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TRANSMISSION-LINE MISSILE ANTENNAS

by

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ABSTRACT

A class of protruding rocket antennas of low silhouette is analyzed. The approximate equivalent circuit of these structures is a shunt-driven transmission line terminated in reactors of arbitrary value at each end. Expressions derived for the currents in the circuit, as functions of the reactive terminations, are employed in calculating the radiation vectors of the antenna. The radiation resistance is obtained by integrating the Poynting vector over the surface of a great sphere enclosing the structure. The driving-point reactance of the antenna is determined from transmission-line formulas, which may be modified appropriately to include terminal zone effects.

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TRANSMISSION-LINE MISSILE ANTENNAS

Introduction

As the Mach number of missiles increases, it becomes necessary to lower the silhouette of externally carried telemetry antennas to reduce drag and not further degrade the aerodynamical characteristics of the missile.

An important class of externally carried missile telemetry antennas of low silhouette may be analyzed in terms of a shunt-driven transmission line terminated at each end in reactors of arbitrary value. Several common rocket antennas of transmission-line type are shown in Figs. 1-4. Figure 1 illustrates a simple inverted L-antenna; its circuit is approximated by a section of radiating transmission line driven at one end and open-circuited at the other. Figures 2 and 3 portray shunt-driven inverted L-antennas, also of transmission-line type. The structure pictured in Fig. 3 has adjustable capacitive end loading that makes some reduction in size possible and also permits a satisfactory driving-point impedance over the required frequency band. Figure 4 shows an m-antenna, so designated because it resembles a lower case m.

It is evident from the four transmission-line antennas that have been mentioned specifically that a general theory of a reactively terminated, shunt-driven transmission line will permit the calculation of the driving-point impedances of a large number of possible rocket-antenna types with low silhouettes.

The theory of transmission-line antennas set forth in this paper applies strictly to relatively thin conductors located over an infinite perfectly conducting ground plane. The dimensions of the circuit, expressed in terms of the wavelength, must satisfy transmission-line criteria. When the antennas are mounted on the body of a missile having a diameter that is not large compared to the wavelength, the computed impedances may be somewhat in error. However, such an error decreases when the height of the structure is reduced. In the event the height of the horizontal member above the ground plane greatly exceeds that permitted by transmission-line theory, more general methods of analysis are required.

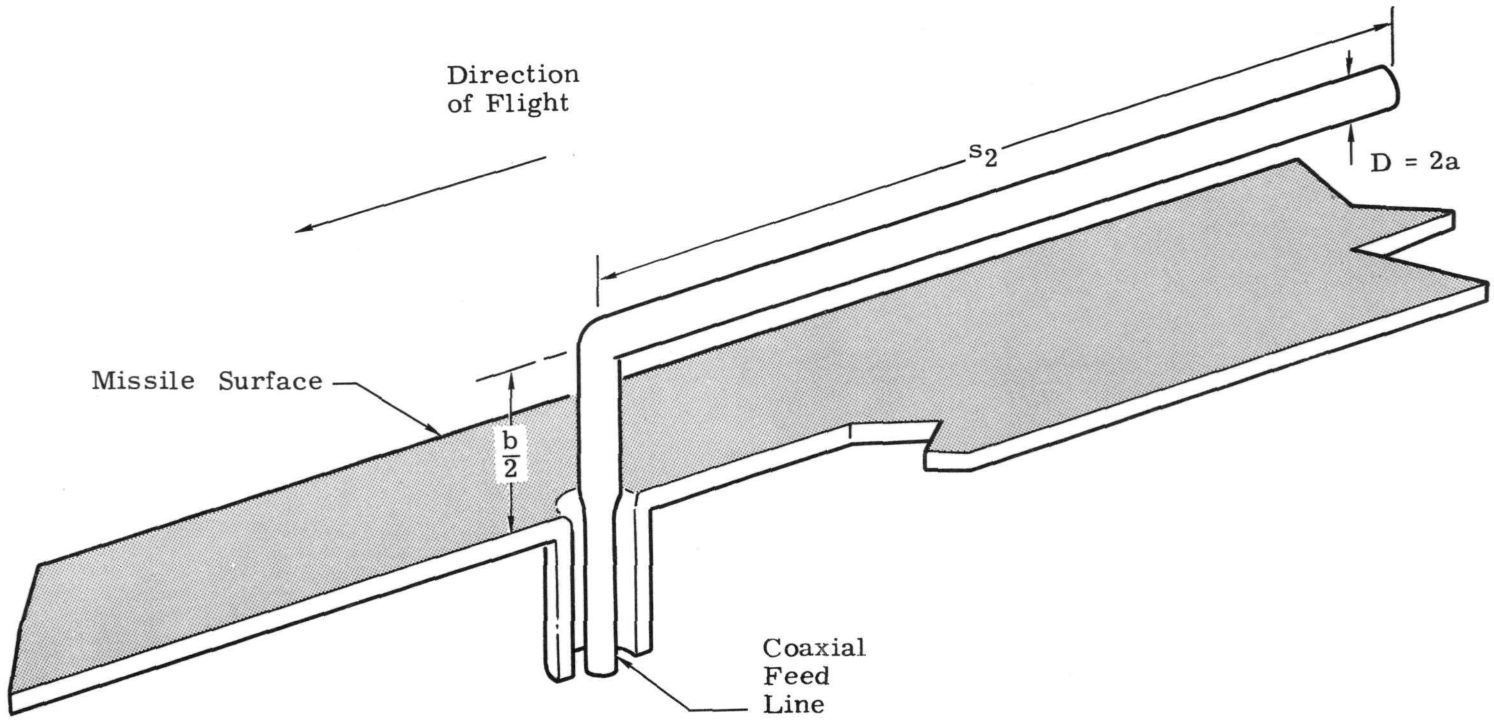

Fig. 1: Inverted L - Antenna-Transmission Line

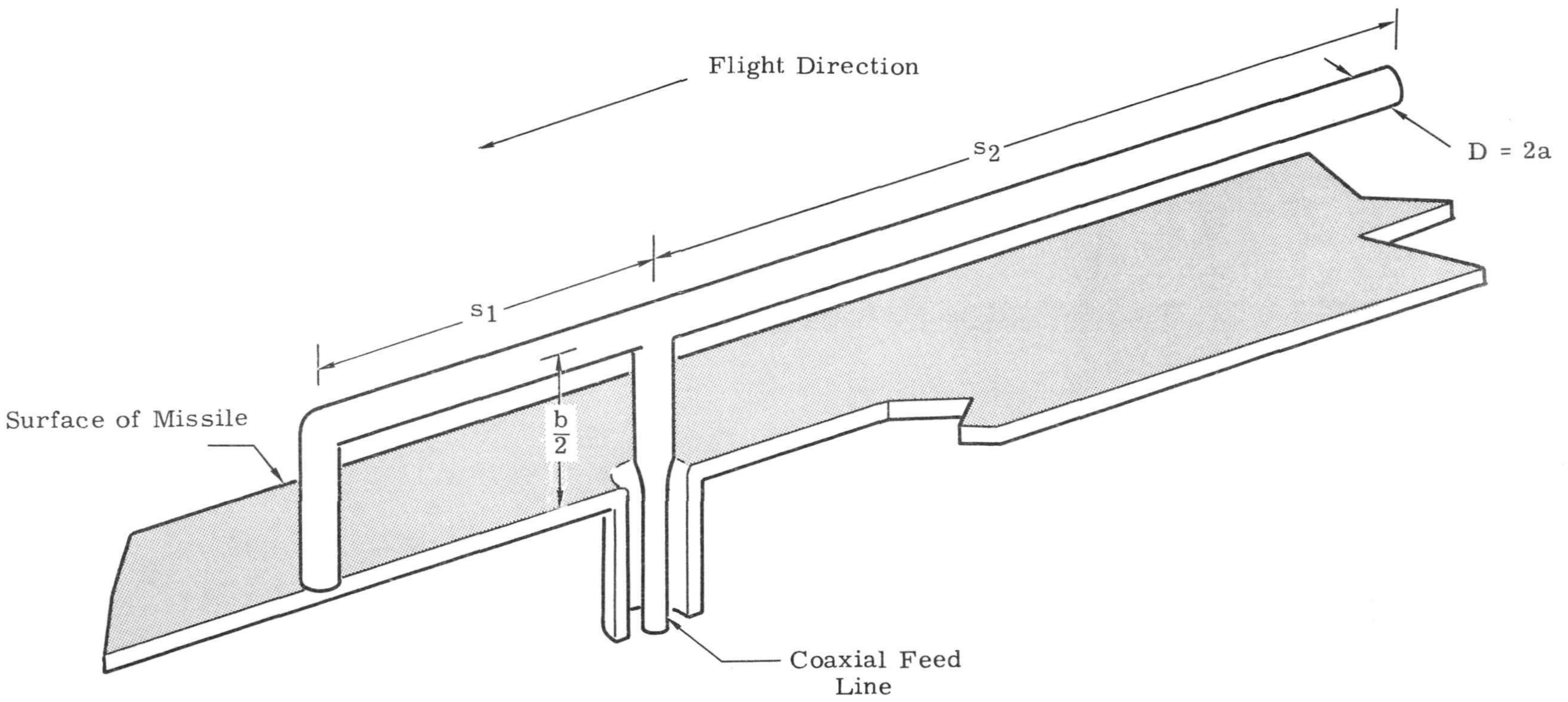

Fig. 2: Shunt-Driven Inverted L Antenna - Transmission Line Without Capacitive End Loading

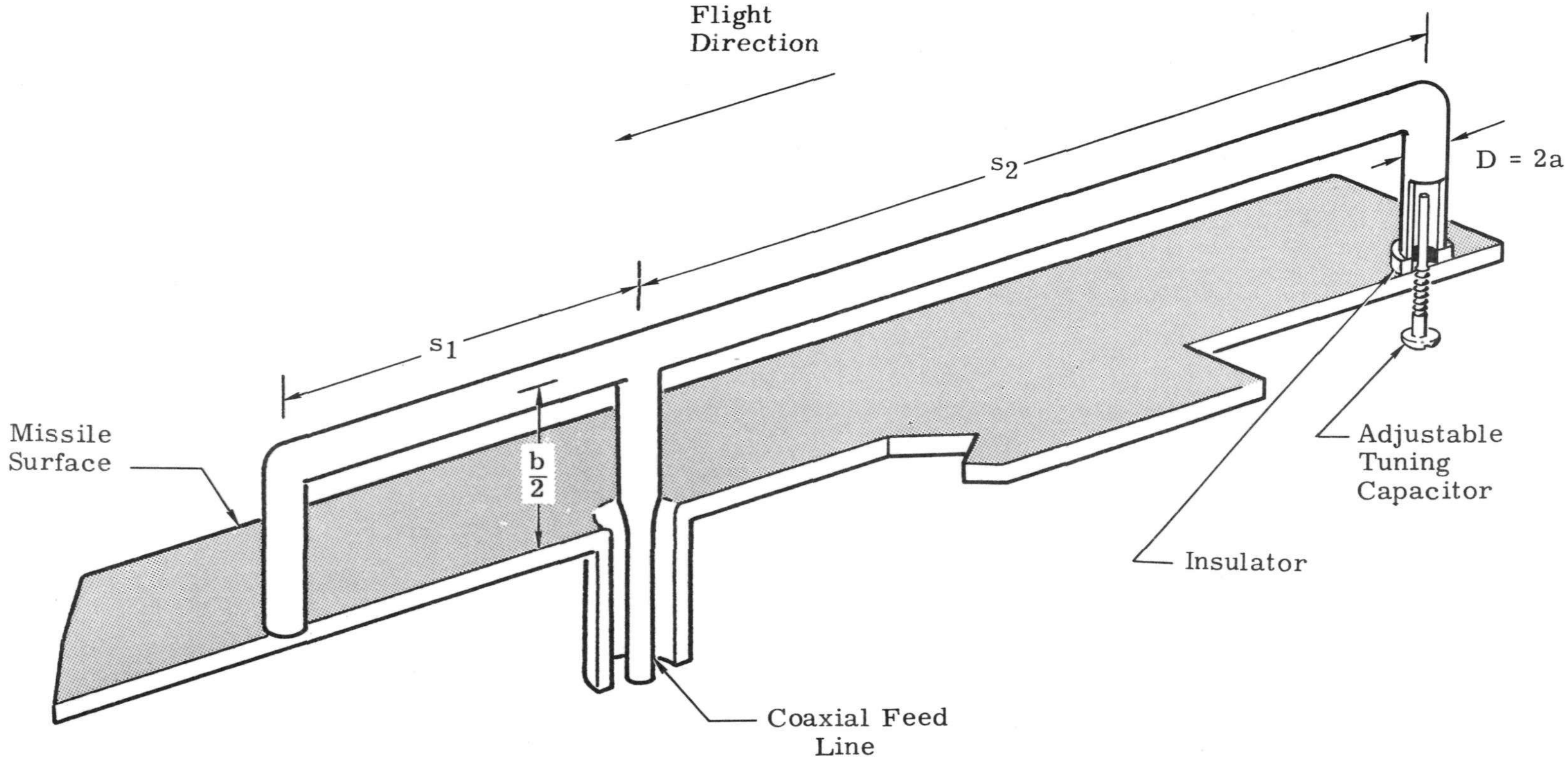

Fig. 3: Shunt-Driven Inverted L-Antenna - Transmission Line with Capacitive End Loading

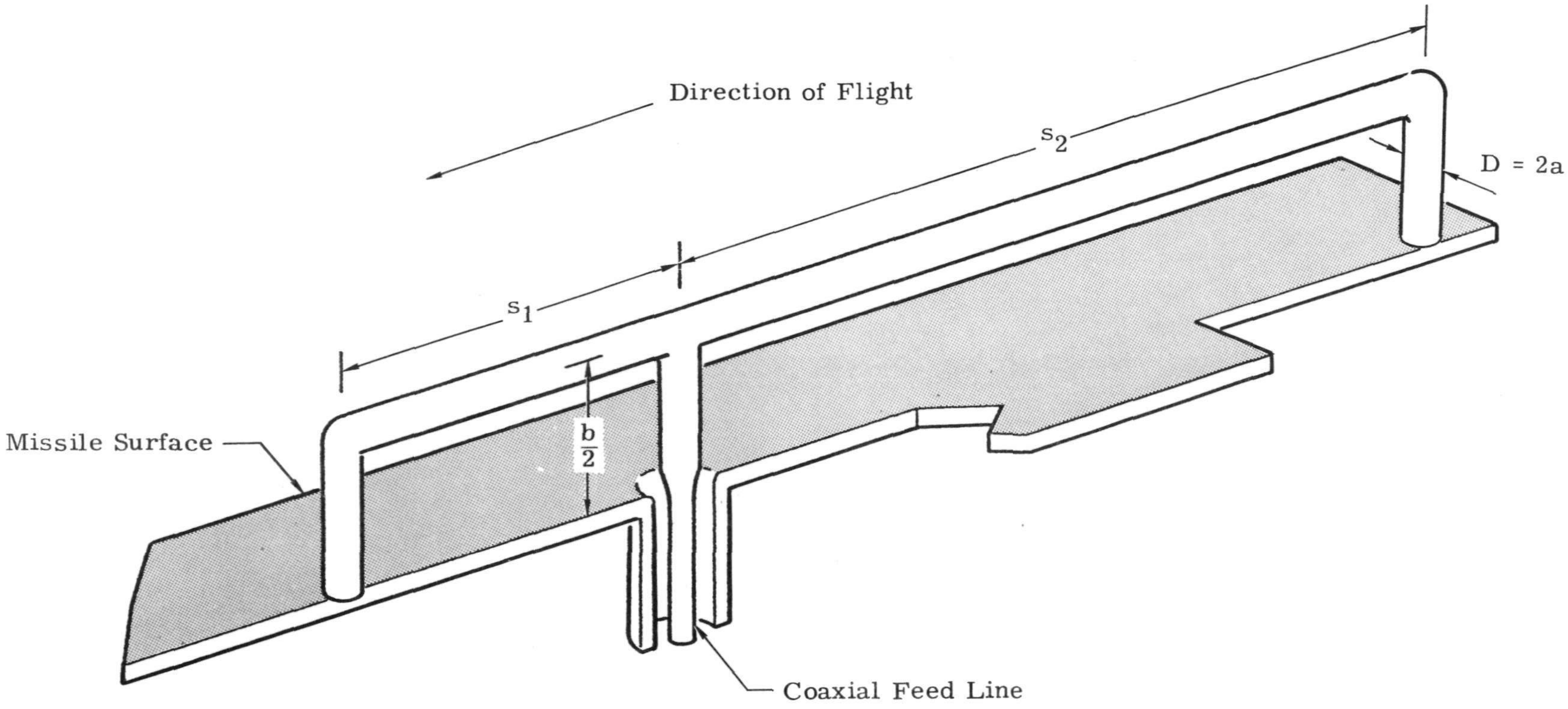

Fig. 4: m - Antenna - Transmission Line

Although the various drawings show antennas constructed of circular wires, conductors of other cross sections may be used. Methods are available for determining the correct sizes of the "equivalent" circular conductors. A conductor of triangular cross section has the aerodynamical advantage of a sharp leading edge.

It is to be emphasized that the radiation resistance of a transmission line is small. However, if the line consists of low-loss conductors, terminated in reactive elements, such as capacitors, conducting bridges, or open ends, the power radiated may greatly exceed the power dissipated in heat, provided the spacing between conductors is not much less than about a hundredth of a wavelength or so.

The radiation resistance of a two-wire line, driven at one end and terminated at the other, has been discussed by Storer and King.[1/] With the exception of the inverted L-antenna with reactive end loading, the theory does not include the antenna types discussed in this paper. A more general circuit must be analyzed in order to permit the calculation of, for example, the impedance of a shunt-driven inverted L-antenna of transmission-line type. Antennas of this class, sometimes with capacitive end loading, are widely used as test vehicle antennas for telemetry.

An extension of the work of Storer and King[1/] permits the development of a formula for the radiation resistance of a shunt-driven transmission line having reactive terminations. The ordinary transmission-line equations may be employed to calculate the reactance of the circuit. Accordingly, after expressions for the currents in the various wires comprising the generalized structure have been obtained, the succeeding analysis will be outlined only briefly since the various steps are straightforward, although quite laborious.

The Current Distribution

Figure 5 represents the generalized equivalent circuit considered in this paper. Parallel wires of radius a, length s, and spaced at a distance b between centers are terminated in impedances Z_o at $z = 0$, $w = s - z = s$, and Z_s at $z = s$, $(w = 0)$. An impedanceless generator of voltage V^e is connected across the line at $z = s_1$ $(w = s_2)$. Ordinarily, $s_2 > s_1$. The currents in the generator and the terminations Z_o and Z_s are, respectively, I_g, I_o, and I_s. The currents in the line are I_{z1} when $0 \leq z \leq s_1$, and I_{z2} when $0 \leq w \leq s_2$.

[1/] Storer, James E., and King, Ronold, "Radiation Resistance of a Two Wire Line," Proc. IRE, Vol. 39, No. 11, pp. 1408-1412, November 1951.

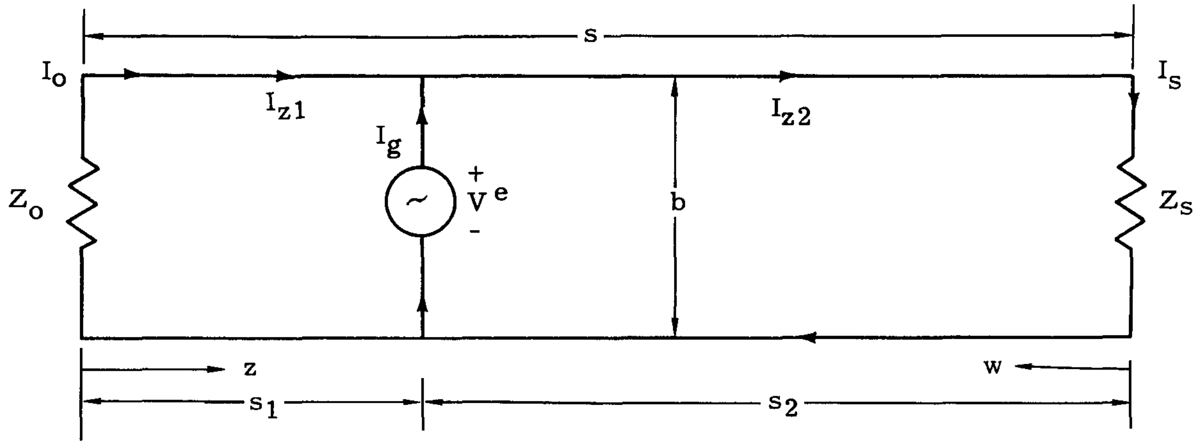

Fig. 5: Shunt-Driven Transmission Line Terminated in Impedances of Arbitrary Value

Analytical expressions for the currents I_{z1} and I_{z2} may be obtained from conventional transmission-line theory.[2/] These are:

$$I_{z1}(z) = \frac{V^e}{D_1}(Z_o \sinh \gamma z + Z_c \cosh \gamma z) \qquad 0 \leq z \leq s_1 \tag{1}$$

$$I_{z2}(w) = \frac{V^e}{D_2}(Z_s \sinh \gamma w + Z_c \cosh \gamma w) \qquad 0 \leq w \leq s_2 \tag{2}$$

where

$$D_1 = Z_c^2 \sinh \gamma s_1 + Z_c Z_o \cosh \gamma s_1 \tag{3}$$

$$D_2 = Z_c^2 \sinh \gamma s_2 + Z_c Z_s \cosh \gamma s_2 \, . \tag{4}$$

The complex propagation constant is $\gamma = \alpha + j\beta$. Z_c is the characteristic impedance.

An inspection of Fig. 5 reveals that

$$I_g = I_{z2}(s_2) - I_{z1}(s_1) \, . \tag{5}$$

[2/] King, Ronold, Transmission Line Theory, McGraw-Hill Book Co., Inc. (1955), Chapter 2, Section 8, p. 83.

This relation, with (1) and (2), makes it possible to express V^e as a function of I_g. When this has been done and the substitutions

$$\theta_o = \rho_o + j\Phi_o = \coth^{-1}\left(\frac{Z_o}{Z_c}\right) \tag{6}$$

$$\theta_s = \rho_s + j\Phi_s = \coth^{-1}\left(\frac{Z_s}{Z_c}\right) \tag{7}$$

made, it is readily shown that

$$I_{z1}(z) = -I_g\left\{\frac{\cosh(\gamma s_2 + \theta_s)\sinh(\gamma z + \theta_o)}{\sinh(\gamma s + \theta_o + \theta_s)}\right\} \qquad 0 \leq z \leq s_1 \tag{8}$$

and

$$I_{z2}(w) = I_g\left\{\frac{\cosh(\gamma s_1 + \theta_o)\sinh(\gamma w + \theta_s)}{\sinh(\gamma s + \theta_o + \theta_s)}\right\} . \qquad 0 \leq w \leq s_2 \tag{9}$$

The currents in the terminating impedances Z_o and Z_s are

$$I_o = I_{z1}(o) = -I_g\left\{\frac{\cosh(\gamma s_2 + \theta_s)\sinh \theta_o}{\sinh(\gamma s + \theta_o + \theta_s)}\right\} \tag{10}$$

$$I_s = I_{z2}(o) = I_g\left\{\frac{\cosh(\gamma s_1 + \theta_o)\sinh \theta_s}{\sinh(\gamma s + \theta_o + \theta_s)}\right\} . \tag{11}$$

Missile antennas of the transmission-line variety are generally designed so that ohmic losses are minimized. To achieve this the line attenuation must be negligible and the terminations predominately reactive. Of course, zero and infinite values of Z_o and Z_s are not excluded. Under these conditions

$$\left.\begin{aligned} \gamma &= \alpha + j\beta \approx j\beta \\ \theta_o &= \rho_o + j\Phi_o \approx j\Phi_o \\ \theta_s &= \rho_s + j\Phi_s \approx j\Phi_s \,. \end{aligned}\right\} \tag{12}$$

Subject to (12), and with the notation

$$K_o = \frac{\cos(\beta s_2 + \Phi_s)\sin\Phi_o}{\sin(\beta s + \Phi_o + \Phi_s) - j\delta} \tag{13}$$

$$K_s = \frac{\cos(\beta s_1 + \Phi_o)\sin\Phi_s}{\sin(\beta s + \Phi_o + \Phi_s) - j\delta}\,, \tag{14}$$

where the quantity

$$\delta = (\alpha s + \rho_o + \rho_s)\cos(\beta s + \Phi_o + \Phi_s) \tag{15}$$

may be neglected except when $\sin(\beta s + \Phi_o + \Phi_s) = 0$, Equations (8) through (11) may be written as follows:

$$I_{z1}(z) = I_o\left(\frac{\sin(\beta z + \Phi_o)}{\sin\Phi_o}\right) \qquad 0 \le z \le s_1 \tag{16}$$

$$I_{z2}(w) = I_s\left(\frac{\sin(\beta w + \Phi_o)}{\sin\Phi_s}\right) \qquad 0 \le w \le s_2 \tag{17}$$

$$I_o = -I_g K_o \tag{18}$$

$$I_s = I_g K_s \,. \tag{19}$$

It will be noticed that the currents I_o, I_g, and I_s are of constant value through the conductors of length b. This is a consequence of the fact that the implicit assumption made in transmission-line analysis is that $\beta^2 b^2 << 1$. The "simplified" expressions for the currents, (16) through (19), are employed in calculating the radiation resistance of the shunt-driven, reactively terminated transmission line with negligible attenuation.

The Vector Potential in the Far Zone of the Radiating Circuit

Figure 6 illustrates the orientation of the shunt-driven antenna with respect to the coordinate systems employed in calculating the radiation vectors of the circuit. In developing the theory of linear radiators it has been customary to employ z' as the source variable, with origin at the driving point. It is deemed desirable to continue this convention in the present paper. This is accomplished by writing $z' + s_1$ for z. Then $w = s_2 - z'$. With this change (16) and (17) become

$$I_z(z') = \frac{I_o}{\sin \Phi_o} \sin\left[\beta(z' + s_1) + \Phi_o\right] \qquad 0 \geq z' \geq -s_1 \tag{20}$$

$$I_z(z') = \frac{I_s}{\sin \Phi_s} \sin\left[\beta(s_2 - z') + \Phi_s\right] \quad . \qquad s_2 \geq z' \geq 0 \tag{21}$$

An inspection of Fig. 6 reveals that the various distances from the current elements to a point $P(R, \theta, \phi)$ in the far zone of the radiating circuit are

$$\left.\begin{aligned} R_s &= R - s_2 \cos\theta \\ R_o &= R + s_1 \cos\theta \\ R_z &= R - z' \cos\theta \\ R_{z1} &= R_z + \frac{b}{2} \sin\theta \cos\phi \\ R_{z2} &= R_z - \frac{b}{2} \sin\theta \cos\phi . \end{aligned}\right\} \tag{22}$$

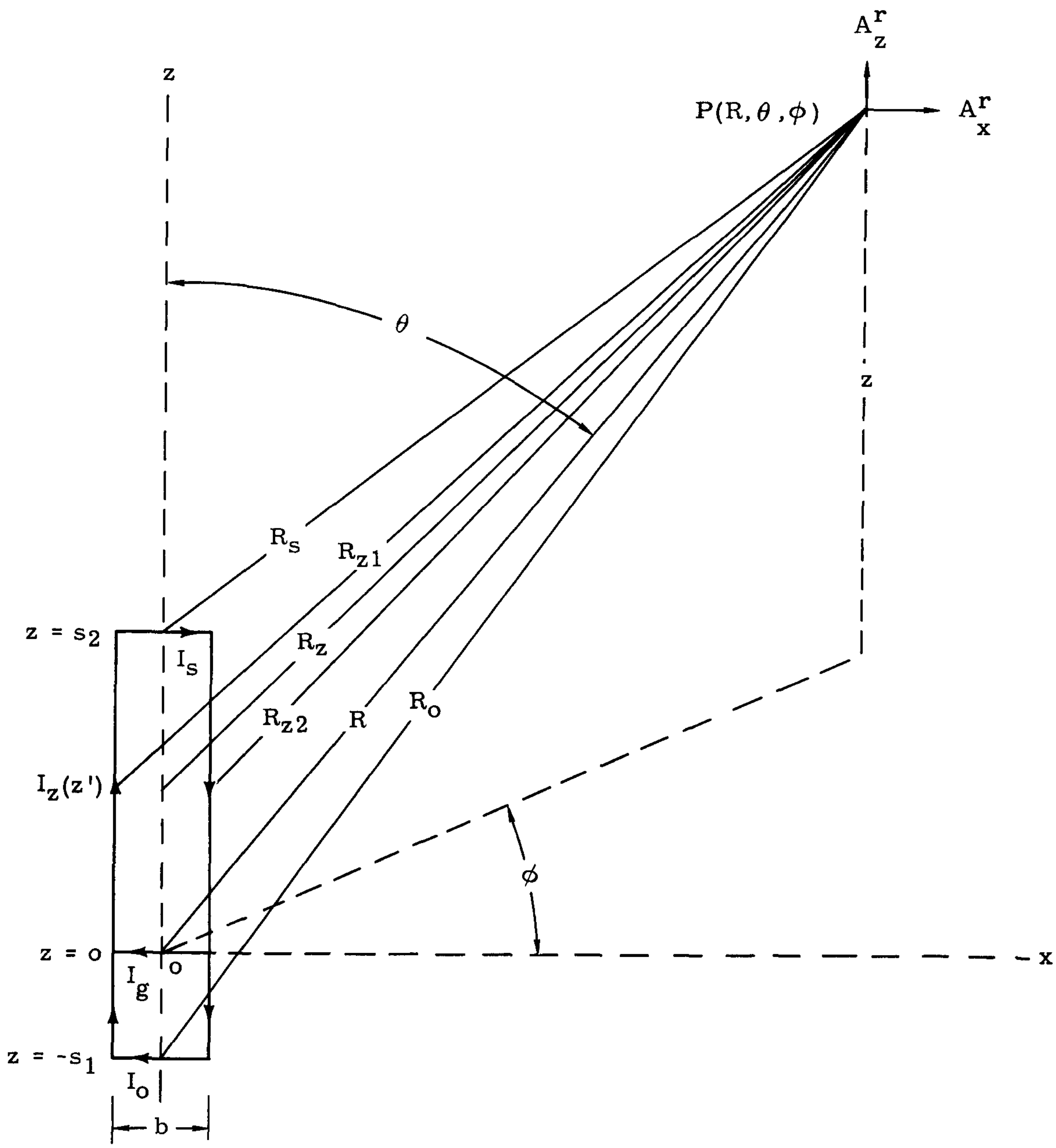

Fig. 6: Exposition of the Coordinate System Utilized in Calculating the Radiation Resistance of a Shunt-Driven Transmission Line.

The two components of the vector potential A_x and A_z are to be determined at point P. It is readily verified that

$$A_x = -\frac{b}{4\pi\nu_o}\left\{I\frac{e^{-j\beta R}}{R} + I_o\frac{e^{-j\beta R_o}}{R_o} - I_s\frac{e^{-j\beta R_s}}{R_s}\right\}$$

$$\approx -\frac{b}{4\pi\nu_o}\frac{e^{-j\beta R}}{R}\left\{1 - K_o e^{-j\beta s_1 \cos\theta} - K_s e^{j\beta s_2 \cos\theta}\right\} \quad (23)$$

and

$$A_z = \frac{1}{4\pi\nu_o}\int_{-s_1}^{s_2} I_z(z')\left\{\frac{e^{-j\beta R_{1z}}}{R_{1z}} - \frac{e^{-j\beta R_{2z}}}{R_{2z}}\right\}dz'$$

$$\approx -j\frac{\beta b \sin\theta \cos\phi}{4\pi\nu_o}\frac{e^{-j\beta R}}{R}\int_{-s_1}^{s_2} I_z(z')e^{j\beta z'\cos\theta}\,dz' . \quad (24)$$

Here

$$\nu_o = \frac{1}{\mu_o} = \frac{1}{4\pi} \times 10^7 \,\frac{\text{meters}}{\text{henry}} .$$

When the integration of (24) with (20) and (21) has been effected and the values of I_o and I_s in terms of I_g introduced, the following result is obtained:

$$A_z^r = \frac{jbI_g}{4\pi\nu_o}\frac{e^{-j\beta R}}{R}\frac{\cos\phi}{\sin\theta}\left\{j\cos\theta + \frac{\cos(\beta s_2 + \Phi_s)}{\sin(\beta s + \Phi_o + \Phi_s)}\left[\cos\Phi_o - j\cos\theta\sin\Phi_o\right]e^{-j\beta s_1\cos\theta}\right.$$

$$\left. - \frac{\cos(\beta s_1 + \Phi_o)}{\sin(\beta s + \Phi_o + \Phi_s)}\left[\cos\Phi_s + j\cos\theta\sin\Phi_s\right]e^{j\beta s_2\cos\theta}\right\} . \quad (25)$$

(In this expression it has been assumed that δ as defined by (15) is zero.)

A_x^r and A_z^r, as given by (23) and (25), are to be used in evaluating the radiation resistance of the shunt-driven, reactively terminated antenna.

Radiation Resistance of the Shunt-Driven Antenna

The Poynting vector S is given by the expression[3/]

$$S = \frac{\omega^2}{2\zeta_o}\left\{A_\theta^r A_\theta^{r*} + A_\phi^r A_\phi^{r*}\right\} . \tag{26}$$

Here $\omega = 2\pi f$ and $\zeta_o \approx 120\pi$ ohms. A_θ^r and A_ϕ^r are the spherical components of the vector potential at $P(R, \theta, \phi)$. These are related to the Cartesian components A_x^r and A_z^r by the formulas[4/]

$$A_\theta^r = A_x^r \cos\theta \cos\phi - A_z^r \sin\theta \tag{27}$$

$$A_\phi^r = -A_x^r \sin\phi . \tag{28}$$

It follows that

$$A_\theta^r A_\theta^{r*} = A_x^r A_x^{r*} \cos^2\theta \cos^2\phi$$

$$+ A_z^r A_z^{r*} \sin^2\theta - \left(A_z^r A_x^{r*} + A_x^r A_z^{r*}\right)\sin\theta \cos\theta \cos\phi \tag{29}$$

and

$$A_\phi^r A_\phi^{r*} = A_x^r A_x^{r*} \sin^2\phi . \tag{30}$$

[3/] King, Ronold, Theory of Linear Antennas, Harvard University Press (1956), Chapter 1, Section 10, p. 21.

[4/] Schelkumoff, S. A., "A General Radiation Formula," Proc. IRE, Vol. 27, No. 10, October 1939, p. 662, Equation 8.

With the shorthand notation

$$a(\theta) = 1 + K_o^2 + K_s^2 - 2K_o \cos(\beta s_1 \cos\theta) - 2K_s \cos(\beta s_2 \cos\theta)$$

$$+ 2K_oK_s \cos(\beta s \cos\theta) \tag{31}$$

$$b(\theta) = -K_o \cot\Phi_o \sin(\beta s_1 \cos\theta) - K_s \cot\Phi_s \sin(\beta s_2 \cos\theta)$$

$$+ K_oK_s(\cot\Phi_o + \cot\Phi_s)\sin(\beta s \cos\theta) \tag{32}$$

$$c(\theta) = K_o^2 \cot^2\Phi_o + K_s^2 \cot^2\Phi_s - 2K_oK_s \cot\Phi_o \cot\Phi_s \cos(\beta s \cos\theta) \tag{33}$$

and

$$g = \frac{bI_g}{4\pi v_o}, \tag{34}$$

it is a straightforward but laborious process to show that

$$A_x^r A_x^{r*} = \frac{g^2}{R^2} a(\theta) \tag{35}$$

$$A_z^r A_z^{r*} = \frac{g^2}{R^2} \frac{\cos^2\phi}{\sin^2\theta}\left\{a(\theta)\cos^2\theta + 2b(\theta)\cos\theta + c(\theta)\right\} \tag{36}$$

$$A_z^r A_x^{r*} + A_x^r A_z^{r*} = \frac{2g^2}{R^2} \frac{\cos\phi}{\sin\theta}\left\{a(\theta)\cos\theta + b(\theta)\right\}. \tag{37}$$

When these expressions are substituted in (29) and (30), and these then added, the result is

$$A_\theta^r A_\theta^{r*} + A_\phi^r A_\phi^{r*} = \frac{g^2}{R^2}\left\{a(\theta)\sin^2\phi + c(\theta)\cos^2\phi\right\}. \tag{38}$$

This expression may be substituted in (26) to obtain the Poynting vector

$$S = \frac{\omega^2 b^2 I_g^2}{32\pi^2 \zeta_o v_o^2 R^2}\left\{a(\theta)\sin^2\phi + c(\theta)\cos^2\phi\right\}. \tag{39}$$

The radiation resistance of the circuit referred to the current I_g is[5/]

$$R^e = \frac{2R^2}{I_g^2}\int_0^{2\pi}\int_0^{\pi} S \sin\theta d\theta d\phi \tag{40}$$

where S is given by (39). It is an interesting and pleasant surprise that the integrals involved in evaluating (40) are trivial for such a complicated circuit! The result of the integration is

$$R^e = 30\beta^2 b^2\left\{\frac{1}{2}\left[1 + \left(\frac{K_o}{\sin\Phi_o}\right)^2 + \left(\frac{K_s}{\sin\Phi_s}\right)^2\right] - \left[K_o\frac{\sin\beta s_1}{\beta s_1} + K_s\frac{\sin\beta s_2}{\beta s_2}\right]\right.$$
$$\left. - \left(\frac{K_o}{\sin\Phi_o}\right)\left(\frac{K_s}{\sin\Phi_s}\right)\cos(\Phi_o + \Phi_s)\frac{\sin\beta s}{\beta s}\right\} \tag{41}$$

where

$$\left(\frac{K_o}{\sin\Phi_o}\right) = \frac{\cos(\beta s_2 + \Phi_s)}{\sin(\beta s + \Phi_o + \Phi_s)} \tag{42}$$

$$\left(\frac{K_s}{\sin\Phi_s}\right) = \frac{\cos(\beta s_1 + \Phi_o)}{\sin(\beta s + \Phi_o + \Phi_s)}. \tag{43}$$

There relations are like K_o and K_s in (13) and (14) with $\delta = 0$.

Equation (41) is the final formula for the radiation resistance of a shunt-driven transmission line with negligible ohmic attenuation when terminated in pure reactances.

[5/] Reference 3.

The following radiation resistances are obtained directly from (41).

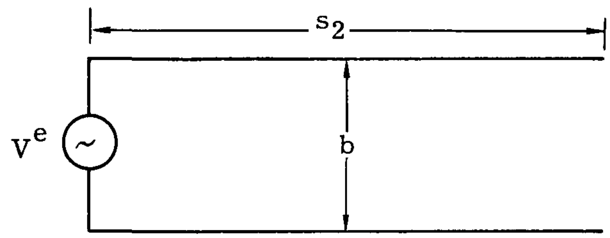

Fig. 7a: End-Driven Open-End Section of Line

a. End-Driven Open-End Section of Line -- For this structure $\Phi_o = 0$, $\Phi_s = 0$, $s_1 = 0$, and $s = s_2$. It follows that

$$\left(\frac{K_o}{\sin \Phi_o}\right) = \cot \beta s_2, \quad K_o = 0$$

$$\left(\frac{K_s}{\sin \Phi_s}\right) = \csc \beta s_2, \quad K_s = 0$$

so that

$$R^e = \frac{30\beta^2 b^2}{\sin^2 \beta s_2}\left\{1 - \frac{\sin 2\beta s_2}{2\beta s_2}\right\}. \tag{44}$$

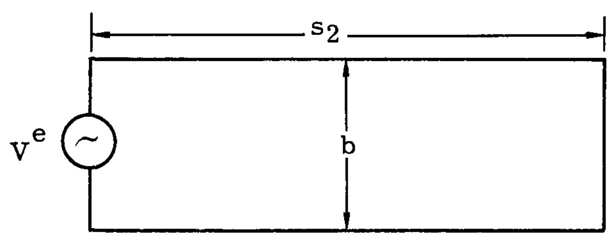

Fig. 7b: End-Driven Closed-End Section of Line

b. End-Driven Closed-End Section of Line -- For this circuit $\Phi_o = 0$, $\Phi_s = \pi/2$, $s_1 = 0$, and $s = s_2$. It follows that

$$\left(\frac{K_o}{\sin \Phi_o}\right) = -\tan \beta s_2, \quad K_o = 0$$

$$\left(\frac{K_s}{\sin \Phi_s}\right) = \sec \beta s_2, \quad K_s = \sec \beta s_2$$

so that

$$R^e = \frac{30\beta^2 b^2}{\cos^2 \beta s_2} \left\{1 - \frac{\sin 2\beta s_2}{2\beta s_2}\right\} . \qquad (45)$$

This formula and (44) agree with the results previously obtained by Storer and King.[6]

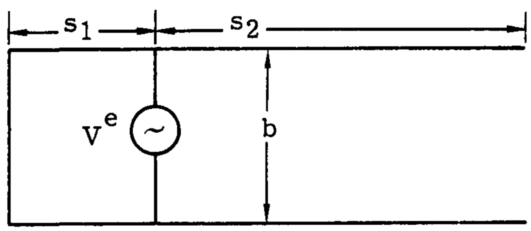

Fig. 7c: Shunt-Driven Line with an Open and a Short-Circuited Termination

c. Shunt-Driven Line with an Open and a Short-Circuited Termination -- For this configuration of wires, $\Phi_o = \pi/2$ and $\Phi_s = 0$. It follows that

$$\left(\frac{K_o}{\sin \Phi_C}\right) = \frac{\cos \beta s_2}{\cos \beta s}, \quad K_o = \frac{\cos \beta s_2}{\cos \beta s}$$

$$\left(\frac{K_s}{\sin \Phi_s}\right) = -\frac{\sin \beta s_1}{\cos \beta s}, \quad K_s = 0$$

[6] Reference 1.

so that

$$R^e = \frac{30\beta^2 b^2}{\cos^2 \beta s}\left\{\frac{1}{2}\left[\cos^2 \beta s + \sin^2 \beta s_1 + \cos^2 \beta s_2\right]\right.$$

$$\left. - \cos \beta s \cos \beta s_2 \frac{\sin \beta s_1}{\beta s_1}\right\} \quad . \qquad (\cos \beta s \neq 0) \tag{46}$$

If $\beta s_2 = \pi/2$, (46) becomes

$$R^e = 30\beta^2 b^2 \tag{47}$$

for all values of s_1. Hence s_1 may be adjusted for the desired input reactance independently of the driving-point (radiation) resistance.

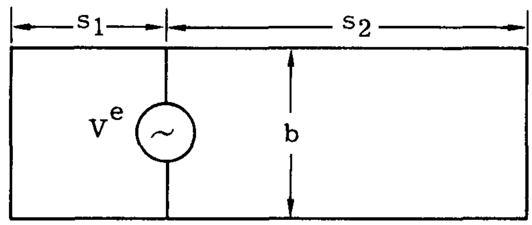

Fig. 7d: Shunt-Driven Line with Short-Circuited Terminations

d. Shunt-Driven Line with Short-Circuited Terminations -- For this arrangement of conductors, $\Phi_o = \Phi_s = \pi/2$. It follows that

$$\left(\frac{K_o}{\sin \Phi_o}\right) = \frac{\sin \beta s_2}{\sin \beta s}, \quad K_o = \frac{\sin \beta s_2}{\sin \beta s}$$

$$\left(\frac{K_s}{\sin \Phi_s}\right) = \frac{\sin \beta s_1}{\sin \beta s}, \quad K_s = \frac{\sin \beta s_1}{\sin \beta s}$$

so that

$$R^e = \frac{30\beta^2 b^2}{\sin^2 \beta s} \left\{ \frac{1}{2} \left(\sin^2 \beta s + \sin^2 \beta s_1 + \sin^2 \beta s_2 \right) \right.$$

$$\left. - \left(\frac{s_1^2 + s_1 s_2 + s_2^2}{\beta s s_1 s_2} \right) \sin \beta s \sin \beta s_1 \sin \beta s_2 \right\} \qquad (48)$$

$$(\sin \beta s \neq 0)$$

In the event that reactive terminations other than open or short circuits are employed, Φ_o and Φ_s may be evaluated from (6) and (7) with (12). Details are given in the literature.[7]

The Driving-Point Impedance of Missile Antennas of Transmission-Line Type

The equivalent lumped circuit of a shunt-driven antenna of transmission-line type is shown by Fig. 8. The driving-point impedance is

$$Z_{in} = R^e + \frac{X_1 X_2}{X_1 + X_2} \qquad (49)$$

where X_1 is the input reactance of the line of length s_1 terminated in X_o; X_2 is the input reactance of the line of length s_2 terminated in X_s. X_1 and X_2 may be computed from the formulas

$$X_1 = -jZ_c \cot(\beta s_1 + \Phi_o) \qquad (50)$$

$$X_2 = -jZ_c \cot(\beta s_2 + \Phi_s) , \qquad (51)$$

respectively. The characteristic impedance of a lossless line (in the ohmic sense) is real. It is given by the relation

$$Z_c = \frac{\zeta_o}{\pi} \ln\left[\frac{b}{2a}\left(1 + \sqrt{1 - \left(\frac{2a}{b}\right)^2}\right)\right]. \qquad (52)$$

[7] Reference 2, Section 19, Chapter 2.

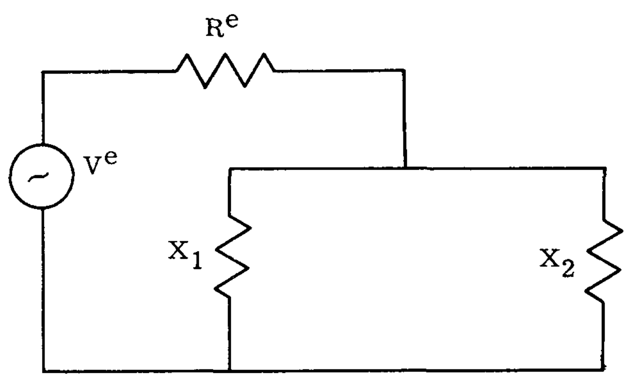

Fig. 8: Equivalent Lumped Circuit of a Shunt-Driven Antenna-Transmission Line of Negligible Ohmic Losses Terminated in Pure Reactors

Example: Compute the input impedance of the shunt-driven, inverted L-antenna transmission line pictured in Fig. 2.

Refer to Fig. 7c $\Phi_o = \pi/2$ and $\Phi_s = 0$, so that (50) and (51) become $X_1 = jZ_c \tan \beta s_1$ and $X_2 = -jZ_c \cot \beta s_2$, respectively. R^e may be calculated using (46), and (49) is then employed to determine Z_{in}. The input impedance of the structure portrayed by Fig. 2 is one-half the impedance of the circuit shown in Fig. 7c.

Conclusions

A class of protruding rocket antennas of low silhouette has been analyzed, using transmission-line concepts. Although in the interest of simplicity the theory was developed specifically for nondissipative terminations and line sections, the formulas are readily generalized to include ohmic losses in the lines and terminations by retaining (6) and (7) without imposing (12).

If the antennas are constructed of conductors of other than circular cross section, the equivalent cylindrical radius may be computed.[8/] When thick wires are employed, the physical dimensions of a given structure may differ considerably from the electrical dimensions. This uncertainty is brought about by the fact that the currents tend to bypass the corners. It is to be noted that X_1 and X_2 as defined in (50) and (51) are the apparent input impedances that include terminal-zone corrections where required.[9/]

[8/] Flammer, Carson, "Equivalent Radii of Thin Cylindrical Antennas with Arbitrary Cross Sections," Technical Report No. 4, SRI Project No. 188, March 15, 1950.

[9/] Reference 2, Section 20, Chapter 2.

Quite accurate results may be expected from the formulas developed in this paper when $b \leq 0.1\lambda$. For greater spacings the accuracy diminishes and a more general attack on the problem is required since the conditions of transmission-line theory and the assumption that currents in shunt members have uniform amplitudes cease to be valid.

The present theory may be used to analyze other missile antennas. For example, by use of the principle of superposition, the impedance of a bent folded unipole (hairpin antenna) may be expressed in terms of the impedance of an inverted L-antenna with image (Fig. 7a) and the impedance of a section of open-wire transmission line.

APPENDIX A

Sandia Corporation Technical Memoranda in this Series

Theory of End-Loaded Monopole, SCTM 275-57(14), October 8, 1957.

Theory of Shunt-Tuned Three-Wire Monopole, SCTM 301-57(14), October 23, 1957.

Antenna Synthesis, SCTM 37-58(14), March 10, 1958.

Theory of Inverted L-Antenna with Image, SCTM 11-58(14), April 8, 1958.

Radiation from an Inverted L-Antenna with Image, SCTM 51-58(14), April 16, 1958.

Hybrid Transmission Lines, SCTM 157-58(14), May 15, 1958.

Antenna Analysis by Circuit Superposition, SCTM 250-58(14), June 12, 1958.

Folded Wire Structures as Receiving Antennas, SCTM 253-58(14), June 18, 1958.

Impedance of a T-Antenna with Image, SCTM 257-58(14), July 7, 1958.

Note on the Design of Logarithmically Periodic Antennas, SCTM 286-58(14), July 8, 1958.

Antenna Coupling Error in Interferometer Angle Measuring Systems, SCTM 328-58(14), August 25, 1958.

Theory of Coupled Folded Antennas, SCTM 332-58(14), September 3, 1958.

Approximate Theory of Multituned Antenna for VLF Transmission, SCTM 362-58(14), September 9, 1958.

Receiving Characteristics of Quasi-Shielded Antennas, SCTM 396-58(14), October 15, 1958.

Transmission Line Missile Antennas, SCTM 436-58(14), November 20, 1958.

Response of an Impedance Loaded Electric Dipole Symmetrically Oriented Within an Imperfectly Conducting Cylinder, SCTM 457-58(14), December 5, 1958.

Impedance of a Folded Loop Antenna, SCTM 15-59(14), January 20, 1959.

Radio Frequency Shielding of Cables, SCTM 45-59(14), February 28, 1959.

Survey of Missile Antennas, SCTM 71-59(14), March 4, 1959.