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THE DERIVATION OF REACTOR HEAT TRANSFER TRANSIENT EQUATIONS FOR GAS COOLED GRAPHITE MODERATED THERMAL REACTORS.

by

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1. SUMMARY.

In making transient studies on the behaviour of gas cooled graphite moderated thermal reactors, one of the sets of mathematical equations involved is that for reactor heat transfer.

These equations have for inputs: reactor power, reactor gas flow and reactor inlet gas temperature; and produce as output, reactor outlet gas temperature, the uranium and graphite reactivity affecting variables, and the maximum fuel element temperatures.

The paper considers the derivation of such equations. From the consideration of a unit length of one fuel element channel in the core, partial differential equations of the system are determined. These are integrated in space, to give the form required for analogue computer studies, i.e. simultaneous ordinary non-linear differential equations.

The final equations are given for two variants in moderator design, viz: a solid-block type of moderator, and a moderator involving in part a graphite fuel element supporting sleeve.

Account is taken of heat transfer by conduction in the solids, and by convection and thermal radiation in and between the gas spaces.

2. SYMBOLS.

b_1 a constant equal to υ_1/W_c

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b_2	a constant equal to ν_2/W_c
C_a	heat capacity per unit length of can
C_{c1}	heat capacity per unit length of gas inside sleeve
C_{c2}	heat capacity per unit length of gas outside sleeve
C_{m1}	heat capacity per unit length of sleeve
C_{m2}	heat capacity per unit length of main moderator
C_{mS}	heat capacity per unit length of surface region of solid moderator
C_{mb}	heat capacity per unit length of bulk region of solid moderator
C_u	heat capacity per unit length of uranium
f_r	variation of heat transfer coefficient of convection with gas flow
H_ℓ	heat generated per unit length of channel at the point of maximum can temperature
Hmax	maximum value of heat generation per unit length in the channel
k	a constant equal to $\dfrac{5+\cos\frac{\pi L}{L_1}}{3\left(1+\frac{L_1}{\pi L}\sin\frac{\pi L}{L_1}\right)}$

k_2, k_3, k_5, k_7 are defined by equation (29)

k_8, k_9 constants defined by equations (23), (26)

ℓ	length along channel from cold end
L	total length of uranium in channel
L_1	effective extrapolated length of axial flux
m	the ratio W/Wc
n	the ratio P/P_c
P	reactor power production
P_c	channel power production
r	radius in uranium at point ℓ
R_A	heat transfer coefficient for radiation between can and sleeve

R_B	heat transfer coefficient for radiation between sleeve and main moderator

$Raml = R_A (Ta^4 - Tm_1{}^4)$
$Rm1m2 = R_B (Tm1^4 - Tm2^4)$
$RmamL = R_A (Tma^4 - Tm\ell^4)$
$RmacL = R_A (Tma^4 - Tc\ell^4)$

All temperatures in absolute degrees

$\overline{R}aml$, etc. are defined by assumption 16

T_a	can temperature at point ℓ
T_{m1}	mean sleeve temperature at point ℓ
T_{m2}	mean main moderator temperature at point ℓ
T_U	mean uranium temperature at point ℓ
T_{c1}	gas temperature inside sleeve at point ℓ
T_{c2}	gas temperature outside sleeve at point ℓ
T_{ci}	gas inlet or cold duct temperature
T_{c10}	outlet temperature of gas inside sleeve
T_{c20}	outlet temperature of gas outside sleeve
T_{c0}	mixed gas outlet or hot duct temperature
T_{ma}	maximum can temperature in the reactor
T_{mU}	maximum uranium temperature in the reactor
$T_{U\ell}$	local value of T_U at point where T_{ma} occurs
$T_{c\ell}$	local value of T_{c1} at point where T_{ma} occurs
$T_{m\ell}$	local value of T_{m1} at point where T_{ma} occurs
$\overline{T}$	statistically weighted mean value of temperature T
U_1	heat transfer coefficient for conduction in uranium
U_2	heat transfer coefficient for convection between can and gas
U_3	heat transfer coefficient for convection between sleeve and gas inside sleeve
U_4	heat transfer coefficient for convection between sleeve and gas outside sleeve

U_5 heat transfer coefficient for convection between main moderator and gas

U_6 heat transfer coefficient for conduction in uranium where T_{ma} occurs

U_7 heat transfer coefficient for convection between can and gas at point where T_{ma} occurs

U_{10} heat transfer coefficient for conduction in solid moderator between surface and bulk regions

υ_1, υ_2 gas velocities inside and outside sleeve, respectively

W reactor gas flow

W_c channel total gas flow

α, β, γ proportions of power generated in fuel, sleeve and main moderator respectively $\alpha + \beta + \gamma = 1$.

δ the ratio $\frac{L.H\ell}{Pc}$

3. INTRODUCTION.

1. In simulating the transient behaviour of a nuclear reactor and its heat removal and power generation system, the various components are treated as separate blocks with appropriate interconnecting variables. Typical blocks are: Reactor Neutron Kinetics, Reactor Heat Transfer, Steam Raising Unit Heat Transfer, Steam Turbine and Generator.

2. The Neutron Kinetics block accepts reactivity as a variable, and produces reactor power as output. The Reactor Heat Transfer block simulates the remaining important variables in the reactor.

3. Three important types of information are provided by the equations comprising the Reactor Heat Transfer block:

(a) The relationship between the nuclear power produced and the heat removed from the reactor. The heat removed is given by the gas flow rate multiplied by the enthalpy gain. In practice the temperature rise and a mean specific heat is used for enthalpy gain.

(b) The temperature variations which affect reactivity. For a CO_2 cooled reactor, only the uranium and graphite temperature changes produce significant reactivity changes.

The perturbation theory of neutrons indicates that in general the overall reactivity in a reactor should be obtained by weighting the local reactivity by the square of the local flux. This is known as statistical weighting. Thus to obtain a single temperature to indicate reactivity change, it is necessary to weight the local temperatures in the same way, along the channel and across the core radius, although arithmetic mean values are appropriate across a single fuel element.

The appropriate temperature for the uranium effect is not known exactly, but as it is probably between the mean uranium surface and the overall bulk mean uranium temperature, both these cases are always considered.

The reactivity change in the moderator is more complex, since the temperature coefficient varies with temperature, and with distance from the fuel element. Experiments have been carried out by the U.K.A.E.A. on a moderator system consisting of two concentric graphite cylinders per fuel element, and relative reactivity coefficients have been measured for this system.

Thus to determine the reactivity changes due to temperature in the reactor, it is necessary to produce:-

For the uranium, a constant coefficient operating on either the statistically weighted mean uranium temperature, or the statistically weighted surface uranium temperature; and for the graphite, two appropriate statistically weighted mean temperatures, with separate non-linear temperature coefficients.

(c) The maximum temperatures which occur in the core. The important temperatures are the maximum uranium and maximum can temperature, and the gas outlet temperature. Other temperatures are not in general of sufficient importance to warrant their deliberate inclusion in the equations.

It has been found that representing the maximum temperatures as simple ratios of other appropriate temperatures does not give an adequate accuracy for these temperatures and separate equations have to be obtained for the maximum temperatures.

4. The required form of outputs from the reactor heat transfer equations is obtained from the above considerations. It is found convenient to use the mean can temperature instead of the mean surface uranium temperature, with little error,

considering the uncertainty of the uranium coefficient. This saves introducing another variable, and does not result in any reduced accuracy of the simulation. The final block diagram for the equations is then as in fig. 1.

5. The input variables are the reactor power (P), the gas flow (W), and the cold duct temperature (T_{ci}).

6. The outputs are the hot duct temperature (T_{co}), of the mixed gas; the reactivity affecting temperatures, mean uranium ($\overline{T}_u$), mean can ($\overline{T}_a$) and the two moderators ($\overline{T}_{m1}$), ($\overline{T}_{m2}$); and the maximum temperatures of the uranium (T_{mu}) and the can (T_{ma}).

7. The object of the present paper is to present equations which will enable the various output variables mentioned above to be determined as time-varying functions of P, W and T_{ci}.

8. Two cases are considered, one with a reactor having graphite sleeves to support the fuel element, and the other for a reactor with a solid moderator and no sleeves.

4. BASIC PRINCIPLES.

9. As the reactor heat transfer equations are only one part of the complete reactor simulation, it is desirable that these equations are not more complicated or extensive than necessary. Because of this, it is essential to reduce the three-dimensional time varying basic partial differential equations to a set of ordinary differential equations which eliminate spatial effects.

10. This is accomplished by considering first a unit length of one channel of the reactor; producing the partial differential equations for this point; and integrating these along the length of that channel; then the resulting equations are extended to include the entire core.

11. The numerical values of the various parameters are then determined from a knowledge of the steady state full power value of these parameters. In order to make the best use of the fuel element cans, it is essential that the specified maximum can temperature shall be attained in each channel. As the reactor power output per channel is less at the edges of the core than the centre, it is necessary either to fit a gas throttle to each channel, or to vary the channel dimensions across a core, or both.

12. One result of this design feature is that the gas outlet temperature from the channels varies only slightly across the core, being higher at the edges.

13. Assumption 1.

Reactor heat transfer equations applying satisfactorily to one channel will apply equally to the entire core, provided that suitable mean values are used for the parameters in the equations.

In a power producing reactor the radial flux shape is usually flattened in the central regions by the introduction of absorbing material; thus a large number of the channels will behave identically. To take the overall behaviour of all channels into account it is usually adequate to use the statistically weighted parameters for the entire reactor to define a single "effective channel", representative of the whole core.

14. The steady state temperature distributions of the gas and fuel elements in one channel are indicated in fig. 2. The positions along the channel where the maximum uranium and maximum can temperature occur are not the same, but the difference is small.

15. Consider a unit length of a single channel. The arrangement of the components is shown in fig. 3.

The channel consists of a fuel element (uranium bar and can), the cylindrical graphite-supporting sleeve for the fuel element, and the main graphite. The latter is arranged for convenience of calculation in a cylindrical geometry, and the resulting "cell" is identical with that taken for nuclear physical calculations.

The steady state temperature profile at one section of the channel is indicated on the figure, and also the local mean temperature of the components.

16. Assumptions 2 - 15.

2. The heat generation along the channel is equivalent to a cosine distribution based on a length equal to the total uranium length plus the flux extrapolation distance at each end of the channel.

   It can be shown from a consideration of the actual discontinuous heat production that this is very nearly correct for heat transfer. The flux distortions due to temperature, poisoning, control rods and end effects will not in general have a large effect on the averaged temperatures.

3. The heat generation is divided between the uranium and the moderator in a ratio which is constant along the channel.

4. The radial heat flow out of the graphite surface of a cell is zero. This will be true in the flattened flux region, and nearly true elsewhere, in view of assumption 1.

5. All axial heat conduction in the solids is negligible compared to the radial heat transfer. This may be demonstrated numerically to be reasonable.

6. The convective heat transfer from the can and the moderators to the gas is proportional to the difference between the mean temperatures of the solid concerned and the gas.

7. The radiative heat transfer between the can and the moderators is proportional to the difference of the fourth powers of the appropriate absolute mean temperatures, at a point in the channel. Provided that the surface temperatures are close to the mean temperatures, this will be reasonable.

8. The radiation absorption in the gas is negligible.

9. The uranium temperature distribution at a point in the channel retains its steady state shape throughout any transient. The effective time-constant for the change in temperature profile is of the order of 10 seconds, and consequently for most transients, this assumption will hold reasonably.

10. The can temperature is substantially uniform throughout its thickness. The error produced by this assumption is small.

11. The gas temperatures inside and outside the graphite sleeve are each uniform in the radial direction. Experimental checks have been made on this assumption.

12. The temperature distributions of the moderators at a point in the channel each retain their steady state shape during a transient.

    For the sleeve, the shape will be very nearly linear under all conditions. For the main moderator, the shape will probably change somewhat during transients, and the assumption could be eliminated at the expense of complexity by considering a "many-regioned" moderator in the same way that the "two-regioned" system is devised (see Section 8).

13. All specific heats, thermal conductivities, heat transfer coefficients, gas densities and velocities are independent of heat rate and temperature.

The change with temperature of most of these parameters is not large, and suitable average values taken along the channel will give reasonable simulation. For transients involving large temperature changes, the constants (particularly specific heats) need adjustment to a new average value, or should be included as temperature varying functions.

14. The heat content of each component is adequately represented by the appropriate mean temperature.

As arithmetic mean temperatures across the cell are used, and the radial change in specific heat is small, this assumption is justified.

15. There is no gas leakage through the sleeve.

Some leakage will occur, but not sufficient to affect the heat transfer characteristics.

17. From assumption 9, and noting that the heat loss from the uranium is proportional to the temperature-radius slope $\partial T_u/\partial r$ at the edge, then this loss is also proportional to $(T_u - T_a)$.

5. DERIVATION OF AVERAGE EQUATIONS.

18. The heat balance equations at a section of the channel distance ℓ from the cold end may now be written down, thus:-

Heat gain in uranium	=	heat produced - heat lost by conduction
Heat gain in can	=	heat transferred in by conduction - heat lost to gas by convection - heat lost to sleeve by radiation
Heat gain in sleeve	=	heat produced + heat gained from can by radiation - heat lost to both gas streams by convection - heat lost to main moderator by radiation
Heat gain in main moderator	=	heat produced + heat gain by radiation from sleeve - heat lost by convection to gas

Heat gain in each gas stream = heat transferred to each stream by convection

19. So, symbolically:-

$$C_v \frac{\partial T_v}{\partial t} = \alpha H_{max} \cos \frac{\pi}{L_1}\left(l - \frac{L}{2}\right) - U_1 (T_v - T_a) \quad (1)$$

$$C_a \frac{\partial T_a}{\partial t} = U_1 (T_v - T_a) - U_2 (T_a - T_{c1}) - R_{am1} \quad (2)$$

$$C_{m1} \frac{\partial T_{m1}}{\partial t} = \beta H_{max} \cos \frac{\pi}{L_1}\left(l - \frac{L}{2}\right) - U_3 (T_{m1} - T_{c1}) - U_4 (T_{m1} - T_{c2}) + R_{am1} - R_{m1m2} \quad (3)$$

$$C_{m2} \frac{\partial T_{m2}}{\partial t} = \gamma H_{max} \cos \frac{\pi}{L_1}\left(l - \frac{L}{2}\right) - U_5 (T_{m2} - T_{c2}) + R_{m1m2} \quad (4)$$

$$C_{c1}\left(\frac{\partial T_{c1}}{\partial t} + v_1 \frac{\partial T_{c1}}{\partial l}\right) = U_2 (T_a - T_{c1}) + U_3 (T_{m1} - T_{c1}) \quad (5)$$

$$C_{c2}\left(\frac{\partial T_{c2}}{\partial t} + v_2 \frac{\partial T_{c2}}{\partial l}\right) = U_4 (T_{m1} - T_{c2}) + U_5 (T_{m2} - T_{c2}) \quad (6)$$

20. The axial statistically weighted mean values are determined by multiplying each term by the (flux shape factor)^2, i.e. $\cos^2 \frac{\pi}{L_1}\left(l - \frac{L}{2}\right)$, and integrating along the length of the channel.

Thus, for a variable X, the statistically weighted mean value, $\overline{X}$, is given by

$$\overline{X} \int_0^L \cos^2 \frac{\pi}{L_1}\left(l - \frac{L}{2}\right) dl = \int_0^L X \cos^2 \frac{\pi}{L_1}\left(l - \frac{L}{2}\right) dl \quad (7)$$

21. Performing the operation on equations (1) to (6) above, using the values of the resulting integrals from Appendix 1, gives:-

$$C_v \frac{d\overline{T}_v}{dt} = \frac{\alpha k P_c}{L} - U_1 (\overline{T}_v - \overline{T}_a) \quad (8)$$

$$C_a \frac{d\overline{T}_a}{dt} = U_1 (\overline{T}_v - \overline{T}_a) - U_2 (\overline{T}_a - \overline{T}_{c1}) - \overline{R}_{am1} \quad (9)$$

$$C_{m1} \frac{d\overline{T}_{m1}}{dt} = \beta \frac{k P_c}{L} - U_3 (\overline{T}_{m1} - \overline{T}_{c1}) - U_4 (\overline{T}_{m1} - \overline{T}_{c2}) + \overline{R}_{am1} - \overline{R}_{m1m2} \quad (10)$$

$$C_{m2} \frac{d\overline{T}_{m2}}{dt} = \frac{\gamma k P_c}{L} - U_5 (\overline{T}_{m2} - \overline{T}_{c2}) + \overline{R}_{m1m2} \quad (11)$$

$$C_{c1}\left\{\frac{d\overline{T}_{c1}}{dt} + \frac{k v_1}{L}(T_{c10} - T_{ci}) + \frac{2 v_1 \pi}{L L_1 \left(1 + \frac{L_1}{\pi L} \sin \frac{\pi L}{L_1}\right)} \int_0^L \left[T_{c1} - \frac{(T_{c10} - T_{ci}) \sin \frac{\pi}{L_1}\left(l - \frac{L}{2}\right)}{2 \sin \frac{\pi L}{2 L_1}}\right] \sin \frac{2\pi}{L_1}\left(l - \frac{L}{2}\right) dl\right\} = U_2 (\overline{T}_a - \overline{T}_{c1}) + U_3 (\overline{T}_{m1} - \overline{T}_{c1}) \quad (12)$$

$$C_{c2}\left\{\frac{d\overline{T}_{c2}}{dt}+\frac{k v_2}{L}(T_{c20}-T_{ci})+\frac{2v_2\pi}{LL_1\left(1+\frac{L_1}{\pi L}\sin\frac{\pi L}{L_1}\right)}\int_0^L\left[T_{c2}-\frac{(T_{c20}-T_{ci})\sin\frac{\pi}{L_1}\left(l-\frac{L}{2}\right)}{2\sin\frac{\pi L}{2L_1}}\right]\sin\frac{2\pi}{L_1}\left(l-\frac{L}{2}\right)dl\right\}$$
$$=U_4(\overline{T}_{m1}-\overline{T}_{c2})+U_5(\overline{T}_{m2}-\overline{T}_{c2}) \quad (13)$$

22\. Assumption 16.

The radiation average $\overline{R}aml$ which by definition is given by $R_A\{\overline{(T_a^4)}-\overline{(T_{m1}^4)}\}$, is equal to $R_A\{(\overline{T}_a)^4-(\overline{T}_{m1})^4\}$

This assumption, which has no physical basis, was determined numerically in a variety of cases, and found quite accurate.

23\. The steady states of the equations (1) to (6) may be found as outlined in Appendix 11, and introducing the normal design requirement that Tc10 = Tc20 in any steady state, then, neglecting the radiation terms:-

(a) $$T_{c1}=T_{c2}=\frac{T_{ci}+T_{co}}{2}+\frac{(T_{co}-T_{ci})\sin\frac{\pi}{L_1}\left(l-\frac{L}{2}\right)}{2\sin\frac{\pi L}{2L_1}} \quad (14)$$

(b) $$\overline{T}_{c1}=\overline{T}_{c2}=\frac{T_{ci}+T_{co}}{2} \quad (15)$$

(c) The integrals in equations (12) and (13) both vanish.

24\. Assumptions 17-18.

17\. The presence of radiation has a negligible effect on the above steady states. The error is usually about 3% at full power.

18\. In any transient state, the integrals in equations (12) and (13) both vanish, and the temperatures Tc10 and Tc20 will be given by:

$$\overline{T}c1 = \frac{Tci + Tc10}{2} \quad (16)$$

$$\overline{T}c2 = \frac{Tci + Tc20}{2} \quad (17)$$

Assumption 18 is probably the most important of the assumptions necessary, since without it the axial averaging would not be readily possible. Considerable analytical and computer work has established that for a wide range of conditions the reactivity controlling mean temperatures remain accurate, even if sometimes the outlet gas and maximum temperatures suffer reductions in accuracy. In general, provided that the gas transit time up the channel remains small compared to the time taken for significant changes in the variables, the assumption will be reasonable.

25. From assumption 13, the gas velocities at any point in the channel may be written in terms of the channel gas flow, thus:

$$\left.\begin{aligned} v_1 &= b_1 W_c \\ v_2 &= b_2 W_c \end{aligned}\right\} \qquad (18)$$

26. The mixed gas temperatures T_{co} will result from T_{c10} and T_{c20}, such that

$$(C_{c1}v_1 + C_{c2}v_2)T_{co} = C_{c1}v_1 T_{c10} + C_{c2}v_2 T_{c20}$$

or, using equations (16) and (17),

$$T_{co} = \frac{2\{C_{c1}b_1\bar{T}_{c1} + C_{c2}b_2\bar{T}_{c2}\}}{C_{c1}b_1 + C_{c2}b_2} - T_{ci} \qquad (19)$$

27. Using equations (16) to (18) in (12) and (13) gives:

$$C_{c1}\left\{\frac{d\bar{T}_{c1}}{dt} + \frac{2kb_1W_c}{L}(\bar{T}_{c1} - T_{ci})\right\} = U_2(\bar{T}_a - \bar{T}_{c1}) + U_3(\bar{T}_{m1} - \bar{T}_{c1}) \qquad (20)$$

$$C_{c2}\left\{\frac{d\bar{T}_{c2}}{dt} + \frac{2kb_2W_c}{L}(\bar{T}_{c2} - T_{ci})\right\} = U_4(\bar{T}_{m1} - \bar{T}_{c2}) + U_5(\bar{T}_{m2} - \bar{T}_{c2}) \qquad (21)$$

and the requirement to ensure that $T_{c10} = T_{c20}$ in the steady state:

$$\frac{C_{c1}b_1}{C_{c2}b_2} = \frac{\alpha + \frac{U_3}{U_3+U_4}\beta}{\gamma + \frac{U_4}{U_3+U_4}\beta} \qquad (22)$$

This last equation, as noted above, neglects the effects of radiation.

6. DERIVATION OF MAXIMUM EQUATIONS.

28. Equations (8) to (11) and (19) to (22) are sufficient to produce the reactivity controlling temperatures, and the gas outlet temperature.

In order to find the maximum uranium and can temperatures further equations are necessary.

29\. Assumption 19.

The maximum uranium and maximum can temperatures occur at the same point along the channel. The error here is very small.

30\. Consider a unit length of channel at the point where the maximum temperatures occur, which is shown in fig. (4).

From assumption 9, following the method used before, the heat loss from the uranium is proportional to $(T_{u\ell} - T_{ma})$, and also:

$$\frac{T_{mu} - T_{ma}}{T_{u\ell} - T_{ma}} = k_8 \text{, a constant} \qquad (23)$$

31\. Following the method used for deriving the mean equations, the maximum equations may be written down:

$$C_u \frac{dT_{u\ell}}{dt} = \alpha H_\ell - U_6 (T_{u\ell} - T_{ma}) \qquad (24)$$

$$C_a \frac{dT_{ma}}{dt} = U_6 (T_{u\ell} - T_{ma}) - U_7 (T_{ma} - T_{c\ell}) - R_{ma\,m\ell} \qquad (25)$$

32\. Assumptions 20 - 23.

20\. The ratio $\frac{T_{c\ell} - T_{ci}}{T_{c_1} - T_{ci}}$ is constant in any transient,

$$= k_9 \text{, say} \qquad (26)$$

This is equivalent to assuming that equation (14) holds in the transient state, and that the point where the maximum temperatures occur does not vary with time. The accuracy of the assumption will fall with decrease in both power and gas flow.

21\. The sleeve temperature $T_{m\ell}$ may be taken equal to $T_{c\ell}$ for radiation purposes. The effect of small changes in the lower temperature of the radiation term is not large, in general. This assumption eliminates the need for another equation to represent $T_{m\ell}$, and so avoids a complete duplication of mean and maximum equations.

22\. The convective heat transfer coefficients are functions of local gas flow only.

The dependence on flow is usually measured for heat transfer coefficients based on surface temperatures, and the assumption will only be true using such functions if the mean temperatures $\overline{T}a$, Tma, $\overline{T}m1$, $\overline{T}m2$ are close to their appropriate surface temperatures. This is so, in general.

23. The ratio of gas flows inside and outside the sleeve is constant. This will be so for turbulent flow in both regions. As the ratio is b_1/b_2, this ensures that if equation (22) holds for one power and flow, it will hold very nearly for all powers and flows, as U_3 and U_4 are generally the same functions of flow.

33. From assumption 1, the relationship W = mWc may be written, where m is a constant. Hence, with this and assumptions 22, 23, the convective heat transfer coefficients may be written

$$U_n = k_n f_n(W), \quad n = 2, 3, 4, 5, 7 \tag{27}$$

f_2, f_7 will be identical, and in general f_3, f_4, f_5 will also be identical, the latter being closely represented by $W^{0.8}$, as the moderator surfaces are smooth.

34. From assumption 2, H_ℓ may be written in terms of P_c:

$$H_\ell = \frac{\delta P_c}{L} \tag{28}$$

From assumption 1, P_c may be written in terms of P:

$$P = n P_c \tag{29}$$

where both δ and n are constants.

35. Collecting all the relevant equations gives the final set:-

$$C_v \frac{d\overline{T}_v}{dt} = \frac{\alpha k P}{nL} - U_1(\overline{T}_v - \overline{T}_a) \tag{30}$$

$$C_a \frac{d\overline{T}_a}{dt} = U_1(\overline{T}_v - \overline{T}_a) - k_2 f_2(W).(\overline{T}_a - \overline{T}_{c1}) - \overline{R}_{am1} \tag{31}$$

$$C_v \frac{dT_{v\ell}}{dt} = \frac{\alpha \delta P}{nL} - U_6(T_{v\ell} - T_{ma}) \tag{32}$$

$$C_a \frac{dT_{ma}}{dt} = U_6(T_{v\ell} - T_{ma}) - k_7 f_7(W).(T_{ma} - T_{c\ell}) - R_{mac\ell} \tag{33}$$

$$C_{m1}\frac{d\overline{T}_{m1}}{dt} = \frac{\beta k P}{nL} - k_3 f_3(W)(\overline{T}_{m1}-\overline{T}_{c1}) - k_4 f_4(W)(\overline{T}_{m1}-\overline{T}_{c2}) + \overline{R}_{am1} - \overline{R}_{m1m2} \quad (34)$$

$$C_{m2}\frac{d\overline{T}_{m2}}{dt} = \frac{\gamma k P}{nL} - k_5 f_5(W)(\overline{T}_{m2}-\overline{T}_{c2}) + \overline{R}_{m1m2} \quad (35)$$

$$C_{c1}\left\{\frac{d\overline{T}_{c1}}{dt} + \frac{2kb_1W}{mL}(\overline{T}_{c1}-T_{ci})\right\} = k_2 f_2(W)(\overline{T}_a-\overline{T}_{c1}) + k_3 f_3(W)(\overline{T}_{m1}-\overline{T}_{c1}) \quad (36)$$

$$C_{c2}\left\{\frac{d\overline{T}_{c2}}{dt} + \frac{2kb_2W}{mL}(\overline{T}_{c2}-T_{ci})\right\} = k_4 f_4(W)(\overline{T}_{m1}-\overline{T}_{c2}) + k_5 f_5(W)(\overline{T}_{m2}-\overline{T}_{c2}) \quad (37)$$

$$T_{c\ell} = k_9\overline{T}_{c1} - (k_9-1)T_{ci} \quad (38)$$

$$T_{mu} = k_8 T_{u\ell} - (k_8-1)T_{ma} \quad (39)$$

$$T_{co} = \frac{2\{C_{c1}b_1\overline{T}_{c1} + C_{c2}b_2\overline{T}_{c2}\}}{C_{c1}b_1 + C_{c2}b_2} - T_{ci} \quad (40)$$

7. CALCULATION OF PARAMETERS.

36.. The most straightforward method of obtaining the various constants in the equations (30) to (40) is, where possible, to determine the appropriate full power steady state values of the variables, and solve the steady state equations for the constants with these values of the variables substituted.

37.. The mean values to apply to the equations are determined from the reactor as a whole. Since statistically weighted mean temperatures are required, the temperatures, power and flow must all be weighted appropriately. The temperatures must be arithmetically averaged across the cell, and statistically weighted along the channel and across the core.

38.. The value of n is determined by calculating the statistically weighted mean channel power P_c, and relating this to the total power produced, P.

39.. The constant δ is found by calculating from the steady state data the actual heat output H_ℓ, in the centre channel, and relating this to P_c for the mean channel.

40.. The constants α, β, γ and k are determined from nuclear physical considerations.

41.. Equation (22) gives the ratio b_1/b_2.

42.. The thermal capacities Cu, Ca, Cm1, Cm2, Cc1, Cc2 must be calculated from the unit length of the channel.

43.. The values R_A and R_B are found from the calculated (or measured) heat radiation between the elements at the specified mean temperatures in the full power state.

44.. All the other constants may be determined from the above, by substituting the statistically weighted mean temperatures at full power into the equations. It is usually reasonable and more straightforward to neglect the radiation terms at this stage, the error being small.

45.. It should be noted that using the ratio b_1/b_2 in equations (36) and (37) removes the necessity of calculating m, the term $\frac{2k}{mL}$ being found as a whole.

46.. If the units of temperature are degrees centigrade, power, megawatts and time seconds, then the units of the equations are MW/ft. The various terms are: for the mean equations, the statistically weighted mean MW/ft. for the reactor as a whole; and for the maximum equations, the actual MW/ft. at the maximum point in the centre channel.

8. EXTENSION OF EQUATIONS FOR A SOLID MODERATOR REACTOR.

47.. If the reactor core does not include graphite fuel element sleeves, somewhat different problems arise. The surface of the moderator will display the same reactivity behaviour and temperature transients as a sleeve, particularly when, under high irradiation, the graphite thermal conductivity has fallen considerably.

48.. Since moderator relative reactivity coefficients are known at present only in terms of two cylindrical regions of moderator, it is necessary to produce two mean temperatures of the moderator for such regions.

49.. The unit length of moderator in a channel is shown in Fig. 5.

Two cylindrical regions are defined - the inner diameter is that of the channel, the outer diameter is that of the cell, to give the correct graphite volume, and the interface is arranged so that the ratio of the two regions is within the range for which experimental results are available.

50. For the regions, arithmetic mean temperatures T_{ms} for the "surface" region, and T_{mb} for the "bulk" region are defined, both for heat transfer and thermal capacity, making use of assumption 14.

51. The heat input to the surface region is identical to that of the sleeve, and the heat production in each region is the same as that for the sleeve and main moderator.

52. Assumption 24.

The heat transfer between the regions is proportional to (Tms-Tmb). Analytical checks on the final equations show that this assumption is valid over a wide range of operating conditions.

53. Then the partial differential equations become:

$$C_{ms}\frac{\partial T_{ms}}{\partial t} = \beta H_{max}\cos\frac{\pi}{L_1}\left(\ell-\frac{L}{2}\right) - U_3(T_{ms}-T_{c1}) - U_{10}(T_{ms}-T_{mb}) + R_{ams} \quad (41)$$

$$C_{mb}\frac{\partial T_{mb}}{\partial t} = \gamma H_{max}\cos\frac{\pi}{L_1}\left(\ell-\frac{L}{2}\right) + U_{10}(T_{ms}-T_{mb}) \quad (42)$$

54. These equations are then averaged exactly as for equations (1) to (6) giving, finally

$$C_{ms}\frac{d\bar{T}_{ms}}{dt} = \frac{\beta k P}{nL} - k_3 f_3(w)(\bar{T}_{ms}-\bar{T}_{c1}) - U_{10}(\bar{T}_{ms}-\bar{T}_{mb}) + \bar{R}_{ams} \quad (43)$$

$$C_{mb}\frac{d\bar{T}_{mb}}{dt} = \frac{\gamma k P}{nL} + U_{10}(\bar{T}_{ms}-\bar{T}_{mb}) \quad (44)$$

55. Equations (43), (44) replace equations (34), (35)

Equations (36), (37), (40) are replaced by

$$C_{c1}\left\{\frac{d\bar{T}_{c1}}{dt} + \frac{2kW}{mL}(\bar{T}_{c1}-T_{ci})\right\} = k_2 f_2(w)(\bar{T}_a-\bar{T}_{c1}) + k_3 f_3(w)(\bar{T}_{ms}-\bar{T}_{c1}) \quad (45)$$

$$T_{co} = 2\bar{T}_{c1} - T_{ci} \quad (46)$$

The final heat transfer equations are then (30) to (33), (38), (39) and (43) to (46), with $\bar{R}$aml in (31) changed to $\bar{R}$ams.

9. CONCLUSIONS

56. Two sets of equations have been produced, one for a reactor having graphite sleeves supporting the fuel elements, and the other for a reactor with a solid moderator and no sleeves.

57. The assumptions involved in each case have been detailed, and briefly discussed. They are in general of a physical rather than mathematical nature, and most may be shown to be at least reasonable.

58. Certain of the assumptions are capable of being reduced or modified, and work is still proceeding in the development of these equations.

59. The radiation terms, while of little importance in the full power steady state, become large in certain fault conditions, and their inclusion is essential.

10. ACKNOWLEDGEMENTS AND REFERENCES

60. Very little published work exists on this subject. A simplified derivation of reactor heat transfer equations is given in:

WOODROW, J. 'Thermal Time Lag in Air-Cooled Pile'

A.E.R.E. Report No. E/R.142

61. The author is indebted to Dr. A.J. Hitchcock of U.K.A.E.A. Risley for various comments relating to the derivation of the equations, and to the Directors of the General Electric Company Limited, of England, for permission to publish this paper.

11. APPENDIX 1.

Certain integrals which occur in the text

(i) $$\int_0^L \cos^2\frac{\pi}{L_1}\left(l-\frac{L}{2}\right)dl = \frac{1}{2}\left[l + \frac{L_1}{2\pi}\sin\frac{2\pi}{L_1}\left(l-\frac{L}{2}\right)\right]_0^L$$

$$= \frac{L}{2}\left[1 + \frac{L_1}{\pi L}\sin\frac{\pi L}{L_1}\right] \qquad (47)$$

(ii) $$\int_0^L \cos^3\frac{\pi}{L_1}\left(l-\frac{L}{2}\right)dl = \frac{L_1}{4\pi}\left[\frac{1}{3}\sin\frac{3\pi}{L_1}\left(l-\frac{L}{2}\right) + 3\sin\frac{\pi}{L_1}\left(l-\frac{L}{2}\right)\right]_0^L$$

$$= \frac{L_1}{4\pi}\left[\frac{1}{3}\sin\frac{3\pi L}{2L_1} + 3\sin\frac{\pi L}{2L_1}\right]$$

$$= \frac{L_1}{2\pi}\left[4\sin\frac{\pi L}{2L_1} - \frac{4}{3}\sin^3\frac{\pi L}{2L_1}\right]$$

$$= \frac{L_1}{3\pi}\sin\frac{\pi L}{2L_1}\left(5 + \cos\frac{\pi L}{L_1}\right) \qquad (48)$$

(iii) $$P_c = \int_0^L H_{max}\cos\frac{\pi}{L_1}\left(l-\frac{L}{2}\right)dl = H_{max}.\frac{2L_1}{\pi}\sin\frac{\pi L}{2L_1} \qquad (49)$$

(iv) $$\frac{\int_0^L H_{max}\cos^3\frac{\pi}{L_1}\left(l-\frac{L}{2}\right)dl}{\int_0^L \cos^2\frac{\pi}{L_1}\left(l-\frac{L}{2}\right)dl} = \frac{\frac{H_{max}.L_1}{3\pi}\left(5+\cos\frac{\pi L}{L_1}\right)\sin\frac{\pi L}{2L_1}}{\frac{L}{2}\left(1+\frac{L_1}{\pi L}\sin\frac{\pi L}{L_1}\right)}$$

$$= P_c\,\frac{5+\cos\frac{\pi L}{L_1}}{3\left(1+\frac{L_1}{\pi L}\sin\frac{\pi L}{L_1}\right)}$$

$$= \frac{kP_c}{L} \qquad (50)$$

(v) $$\frac{2v_1}{L\left(1+\frac{L_1}{\pi L}\sin\frac{\pi L}{L_1}\right)}\int_0^L \frac{\partial T_{c1}}{\partial l}\cos^2\frac{\pi}{L_1}\left(l-\frac{L}{2}\right)dl$$

Integrating by parts gives

$$\frac{2v_1}{L\left(1+\frac{L_1}{\pi L}\sin\frac{\pi L}{L_1}\right)}\left[T_{c1}\cos^2\frac{\pi}{L_1}\left(l-\frac{L}{2}\right)\right]_0^L + \frac{2v_1\pi}{LL_1\left(1+\frac{L_1}{\pi L}\sin\frac{\pi L}{L_1}\right)}\int_0^L T_{c1}\sin\frac{2\pi}{L_1}\left(l-\frac{L}{2}\right)dl$$

$$= \frac{2v_1\left[T_{c1o}-T_{ci}\right]\cos^2\frac{\pi L}{2L_1}}{L\left(1+\frac{L_1}{\pi L}\sin\frac{\pi L}{L_1}\right)} + \frac{2v_1\pi}{LL_1\left(1+\frac{L_1}{\pi L}\sin\frac{\pi L}{L_1}\right)}\int_0^L T_{c1}\sin\frac{2\pi}{L_1}\left(l-\frac{L}{2}\right)dl \qquad (51)$$

Now from Appendix II, the steady state value of Tc1 is given by

$$T_{c1} - \frac{T_{ci}+T_{ao}}{2} - \frac{(T_{c10}-T_{ci})\sin\frac{\pi}{L_1}(l-\frac{L}{2})}{2\sin\frac{\pi L}{2L_1}} = 0$$

The integral in (51) can be modified by forming an identity with the above expression.

Thus, multiplying by $\sin\frac{2\pi}{L_1}(l-\frac{L}{2})$ and integrating gives

$$\int_0^L \left\{ T_{c1} - \frac{T_{ci}+T_{c10}}{2} - \frac{(T_{c10}-T_{ci})}{2\sin\frac{\pi L}{2L_1}}\sin\frac{\pi}{L_1}(l-\frac{L}{2}) \right\} \sin\frac{2\pi}{L_1}(l-\frac{L}{2})\,dl$$

$$= \int_0^L \left\{ T_{c1} - \frac{T_{c10}-T_{ci}}{2\sin\frac{\pi L}{2L_1}}\sin\frac{\pi}{L_1}(l-\frac{L}{2}) \right\} \sin\frac{2\pi}{L_1}(l-\frac{L}{2})\,dl$$

as T_{ci} and T_{c10} are constant for the integration

$$= \int_0^L T_{c1}\sin\frac{2\pi}{L_1}(l-\frac{L}{2})\,dl - \frac{(T_{c10}-T_{ci})}{2\sin\frac{\pi L}{2L_1}}\int_0^L 2\sin^2\frac{\pi}{L_1}(l-\frac{L}{2})\cos\frac{\pi}{L_1}(l-\frac{L}{2})\,dl$$

$$= \int_0^L T_{c1}\sin\frac{2\pi}{L_1}(l-\frac{L}{2})\,dl - \frac{L_1(T_{c10}-T_{ci})}{\pi\sin\frac{\pi L}{2L_1}}\left[\frac{\sin^3\frac{\pi}{L_1}(l-\frac{L}{2})}{3}\right]_0^L$$

$$= \int_0^L T_{c1}\sin\frac{2\pi}{L_1}(l-\frac{L}{2})\,dl - \frac{2L_1(T_{c10}-T_{ci})\sin^2\frac{\pi L}{2L_1}}{3\pi}$$

and the identity is:-

$$\int_0^L T_{c1}\sin\frac{2\pi}{L_1}(l-\frac{L}{2})\,dl = \frac{2L_1(T_{c10}-T_{ci})}{3\pi}\sin^2\frac{\pi L}{2L_1}$$

$$+ \int_0^L \left\{ T_{c1} - \frac{(T_{c10}-T_{ci})\sin\frac{\pi}{L_1}(l-\frac{L}{2})}{2\sin\frac{\pi L}{2L_1}} \right\} \sin\frac{2\pi}{L_1}(l-\frac{L}{2})\,dl \qquad (52)$$

Substituting (52) in (51) gives the integral expression (V) as

$$\frac{2v_1(T_{c10}-T_{ci})\cos^2\frac{\pi L}{2L_1}}{L\left(1+\frac{L_1}{\pi L}\sin\frac{\pi L}{L_1}\right)} + \frac{2v_1\pi}{LL_1\left(1+\frac{L_1}{\pi L}\sin\frac{\pi L}{L_1}\right)} \times \frac{2L_1(T_{c10}-T_{ci})\sin^2\frac{\pi L}{2L_1}}{3\pi}$$

$$+\frac{2v_1\pi}{LL_1\left(1+\frac{L_1}{\pi L}\sin\frac{\pi L}{L_1}\right)}\int_0^L\left\{T_{c1}-\frac{(T_{c10}-T_{ci})\sin\frac{\pi}{L_1}\left(l-\frac{L}{2}\right)}{2\sin\frac{\pi L}{2L_1}}\right\}\sin\frac{2\pi}{L_1}\left(l-\frac{L}{2}\right)dl$$

which reduces to

$$\frac{kv_1(T_{c10}-T_{ci})}{L}+\frac{2v_1\pi}{LL_1\left(1+\frac{L_1}{\pi L}\sin\frac{\pi L}{L_1}\right)}\int_0^L\left\{T_{c1}-\frac{(T_{c10}-T_{ci})\sin\frac{\pi}{L_1}\left(l-\frac{L}{2}\right)}{2\sin\frac{\pi L}{2L_1}}\right\}\sin\frac{2\pi}{L_1}\left(l-\frac{L}{2}\right)dl \qquad (53)$$

12. APPENDIX II.

Steady State Solution - outline of approach

Taking the steady states of equations (1) to (6), and eliminating directly all terms except Tc1 and Tc2 gives

$$C_{c1} v_1 \frac{\partial T_{c1}}{\partial \ell} = \left(\alpha + \frac{U_3}{U_3+U_4}\beta\right) H_{max} \cos\frac{\pi}{L_1}\left(\ell - \frac{L}{2}\right) - \frac{U_3 U_4}{U_3+U_4}(T_{c1} - T_{c2}) - \frac{U_4 R_{am1} + U_3 R_{m1m2}}{U_3+U_4} \qquad (54)$$

$$C_{c2} v_2 \frac{\partial T_{c2}}{\partial \ell} = \left(\gamma + \frac{U_4}{U_3+U_4}\beta\right) H_{max} \cos\frac{\pi}{L_1}\left(\ell - \frac{L}{2}\right) + \frac{U_3 U_4}{U_3+U_4}(T_{c1} - T_{c2}) + \frac{U_4 R_{am1} + U_3 R_{m1m2}}{U_3+U_4} \qquad (55)$$

Multiplying (54) by $C_{c2} v_2$ and (55) by $C_{c1} v_1$, subtracting and dividing by $C_{c1} C_{c2} v_1 v_2$ gives, neglecting the radiation terms

$$\frac{\partial}{\partial \ell}(T_{c1} - T_{c2}) = A \cos\frac{\pi}{L_1}\left(\ell - \frac{L}{2}\right) - B(T_{c1} - T_{c2}) \qquad (56)$$

where $A . C_{c1} C_{c2} v_1 v_2 = \left\{ C_{c2} v_2 \left(\alpha + \frac{U_3}{U_3+U_4}\beta\right) - C_{c1} v_1 \left(\gamma + \frac{U_4}{U_3+U_4}\beta\right)\right\} H_{max}$

and $B . C_{c1} C_{c2} v_1 v_2 = \frac{U_3 U_4}{U_3+U_4}(C_{c1} v_1 + C_{c2} v_2)$

Integrating (50),

$$T_{c1} - T_{c2} = F e^{-B\ell} + \frac{A}{B^2 + \frac{\pi^2}{L_1^2}}\left[B \cos\frac{\pi}{L_1}\left(\ell - \frac{L}{2}\right) + \frac{\pi}{L_1}\sin\frac{\pi}{L_1}\left(\ell - \frac{L}{2}\right)\right]$$

and the boundary conditions are Tc1-Tc2 = 0 at ℓ = 0 and ℓ = L

$$\left.\begin{aligned} 0 &= F + \frac{A}{B^2 + \frac{\pi^2}{L_1^2}}\left[B \cos\frac{\pi L}{2L_1} - \frac{\pi}{L_1}\sin\frac{\pi L}{2L_1}\right] \\ 0 &= F e^{-BL} + \frac{A}{B^2 + \frac{\pi^2}{L_1^2}}\left[B \cos\frac{\pi L}{2L_1} + \frac{\pi}{L_1}\sin\frac{\pi L}{2L_1}\right] \end{aligned}\right\} \qquad (57)$$

and the only legitimate solution of equations (57) is F = A = 0

Thus: (a) Tc1 = Tc2 everywhere

$$\text{(b)} \quad \frac{C_{c1} v_1}{C_{c2} v_2} = \frac{\alpha + \frac{U_3}{U_3+U_4}\beta}{\gamma + \frac{U_4}{U_3+U_4}\beta} \tag{58}$$

Then, adding (54) and (55) gives finally

$$\text{(c)} \quad T_{c1} = T_{c2} = \frac{T_{ci} + T_{co}}{2} + \frac{(T_{co} - T_{ci}) \sin \frac{\pi}{L_1}(\ell - \frac{L}{2})}{2 \sin \frac{\pi L}{2L_1}} \tag{59}$$

the last result including radiation if (a) holds in that case.

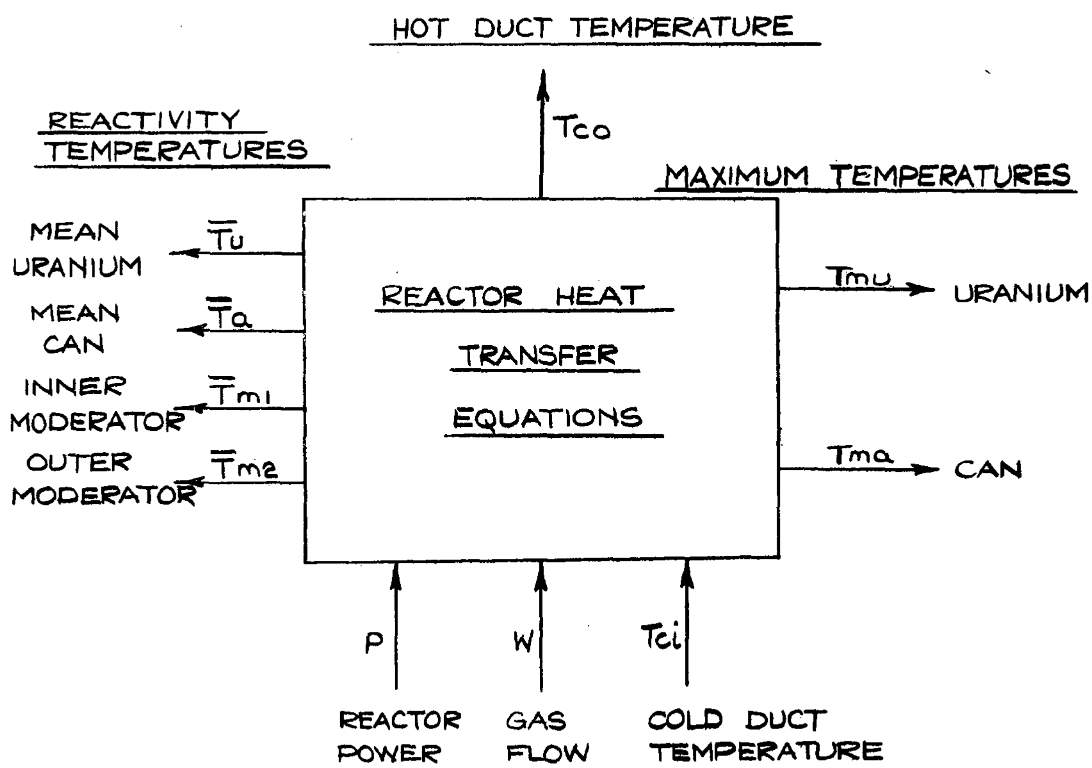
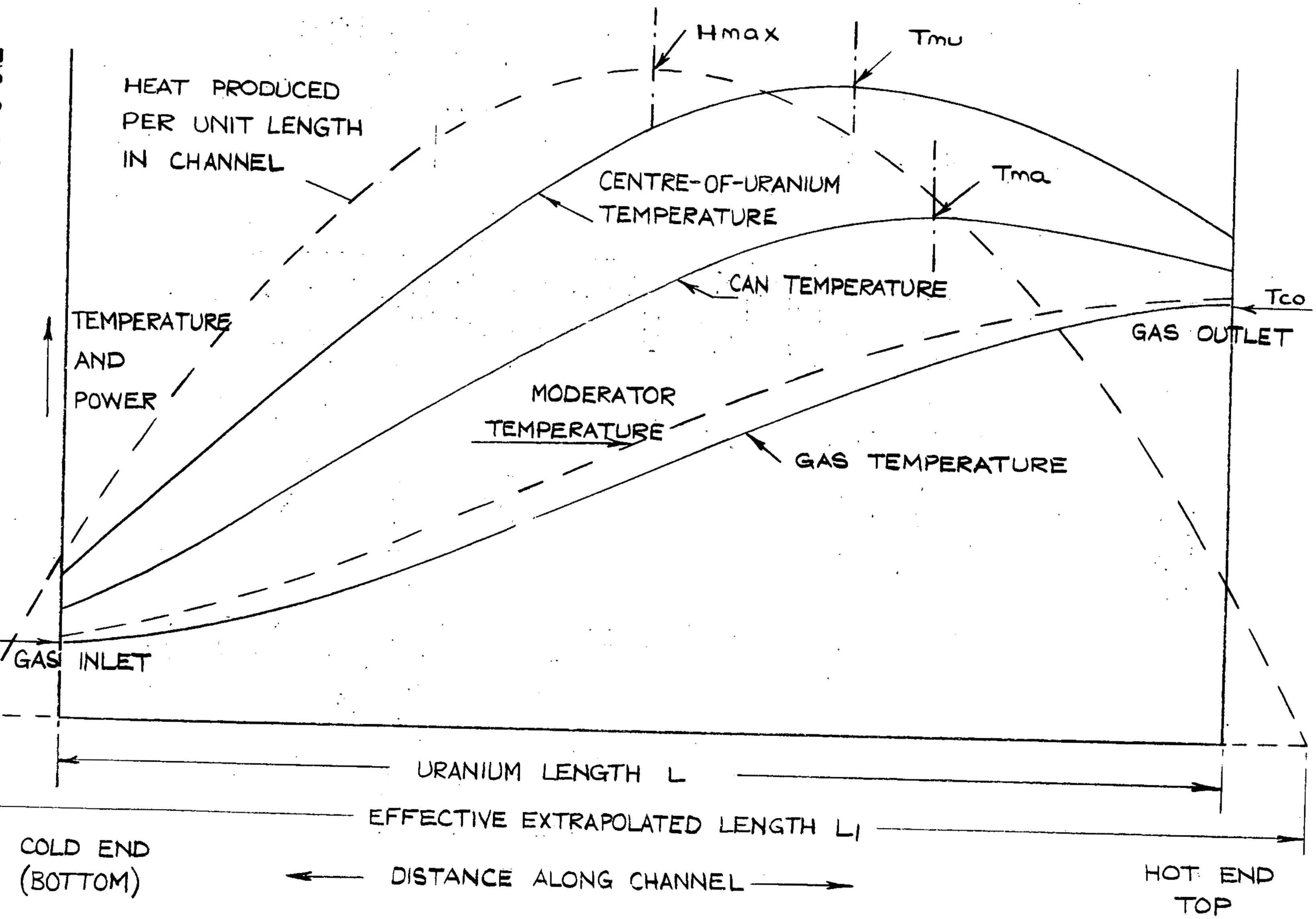


FIG. 1. BLOCK DIAGRAM FOR REACTOR HEAT TRANSFER EQUATIONS

FIG. 2. STEADY STATE TEMPERATURE DISTRIBUTION IN ONE CHANNEL



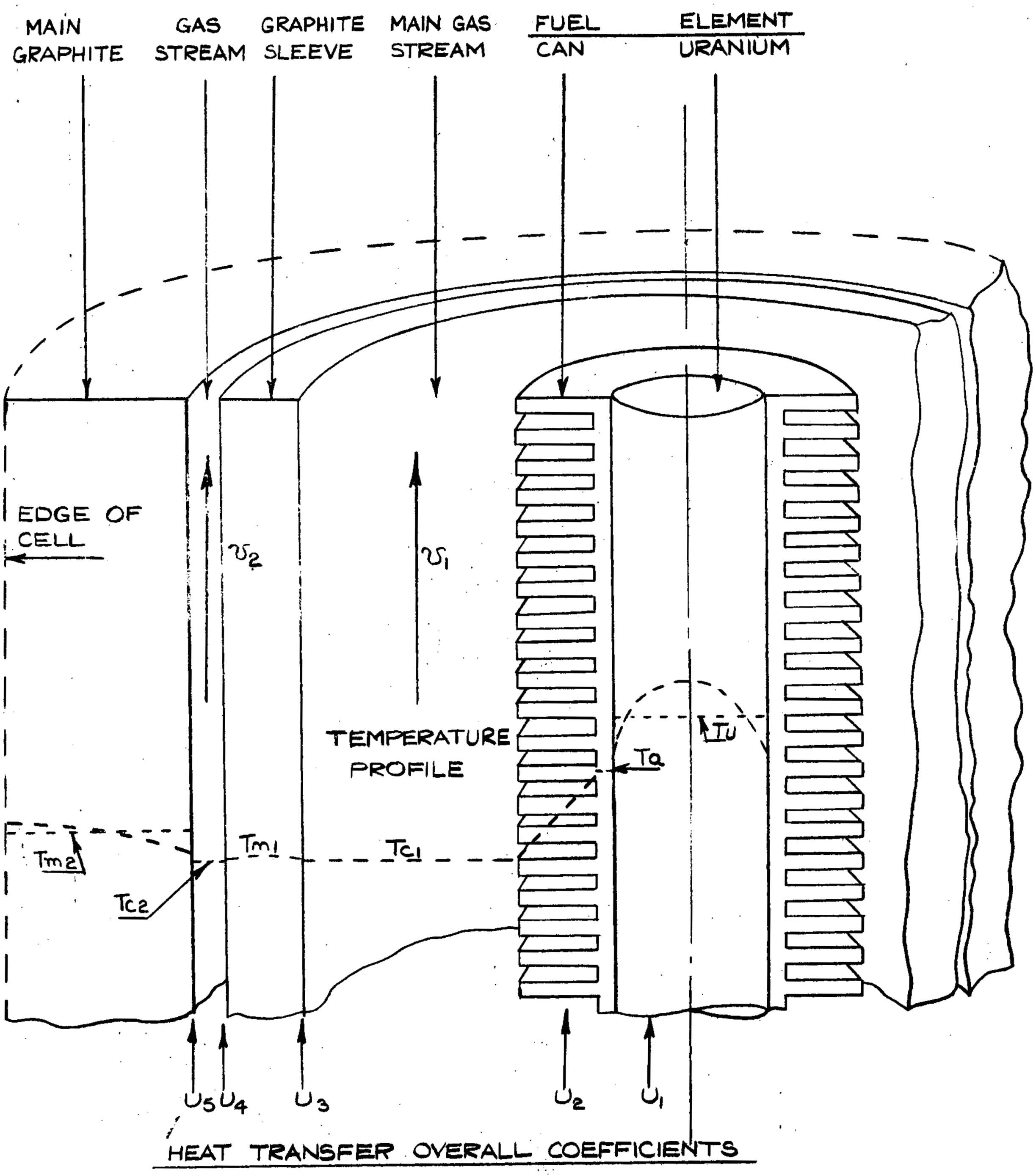


FIG. 3. UNIT LENGTH OF CHANNEL

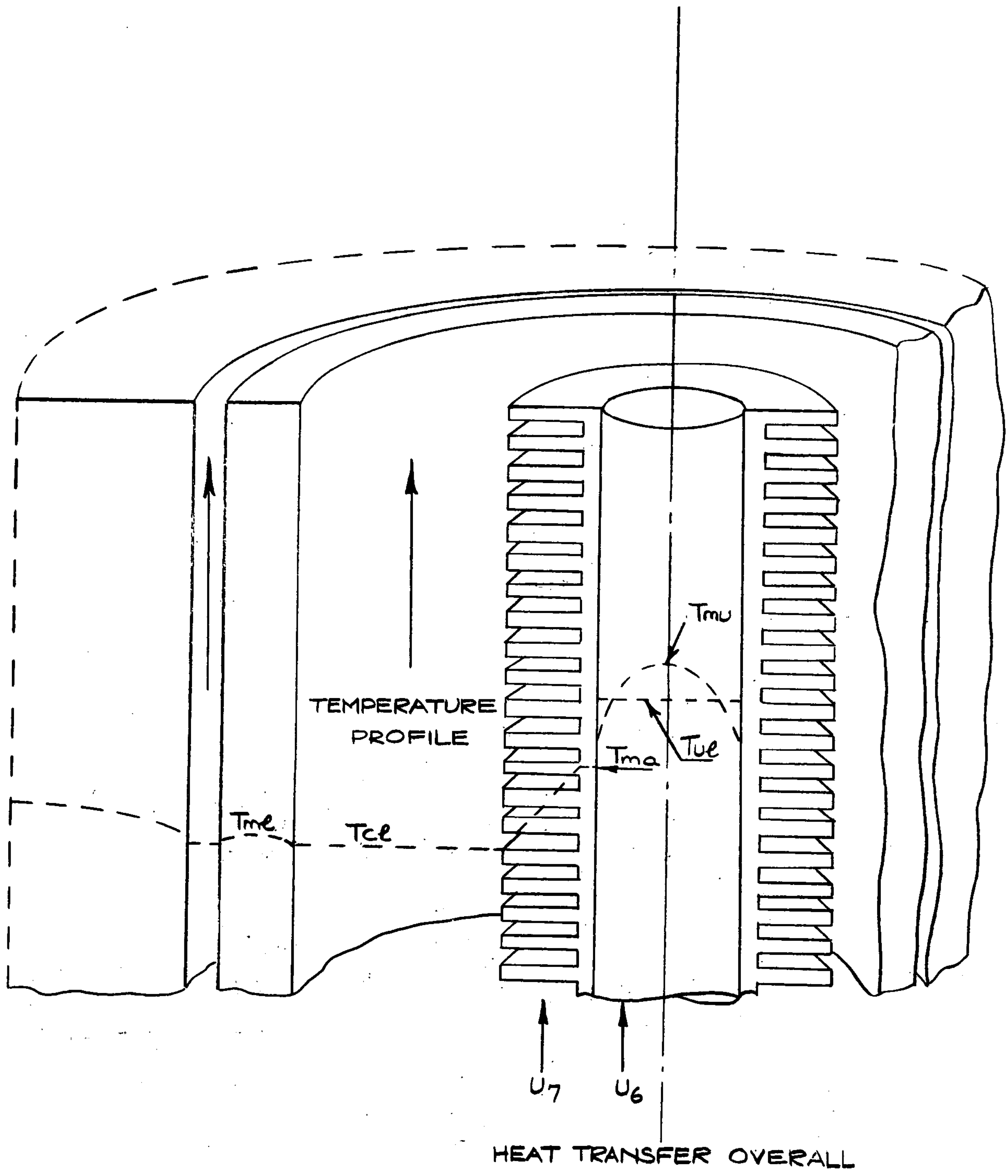


FIG. 4. UNIT LENGTH OF CHANNEL AT POINT OF MAXIMUM TEMPERATURES

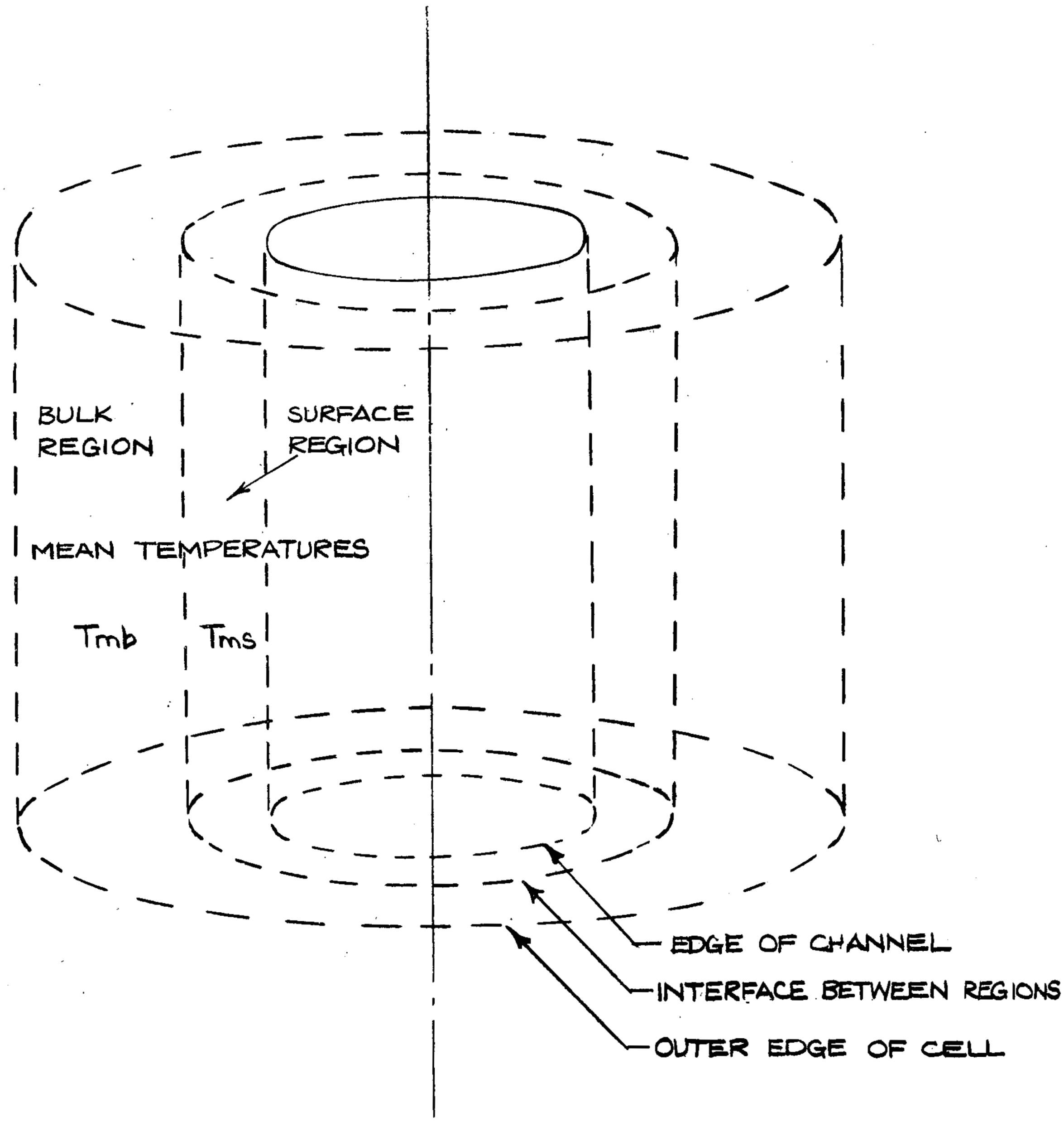


FIG.5. UNIT LENGTH OF SOLID MODERATOR IN A CHANNEL

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