont, Illinois

THE CALCULATION OF RADIAL TEMPERATURE DISTRIBUTIONS IN CYLINDRICAL FUEL SPECIMENS DURING NEUTRON IRRADIATION

by

Frank R. Taraba

Metallurgy Program 6.1.26

November, 1958

Operated by The University of Chicago
under
Contract W-31-109-eng-38

DISCLAIMER

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

THE CALCULATION OF RADIAL TEMPERATURE DISTRIBUTIONS IN CYLINDRICAL FUEL SPECIMENS DURING NEUTRON IRRADIATION

Frank R. Taraba

INTRODUCTION

A knowledge of the temperature distribution within a fuel sample during irradiation is required in order to evaluate the changes that occur in the physical and metallurgical properties of a fuel material upon irradiation. Because it is often impractical to measure temperatures within a fuel sample during irradiation, it is frequently necessary to rely upon computed values of the temperature. Therefore, a brief description and a comparison of the methods most frequently used for computing radial temperature distributions within irradiation samples are presented.

The calculation of temperature distributions in samples containing fissionable isotopes is complicated by a number of variables, among which are the variation of heat production throughout the sample (due to local neutron-flux perturbations) and the variation of thermal conductivity occasioned by unusually steep temperature gradients. Various assumptions, which are considered to fit most closely the conditions at hand, are made for these calculations. As no standard set of assumptions can fit all cases, four illustrative cases are presented, representing four different sets of conditions applied to the heat-conduction equation. The four cases considered may be briefly described as (A) variable thermal conductivity, non-uniform heat production, (B) variable thermal conductivity, uniform heat production, (C) constant thermal conductivity, non-uniform heat production, and (D) constant thermal conductivity, uniform heat production.

THE GENERAL HEAT-CONDUCTION EQUATION

The general form of the equation for the conduction of heat in a solid may be written as follows:

$$\vec{\nabla} \cdot K(T)\, \vec{\nabla} T + S(\vec{r},t) = \rho c \frac{\partial T}{\partial t} \quad , \qquad (1)$$

where $K = K(T)$, ρ, and c are the thermal conductivity, density, and specific heat of the material, respectively.

Case A. <u>Thermal Conductivity a Function of Temperature; Rate of Heat Production a Function of Radial Position:</u>

The temperature distribution within a solid is usually computed from a simplified form of the general heat-conduction equation. If the temperature distribution within a solid does not change with time, that is, if $\frac{\partial T}{\partial t} = 0$ at each point within the solid, the temperature, T, satisfies the following differential equation:

$$\vec{\nabla} \cdot K(T) \ \vec{\nabla} T + S(\vec{r}) = 0 \qquad (2)$$

If the solid is a circular cylinder whose axis coincides with the axis of Z, and the boundary conditions are independent of the coordinates θ and z, the temperature within the cylinder will be a function of r only, and equation (2) reduces to

$$\frac{1}{r} \frac{d}{dr} \left[r \ K(T) \frac{dT}{dr} \right] + S(r) = 0 \quad , \qquad (3)$$

since $\vec{\nabla} \cdot K(T) \vec{\nabla} T \equiv \text{div } K(T) \vec{\nabla} T = \frac{1}{r} \frac{d}{dr} \left[r \ K(T) \frac{dT}{dr} \right]$, in a cylindrical coordinate system. In this case the flow of heat takes place in planes perpendicular to the axis of the cylinder, and the lines of flow are radial.

Equation (3) may be solved quite easily as follows: first multiply the equation through by r. Hence

$$\frac{d}{dr} \left[r \ K(T) \frac{dT}{dr} \right] + r \ S(r) = 0, \text{ for } r \neq 0.$$

Next, integrate each term of the resulting equation with respect to r.

$$r \ K(T) \frac{dT}{dr} + \int_0^r r' \ S(r') \ dr' = C,$$

where C is a constant of integration. Now divide each term of the equation by r:

$$K(T) \frac{dT}{dr} + \frac{1}{r} \int_0^r r' \ S(r') \ dr' = \frac{C}{r} \quad .$$

If the source function S(r) is assumed to be bounded and continuous near r = 0, r S(r) = 0(r), then

$$\int_0^r r' S(r') \ dr' = 0(r^2),$$

and

$$\frac{1}{r}\int_0^r r' S(r') dr' = 0(r). \text{ Therefore}$$

$$\frac{1}{r}\int_0^r r' S(r') dr' \to 0 \text{ as } r \to 0.$$

Since, by hypothesis, $\frac{dT}{dr} = 0$ at $r = 0$ and $K(T)$ is bounded, C must be equal to zero. Hence

$$K(T)\frac{dT}{dr} = -\frac{1}{r}\int_0^r r' S(r') dr',$$

and

$$\int_{T(a)}^{T(r)} K(T)\, dT = -\int_a^r \left(\frac{1}{r'}\int_0^{r'} r'' S(r'') dr''\right) dr' \quad , \tag{4}$$

where a is the radius of the cylinder, and T(a) is the temperature at the surface of the cylinder, i.e., at r = a. Therefore, if the functional forms of K(T) and S(r) are known, equation (4) may be integrated.

Case B. <u>Thermal Conductivity a Function of Temperature; Rate of Heat Production Independent of Radial Position:</u>

If the source function, $S(\vec{r})$, is a constant, equation (2) becomes

$$\vec{\nabla}\cdot K(T)\,\vec{\nabla}T + S = 0. \tag{5}$$

If the temperature distribution within the cylinder is a function of r only, equation (5) reduces to

$$\frac{1}{r}\frac{d}{dr}\left[r\,K(T)\frac{dT}{dr}\right] + S = 0\ . \tag{6}$$

The solution of equation (6) is obtained from the solution of equation (3) by letting S(r) = S. Hence, upon substitution in equation (4),

$$\int_{T(a)}^{T(r)} K(T)\, dT = \frac{S}{4}\,(a^2 - r^2) \quad , \tag{7}$$

where a is equal to the radius of the cylinder.

Case C. Thermal Conductivity Constant; Rate of Heat Production a Function of Radial Position:

If the thermal conductivity, K, is a constant, equation (2) becomes

$$K\vec{\nabla} \cdot \vec{\nabla} T + S(\vec{r}) = 0, \qquad \text{or}$$

$$\nabla^2 T = -\frac{S(\vec{r})}{K} \quad . \tag{8}$$

If the temperature distribution within the cylinder is a function of r only, equation (8) reduces to

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) = -\frac{S(r)}{K} \quad . \tag{9}$$

The solution of equation (9) is obtained from the solution of equation (3) by letting K(T) = K. Hence, from equation (4),

$$T(r) = -\frac{1}{K}\int_a^r \left(\frac{1}{r'}\int_0^{r'} r'' \, S(r'') \, dr''\right) dr' + T(a) \quad , \tag{10}$$

where T(a) is the temperature at the surface of the cylinder, i.e., at r = a.

Case D. Thermal Conductivity Constant; Rate of Heat Production Independent of Radial Position:

If the thermal conductivity function, K(T), and the source function, S(r), are considered to be constants, K and S, respectively, equation (2) becomes

$$\nabla^2 T = -\frac{S}{K} \quad . \tag{11}$$

If the temperature distribution within the cylinder is a function of r only, equation (11) reduces to

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) = -\frac{S}{K} \quad . \tag{12}$$

The solution of equation (12) is

$$T(r) = \frac{S}{4K}(a^2 - r^2) + T(a) \quad . \tag{13}$$

It should be noted that the radial temperature gradient in the sample, ΔT, is simply

$$\Delta T = \frac{Sa^2}{4K} \quad . \tag{14}$$

THE RADIAL HEAT SOURCE FUNCTION, S(r)

Since S(r), the radial heat source function, appears in two of the four preceding cases, an evaluation of this function is necessary. This is the expression which denotes variation in heat production in the sample as a function of radial position.

To evaluate, it is necessary to know the radial thermal neutron flux distribution in the sample. A companion report[1] deals with the experimental determination of neutron flux in cylindrical fuel samples. The empirical equations resulting from that investigation have been used here.

If $\phi(r)$ is a function describing the radial distribution of thermal neutrons in a cylindrically shaped fuel sample during irradiation such that $\phi(r) = 1$ at $r = a$ (i.e., $\phi(r)$ is assumed to be normalized to unity at the surface of the sample), the heat source function becomes

$$S(r) = \beta \phi(r) \quad , \qquad (15)$$

where

$$\beta = 7.648 \times 10^{-12} \phi_s \Sigma_f \quad \frac{\text{cal}}{\text{cm}^3\text{-sec}} \quad ,$$

ϕ_s = Thermal neutron flux at the surface of the sample,

$\frac{\text{neutrons}}{\text{cm}^2\text{-sec}}$, and

Σ_f = Macroscopic fission cross section for thermal neutrons, cm^{-1}.

The radial distribution of thermal neutron flux within a cylindrically shaped fuel sample during irradiation may be obtained from the following empirical equation:[1]

$$\phi(r,\alpha) = A(\alpha)\left[1 + 0.69713\alpha\left(\frac{r}{a}\right)^2 + 0.48598\,\alpha^2\left(\frac{r}{a}\right)^4 + 0.33879\,\alpha^3\left(\frac{r}{a}\right)^6\right], \qquad (16)$$

for $0 \leqslant r \leqslant a$, and $0 \leqslant \alpha \leqslant 2$,

where

$$A(\alpha) = \frac{1}{1 + 0.69713\,\alpha + 0.48598\,\alpha^2 + 0.33879\,\alpha^3} \quad ,$$

$\alpha = a \cdot \Sigma_{abs}$, dimensionless,

Σ_{abs} = macroscopic absorption cross section for thermal neutrons, cm^{-1}, and

a = radius of the sample, cm.

Equation (16) gives the thermal neutron flux at position r along the radius of a rod whose radius is a. The neutron flux is normalized to unity at the surface of the rod, i.e., $\phi(a,\alpha) = 1$.

The ratio of the average-to-surface neutron flux within a fuel sample during irradiation may be obtained from the following equation:[1]

$$\phi_{avg}/\phi_s = A(\alpha)\ [1 + 0.34856\ \alpha + 0.16199\ \alpha^2 + 0.08698\ \alpha^3] \quad , \qquad (17)$$

for $0 \leqslant \alpha \leqslant 2$. It should be noted that the ratio of the axial to surface neutron flux within a fuel sample is simply $A(\alpha)$, i.e.,

$$\phi_{axial}/\phi_s = A(\alpha), \text{ for } 0 \leqslant \alpha \leqslant 2. \qquad (18)$$

The Radial Temperature Distribution within a Fuel Sample as Computed by Each of the Above Methods

Numerical solutions for one metallic and one ceramic specimen for postulates A through D appear in the tables and figures at the end of the report.

A summary of the most frequently used methods for computing radial temperature distributions within a cylindrically shaped fuel sample during irradiation is given in Table I. Tables II, IV, V and VI give the radial temperature distributions within a 0.635-cm dia., U-5 w/o U^{235} fuel sample during irradiation. Tables VII, IX, X and XI give the radial temperature distributions within a 0.600-cm dia., ThO_2-$2\frac{1}{2}$ w/o $U^{235}O_2$ fuel sample during irradiation. The temperature distributions are computed by each of the methods given in Table I. Tables III and VIII give the computed radial temperature gradients within each of the above fuel samples during irradiation assuming various surface temperatures.

The upper portions of Figures 1 and 2 show the radial temperature distributions in a 0.635-cm dia., U-5 w/o U^{235} specimen and a 0.600-cm dia. ThO_2-$2\frac{1}{2}$ w/o $U^{235}O_2$ specimen as computed by Case A., i.e., assuming variable thermal conductivity and variable heat generation. The lower portions of Figures 1 and 2 show the temperature deviation curves $T_B(r) - T_A(r)$, $T_C(r) - T_A(r)$, and $T_D(r) - T_A(r)$. (The subscripts refer to the Case used in computing the temperature distribution T(r).)

Figures 3 and 4 give the thermal conductivity of natural uranium and thorium oxide, respectively, as a function of the temperature.

The results shown in the Tables may be summarized by the following statements:

1. If the thermal conductivity of the fuel material is assumed to depend upon the temperature and the rate of heat production is considered to be independent of position within the sample, the computed central fuel temperature will be too high.

2. If the thermal conductivity increases with increasing temperature, the radial temperature gradient within the sample will become smaller as the surface temperature of the fuel increases.

3. If the thermal conductivity decreases with increasing temperature, the radial temperature gradient within the sample will become larger as the surface temperature of the fuel increases.

4. If the thermal conductivity of the fuel material is assumed to be a constant and the rate of heat production is either uniform throughout the sample or depends upon the position in the sample, the computed central fuel temperature may be either too high or too low depending upon the value assumed for the thermal conductivity.

ACKNOWLEDGEMENTS

The author wishes to acknowledge the help of E. H. Bareiss of the ANL Applied Mathematics Division in arranging to have parts of this problem programmed for machine computation. The author also wishes to acknowledge the assistance of E. M. Loeffler of the ANL Metallurgy Division in carrying out numerous hand computations.

REFERENCE

1. Taraba, F.R. and Paine, S.H., The Radial Distribution of Thermal Neutron Flux within Cylindrical Fuel Specimens during Neutron Irradiation, ANL-5872 (1958).

Table I

A Summary of the Most Frequently Used Methods for Computing Radial Temperature Distributions within Cylindrically Shaped Fuel Samples during Irradiation

Case	Thermal Conductivity	Rate of Heat Production	Heat Conduction Equation	Solution of Heat Conduction Equation
A	$K = K(T)$	$S = S(r)$	$\frac{1}{r}\frac{d}{dr}\left[r\,K(T)\frac{dT}{dr}\right] + S(r) = 0$	$\int_{T(a)}^{T(r)} K(T)\,dT = -\int_a^r \left(\frac{1}{r'}\int_0^{r'} r''\,S(r'')\,dr''\right)dr'$
B	$K = K(T)$	$S =$ Constant	$\frac{1}{r}\frac{d}{dr}\left[r\,K(T)\frac{dT}{dr}\right] + S = 0$	$\int_{T(a)}^{T(r)} K(T)\,dT = \frac{S}{4}(a^2 - r^2)$
C	$K =$ Constant	$S = S(r)$	$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) + \frac{S(r)}{K} = 0$	$T(r) = -\frac{1}{K}\int_a^r \left(\frac{1}{r'}\int_0^{r'} r''\,S(r'')\,dr''\right)dr' + T(a)$
D	$K =$ Constant	$S =$ Constant	$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) + \frac{S}{K} = 0$	$T(r) = \frac{S}{4K}(a^2 - r^2) + T(a)$

The following assumptions are implicit in the solution of the heat conduction equation:

(1) $T = T(r)$

(2) $\frac{dT}{dr} = 0$ at $r = 0$

(3) $T = T(a)$ at $r = a$, where a is the radius of the sample

TABLE II

Radial Temperature Distribution in 0.635-cm Diameter U-5 w/o U^{235} Specimens (Case A).

A. Conductivity and Heat Source Functions.

No.	$K(T)\left(\frac{cal}{sec\text{-}cm\text{-}^\circ C}\right)$	$S(r)\left(\frac{cal}{cm^3\text{-}sec}\right)$	Σ_{abs} (cm^{-1})	Σ_f (cm^{-1})	ϕ_s $\left(\frac{Neutrons}{cm^2\text{-}sec}\right)$
1	$K(T) = \sum_{i=0}^{3} a_i T^i$	$S(r) = \beta\ \phi(r) = \beta \sum_{i=0}^{3} b_i r^{2i}$	1.593	1.251	8.5×10^{13}
2	$K(T) = \sum_{i=0}^{3} a_i T^i$	$S(r) = \beta\ \phi(r) = \beta \sum_{i=0}^{3} b_i r^{2i}$	1.593	1.251	1.0×10^{14}
3	$K(T) = \sum_{i=0}^{3} a_i T^i$	$S(r) = \beta\ \phi(r) = \beta \sum_{i=0}^{3} b_i r^{2i}$	1.593	1.251	1.2×10^{14}
4	$K(T) = \sum_{i=0}^{3} a_i T^i$	$S(r) = \beta\ \phi(r) = \beta \sum_{i=0}^{3} b_i r^{2i}$	1.593	1.251	1.5×10^{14}
5	$K(T) = \sum_{i=0}^{3} a_i T^i$	$S(r) = \beta\ \phi(r) = \beta \sum_{i=0}^{3} b_i r^{2i}$	1.593	1.251	2.0×10^{14}

B. Coefficients Appearing in the Conductivity and Heat Source Functions.

	i = 0	i = 1	i = 2	i = 3
a_i	56×10^{-3}	30.833×10^{-6}	10×10^{-9}	16.666×10^{-12}
b_i	0.65757	2.3000	8.0446	28.138

<u>Note:</u> K(T) Increases as the Temperature Increases.

C. Temperature Distributions.

r, cm	$T_1(r)$, °C	$T_2(r)$, °C	$T_3(r)$, °C	$T_4(r)$, °C	$T_5(r)$, °C
0	421.6	457.8	504.4	571.7	676.9
.025	420.5	456.4	502.8	569.8	674.6
.05	417.0	452.3	498.2	564.3	667.7
.10	402.8	436.0	479.4	541.6	639.7
.15	378.4	408.0	446.4	502.3	590.7
.20	342.6	366.5	397.9	443.7	516.9
.25	293.4	309.4	330.4	361.4	411.8
.30	227.9	233.0	239.3	249.0	265.0
.3175	200.0	200.0	200.0	200.0	200.0

TABLE III

Radial Thermal Gradient in 0.635-cm Diameter U-5 w/o U^{235} Specimen (Case A).

A. Conductivity and Heat Source Functions

$\left(\frac{\text{cal}}{\text{sec-cm-°C}}\right)$ K(T)	$\left(\frac{\text{cal}}{\text{cm}^3\text{-sec}}\right)$ S(r)	Σ_{abs} (cm^{-1})	Σ_f (cm^{-1})	$\left(\frac{\text{Neutrons}}{\text{cm}^2\text{-sec}}\right)$ ϕ_s
$K(T) = \sum_{i=0}^{3} a_i T^i$	$S(r) = \beta\ \phi(r) = \beta \sum_{i=0}^{3} b_i r^{2i}$	1.593	1.251	1.0×10^{14}

B. Coefficients Appearing in the Conductivity and Heat Source Functions

	i = 0	i = 1	i = 2	i = 3
a_i	56×10^{-3}	30.833×10^{-6}	10×10^{-9}	16.666×10^{-12}
b_i	0.65757	2.3000	8.0446	28.138

Note: K(T) Increases as the Temperature Increases.

C. Temperature Gradients

T(a), °C	T(0), °C	ΔT, °C
100	372.8	272.8
200	457.8	257.8
300	542.7	242.7
400	627.8	227.8
500	712.9	212.9
600	798.4	198.4
700	884.2	184.2
800	970.3	170.3
900	1057.1	157.1
1000	1144.5	144.5

TABLE IV

Radial Temperature Distribution in 0.635-cm Diameter U-5 w/o U^{235} Specimens (Cases A and B).

A. Conductivity and Heat Source Functions.

No.	$K(T)\left(\frac{cal}{sec\text{-}cm\text{-}°C}\right)$	$S(r)\left(\frac{cal}{cm^3\text{-}sec}\right)$	Σ_{abs} (cm^{-1})	Σ_f (cm^{-1})	$\phi_s\left(\frac{Neutrons}{cm^2\text{-}sec}\right)$
1	$K(T) = \sum_{i=0}^{3} a_i T^i$	$S(r) = \beta\,\phi(r) = \beta \sum_{i=0}^{3} b_i r^{2i}$	1.593	1.251	1.0×10^{14}
2	$K(T) = \sum_{i=0}^{3} a_i T^i$	$S(r) = \frac{2\beta}{a^2}\int_0^a r\,\phi(r)\,dr = \beta\,\phi_{avg}$	1.593	1.251	1.0×10^{14}

B. Coefficients Appearing in the Conductivity and Heat Source Functions.

	i = 0	i = 1	i = 2	i = 3
a_i	56×10^{-3}	30.833×10^{-6}	10×10^{-9}	16.666×10^{-12}
b_i	0.65757	2.3000	8.0446	28.138

Note: K(T) Increases as the Temperature Increases.

C. Temperature Distributions.

r, cm	$T_1(r)$, °C	$T_2(r)$, °C
0	457.8	484.1
.025	456.4	482.5
.05	452.3	477.6
.10	436.0	458.1
.15	408.0	425.0
.20	366.5	377.4
.25	309.4	314.0
.30	233.0	233.0
.3175	200.0	200.0

TABLE V

Radial Temperature Distribution in 0.635-cm Diameter U-5 w/o U^{235} Specimens (Cases A and C).

A. Conductivity and Heat Source Functions.

No.	$K(T)$ $\left(\frac{cal}{sec\text{-}cm\text{-}°C}\right)$	$S(r)$ $\left(\frac{cal}{cm^3\text{-}sec}\right)$	Σ_{abs} (cm^{-1})	Σ_f (cm^{-1})	ϕ_s $\left(\frac{Neutrons}{cm^2\text{-}sec}\right)$
1	$K(T) = \sum_{i=0}^{3} a_i T^i$	$S(r) = \beta\ \phi(r) = \beta \sum_{i=0}^{3} b_i r^{2i}$	1.593	1.251	1.0×10^{14}
2	K = 0.0632, at 200° C	$S(r) = \beta\ \phi(r) = \beta \sum_{i=0}^{3} b_i r^{2i}$	1.593	1.251	1.0×10^{14}
3	K = 0.0671, at 300° C	$S(r) = \beta\ \phi(r) = \beta \sum_{i=0}^{3} b_i r^{2i}$	1.593	1.251	1.0×10^{14}
4	K = 0.0714, at 400° C	$S(r) = \beta\ \phi(r) = \beta \sum_{i=0}^{3} b_i r^{2i}$	1.593	1.251	1.0×10^{14}

B. Coefficients Appearing in the Conductivity and Heat Source Functions.

	i = 0	i = 1	i = 2	i = 3
a_i	56×10^{-3}	30.833×10^{-6}	10×10^{-9}	16.666×10^{-12}
b_i	0.65757	2.3000	8.0446	28.138

<u>Note:</u> K(T) Increases as the Temperature Increases.

C. Temperature Distributions.

r, cm	$T_1(r)$, °C	$T_2(r)$, °C	$T_3(r)$, °C	$T_4(r)$, °C
0	457.8	477.1	461.0	445.3
.05	452.3	470.9	455.2	439.8
.10	436.0	452.0	437.4	423.1
.15	408.0	420.0	407.2	394.7
.20	366.5	373.9	363.8	353.9
.25	309.4	312.2	305.7	299.3
.30	233.0	232.9	231.0	229.1
.3175	200.0	200.0	200.0	200.0

TABLE VI

Radial Temperature Distribution in 0.635-cm Diameter U-5 w/o U^{235} Specimens (Cases A and D).

A. Conductivity and Heat Source Functions.

No.	$K(T)\left(\frac{cal}{sec\text{-}cm\text{-}°C}\right)$	$S(r)\left(\frac{cal}{cm^3\text{-}sec}\right)$	Σ_{abs} (cm^{-1})	Σ_f (cm^{-1})	$\phi_s\left(\frac{Neutrons}{cm^2\text{-}sec}\right)$
1	$K(T) = \sum_{i=0}^{3} a_i T^i$	$S(r) = \beta\ \phi(r) = \beta \sum_{i=0}^{3} b_i r^{2i}$	1.593	1.251	1.0×10^{14}
2	K = 0.0632, at 200° C	$S(r) = \frac{2\beta}{a^2}\int_0^a r\ \phi(r)\,dr = \beta\ \phi_{avg}$	1.593	1.251	1.0×10^{14}
3	K = 0.0671, at 300° C	$S(r) = \beta\ \phi_{avg}$	1.593	1.251	1.0×10^{14}
4	K = 0.0714, at 400° C	$S(r) = \beta\ \phi_{avg}$	1.593	1.251	1.0×10^{14}

B. Coefficients Appearing in the Conductivity and Heat Source Functions.

	i = 0	i = 1	i = 2	i = 3
a_i	56×10^{-3}	30.833×10^{-6}	10×10^{-9}	16.666×10^{-12}
b_i	0.65757	2.3000	8.0446	28.138

Note: K(T) Increases as the Temperature Increases.

r, cm	$T_1(r)$, °C	$T_2(r)$, °C	$T_3(r)$, °C	$T_4(r)$, °C
0	457.8	508.2	490.3	472.8
.05	452.3	500.6	483.1	466.1
.10	436.0	477.7	461.5	445.8
.15	408.0	439.4	425.5	411.9
.20	366.5	385.9	375.1	364.6
.25	309.4	317.1	310.3	303.7
.30	233.0	233.0	231.0	229.2
.3175	200.0	200.0	200.0	200.0

TABLE VII

Radial Temperature Distribution in 0.600-cm Diameter ThO_2-$2\frac{1}{2}$ w/o $U^{235}O_2$ Specimens (Case A).

A. Conductivity and Heat Source Functions.

No.	$K(T)\ \left(\frac{cal}{sec\text{-}cm\text{-}^\circ C}\right)$	$S(r)\ \left(\frac{cal}{cm^3\text{-}sec}\right)$	Σ_{abs} (cm^{-1})	Σ_f (cm^{-1})	ϕ_s $\left(\frac{Neutrons}{cm^2\text{-}sec}\right)$
1	$K(T) = \sum_{i=0}^{3} c_i T^i$	$S(r) = \beta\phi(r) = \beta \sum_{i=0}^{3} e_i r^{2i}$	0.419	0.252	6.0×10^{13}
2	$K(T) = \sum_{i=0}^{3} c_i T^i$	$S(r) = \beta\phi(r) = \beta \sum_{i=0}^{3} e_i r^{2i}$	0.419	0.252	1.2×10^{14}
3	$K(T) = \sum_{i=0}^{3} c_i T^i$	$S(r) = \beta\phi(r) = \beta \sum_{i=0}^{3} e_i r^{2i}$	0.419	0.252	1.5×10^{14}

B. Coefficients Appearing in the Conductivity and Heat Source Functions.

	i = 0	i = 1	i = 2	i = 3
c_i	2.4056×10^{-2}	-4.0519×10^{-5}	2.8286×10^{-8}	-6.6652×10^{-12}
e_i	0.91245	0.88820	0.86458	0.84160

Note: K(T) Decreases as the Temperature Increases

C. Temperature Distributions.

r, cm	$T_1(r)$, °C	$T_2(r)$, °C	$T_3(r)$, °C
0	378.0	617.2	783.9
.025	377.8	616.8	783.2
.05	372.7	601.4	758.7
.10	357.1	556.2	688.0
.15	331.8	487.4	584.4
.20	298.1	402.4	462.9
.25	257.0	308.3	335.8
.30	210.0	210.0	210.0

TABLE VIII

Radial Thermal Gradient in 0.600-cm Diameter ThO_2-2½ w/o $U^{235}O_2$ Specimens (Case A).

A. Conductivity and Heat Source Functions.

$K(T)\left(\frac{cal}{sec\text{-}cm\text{-}°C}\right)$	$S(r)\left(\frac{cal}{cm^3\text{-}sec}\right)$	Σ_{abs} (cm^{-1})	Σ_f (cm^{-1})	ϕ_s $\left(\frac{Neutrons}{cm^2\text{-}sec}\right)$
$K(T) = \sum_{i=0}^{3} c_i T^i$	$S(r) = \beta \phi(r) = \beta \sum_{i=0}^{3} e_i r^{2i}$	0.419	0.252	8.5×10^{13}

B. Coefficients Appearing in the Conductivity and Heat Source Functions.

	i = 0	i = 1	i = 2	i = 3
c_i	2.4056×10^{-2}	-4.0519×10^{-5}	2.8286×10^{-8}	-6.6652×10^{-12}
e_i	0.91245	0.88820	0.86458	0.84160

Note: K(T) Decreases as the Temperature Increases

C. Temperature Gradients.

T(a), °C	T(0), °C	ΔT, °C
200	450.4	250.4
300	614.0	314.0
400	794.0	394.0
500	985.0	485.0
600	1172.9	572.9
700	1344.2	644.2
800	1495.8	695.8
900	1632.3	732.3
1000	1761.2	761.2

TABLE IX

Radial Temperature Distribution in 0.600-cm Diameter ThO_2-2½ w/o $U^{235}O_2$ Specimens (Cases A and B).

A. Conductivity and Heat Source Functions.

No.	$K(T)\ \left(\frac{cal}{sec\text{-}cm\text{-}°C}\right)$	$S(r)\ \left(\frac{cal}{cm^3\text{-}sec}\right)$	Σ_{abs} (cm^{-1})	Σ_f (cm^{-1})	$\phi_s\ \left(\frac{Neutrons}{cm^2\text{-}sec}\right)$
1	$K(T) = \sum_{i=0}^{3} c_i T^i$	$S(r) = \beta\ \phi(r) = \beta \sum_{i=0}^{3} e_i\ r^{2i}$	0.419	0.252	8.5×10^{13}
2	$K(T) = \sum_{i=0}^{3} c_i T^i$	$S(r) = \frac{2\beta}{a^2}\int_0^a r\phi(r)\,dr = \beta\ \phi_{avg}$	0.419	0.252	8.5×10^{13}

B. Coefficients Appearing in the Conductivity and Heat Source Functions.

	i = 0	i = 1	i = 2	i = 3
c_i	2.4056×10^{-2}	-4.0519×10^{-5}	2.8286×10^{-8}	-6.6652×10^{-12}
e_i	0.91245	0.88820	0.86458	0.84160

Note: K(T) Decreases as the Temperature Increases

C. Temperature Distributions

r, cm	$T_1(r)$, °C	$T_2(r)$, °C
0	434.8	442.0
.025	432.8	439.9
.05	426.4	433.3
.10	402.4	407.8
.15	364.2	367.7
.20	314.2	316.0
.25	255.2	255.8
.30	190.0	190.0

TABLE X

Radial Temperature Distribution in 0.600-cm Diameter ThO_2-2½ w/o $U^{235}O_2$ Specimens (Cases A and C).

A. Conductivity and Heat Source Functions

No.	$K(T)\ \left(\frac{cal}{sec\text{-}cm\text{-}°C}\right)$	$S(r)\ \left(\frac{cal}{cm^3\text{-}sec}\right)$	Σ_{abs} (cm^{-1})	Σ_f (cm^{-1})	ϕ_s $\left(\frac{Neutrons}{cm^2\text{-}sec}\right)$
1	$K(T) = \sum_{i=0}^{3} c_i T^i$	$S(r) = \beta\ \phi(r) = \beta \sum_{i=0}^{3} e_i r^{2i}$	0.419	0.252	8.5×10^{13}
2	K = 0.0168 at 200° C	$s(r) = \beta\ \phi(r) = \beta \sum_{i=0}^{3} e_i r^{2i}$	0.419	0.252	8.5×10^{13}
3	K = 0.0141 at 300° C	$S(r) = \beta\ \phi(r) = \beta \sum_{i=0}^{3} e_i r^{2i}$	0.419	0.252	8.5×10^{13}
4	K = 0.0120 at 400° C	$S(r) = \beta\ \phi(r) = \beta \sum_{i=0}^{3} e_i r^{2i}$	0.419	0.252	8.5×10^{13}

B. Coefficients Appearing in the Conductivity and Heat Source Functions.

	i = 0	i = 1	i = 2	i = 3
c_i	2.4056×10^{-2}	-4.0519×10^{-5}	2.8286×10^{-8}	-6.6652×10^{-12}
e_i	0.91245	0.88820	0.86458	0.84160

Note: K(T) Decreases as the Temperature Increases.

C. Temperature Distributions.

r, cm	$T_1(r)$, °C	$T_2(r)$, °C	$T_3(r)$, °C	$T_4(r)$, °C
0	434.8	405.0	444.3	487.0
.05	426.4	399.4	437.6	479.2
.10	402.4	382.7	418.7	455.7
.15	364.2	354.6	384.2	416.5
.20	314.2	315.0	337.1	361.1
.25	255.2	263.6	275.8	289.1
.30	190.0	190.0	190.0	190.0

TABLE XI

Radial Temperature Distribution in 0.600-cm Diameter ThO_2-2½ w/o $U^{235}O_2$ Specimens (Cases A and D)

A. Conductivity and Heat Source Functions.

No.	$K(T)$ $\left(\frac{cal}{sec\text{-}cm\text{-}°C}\right)$	$S(r)$ $\left(\frac{cal}{cm^3\text{-}sec}\right)$	Σ_{abs} (cm^{-1})	Σ_f (cm^{-1})	ϕ_s $\left(\frac{Neutrons}{cm^2\text{-}sec}\right)$
1	$K(T) = \sum_{i=0}^{3} c_i T^i$	$S(r) = \beta\ \phi(r) = \beta \sum_{i=0}^{3} e_i r^{2i}$	0.419	0.252	8.5×10^{13}
2	K = 0.0168 at 200° C	$S(r) = \frac{2\beta}{a^2}\int_0^a r\phi(r)\,dr = \beta\ \phi_{avg}$	0.419	0.252	8.5×10^{13}
3	K = 0.0141 at 300° C	$S(r) = \beta\ \phi_{avg}$	0.419	0.252	8.5×10^{13}
4	K = 0.0120 at 400° C	$S(r) = \beta\ \phi_{avg}$	0.419	0.252	8.5×10^{13}

B. Coefficients Appearing in the Conductivity and Heat Source Functions.

	i = 0	i = 1	i = 2	i = 3
c_i	2.4056×10^{-2}	-4.0519×10^{-5}	2.8286×10^{-8}	-6.6652×10^{-12}
e_i	0.91245	0.88820	0.86458	0.84160

Note: K(T) Decreases as the Temperature Increases.

C. Temperature Distributions.

r, cm	$T_1(r)$, °C	$T_2(r)$, °C	$T_3(r)$, °C	$T_4(r)$, °C
0	434.8	399.8	439.9	483.7
.05	426.4	393.9	433.0	475.5
.10	402.4	376.5	412.2	451.0
.15	364.2	347.3	377.4	410.2
.20	314.2	306.5	328.8	353.1
.25	255.2	254.1	266.4	279.7
.30	190.0	190.0	190.0	190.0

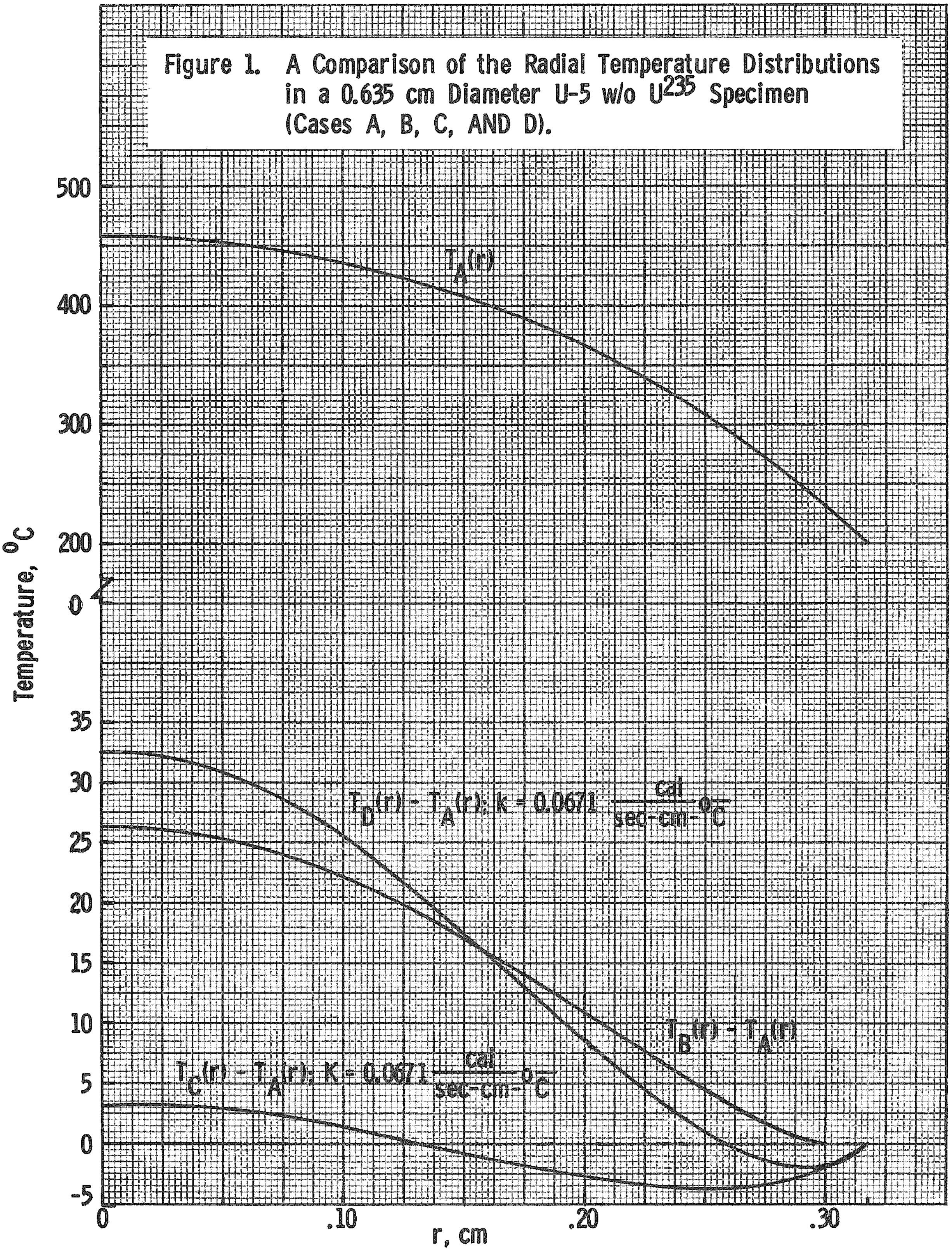

Figure 1. A Comparison of the Radial Temperature Distributions in a 0.635 cm Diameter U-5 w/o U^{235} Specimen (Cases A, B, C, AND D).

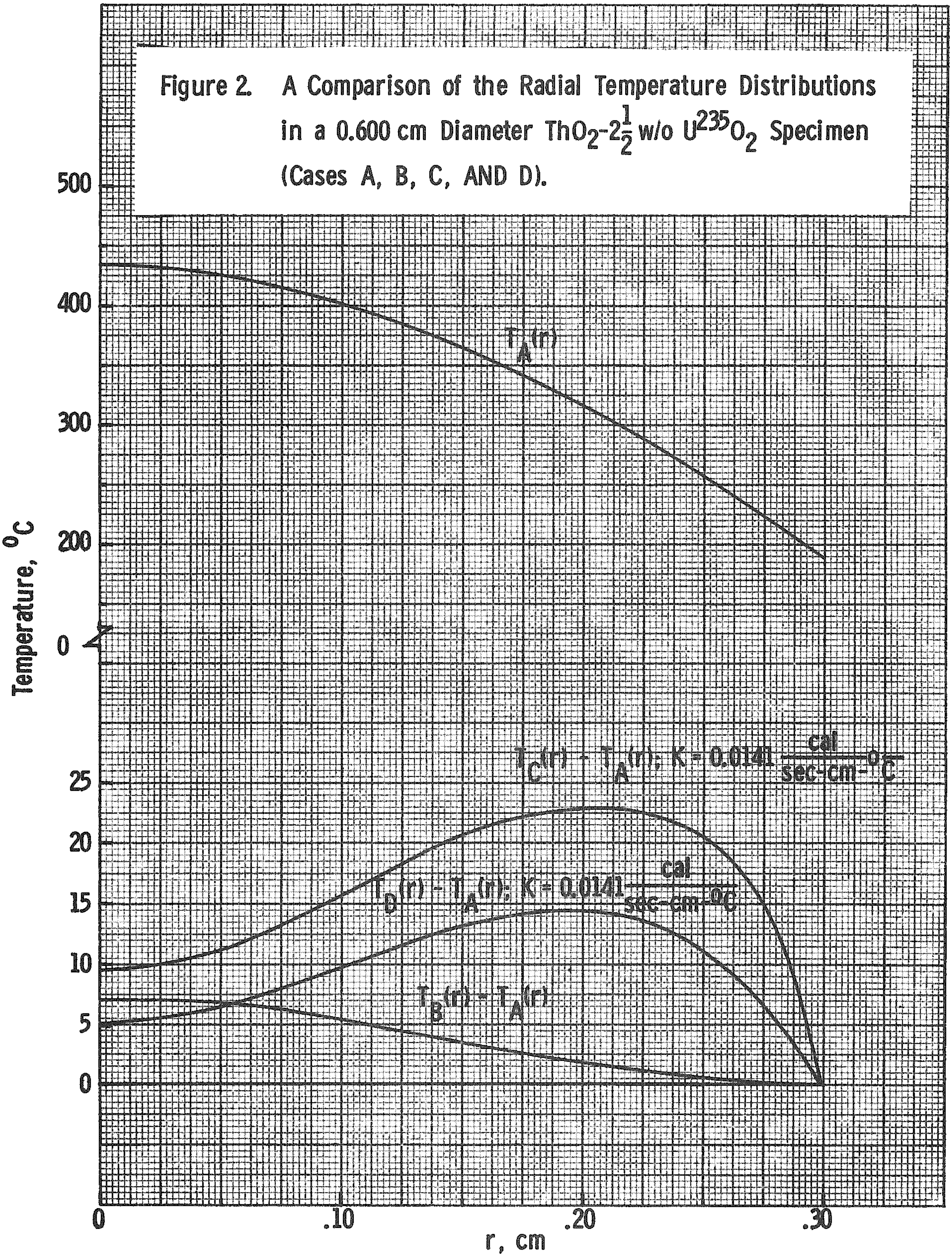

Figure 2. A Comparison of the Radial Temperature Distributions in a 0.600 cm Diameter ThO_2-$2\frac{1}{2}$ w/o $U^{235}O_2$ Specimen (Cases A, B, C, AND D).

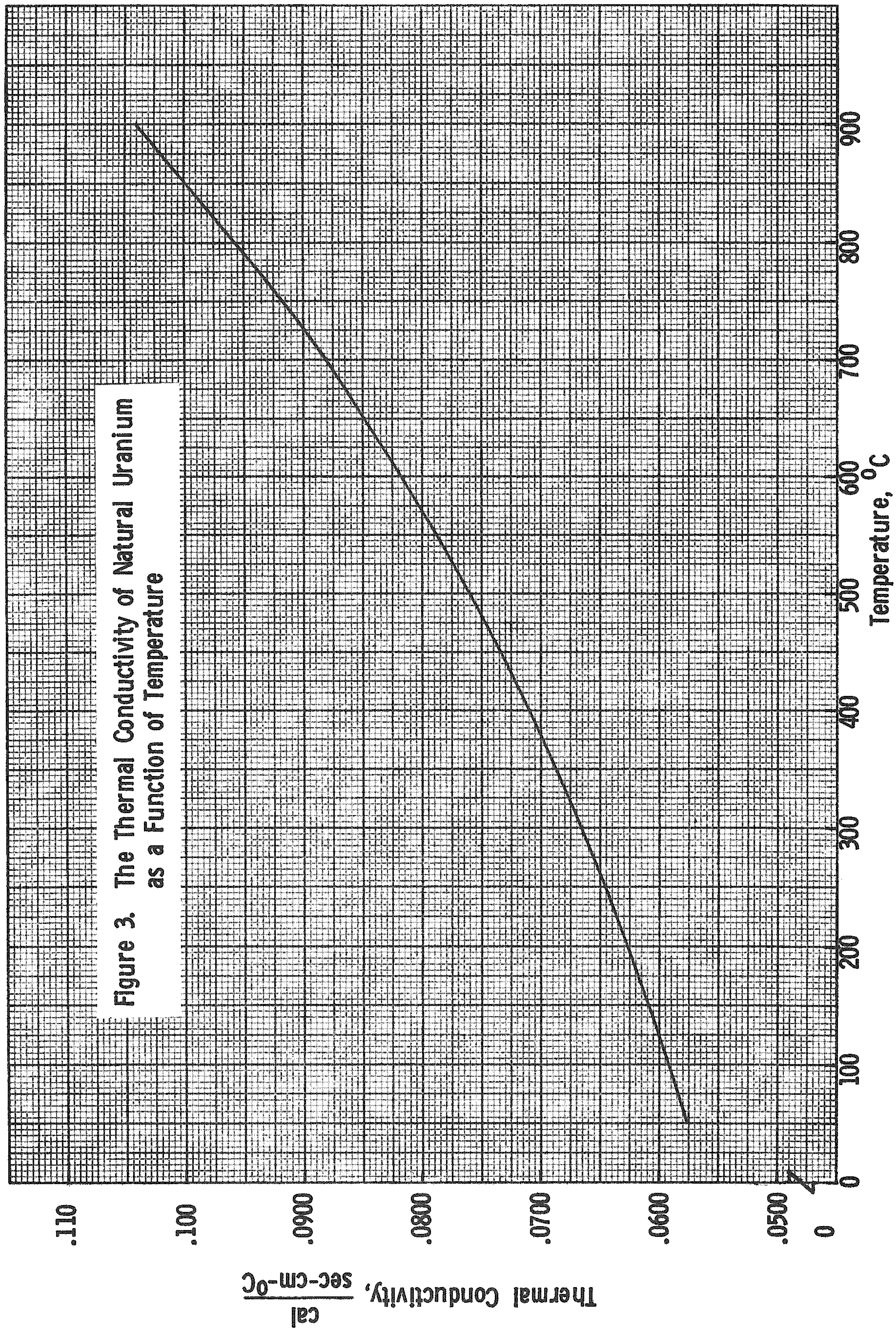

Figure 3. The Thermal Conductivity of Natural Uranium as a Function of Temperature

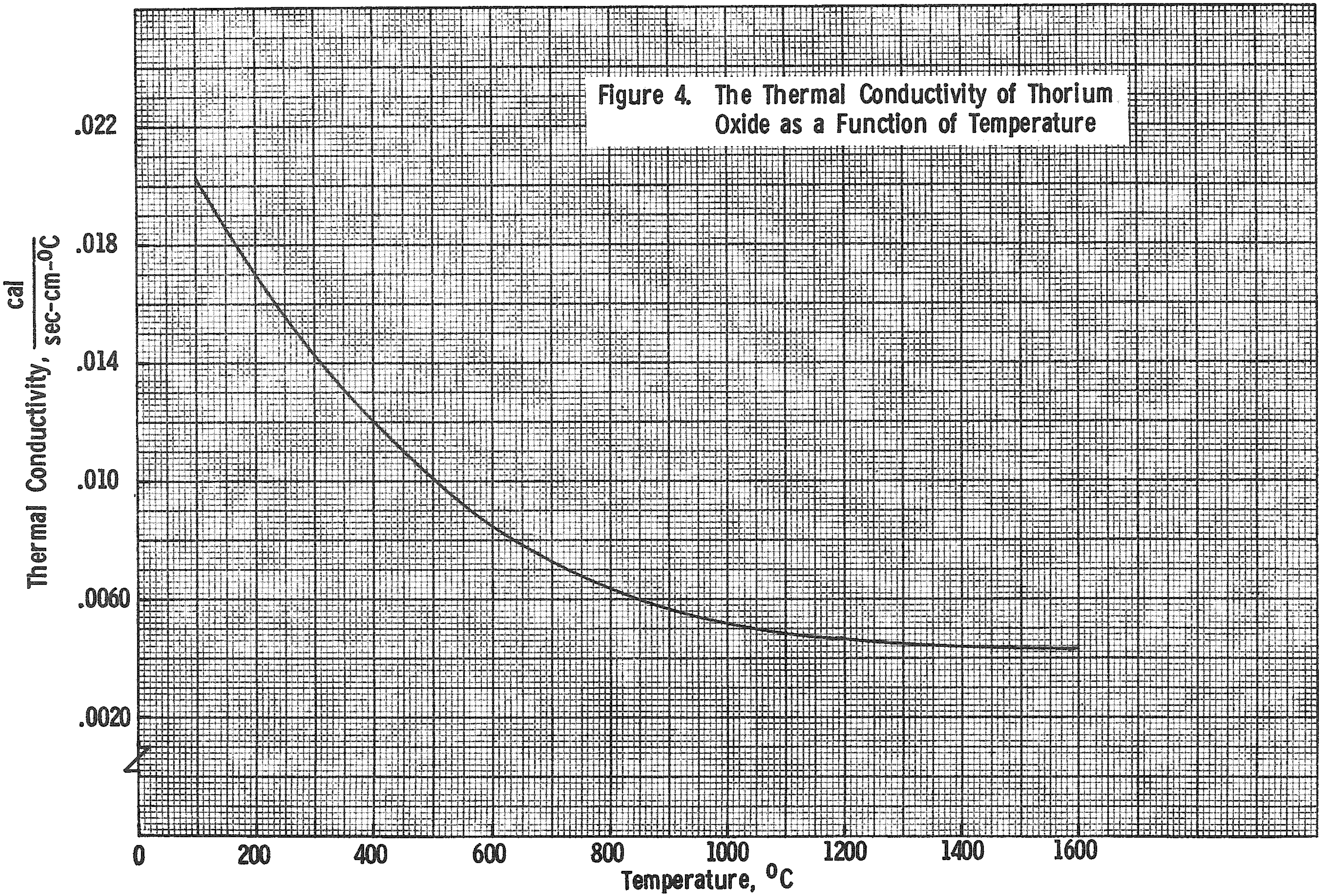

Figure 4. The Thermal Conductivity of Thorium Oxide as a Function of Temperature