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THE WORTH OF SINGLE CONTROL RODS IN HYDROGENOUS REACTORS

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ABSTRACT

Reactivity measurements of the worth of single slab type control rods fabricated from a variety of materials have been made in five Flexible Plastic Reactor assemblies. These assemblies cover what is believed to be a reasonable spectral range for hydrogen moderated reactors ($\beta\gamma$ = 0.08 to 0.71). An analysis of these experiments shows that the relative worths, which are sensitive functions of the spectrum, can be described by a two group absorption area scheme. The measured ratio of the worth of an absorber having complicated resonances to that of a simple material with a slowly varying cross section is used to evaluate the epithermal transmission probability of the former. It is shown that when this parameter is used to calculate the relative worth in a different spectrum, good agreement with experiment is obtained. That is, in view of the agreement with the measurements, the epithermal transmission probability is a constant and can be used over the entire range of spectra examined.

Additional measurements of the dependence of control rod worth on radial position have been made. A comparison of these measurements with statistical weight calculations shows good agreement. A perturbation theory approach which employs the absorption area is then used to evaluate the absolute worth of a single cadmium slab rod in each of the five reactors. Comparison with experimental worths indicates a maximum discrepancy of eight percent in $\Delta k/k$.

INTRODUCTION

The present difficulties in calculating the epithermal worth of control materials with complicated cross sections are well known. These methods invariably involve transmission calculations of various types using the experimental microscopic cross sections and the reactor neutron spectrum. Since it is not unreasonable to assume that the relative worth of different control materials is a function of the neutron spectrum alone, it seemed advisable to make a survey of a variety of materials over a spectral range likely to occur in practice in hydrogen moderated reactors.

Primarily, these experimentally determined ratios could be put to practical use in critical assemblies to obviate the use of expensive materials (e.g., hafnium) in mockups. In addition, the measurements can evaluate the adequacy of present analytical techniques in predicting the spectrum dependence of the relative worth of control rod materials. A reasonable fit with an analytical description in which a fixed rod parameter is used to describe the effect over the resonance region would imply that this parameter is of more value in reactor applications than the microscopic cross sections.

In this report, measurements in five Flexible Plastic Reactors (FPR) assemblies on the effectiveness of hafnium, silver, indium, boron, europium oxide, samarium oxide and cadmium control rods are described. Three reactors have essentially the same spectrum and have U^{235} cadmium fractions of 0.80. However, they differ markedly in size and composition. Additional measurements were made in a thermal reactor with a cadmium fraction of 0.957 and a faster reactor having a cadmium fraction of 0.66. These measurements are

analyzed using the absorption area concept (Reference 1) applied in a three group scheme.

There is continuing interest in the problem of calculating and comparing with experiment the absolute worth of a single rod in clean geometry. Experiments which measure the worth of a single control rod as a function of radial position can answer the question as to whether the statistical weight as predicted by a perturbation theory calculation is in agreement with experiment. Such measurements were made in three of the assemblies and are compared with calculations.

Our general experimental procedure has been to examine independently the dependence of control rod worth on shape (Reference 2), spectrum, radial position, and reactor composition. Without reference to any calculational scheme, the degree of separability of control rod worths as functions of these parameters can be evaluated. If separability is evident, comparisons with calculations for each functional dependence can be made. It is only after these have been carried out that confidence can be placed in the calculation of the absolute worth. In addition, considerably more weight can be placed in some of the separate experiments than in the measurement of the absolute worth. For these reasons, the analysis of these experiments has been separated into the relative worth as a function of spectrum, worth as a function of position, and finally, the absolute worth.

A major difficulty in the analysis is that absolute rod calculations are made in terms of reactivity, whereas rod worths are measured in terms of the effective delayed neutron fraction, which is reactor dependent. A second

difficulty is that the experimental method itself is subject to uncertainties which are not simple to evaluate. A point in favor of the type of measurement described here is that the measurements are differential, and hence, despite the uncertainties, are believed to give a more sensitive comparison than the technique of calculating k_{eff} for a critical rodded reactor. In addition, both clean and rodded reactors have the same composition and the effects of rod insertion are not masked by other changes which may tend to be self compensating.

EXPERIMENTAL METHOD AND RESULTS

The measurements were made in polyethylene moderated, highly enriched U^{235} assemblies containing aluminum as structural material and borated polyethylene tape as distributed poison. The geometry and composition of the reactors are given in Appendix I.

In each reactor, a 3.0" x 0.3" x 18" void slot was created along the unique axis to accommodate the control slabs. Since the assemblies are 36" long and built in two separable halves, the rods extend from midplane to reflector. The compositions of the assemblies were arranged so as to have the standard reactor shim rods inserted as little as practicable into the critical core with the sample removed.

The worth of each sample was measured by means of the inverse multiplication method calibrated by the rod drop technique and has been described previously (Reference 2). Since the overall flux distribution is distorted in the same way in every measurement, it is believed that the relative measurements are as accurate as the counting statistics indicate. The measured

reactivity value as given by the average of two end counters placed outside the reflector on the unique axis of the reactor was always found to be within two and one half percent of the value obtained by a side counter. Consequently, this has been taken as the value of the probable error for the absolute measurements. This suggests that more confidence can be placed in the absolute value measurement than one would normally expect of a similar measurement in less clean geometry. It is also important to note that the relative measurements are independent of the value of the effective delayed neutron fraction, since this is essentially constant for any particular assembly.

As a further check on the validity of the absolute value as given by the inverse multiplication measurements, a pulse measurement was made by B. E. Simmons (Reference 3) on the worth of three 0.850" x 0.027" x 18" Cd slabs placed in adjacent channels in one half of FPR-13. This constitutes an approximately three inch wide half rod placed on the unique axis and was chosen for the pulse-1/M comparison in order to accentuate the geometrical difficulties encountered in the 1/M measurement. The pulse measurement gave a value of $2.37 $\pm$ 0.12 which is in good agreement with the 1/M value of $2.34 $\pm$ 0.06.

The rod sizes were made as large as possible (2.8" x 18") to minimize edge absorption and small enough to make accurate measurement possible. Identical samples were used in each reactor so as to prevent slight differences in composition from affecting the results.

The inverse multiplication measurement was calibrated at least once during each running day and the standard 0.027" thick cadmium slab was

measured every day. The reproducibility of the cadmium measurement was found to be compatible with the standard deviation indicated by counting statistics alone.

In Table I are listed the materials measured in FPR-11, FPR-12, and FPR-13, together with the observed reactivity worth of each material. The error quoted with each measurement is the standard deviation due to counting statistics alone. The compositions of the materials in the form of dispersions are nominal unless otherwise indicated. More confidence should be placed in the measurements of the pure materials (i.e., Hf, Cd, Ag, and In) than in the dispersions due to the difficulty in fabrication and chemistry of the latter. These data are exhibited in graphical from in Figures 1 and 2 where the ratio of the worth of each slab relative to the worth of 0.027" cadmium is shown as a function of cadmium fraction. Some of the data are also plotted in Figure 3 as a function of $\beta\gamma = \Sigma_a(kT) \,/\, \xi\Sigma_s(\infty)$. For $\beta\gamma = 0$, the spectrum is pure Maxwellian, and materials black over the entire spectrum must exhibit the same worth. That is, in Figure 3 the curves for thick materials should extrapolate to unity at $\beta\gamma = 0$. This is seen to be approximately true for the thicker samples of hafnium, silver, and boron. Some of the data are also given as a function of thickness in Figures 4, 5, and 6.

Measurements of the worth of control rods as a function of radial position were made in FPR-11, 12, and 13. Since every one inch square matrix tube of the FPR contains a 0.900" x 0.150" void channel, the position measurements were made by using 18" long samples cut to fit the channel. In

case a larger rod value was desired, adjacent channels were also loaded. In FPR-11, measurements were made over one eighth of the core in one half, so as to obtain a sampling representative of the entire core. These measurements were made by compensation with calibrated control rods situated in the opposite half of the reactor and are listed in Table III. The remaining position measurements were made by the inverse multiplication method and are shown in Figures 7 through 10. The errors given for the critical measurements are based on reproducibility. Those given for the 1/M measurements are based on the side-end counter discrepancy.

Since in the calculation of the absolute worth it was found necessary to calculate the worth of a rod inserted only half way into the core, a series of measurements were made which evaluate the worth of a half rod compared to that of a fully inserted rod. These are listed in Table II. For a variety of situations, the half rod has one half the value of the fully inserted rod.

TABLE I

Worth of 2.814" x 18" Slab Rods in FPR-11, 12, 13

Type of Rod	FPR-13($) C.F. = 0.957	FPR-11($) C.F. = 0.80	FPR-12($) C.F. = 0.66
0.027" Cd	2.555 ± .010	0.922 ± .006	0.609 ± .003
0.050" Hf	2.227 ± .009	1.040 ± .006	0.812 ± .004
0.100" Hf	2.677 ± .011	1.378 ± .008	1.097 ± .005
0.150" Hf	2.968 ± .012	1.576 ± .009	1.273 ± .006
0.200" Hf	3.164 ± .013	1.700 ± .010	1.404 ± .007
0.250" Hf	3.267 ± .013	1.815 ± .011	1.504 ± .008
0.050" Ag	1.881 ± .008	0.752 ± .005	0.553 ± .003
0.100" Ag	2.423 ± .010	1.065 ± .006	0.807 ± .004
0.150" Ag	2.711 ± .011	1.269 ± .008	0.986 ± .005
0.200" Ag	2.922 ± .012	1.433 ± .009	1.122 ± .006
0.250" Ag	3.066 ± .012	1.557 ± .009	1.232 ± .006
0.027" Cd + 0.200" Ag	3.116 ± .013	1.622 ± .010	1.274 ± .006
0.027" Cd + 0.100" Ag	2.900 ± .012	1.384 ± .008	1.057 ± .005
0.027" Cd + 0.050" Ag	2.786 ± .011	1.234 ± .007	0.904 ± .005
1.02 w/o B^{10} in SS (0.250" thick)	3.250 ± .013	1.783 ± .011	1.434 ± .007
0.036 gm/cm^2 (chem) B^{nat} in Al (0.100" thick)	2.442 ± .011	0.950 ± .006	0.668 ± .003
1.36 w/o Eu_2O_3 in SS (0.100" thick)	1.503 ± .006	0.468 ± .003	0.308 ± .002
13.85 w/o Eu_2O_3 in SS (0.100" thick)	2.789 ± .011	1.254 ± .007	0.958 ± .005
9.56 w/o Sm_2O_3 in Cu (0.100" thick)	2.602 ± .010	0.916 ± .006	0.644 ± .003

TABLE I (Con't.)

Worth of 2.814" x 18" Slab Rods in FPR-11, 12, 13

Type of Rod	FPR-13(¢) C.F. = 0.957	FPR-11(¢) C.F. = 0.80	FPR-12(¢) C.F. = 0.66
0.050" In	2.358 ± .010	1.005 ± .006	0.745 ± .004
0.100" In	2.781 ± .011	1.276 ± .008	0.958 ± .005
0.150" In	2.976 ± .012	1.426 ± .009	1.085 ± .005
0.200" In	3.117 ± .013	1.535 ± .009	1.167 ± .006

TABLE II

Comparison of the Worth of Half Rods to Fully Inserted Rods

Type of Rod	Worth of Half Rod ($)	One-Half Worth of Fully Inserted Rod
FPR-11		
Three 0.032" x 0.938" x 18" Cd slabs at x = 0	0.919 ± .009	0.926 ± .009
Three 0.032" x 0.850" x 18" Cd slabs at x = 0	0.873 ± .009	0.911 ± .009
FPR-12		
Three 0.032" x 0.850" x 18" Cd slabs at x = 0	0.585 ± .006	0.601 ± .006
Three 0.032" x 0.850" x 18" Cd slabs at x = 4"	0.423 ± .004	0.417 ± .004
Three 0.032" x 0.850" x 18" Cd slabs at x = 6	0.290 ± .003	0.281 ± .003
FPR-13		
Two 0.032" x 0.850" x 18" Cd slabs at x = 0	1.712 ± .017	1.683 ± .017

ANALYSIS AND CONCLUSIONS

A. Relative Worths

In view of the success of the absorption area concept (Reference 1) in describing the worth of control rods as a function of shape (Reference 2) and the fact that the absorption area provides a basis for many current rod calculational method (References 4 - 9), the analysis here will make use of the absorption area. The method used is essentially that described by Deutsch (Reference 4), except that due to some difficulties inherent in the Deutsch cross section prescription (Reference 10), the data have also been analyzed using the MUFT-SOFOCATE (Reference 11) (MS) codes in three energy groups with a thermal cutoff at 0.625 ev and an epithermal cutoff at 180 kev.

In the method of Deutsch (Reference 4), diffusion theory is applied to calculate a thermal and epithermal absorption area where the characteristic lengths are given by $L_3 = \left(D_3/\Sigma_{a_3}\right)^{1/2}$ and $L_2 = \left[D_2/(\Sigma_{a_2}+\Sigma_{sl_2})\right]^{1/2}$, respectively.

Making use of the Hurwitz-Roe small L limit expression which includes a corner correction, and using an extrapolated endpoint correction which contains a diffusion theory correction for grayness, Deutsch finds for the absorption area of a slab of width 2a:

$$C = 4a\left[\left(\frac{1}{1+\Sigma_{a_3}/\Sigma_{sl_2}}\right)\left(\frac{L_3}{1+d_3/L_3}\right)\left(1-\frac{1}{(1+L_3/L_2)(1+d_2/L_2)}\right)\left(1+\frac{1}{1+d_3/L_3}\,\frac{L_3}{2a}\right)\right. \quad (1)$$

$$\left. + \frac{L_2}{1+d_2/L_2}\left(1+\frac{1}{1+d_2/L_2}\cdot\frac{L_2}{2a}\right)\right]$$

where $d_i = 2\ D_i\left(\frac{1 + P_i}{1 - P_i}\right)$ is the extrapolation length for the rod in the ith group

and P_i is the group averaged rod transmission probability.

P_3 for the thermal group is given by:

$$P_3 = 2\ E_3\ (N\bar{\sigma}t) \qquad (2)$$

$$\text{where } E_3 = \int_1^\infty \frac{e^{-N\bar{\sigma}tx}}{x^3}\,dx$$

and $\bar{\sigma}$ is the average thermal group microscopic absorption cross section for the rod material.

The transmission probability for the epithermal group P_2 is calculated by the following numerical integration, which assumes a 1/E spectrum with an upper cutoff at 180 kev:

$$P_2 = \frac{2}{u_c - 4}\int_4^{u_c} E_3\left(N\sigma(u)t\right)\,du \qquad (3)$$

where $u = \ln\left(\frac{10\ \text{Mev}}{E}\right)$ and u_c is the thermal cutoff lethargy.

It is assumed that P_2 and P_3 can be calculated for materials having cross sections that vary slowly with energy. The materials for which explicit calculations have been carried out here are cadmium and boron, although it is believed that the values calculated for cadmium are somewhat more useful, due to the experimental difficulty of accurate boron content determination.

For materials which have appreciable resonance absorption, P_3 is calculated by Eq. 2. The ratio of the observed worth of the complex rod

where $d_i = 2\,D_i\left(\frac{1 + P_i}{1 - P_i}\right)$ is the extrapolation length for the rod in the ith group

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$$\text{where } E_3 = \int_1^\infty \frac{e^{-N\bar{\sigma}t\chi}}{\chi^3}\,d\chi$$

and $\bar{\sigma}$ is the average thermal group microscopic absorption cross section for the rod material.

The transmission probability for the epithermal group P_2 is calculated by the following numerical integration, which assumes a 1/E spectrum with an upper cutoff at 180 kev:

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It is assumed that P_2 and P_3 can be calculated for materials having cross sections that vary slowly with energy. The materials for which explicit calculations have been carried out here are cadmium and boron, although it is believed that the values calculated for cadmium are somewhat more useful, due to the experimental difficulty of accurate boron content determination.

For materials which have appreciable resonance absorption, P_3 is calculated by Eq. 2. The ratio of the observed worth of the complex rod

to that of the simple rod (e.g. Cd or B) is then equated to the ratio of absorption areas as given by Eq. 1 in order to determine P_2 for the complex rod.

WANDA (Reference 12) one dimensional calculations were carried out in order to obtain the k_{eff} of the no rodded reactor. The boron and fuel densities were adjusted in order to attain criticality and the group constants so determined were used in the absorption area calculation (Eq. 1). The group constants for the five reactors are listed in Appendix II and the SOFOCATE thermal spectra are shown in Figure 11. It is seen that the calculated spectra for FPR-3, 4 and 11 are essentially identical.

In Table III are given the FPR-11 experimental ratios of the worth of each material relative to that of 0.027" cadmium together with the calculated P_3 and evaluated P_2 values. The quoted errors are those due to the standard deviation of counting statistics alone. Table IV shows the calculated and experimental ratios for FPR-12 and FPR-13. The experimental and calculated ratios are in quite good agreement. Since there is a considerable change in relative rod worth between FPR-13 and FPR-12, it is perhaps more meaningful to compare the predicted and experimental change in relative worth in these two reactors. For 0.250" hafnium, the ratio: worth Hf/worth Cd is predicted to change by the factor 1.926 as compared to measured change 1.93 $\pm$ .02.

TABLE III

Measured Rod Worth Ratios and MS Transmission Probabilities for FPR-11

Material	Rod Worth/Worth 0.027" Cd (Measured)	P_3	P_2 (From Boral Calculation)	P_2 (From Cd Calculations)
0.027" Cd	1.000	0	0.975	0.991
0.036 gm/cm^2 B^n in Al	1.030 ± .009	0.234	0.922	0.939
1 w/o B^{10} in SS	1.934 ± .017	0	0.715	0.718
1 w/o B^{10} in SS	1.934 ± .017	0	0.700 (Absolute Calc. of P_2)	
0.050" Hf	1.128 ± .010	0.510	0.842	0.859
0.100" Hf	1.495 ± .013	0.284	0.789	0.813
0.150" Hf	1.709 ± .015	0.163	0.752	0.780
0.200" Hf	1.844 ± .017	0.0964	0.726	0.756
0.250" Hf	1.968 ± .018	0.0578	0.697	0.730
0.050" In	1.090 ± .010	0.362	0.881	0.899
0.100" In	1.384 ± .012	0.151	0.842	0.865
0.150" In	1.547 ± .014	0.0664	0.812	0.838
0.200" In	1.663 ± .015	0.0302	0.785	0.813
0.050" Ag	0.815 ± .008	0.608	0.900	0.913
0.100" Ag	1.154 ± .010	0.392	0.858	0.877
0.150" Ag	1.375 ± .012	0.260	0.825	0.847
0.200" Ag	1.553 ± .014	0.176	0.792	0.817
0.250" Ag	1.687 ± .015	0.120	0.765	0.792

TABLE III (Con't.)

Measured Rod Worth Ratios and MS Transmission Probabilities for FPR-11

Material	Rod Worth/Worth 0.027" Cd (Measured)	P_3	P_2 (From Boral Calculation)	P_2 (From Cd Calculations)
1.36 w/o Eu_2O_3 in Fe	0.508 ± .005	0.626	0.974	0.982
13.85 w/o Eu_2O_3 in Fe	1.359 ± .012	0.0252	0.870	0.893
9.56 w/o Sm_2O_3 in Cu	1.026 ± .009	0.00093	0.965	0.985
0.027" Cd + 0.050" Ag	1.337 ± .012	0	0.881	0.903
0.027" Cd + 0.100" Ag	1.499 ± .015	0	0.836	0.861
0.027" Cd + 0.200" Ag	1.757 ± .016	0	0.764	0.794

TABLE IV

Comparison of MS Calculated and Measured Rod Worth Ratios

Material	Rod Worth/ Worth 0.027" Cd (Measured)	Absorption Area Ratio (From Boral Calculation)	Absorption Area Ratio (From Cd Calculation)	P_3
FPR-12				
0.036 gm/cm^2 B^n in Al	1.097 ± .008	1.091	1.089	0.278
1 w/o B^{10} in SS	2.354 ± .016	2.424	2.457	0
0.050" Hf	1.333 ± .009	1.348	1.368	0.542
0.100" Hf	1.801 ± .013	1.814	1.837	0.318
0.150" Hf	2.090 ± .015	2.105	2.134	0.192
0.200" Hf	2.305 ± .016	2.294	2.330	0.119
0.250" Hf	2.469 ± .017	2.479	2.516	0.0744
0.050" In	1.223 ± .009	1.236	1.241	0.394
0.100" In	1.573 ± .011	1.612	1.624	0.177
0.150" In	1.781 ± .013	1.842	1.858	0.083
0.200" In	1.916 ± .013	2.018	2.040	0.0404
0.050" Ag	0.908 ± .006	0.926	0.930	0.646
0.100" Ag	1.325 ± .009	1.334	1.340	0.440
0.150" Ag	1.619 ± .011	1.619	1.636	0.306
0.200" Ag	1.842 ± .013	1.866	1.889	0.216
0.250" Ag	2.022 ± .014	2.057	2.088	0.154
1.36 w/o Eu_2O_3 in Cu	0.506 ± .004	0.500	0.496	0.636
13.85 w/o Eu_2O_3 in Cu	1.577 ± .011	1.557	1.566	0.0288

TABLE IV (Con't.)

Comparison of MS Calculated and Measured Rod Worth Ratios

Material	Rod Worth/ Worth 0.027" Cd (Measured)	Absorption Area Ratio (From Boral Calculation)	Absorption Area Ratio (From Cd Calculation)	P_3
FPR-12 (Con't.)				
9.56 w/o Sm_2O_3 in Cu	1.060 ± .007	1.053	1.034	.00214
0.027" Cd + 0.050" Ag	1.484 ± .011	1.515	1.526	0
0.027" Cd + 0.100" Ag	1.736 ± .012	1.763	1.780	0
0.027" Cd + 0.200" Ag	2.092 ± .014	2.158	2.182	0
FPR-13				
0.036 gm/cm^2 B^n in Al	0.955 ± .007	0.980	0.977	0.163
1 w/o B^{10} in SS	1.27 ± .009	1.280	1.265	0
0.050" Hf	0.872 ± .006	0.904	0.898	0.446
0.100" Hf	1.048 ± .007	1.101	1.091	0.222
0.150" Hf	1.162 ± .008	1.193	1.181	0.115
0.200" Hf	1.238 ± .009	1.244	1.231	0.0616
0.250" Hf	1.279 ± .009	1.287	1.271	0.0366
0.050" In	0.922 ± .006	0.952	0.951	0.300
0.100" In	1.088 ± .008	1.096	1.088	0.107
0.150" In	1.164 ± .008	1.157	1.148	0.0408
0.200" In	1.219 ± .008	1.197	1.186	0.0161
0.050" Ag	0.736 ± .005	0.763	0.759	0.536
0.100" Ag	0.948 ± .007	0.972	0.965	0.312

TABLE IV (Con't.)

Comparison of MS Calculated and Measured Rod Worth Ratios

Material	Rod Worth/ Worth 0.027" Cd (Measured)	Absorption Area Ratio (From Boral Calculation)	Absorption Area Ratio (From Cd Calculation)	P_3
FPR-13 (Con't.)				
0.150" Ag	1.061 ± .007	1.077	1.070	0.187
0.200" Ag	1.143 ± .008	1.148	1.139	0.115
0.250" Ag	1.200 ± .008	1.197	1.186	0.0714
1.36 w/o Eu_2O_3 in SS	0.588 ± .004	0.624	0.624	0.586
13.85 w/o Eu_2O_3 in SS	1.091 ± .008	1.105	1.098	0.0153
9.56 w/o Sm_2O_3 in Cu	1.018 ± .007	1.010	1.006	0
0.027" Cd + 0.050" Ag	1.090 ± .008	1.099	1.094	0
0.027" Cd + 0.100" Ag	1.135 ± .008	1.148	1.135	0
0.027" Cd + 0.200" Ag	1.219 ± .008	1.226	1.219	0

In Table IV there are two sets of calculated ratios: one is derived from a calculation of both P_2 and P_3 for the Boral rod, and the other is obtained from a similar calculation for the 0.027" cadmium rod. The degree of agreement with the measured ratios in FPR-12 and FPR-13 is not significantly different for the two sets of calculations. However, the P_2 values as given by the boron calculation are from three to five percent lower than those derived from cadmium. Absolute calculations of P_2 and P_3 were also carried out for the B^{10} in stainless steel rod. Using these values in Eq. (1), the calculated B^{10} in SS/Boral worth ratio is found to be 1.894 as compared to a measured ratio of $1.877 \pm .013$ in FPR-11.

In a similar analysis using the WOXX (Reference 13) (W) cross section code (based on the Deutsch prescription), a first examination showed that the calculated relative worths were not at all compatible with the measured values. Some of the data were analyzed in this way and are given in Table V (a). It was pointed out (Reference 14) that the difficulty might be due to the variable thermal cutoff inherent in the Deutsch prescription. For cross sections which are slowly varying about the point of thermal cutoff, no real difficulty exists. However, since the cutoff is in the region of the cadmium resonance peak, it is not surprising that it is impossible to fit the data with a constant P_2. It was found that satisfactory agreement is obtained when the P_2 value for cadmium is treated as a variable depending on the spectrum and is recalculated by means of Eq. (3) for each reactor. (See Table V (b)). In addition, the FPR-11 P_2 value (0.913) formed from the Boral calculation and the measured Boral/cadmium ratio is in good agreement with the value (0.916) given by Eq. (3).

TABLE V (a)

Comparison of WOXX Calculated and Measured Rod Worth Ratios
(Fixed P_2 - using Boral Calculations in FPR-11)

Material	P_2	FPR-12 Worth/Worth 0.027" Cd (P_2 Boral = .894)		FPR-13 Worth/Worth 0.027" Cd (P_2 Boral = .867)	
		Measured	Calculated	Measured	Calculated
0.027" Cd	0.913	1.0	1.0	1.0	1.0
.036 gms/cm^2 B^{nat} in Al	0.883	1.097 ± .008	0.977	0.955 ± .007	1.019
0.100" Hf	0.760	1.801 ± .013	1.605	1.048 ± .007	1.167
0.150" Hf	0.718	2.090 ± .015	1.861	1.162 ± .008	1.271
0.200" Hf	0.692	2.305 ± .016	2.014	1.238 ± .009	1.329
0.250" Hf	0.664	2.469 ± .017	2.163	1.279 ± .009	1.382
0.150" Ag	0.794	1.619 ± .011	1.451	1.061 ± .007	1.127
0.200" Ag	0.758	1.842 ± .013	1.664	1.143 ± .008	1.210
0.250" Ag	0.729	2.022 ± .014	1.828	1.200 ± .008	1.269
0.100" Ag	0.803	1.573 ± .011	1.363	1.088 ± .008	1.141

TABLE V (b)

Comparison of WOXX Calculated and Measured Rod Worth Ratios

(P_2 for Cadmium and Boral Recalculated by Eq. (3) for each Reactor; P_2 for Other Materials Evaluated from Measurements and Boral Calculation in FPR-11)

Material	FPR-12 (P_2 Cd = 0.938)			FPR-13 (P_2 Cd = 0.890)		
	P_3	Worth/Worth 0.027" Cd Measured	Calculated	P_3	Worth/Worth 0.027" Cd Measured	Calculated
0.036 gms/cm^2 B^{nat} in Al	0.245	1.097 ± .008	1.109	0.141	0.955 ± .007	0.959
0.100" Hf	0.301	1.801 ± .013	1.822	0.202	1.048 ± .007	1.053
0.150" Hf	0.179	2.090 ± .015	2.112	0.101	1.162 ± .008	1.167
0.200" Hf	0.108	2.305 ± .016	2.287	0.0518	1.238 ± .009	1.241
0.250" Hf	0.0666	2.469 ± .017	2.455	0.0271	1.279 ± .009	1.284
0.150" Ag	0.272	1.619 ± .011	1.647	0.163	1.061 ± .007	1.052
0.2002 Ag	0.187	1.842 ± .013	1.888	0.0963	1.143 ± .008	1.134
0.250" Ag	0.129	2.022 ± .014	2.075	0.0578	1.200 ± .008	1.190
0.100" In	0.157	1.573 ± .011	1.547	0.0892	1.088 ± .008	1.092

There are three major possibilities for the discrepancy between the absolute Boral and cadmium P_2 values as given in MS scheme: 1) There is a significant error in the Boron content of the Boral rod. 2) The few group treatment is inadequate in calculating the thermal source due to epithermal absorption. 3) The absorption area concept is being employed in cadmium over a region of too great rod transparency in the epithermal group.

Possibility (1) is ruled out since the Boral/cadmium ratio is calculated correctly in the W analysis for FPR-11. In addition, the B^{10} in SS/Boral ratio, which involves absolute calculations of P_2 and P_3 for widely different boron concentrations, is calculated correctly for FPR-11 in the MS analysis. As well as assisting in discarding (1), this also suggests that (2) is not a valid objection.

A stronger argument for removing (2) can be found by an examination of the cadmium and cadmium-silver analyses. If all the absorption in cadmium occurs in the thermal group, an experimental evaluation of the P_2 for a silver rod should require the same P_2 as is required for the same thickness of silver plus cadmium. This can only hold if the thermal absorption of the pure silver is calculated correctly, since an error would require a different P_2 for the Ag rod than for the Cd-Ag rod. Measurements made in FPR-11 show that the P_2 values required for the Cd-Ag rods are one percent lower than those needed for the Ag rods using the normalization to cadmium and about two and one half percent lower using the Boral normalization. It can be concluded, therefore, that the P_2 values required for the two types of rod are essentially the same; the difference can be attributed to the fact that there is some cadmium absorption in the epithermal group.

We are, therefore, left with (3) as the most likely reason for the discrepancy in the MS analysis. It is intuitively unreasonable to assume that the absorption area, which was originally calculated as a geometric problem for a black absorber, can be applied to groups in which the rod is essentially transparent. A more reasonable manner of employing the absorption area has been suggested by B. Wolfe (Reference 7) in which the upper energy cutoff of the epithermal group is taken to be where the rod becomes one absorption mean free path thick. However, for absorbers having complicated cross sections, this energy is not well defined.

In view of the above, it would seem that the following statements can be made for hydrogenous reactors having compositions which lead to spectra within the range examined:

1. Using a scheme having a fixed cutoff between the thermal and epithermal groups, epithermal transmisstion probabilities can be found experimentally for materials having complicated cross sections. These quantities, for all practical purposes, are constants which can be used to describe the epithermal absorption in the rod in any reasonable spectrum. Obviously, this constant depends on the type of analysis used.
2. When a variable cutoff is employed, the epithermal transmission probabilities are not constant for materials which have cross sections which are rapidly varying in the cutoff region.
3. The epithermal group must be chosen so as to limit the transparency of the rod in this group. Reasonable bounds can be found by an examination of the experimental results given here by any proposed method.

4. Since it is possible to fit the relative worths over a wide range in spectrum with the method outlined above, the single rod problem has evidently been reduced to the problem of calculating the absolute worth of a cadmium rod.

B. Worth as a Function of Position and Absolute Worth

Making use of the assumption of first order perturbation theory in which both unperturbed fluxes and adjoints are used, and calculating the worth of a single rod wholly inserted into the core so that only two dimensions need be considered, we have for the change in reactivity resulting from rod insertion at a radial position r:

$$\Delta k/k = \frac{\left(\begin{array}{l}\text{number of thermal neuts absorbed}\\ \text{by rod sec}^{-1}\ \text{cm}^{-1}\end{array}\right) \times \left(\begin{array}{l}\text{relative importance of}\\ \text{thermal neuts absorbed}\end{array}\right)}{\text{Total Fissions sec}^{-1}\ \text{cm}^{-1}}$$

$$= \frac{C\left[\Sigma_{a_2}\varphi_2\,\varphi_2^{\dagger}\right]_r}{\int_{core}\Sigma_{a_2}\varphi_2\,\varphi_2^{\dagger}\,dA}$$

where C is the absorption area of the rod and the subscripts 1 and 2 refer to fast and thermal group fluxes and adjoints respectively.

Since: $$\frac{\varphi_2^{\dagger}}{\varphi_1^{\dagger}} = \frac{\nu\Sigma_f}{\Sigma_{a_2}}$$

$$\Delta k/k = \frac{C\left[\varphi_1^{\dagger}\Sigma_f\varphi_2\right]_r}{\int\varphi_1^{\dagger}\Sigma_f\varphi_2\,dA} = \left(\frac{C}{A_{core}}\right)\frac{\left[\varphi_1^{\dagger}\Sigma_f\varphi\right]_r}{\overline{\varphi_1^{\dagger}\Sigma_f\varphi}} \qquad (4)$$

where the subscript 2 has been dropped since $\Sigma_f = 0$ in group 1. Group 1 is assumed to contain all fission neutrons.

Since the only position dependent term in Eq. (4) is the product $\varphi_1^+ \Sigma_a \varphi$, a straightforward comparison of this quantity with measurements of the worth of a single rod as a function of position can be made. This comparison is independent of the delayed neutron fraction.

In order to determine $\varphi_1^+ \Sigma_a \varphi$, flux adjoint calculations were carried out using W cross sections and making use of WANDA (Reference 12) one dimensional and CURE (Reference 15) two dimensional calculations in three groups. In using Eq. (4) and Eq. (1) in the three group scheme we are, therefore, assuming equality of importance of the epithermal and thermal neutrons absorbed by the rod. Comparison of the positional dependence as calculated in this manner with measurements made in FPR-11, 12 and 13 are shown in Table VI and Figures 7, 8 and 9. In all cases, the calculations have been normalized to the measurements at x, y, = 0. Over the greater part of the core, the agreement is seen to be satisfactory; in fact, agreement close to the core reflector interface should not be expected, since the spectrum there is not typical of the core as a whole. The absorption area cannot be considered a constant in this transition region, as the spectrum is changing and, therefore, requires recalculation of P_3 at every point. In principle, it is of course possible to take this effect into account.

Since it is conceivable, although unlikely, that the positional dependence is material dependent, measurements were made of the worth of a thin (0.025" hafnium slab as a function of position as well as 0.027"

cadmium. A thin sample was chosen so as to accentuate the epithermal absorptions relative to the thermal absorptions. These data are shown in the form of the ratio of cadmium compared to hafnium in Figure 10. The ratio is seen to be essentially constant except near the reflector. Again, the reason for the departure from constancy in the changing spectrum.

TABLE VI

Comparison of Experimental and Calculated Positional Dependence of Rod Worth in FPR-11 (0.032" x 0.0850" x 18" Cd)

Coordinate (Inches)		Measured Worth (Dollars)	Calculated Worth (Norm. at x, y = 0)
x	y		
0	0	0.307 ± .003	.307
2	0	0.282 ± .003	.284
4	0	0.222 ± .002	.223
6	0	0.139 ± .002	.149
3	1	0.257 ± .003	.252
5	1	0.179 ± .002	.181
7	1	0.125 ± .002	.152
2	2	0.263 ± .003	.263
4	2	0.202 ± .002	.206
6	2	0.130 ± .002	.138
5	3	0.151 ± .002	.154
7	3	0.108 ± .001	.129
4	4	0.160 ± .002	.161
6	4	0.100 ± .001	.108
5	5	0.106 ± .001	.110
7	5	0.074 ± .001	.091
6	6	0.060 ± .001	.071
7	7	0.046 ± .001	.061

Having shown that Eq. (4) is adequate for independently predicting the effectiveness of control rods as pertains to simple shapes, (Reference 2) relative worths as a function of the reactor spectrum, and worth as a function of position, we are in a position to calculate and compare with experiment the absolute worth of a simple slab rod in the five assemblies.

In addition to the calculations already described in some detail, it is obviously necessary to calculate the effective delayed neutron fraction in order to convert from reactivity to dollars. Two methods have been employed which give results that differ by only one or two percent. The first is a simple approach (Reference 16), which uses the following expression:

$$\beta_{eff} = \beta\left[1 + B^2(\tau_p - \tau_d)\right]$$

where τ_p is the prompt neutron age as given by a three group machine calculation and τ_d is found from this quantity by using a calculated relationship between prompt and delayed neutron ages. (Reference 17). The second method employs the MS code and a pair of criticality calculations in which the fission spectrum in one calculation is modified to contain the delayed neutrons. (Reference 18)

The absorption area is calculated directly from the cross section for cadmium using Eq. (1), and the peak to average flux adjoint ratio is found by utilizing the W, WANDA, and CURE codes. Since the measured rods are half rods, and it has been shown that the factor one-half can be used with some precision, this factor is used in Eq. (4).

Stewart (Reference 19) has compared the absorption area of a black absorber as given by diffusion theory to that given by a transport variational calculation. It is, therefore, possible to take account of finite core absorption and it is found that the diffusion theory absorption area is too large by from three to five percent over the range of blackness involved in the assemblies examined here.

In Table VII are given the measured and calculated absolute worth values for a single 0.027" x 2.81" x 18" cadmium slab situated at the center of each of the five assemblies. In view of the difficulties mentioned above concerning the MS cadmium absorption area, the W absorption area for cadmium has been used. Considering that the rod worth varies by as much as a factor of six and that there is a wide range of reactor composition and size, it is believed that the correlation between the measured and calculated values is not fortuitous.

The authors would like to point out that they do not consider the analysis of this series of experiments to be complete. We believe that the simple analysis has demonstrated that some features of the theory are adequate for practical purposes; however, there is certainly room for improvement in the absolute value calculation. It is hoped that the experimental results will provide a useful set of data for comparison with the variety of methods which are being used in control rod calculations at the present time.

TABLE VII

Comparison of Absolute Value Measurements and Calculations For Single 0.027" x 2.81" x 18" Cadmium Slab Rods

Assembly	3	4	11	12	13
β_{eff} (machine calc)	0.00733	0.00796	0.00777	0.00800	0.00765
β_{eff} (hand calc)	0.0072	0.0081	0.0078	0.0081	0.0076
Core Area (cm^2)	5606	5606	1452	1452	1755
Absorption Area (cm^2) (From Eq. 1)	17.94	30.84	12.94	10.14	34.11
Transport Correction (T)	0.96	0.96	0.96	0.95	0.97
$\frac{\varphi_1^+ \Sigma_c \varphi}{\varphi_1^+ \Sigma_a \varphi}$ (Calc)	2.261	1.913	1.70	1.59	2.083
$\Delta k/k = \frac{1}{2}\left(\frac{C\,T}{A}\right)\frac{\varphi_1^+ \Sigma_c \varphi}{\varphi_1^+ \Sigma_a \varphi}$	0.00347	0.00505	0.00727	0.00527	0.00196
$\Delta k/k$ (¢) (calc)	0.474	0.634	0.936	.659	2.56
$\Delta k/k$ (¢) (meas)	0.437 ± .011	0.617 ± .015	0.922 ± .023	0.609 ± .015	2.55 ± .07
Ratio: calc/meas	1.08	1.03	1.02	1.08	1.00

It is a pleasure to acknowledge the careful work of our reactor operator, A. E. Boshoff. The pulse measurement made by B. E. Simmons lends considerable confidence to the absolute value measurements. Discussions with H. Hurwitz, Jr., W. Skolnik, M. L. Storm, and especially L. S. Bohl and J. C. Stewart who have been most helpful. We are indebted to J. S. King and R. G. Luce for making these experiments possible.

Figure 1

Worth of 2.8" x 18" slabs relative to 0.027" Cadmium as a function of spectrum

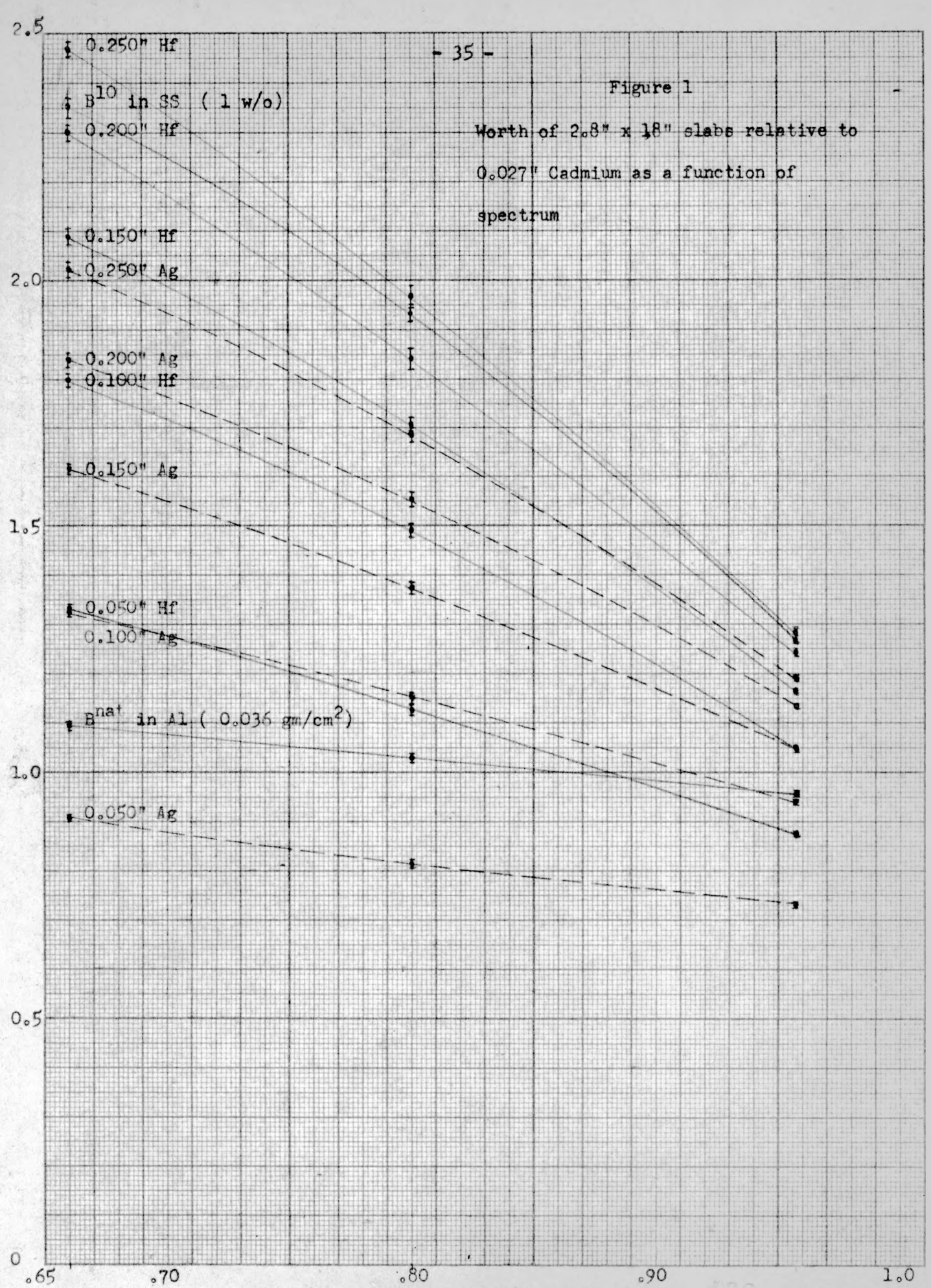


Figure 2

Worth of 2.8" x 18" slabs relative to 0.027" Cadmium as a Function of Spectrum

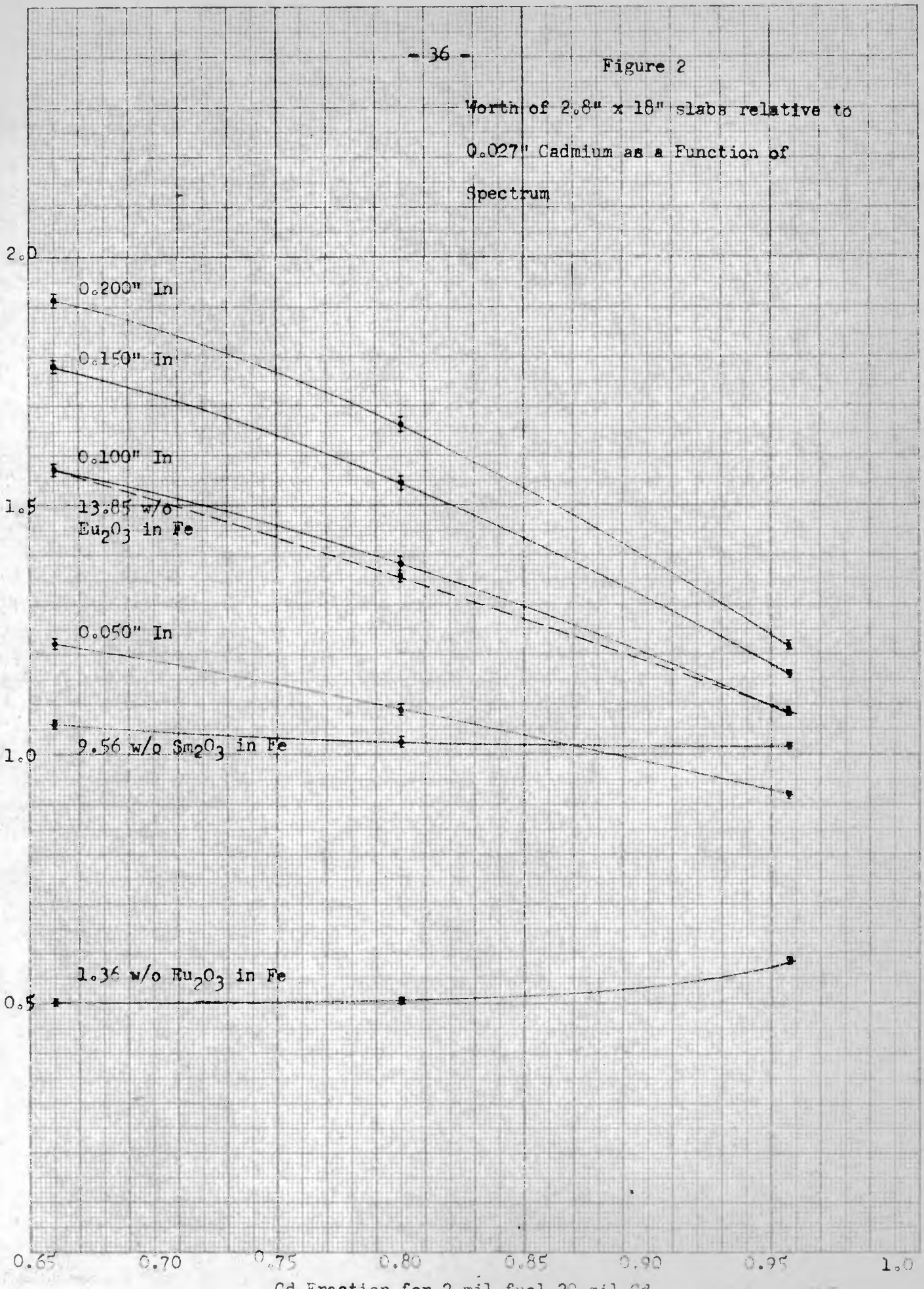


Figure 3

Worth of 2.8" x 18" slabs relative to 0.027" cadmium as a function of

$\beta\gamma = \Sigma_a(kT)/\xi\Sigma_s(\alpha)$

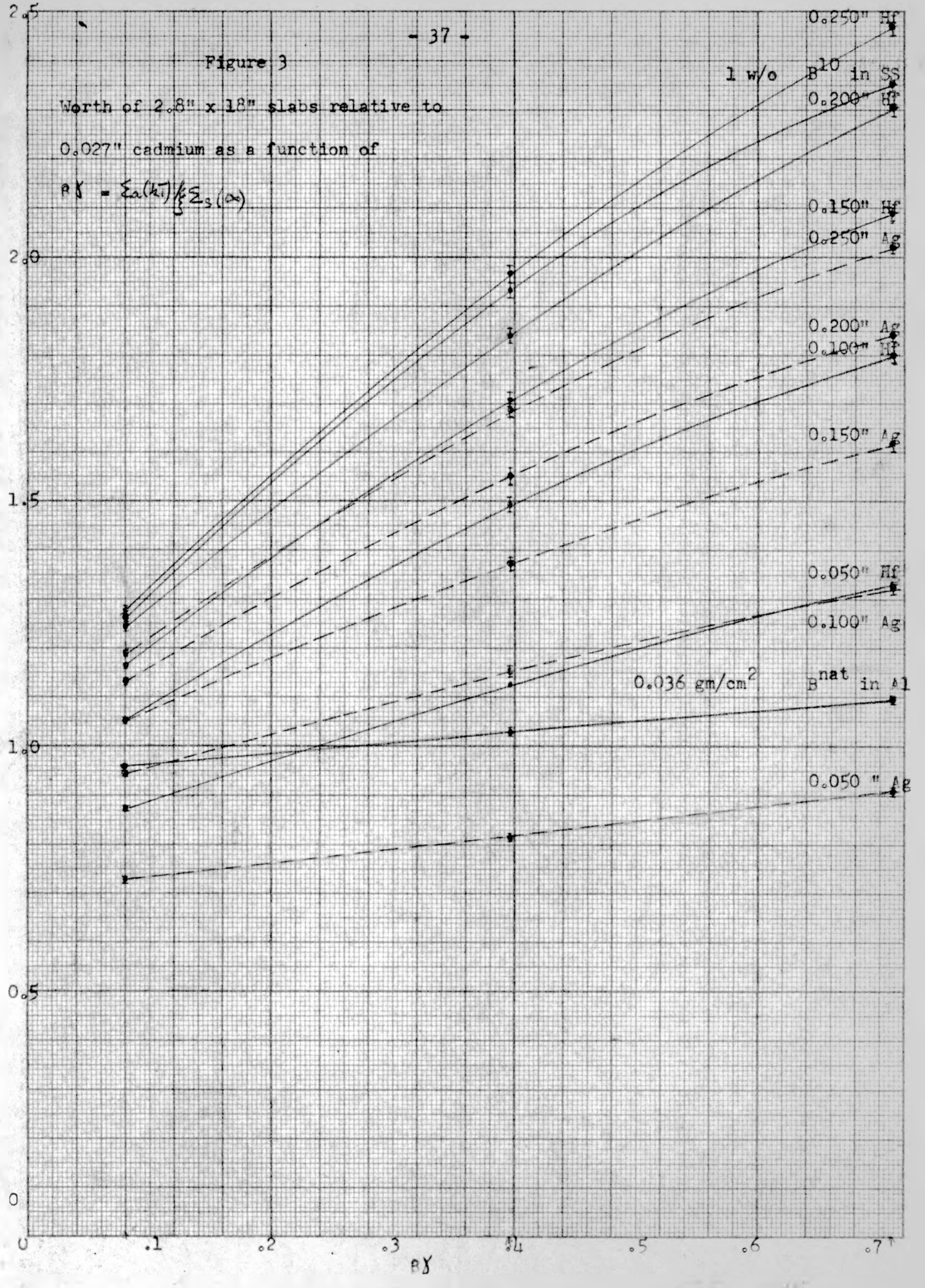


Figure 4

Worth of 2.8" x 18" Slab Rods

as a Function of Thickness

FPR-11 ($\rho\infty$ = 0.396)

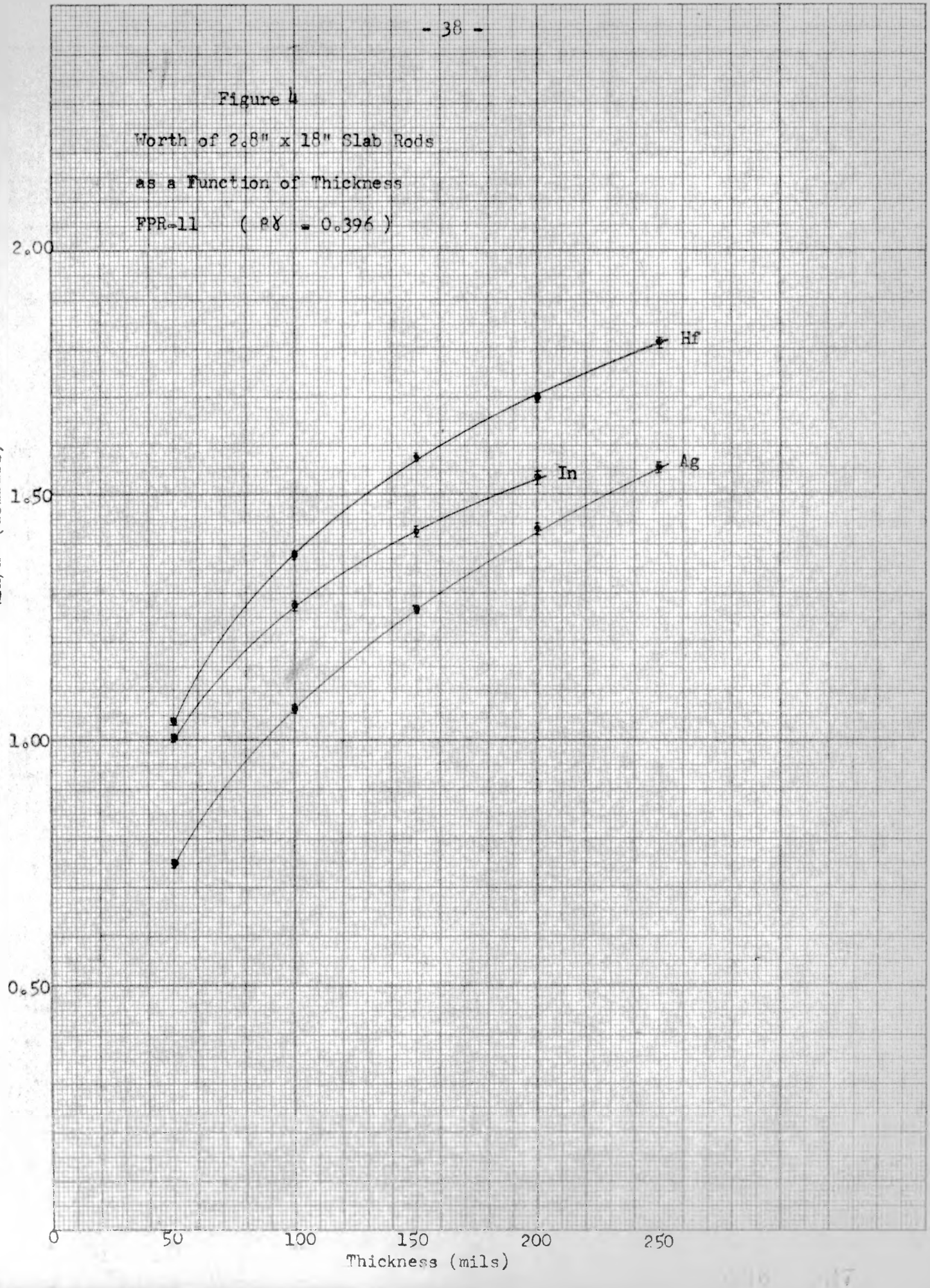


Figure 5

Worth of 2.8" x 18" Slab Rods

as a Function of Thickness

FPR-12 ($\bar{\nu}$ = 0.71)

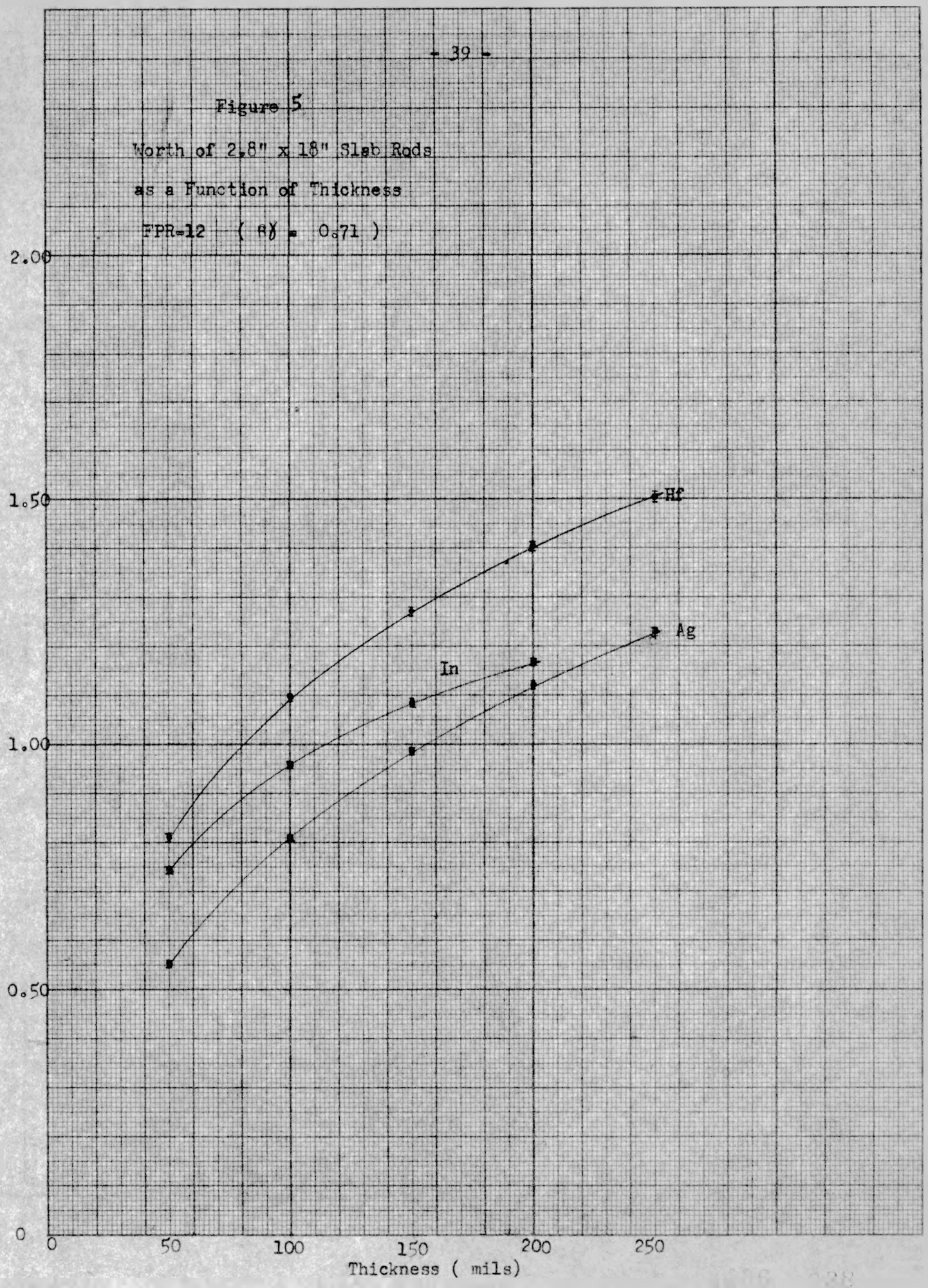


Figure 6

Worth of 2.8" x 18" Slab Rods

as a Function of Thickness

FPR-13 ($\beta\gamma$ = 0.08)

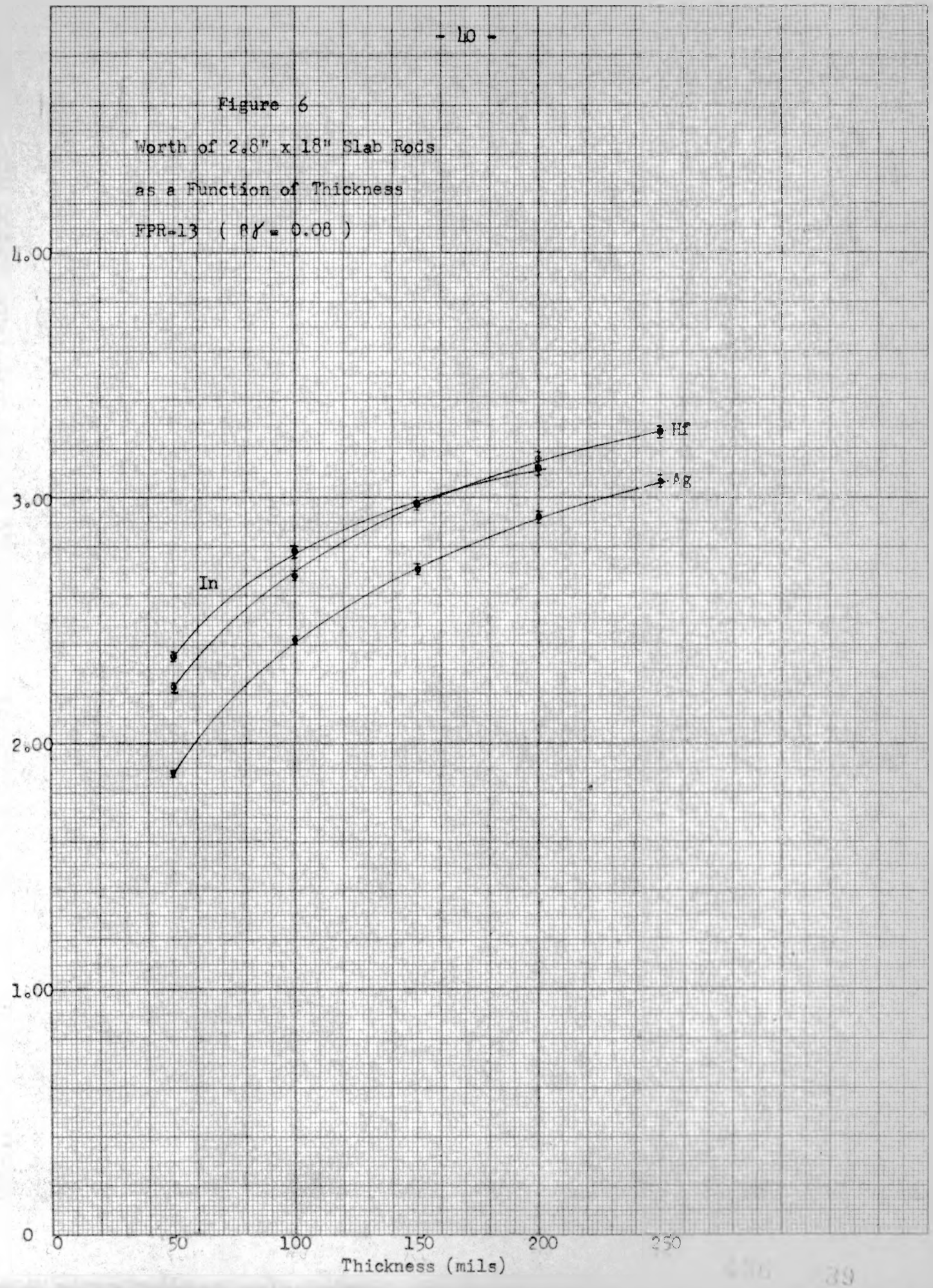


Figure 7

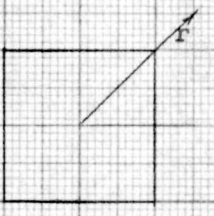
Worth of a 0.032" x 0.850" x 18" Cd slab
as a function of position in FPR 11

Φ Experiment

Calculated curves - normalized at r = 0

——— $\varphi_1^* \Sigma_f \varphi$

- - - - φ_1^2



Core - deflector interface

Δk/k (dollars)

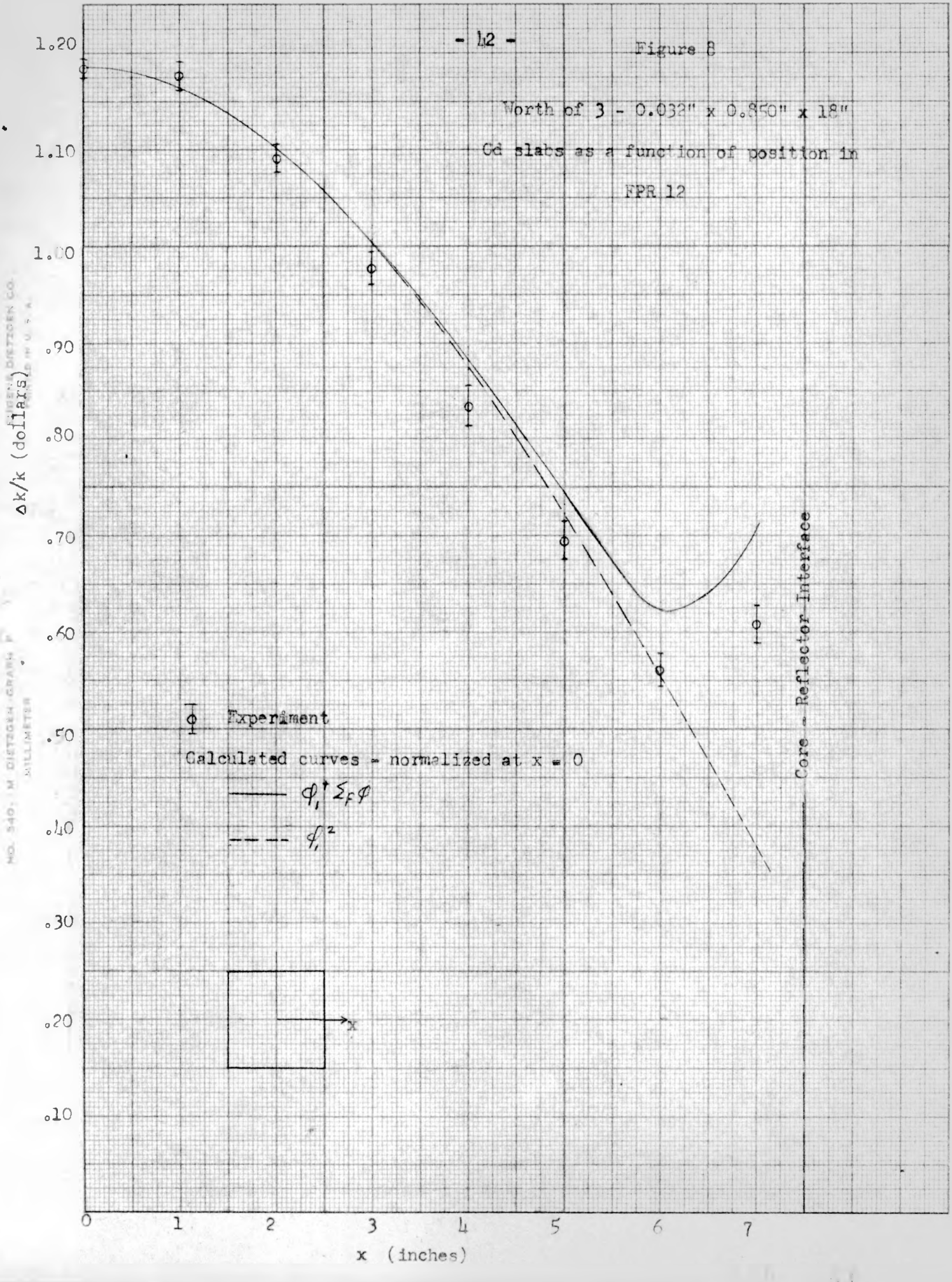
.40
.30
.20
.10

0 1 2 3 4 5 6 7

r (in units of 1.41 inches)

Figure 8

Worth of 3 - 0.032" x 0.850" x 18" Cd slabs as a function of position in FPR 12



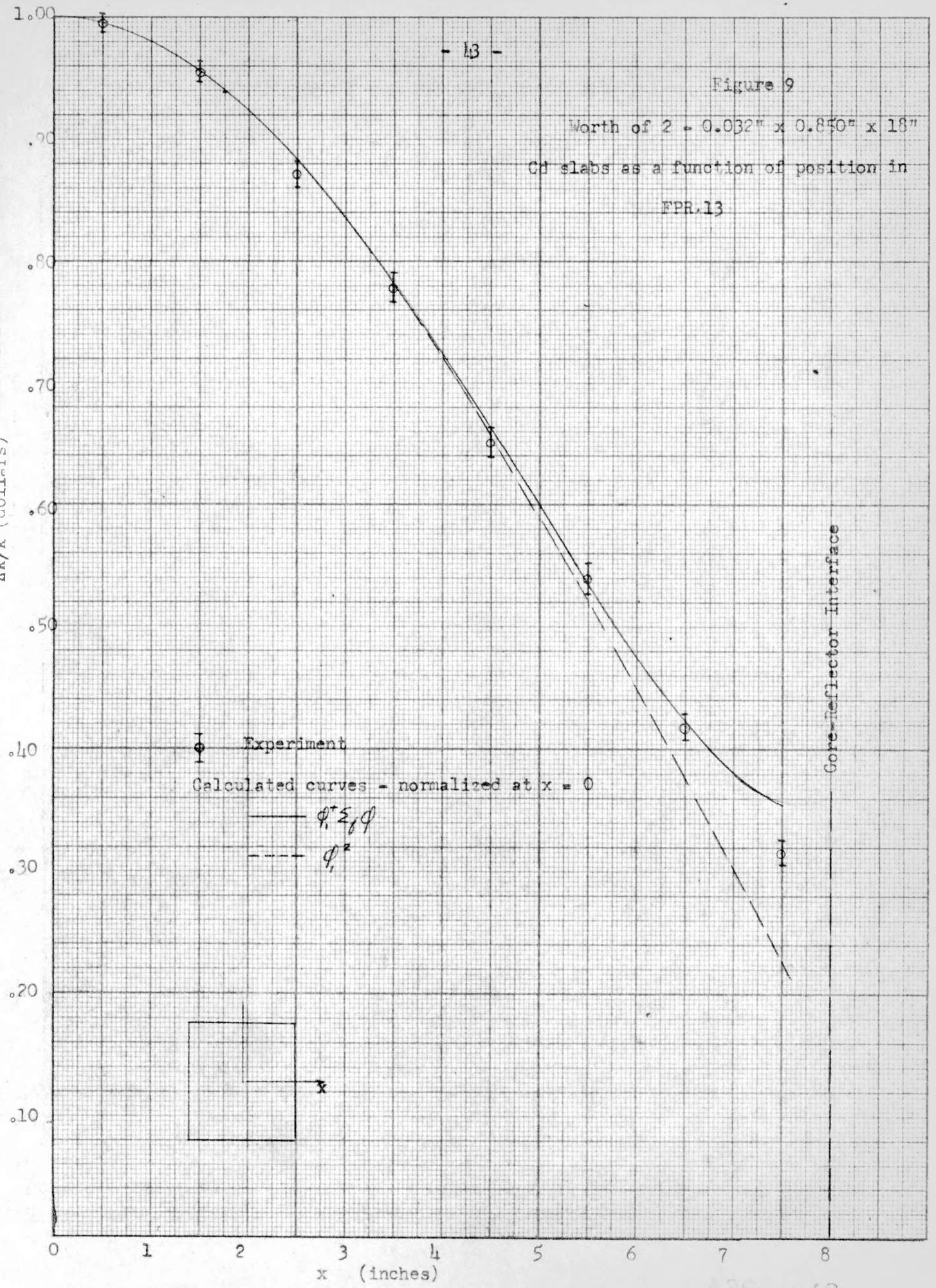


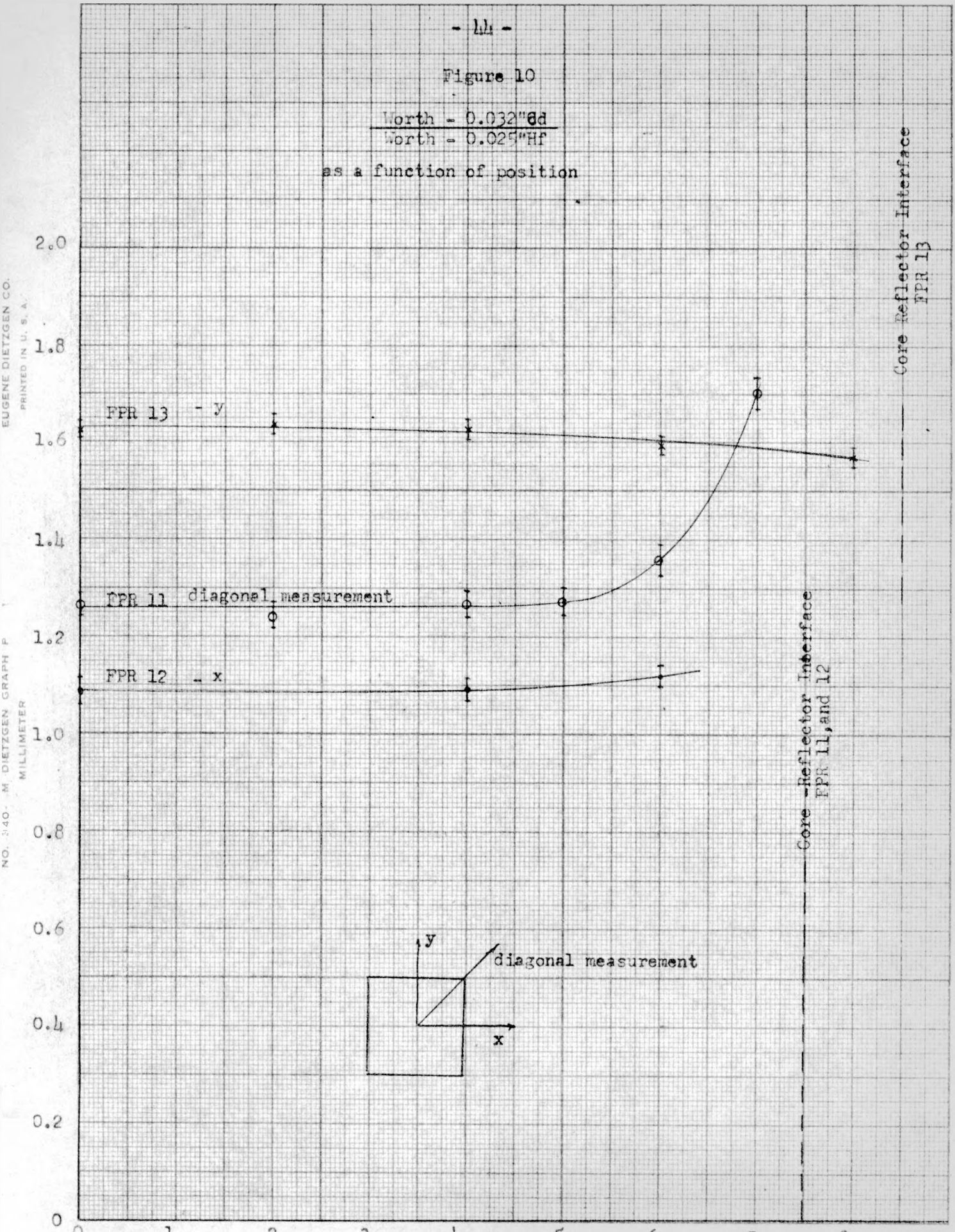
Figure 9

Worth of 2 - 0.032" x 0.850" x 18" Cd slabs as a function of position in FPR.13

Figure 10

$$\frac{\text{Worth} - 0.032\text{"Cd}}{\text{Worth} - 0.025\text{"Hf}}$$

as a function of position



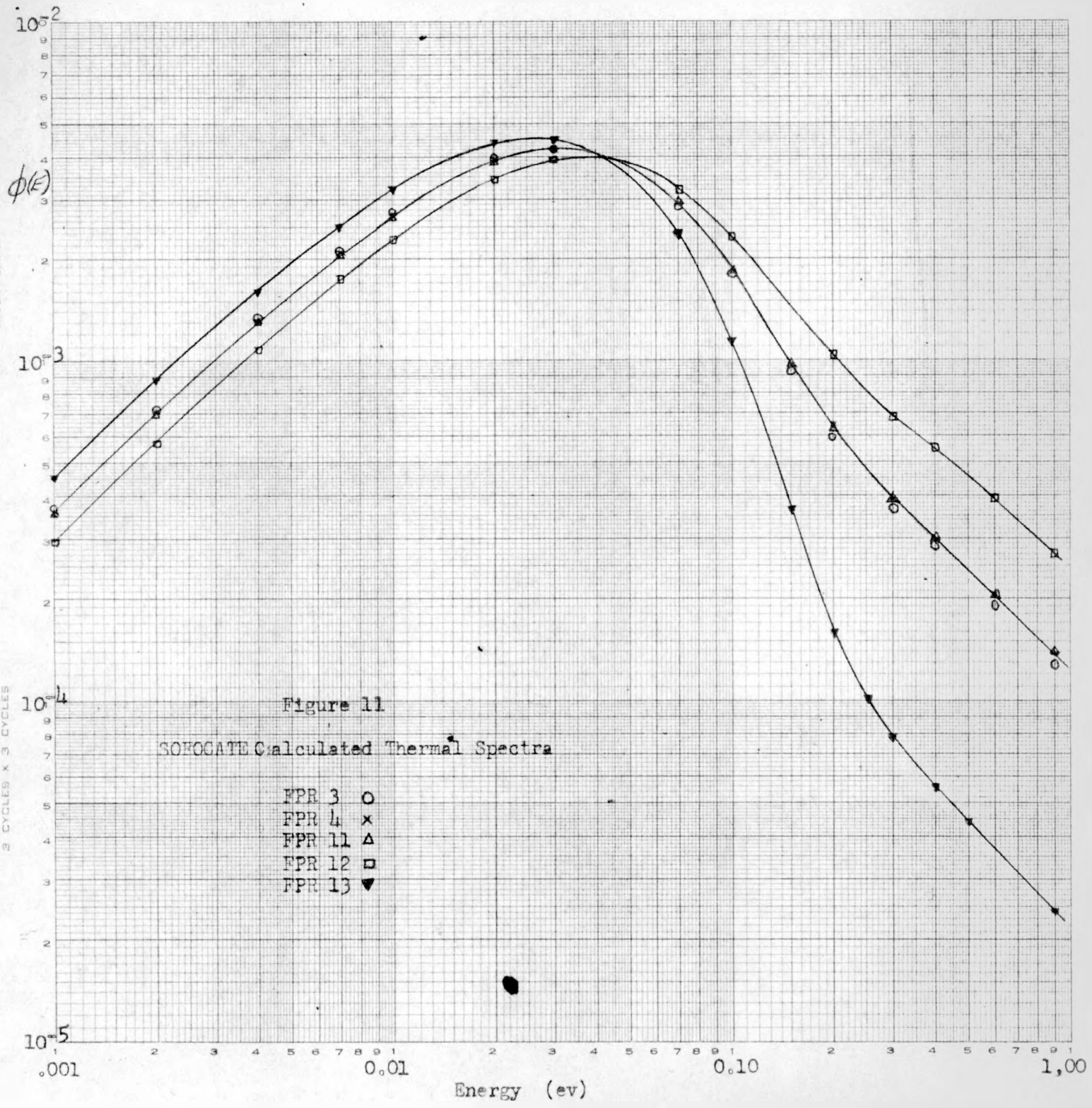


Figure 11

SOKOGATE Calculated Thermal Spectra

APPENDIX I

Assembly	Composition (Kilograms)					Geometry
	U^{235}	Al	CH_2	CF_2	$B_4^{nat}C$	
FPR - 3	44.89	568.13	178.99	3.21	1.103	869 - 1" square rods 36" long arranged as cylinder
4	32.87	652.64	107.81	1.96	0.41	
11	19.151	90.12	62.28	1.15	.326	225 - 1" square rods 36" long arranged as square parallelepiped (15" x 15" x 36")
12	29.578	121.54	52.78	1.77	.478	
13	5.148	119.42	77.85	.31	0	272 - 1" square rods 36" long arranged as parallelepiped (16" x 17" x 36")

All assemblies are surrounded by a 6" polyethylene reflector.

APPENDIX II

MUFT SOFOCATE Group Constants

	Σ_{s1_1}	D_1	Σ_{s1_2}	Σ_{a_2}	Σ_{f_2}	D_2	Σ_{a_3}	Σ_{f_3}	D_3
FPR-3	.04263	3.440	.04061	.006702	.003453	1.055	.1313	.07154	.4316
4	.02660	4.555	.02382	.004272	.002485	1.402	.08241	.05154	.6878
11	.05735	2.900	.05307	.009904	.005609	.9319	.1862	.1157	.3333
12	.04910	3.120	.04062	.011472	.008741	1.002	.2446	.1604	.3976
13	.05919	2.828	.06060	.001983	.001106	.8995	.04847	.02879	.2708

References

1. H. Hurwitz, Jr., and G. M. Roe, Absorption of Neutrons by Black Control Rods, J. Nuclear Energy, 2, 85 (1955)
2. D. R. Bach and S. W. Kitchen, Effects of Rod Shape on Control Rod Worth, TID-7532 (Part 1), October, 1957
3. B. E. Simmons - Private Communication
4. R. W. Deutsch, Nuclear Evaluation of Slab Type Control Rods, KAPL-M-RWD-17, December 19, 1955
5. S. Pearlstein, Application of the Absorption Area Method to Three Group Diffusion Theory Problems, KAPL-M-SP-10, July 2, 1957
6. J. C. Stewart and J. H. Smith, Interaction of Black Control Rods, KAPL-1471, January 8, 1956.
7. B. Wolfe, Epithermal Effects on Control Element Worth, GEAP-1061, July, 1957
8. B. Wolfe and D. L. Fischer, Perturbation Theory of Control Elements Part I, GEAP-2039
9. B. Wolfe and D. L. Fischer, Perturbation Theory of Control Elements Part II GEAP-2059, December, 1957
10. R. W. Deutsch, Computing 3-Group Constants for Neutron Diffusion, Nucleonics 15, No. 1, 47 (1957)
11. H. Amster and R. Suarez, The Calculations of Thermal Constants Averaged Over a Wigner-Wilkins Flux Spectrum: Description of the SOFOCATE Code, WAPD-TM-39, January, 1957

    H. Bohl, Jr., E. M. Gelbard, and G. H. Ryan, MUFT-4: Fast Nuetron Spectrum Code For the IBM-704, WAPD-TM-72, July, 1957

References (Con't.)

12. O. J. Marlowe et al, WANDA-A One Dimensional Few Group Diffusion Equation Code for the IBM-704, WAPD-TM-28, November, 1956

13. E. B. VanDriesen, A Comparison of WOXX, WOX-2, and WOX-3 KAPL 3-Group Microscopic Cross Section Codes for the IBM-704, KAPL-M-EVD-1, August 16, 1957

14. L. S. Bohl, Private Communication

15. E. L. Wachspress, CURE: A Generalized Two-Space Dimension Multigroup Coding for the IBM-704, KAPL-1724, April 30, 1957

16. W. A. Northrop, Private Communication

17. P. F. Zweifel and S. R. Bigelow, Ages and Slowing Down Densities For Prompt and Delayed Fission Neutrons in Zr-H_2O Mixtures, KAPL-M-PEZ-2, September 16, 1955

18. W. Skolnik, Private Communication

19. J. C. Stewart, Private Communication