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Survey of Possible Plasma Oscillations in the Stellarator

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Report written by:

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Survey of Possible Plasma Oscillations in the Stellarator

by

Lyman Spitzer, Jr.

ABSTRACT

Low-frequency ion oscillations in a normal plasma are known to produce much more rapid diffusion across a magnetic field than would be expected from collisions in a quiescent medium. In the electric arc these oscillations have a frequency of about 10^5 per second. An analysis of these oscillations shows that under conditions anticipated in the Stellarator the frequency of these oscillations, for the wave lengths of interest, is increased by a factor of at least 10^2 as compared to normal laboratory conditions. There is some question whether oscillations will develop in a plasma where no ordered electron beams are present. A combined program of theory and observation should indicate conclusively whether or not plasma oscillations exclude the successful operation of a Stellarator.

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1. Introduction

The basic design and underlying theory of the Stellarator have been presented in a previous report[1], referred to

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1. L. Spitzer, Jr., A Proposed Stellarator, Department of Astronomy, Princeton University, May 12, 1951.

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subsequently as APS. This earlier work was based entirely on the assumption that the plasma of electrons and positive ions was quiescent, and that diffusion of particles took place entirely by collisions between them. The present paper investigates the nature of the oscillations that may be expected in the Stellarator plasma, and the effects these may have on the performance of this proposed device.

One of the most thorough experimental surveys of plasma oscillations in a magnetic field has been reported and and analyzed by D. Bohm[2]. Observations were made on a plasma

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2. Characteristics of Electrical Discharges in Magnetic Fields, edited by A. Guthrie, and R. K. Wakerling, National Nuclear Energy Series, Manhattan Project Technical Section, Division I, Volume 5. (McGraw-Hill, New York, 1949)

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surrounding an electric arc. It was observed that diffusion of electrons and ions across the magnetic lines of force was about a hundred times as rapid as the diffusion predicted from collisions alone. This effect is attributed to electrical oscillations with frequencies between 4×10^4 and 2×10^5 cycles per second, which were observed to be present in the plasma. In his theoretical discussion (chapter 2), Bohm gives the following expression for the collisional diffusion coefficient D perpendicular to the magnetic field

$$D = \frac{u}{4L} \rho^2 , \qquad (1)$$

where ρ is the radius of gyration in the Larmor circle about the lines of force, u is the mean velocity perpendicular to H, and L is the mean free path between collisions. When departures from thermodynamic equilibrium are present, Bohm states that the plasma oscillations will increase exponentially until the transverse diffusion which they produce damps out any further increase. At this point, according to Bohm, D becomes

$$D = \frac{u \rho}{32} . \qquad (2)$$

Since the coefficient of diffusion along the field is of the order uL, evidently the value of D in equation (2) is about the geometical mean between the value of D along the field and its theoretical value across the field in a quiescent plasma. The observed values of D seem to agree with equation (2) to within the error of observation. Thus oscillations increase D by a factor of $L/8\rho$ above its value in a quiescent medium.

In a Stellarator, ρ is about 1 cm for ions, while L is $u\tau$, or 3×10^5 cm (see eq. 17, APS). Thus if equation (2) is really applicable, the loss of energy to the wall increases by a factor of 10^5. In this case a very great increase of Stellarator dimensions, to probably quite impractical values, would be necessary for successful operation. However, it is not at all certain that equation (2) is applicable in a Stellarator plasma. In the next section we shall discuss the type of plasma oscillation to be expected in a Stellarator, while the final section treats briefly the instabilities that may give rise to oscillations.

2. Oscillations of a Plasma

Plasma oscillations may be divided into two categories, electron oscillations and positive-ion oscillations. The former category have been extensively studied by Langmuir, Tonks, Bohm, Vlassov and others. It is well known that the angular frequency of such oscillations is approximately given by the equation

$$\omega_e^2 = \frac{4\pi n_e e^2}{m_e} \tag{3}$$

where n_e is the electron density, while e and m_e are the electronic charge and mass, respectively. In a magnetic field ω_e^2 is somewhat increased for longitudinal oscillation[3].

3. E. P. Gross, Phys. Rev. 82,232, 1951.

For standard Stellarator collisions we find

$$\omega_e = 9 \times 10^{11} \text{ sec}^{-1} \tag{4}$$

This value is about three times the Larmor frequency $eH/m_e c$, and the magnetic field has only a small effect on these

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longitudinal oscillations. Transverse plasma oscillations also exist; these are essentially electromagnetic waves, and have higher frequencies than the longitudinal waves, in general.

No mechanism has yet been found whereby electron oscillations of this type can directly cause diffusion perpendicular to the magnetic field. Each electron in the plasma oscillates about its equilibrium position, with no net displacement. Positive ions, of greater mass, are completely unaffected. Possibly in a higher approximation, including shock waves, etc., electron plasma oscillations might be responsible for a net flow, at least of electrons, but this subject still remains to be explored.

The ion oscillations are of much lower frequency. If motions of the positive ions did not affect the electrons, the ion oscillations would have a frequency given by equation (3), with m_i replacing m_e and $Z^2 n_i$ replacing n_e; in a deuteron plasma ω_i would be given by

$$\omega_i = 1.5 \times 10^{10} \text{ sec}^{-1} . \qquad (5)$$

In fact, however, the electrostatic fields produced by the oscillating positive ions are largely neutralized by the electrons. If the electrons had no random velocities, the neutralization would be exact, and there would be no restoring force whatever on the positive ions. At any finite electron temperature, however, the neutralization is not quite perfect, and a very small restoring force tends to bring the ions back to uniform distribution. Hence oscillations can proceed at

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much lower frequencies than given by equation (5).

In view of the importance of these ion oscillations, we give here an analysis of their frequency, adapted largely from R. Rompe and H. Steenbeck[4]. If φ denotes the

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4. Ergebnisse der exacten Naturwissenschaften, Bd. 18, 303, 1939.

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electrostatic potential, then, according to Poisson's equation

$$\nabla^2 \varphi = 4\pi e\,(n_e - Z_1 n_1)\,, \tag{6}$$

where Z_1 is the number of unit charges per positive ion. We assume that on the average

$$\bar{n}_e = Z_1 \bar{n}_1 = n_o\,, \tag{7}$$

and we let

$$n_1 = n_o + \Delta n_1\,. \tag{8}$$

Now we assume that the ionic motion is sufficiently slow so that the electrons obey at all times the Boltzmann destribution law

$$n_e = n_o e^{e\varphi/kT}\,, \tag{9}$$

where k is the gas constant. If we expand the exponential in equation (9), and retain only the first two terms, then equation (6) becomes, for small oscillations,

$$\nabla^2 \varphi - \frac{4\pi e^2 n_o}{kT} = -4\pi e Z_1 \Delta n_1\,. \tag{10}$$

If we assume a sinusoidal progressive wave,

$$\varphi \propto e^{i(kx - \omega t)}\,, \tag{11}$$

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then equation (10) becomes

$$\varphi = \frac{4\pi e Z_1 \Delta n_1}{k^2 + 1/h^2}, \tag{12}$$

where h is the Debeye shielding distance, given by

$$h^2 = \frac{k_B T}{4\pi e^2 n_o}. \tag{13}$$

Equation (12) gives the electrostatic potential resulting from the displacement of the positive ions, taking into account electron shielding. If h were infinite, equation (12) would give the potential in the absence of shielding. Fvidently for wave lengths long compared to the shielding distance, φ is much reduced by the presence of electrons.

We now consider the equation of motion of the positive ions, which is

$$m_1 \frac{\partial \underset{\sim}{v}}{\partial t} = - Z_1 e \underset{\sim}{\nabla} \varphi. \tag{14}$$

The equation of continuity yields

$$n_o \underset{\sim}{\nabla} \cdot \underset{\sim}{v} + \frac{\partial}{\partial t}(\Delta n_i) = 0 \tag{15}$$

where we neglect a second order term in $\underset{\sim}{v} \cdot \underset{\sim}{\nabla} n_i$

If we take the divergence of equation (14) and combine with the time derivative of equation (15), then we obtain

$$\frac{\partial^2}{\partial t^2}(\Delta n_1) = \frac{Z_1 e n_o}{m_1} \nabla^2 \varphi. \tag{16}$$

From equation (12) we know φ in terms of Δn_1. Hence we obtain a wave equation for Δn_1. Considering again a variation of the form (11), we obtain, finally,

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$$\omega^2 = \frac{\omega_i^2}{1 + 1/h^2 k^2} \quad . \qquad (17$$

In most cases of interest, hk is small compared with one and

$$\omega = \omega_1 h k \quad . \qquad (18$$

The wave velocity V equals ω/k, and becomes

$$V = \omega_1 h = \left(\frac{Z_1 \; k T}{m_1} \right)^{1/2} \quad . \qquad (19$$

Since the wave velocity is of the order of the thermal velocity, these low-frequency ion waves have been called acoustic plasma waves.

For low frequencies these ion waves are obviously affected strongly by the magnetic field. The Larmor frequency of a deuteron for standard Stellarator conditions is about 10^8, and for frequencies less than this, no ion velocities across the lines of force are possible. In particular, a transverse electric field does not produce a current in the field direction but only a drift at right angles. Under these conditions, ion waves can travel along the magnetic field, but they cannot travel perpendicularly across the field.

It is these ion oscillations which, according to Bohm[2], produce the observed rapid diffusion of electrons and ions across the field. We may imagine many such waves travelling along the field, with different amplitudes and phases at different points on a transverse plane, perpendicular to H. In such a plane there will, therefore, be oscillating electrical fields, which will produce drifts across the field, and

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may well be responsible for the observed diffusion. The wave lengths of such disturbances must be small compared to the dimensions of the arc, if different waves are to be present at different points in a transverse plane, with about 1 cm as a reasonable maximum value for the roughly 10 cm arcs investigated. Since the kinetic temperature of the argon ions in the plasma was probably in the neighborhood of 0.1 to 1 volt, the wave velocity V in these measurements was about 10^5 cm/sec, and a 1 cm wave length would give a frequency of 10^5 per second, in agreement with the observations. Since the electron shielding distance h is about 10^{-3} cm under these conditions, equation (18) is a valid approximation.

We now examine these acoustic waves under the conditions to be expected in the Stellarator. The shielding distance is increased to only 4×10^{-3} cm, since the increase in n_o almost counterbalances the increase in T. However, the ion velocity is increased by a factor of 10^3, and for a wave length of one centimeter, the oscillation frequency is 10^8 sec^{-1}. Since the Stellarator tube would be some ten times as wide as the arcs investigated by Bohm and his co-workers, a wave length of 10 cm may be assumed for the plasma oscillations, but even for this long a wave length, the frequency is 10^7 sec^{-1}. Waves of much smaller frequency would tend to be nearly uniform over each cross-section of the Stellarator tube; no electric fields in the transverse direction could develop, and no transverse diffusion could be produced by oscillations of substantially lower frequency.

It is interest that this computed frequency of 10^7 per second is substantially greater than the electron collision

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frequency in the Stellarator, which is about 2×10^4 per second. For comparison, in the experiments discussed by Bohm the electric collision frequency was between 10^7 and 10^8 per second, as compared to a positive ion plasma frequency of about 10^5. It is possible that this change may produce important alterations in the plasma oscillations. The distribution function f for the electrons must be determined from the solution of the Boltzmann partial differential equation in this case, and the shielding effect of the electrons will depend on the past history of the wave. Both the frequency and the damping of the positive ion oscillations may be affected, but detailed computations are needed to indicate the effects to be expected.

We may conclude that positive ion oscillations in the Stellarator appear at considerably higher frequencies than in familiar low-temperature arcs. If the increase in frequency is sufficiently great, transverse diffusion may not appear at all. In any case, the transverse diffusion produced by such oscillations will tend to be reduced by the increase in frequency, since a greater number of smaller random drifts would produce a smaller effect, other things being equal.

3. Generation of Plasma Oscillations

There remains the question whether oscillations will in fact be generated in a Stellarator plasma. It is not impossible that under practical operating conditions oscillations will not appear. We consider briefly the possible sources of oscillations.

It is well known that a beam of electrons passing through a plasma is inherently unstable if all the electrons in the

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beam have about the same energy. A small oscillation in the distribution of the plasma electrons produce a small oscillating electrostatic field which modulates the electron beam, produces large oscillations in the spatial distribution of electrons in the beam and thus produces a greatly amplified oscillating electrical field. In most plasmas the high ionization level is maintained by electron beams, and thus the strong oscillations observed in most plasmas are not surprising.

Little is known about the instability of plasma oscillations in otherwise quiescent plasma. Bohm[2] has stated that whenever there exist density gradients or electrical fields across a magnetic field, instability is necessarily present. However, it appears[5] that the proof of this statement was not

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5. Informal conversation with D. Bohm.

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very rigorous, was never written up, and is not now available. It has been demonstrated that plasma waves will increase in amplitude when travelling in the direction of decreasing density, but they will suffer a corresponding decrease on the way back, and it is not clear how the amplitude of the oscillations at a given point can increase exponentially with time. However, in a system with so many particles and with so few collisions as a Stellarator plasma, almost any type of behavior would not be too surprising.

If Bohm's statement is correct, and essentially all plasmas not in thermodynamic equilibrium are unstable, one may ask what happens to the plasma oscillations in a Stellarator.

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If, as suggested in the proceeding section, transverse diffusion cannot be produced under such conditions, what will limit the amplitude of the plasma oscillations? One may conjecture that other non-linear phenomena become limiting. Possibly plasma shock waves may appear, and limit the further growth of the oscillations.

It is evident that much further work is needed on plasma oscillations to yield conclusive results. In particular, experiments should be carried out with plasmas where the ionization is maintained by the photoelectric effect, or by some other process that does not, in itself, produce instabilities. The nature of the oscillations appearing in such a plasma could then be examined. Possibly the nature of plasma waves generated by an external source could be studied. A thorough theoretical analysis, carried out in conjunction with this experimental program, should show conclusively whether or not plasma oscillations may provide an essential barrier to the practical functioning of a Stellarator.

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