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Some Monte Carlo Calculations For Cylindrical and Cruciform Control Rods

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I. INTRODUCTION

A limited investigation of neutron absorption has been conducted to examine the relative effectiveness of cruciform and cylindrical control rods. If such data could be firmly established, reactors using cruciform rods could be readily analysed by transforming the rods to their cylindrical equivalent then applying traditional theories.[1] Conversely, the number of rods required in a reactor and their location could be determined for cylindrical rods then transformed to equivalent cruciform rods. In the present context, only the cruciform rods are of engineering interest; the cylindrical rods are merely an analytical device.

The Monte Carlo method was selected for this study to minimize the number of approximations required.[2] The ease with which it can handle complex geometries makes this method particularly attractive for the control rod problem, but it often becomes very expensive to follow a sufficient number of neutrons to achieve satisfactory statistics. The project was, in part, an experiment to determine if this type of problem could be solved with reasonable accuracy at a reasonable cost with Monte Carlo.

II. SCOPE

Variables included in the investigation were:

a) Flange length for cruciform rods (ℓ),

b) Flange thickness (t),

c) Radius of cylindrical rods (r),

d) Macroscopic absorption cross section of the rod material (Σ_a^r),

e) Macroscopic absorption cross section of the surrounding medium (Σ_a^m),

f) Macroscopic scattering cross section of the rod material (Σ_s^r), and

g) Macroscopic scattering cross section of the medium (Σ_s^m).

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[1] Footnotes and references are assembled at the end of the report.

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Cross sections for control rods were selected to represent thermal and epithermal absorption for pure elements and dispersions currently being considered as control materials.[3] Cross sections for the medium were based on homogenized fuel, structural material, and moderator for slightly enriched uranium either carbon or water moderated. Study of certain cases, however, involving weak absorption and strong scattering was suspended after discovering that 'neutrons' wandered around too long in the moderator and wasted precious machine time. Further limitations were imposed upon the size of the moderator region for the same reason. So as to eliminate this dimension as an additional variable, it was assumed that the relative effectiveness of the cruciform and cylindrical rods would be insensitive to the size of the cell if the rods occupied only a small percentage of the volume and if the boundaries were removed by many mean-free paths from the rods. Calculations for such geometry were too lengthy because too few neutrons hit the rods, consequently, the volume of the medium was greatly reduced. Results, therefore, apply rigorously only to the specific cell size studied.

Without loss of generality, degradation of neutrons was prohibited to obtain a significant increase in the computing speed. The calculations therefore apply to an arbitrary lethargy group in which the slowing down cross section is included in the absorption cross section; it is here a matter of indifference whether a neutron is removed from a lethargy group by absorption or scattering out. Thus the concept of removal cross section, denoted simply by Σ, is henceforth employed to emphasize this point. For thermal neutrons, the removal cross section is naturally pure absorption.

Measures of effectiveness were obtained for the cruciform and cylindrical rods shown in Table I embedded in an infinitely long cylindrical cell of 8-inch radius. All permutations of the cross section values were investigated except for the combination of weakest absorption and strongest scattering in the medium. Particular combinations of these parameters are identified by a six digit number for cruciform rods and a five digit number for cylindrical rods that designate the level of the variables from left to right in the table. For example, the number 111122 denotes the small, thin, black, cruciform rod with low scattering embedded in a medium of intermediate absorption and scattering.

TABLE I. PARAMETERS OF GEOMETRY AND MATERIALS

Geometry				Materials			
				Control Rod		Surrounding Medium	
Level	Flange Length (in)	Flange Thickness (in)	Rod Radius (in)	$\Sigma (cm^{-1})$	$\Sigma_s (cm^{-1})$	$\Sigma (cm^{-1})$	$\Sigma_s (cm^{-1})$
1	3.0	0.10	2.5	100	0.3	0.01	0.35
2	5.0	--	4.0	1.0	0.8	0.035	1.00
3	--	0.30	5.5	0.1	--	0.1	3.50

III. DISCUSSION OF RESULTS

The fraction of all neutrons that are absorbed by the control rods was used as the measure of their effectiveness. While many possible alternatives exist, this criterion was selected to allow more direct comparison with the popular and successful notion of equivalent homogeneous poison.[4]

Capture fractions for black cylindrical and thin, black, cruciform rods are plotted in Figure 1 against their radii and flange lengths respectively. Values of other parameters are as noted. Since a rod of zero radius or flange length would capture no neutrons, the origin was used as an additional point in determining the shape of the curves. Whenever the surrounding medium has a small removal cross section, the effect of a rod is enhanced as shown by the vertical displacement of the curves. From physical reasoning, one would expect the curves to be monotonically increasing away from the origin with positive curvature. Hence the inflection in the curve for cylinders in a weakly absorbing medium is almost certainly unreal. This discrepancy can be attributed to variations in the results which, in some instances, are shown as short vertical lines through the points. The length of these lines is twice the probable error of the mean, that is, the probability is 0.50 that the true capture fraction lies between the extremeties of the line. Statistical uncertainty is also the most likely explanation for the negative curvature of the curve for cruciform rods in a weakly absorbing medium. For the 3-inch cruciform rod with $\Sigma^m = 0.035$, the capture fraction is 10.5% with a probable error

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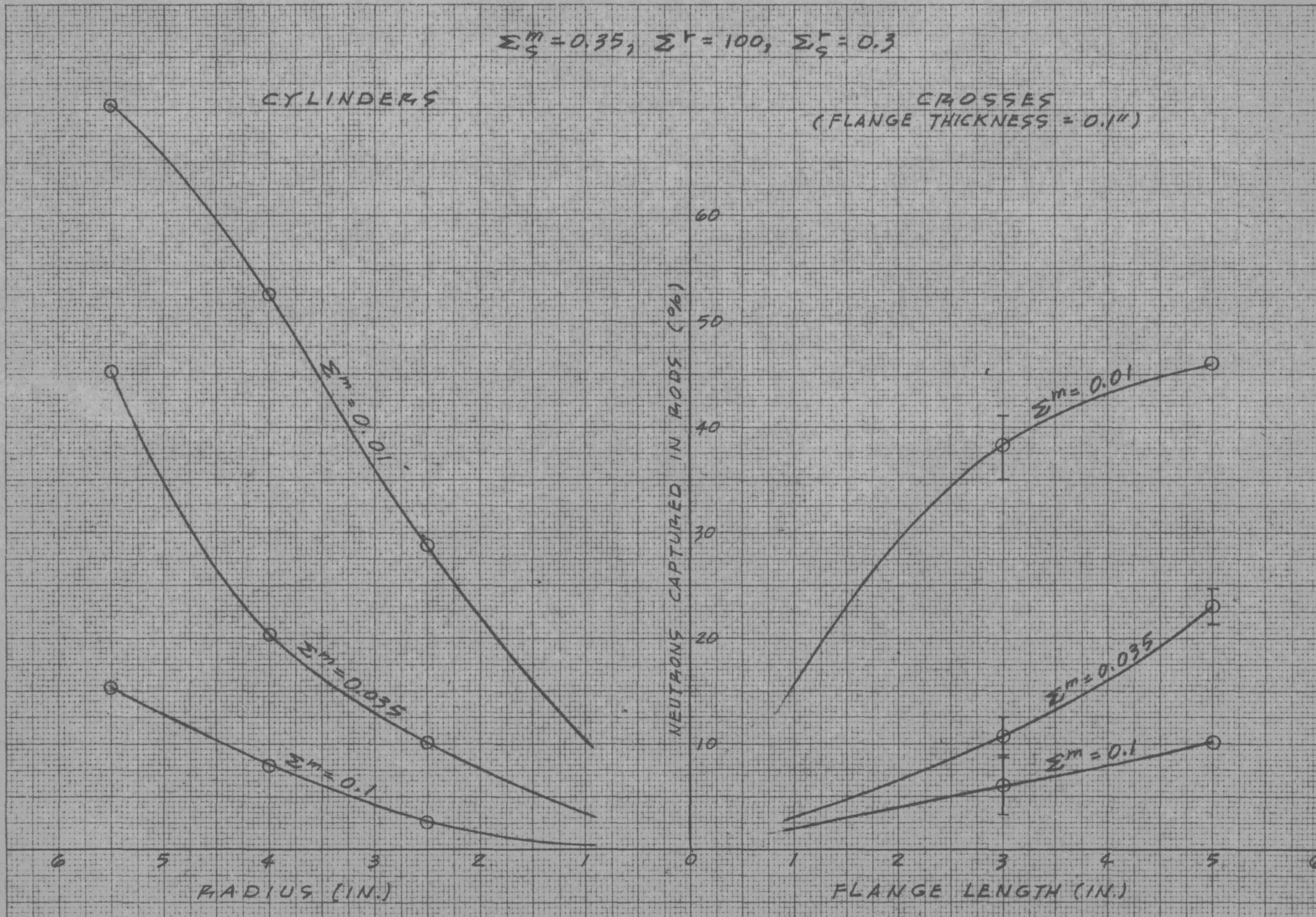

FIG. I PERCENTAGE OF NEUTRONS CAPTURED IN ROD VS. RADIUS OF ROD & FLANGE LENGTH of ± 2.1%. This degree of uncertainty, a consequence of following a very limited number of neutron histories, is typical of the present results. A detailed discussion of the statistical significance of these data appears in the Appendix.

In Figures 2 through 4, similar results are presented for different values of scattering in the rod and the medium. Distortion of the curve for the cylindrical rods in a weak absorber (Figure 3) is again observed and attributed to statistical variation. A more startling anomaly occurs for all instances of the larger cruciform rods in an intermediately absorbing medium. The results show an insensitivity to increased flange length which is unreasonable.

A meager amount of data on absorption by gray rods is presented in Figure 5. As only two degrees of grayness were investigated, the points have been connected by a straight line merely to indicate increasing or decreasing values. These results seem quite reasonable. The change in capture fraction for the black and gray rods is roughly proportional to the strength of the rod. In a weakly absorbing medium the gray rod studied ($t\ \Sigma^r = 0.76$) captured about 41% of the neutrons whereas a black rod under otherwise identical conditions captured about 45%. For a highly absorbing medium, the grayness has little effect.

Some additional data for strong scattering in the rod and the medium has such poor statistics that it was not included.

IV. CONCLUSIONS

While the present results have too much statistical scatter to be very reliable, the Monte Carlo approach to the problem of assessing the effects of control rods does hold promise. By extending the computations to include an adequate number of neutrons, any desired precision could be obtained. The cost of Monte Carlo calculations, however, is probably too great to warrant its extensive application to surveys. About 13 hours of IBM 704 time, or about $4000 of computing, were used in this study and an analysis of the results indicates that about ten times this sum would be required for acceptable accuracy and reliability. In order that the power of the Monte Carlo methods is not misrepresented, it should be noted that this study followed about 14,000 particle histories and covered 96 different problems.

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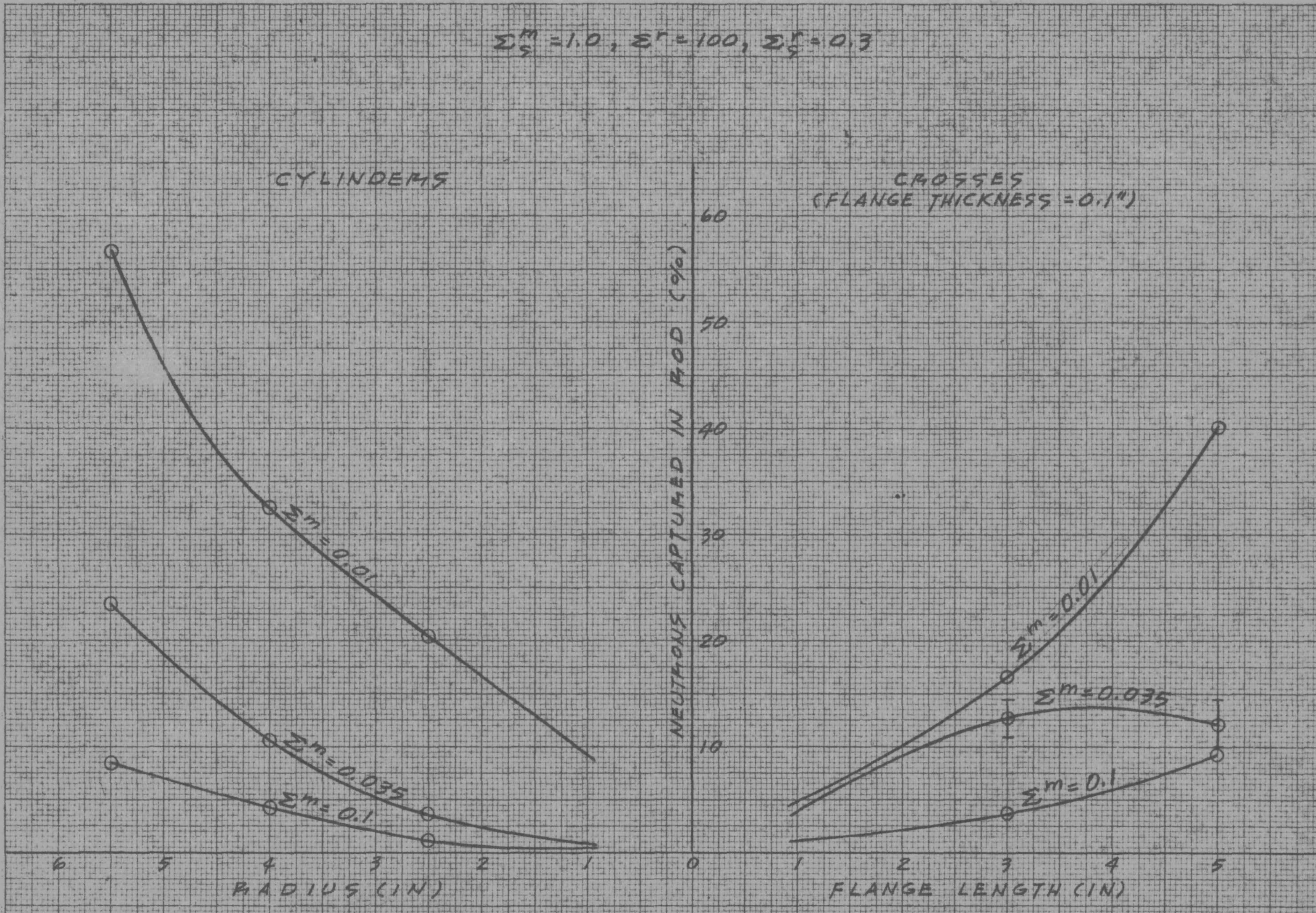

FIG. 2 PERCENTAGE OF NEUTRONS CAPTURED IN ROD VS. RADIUS OF ROD & FLANGE LENGTH

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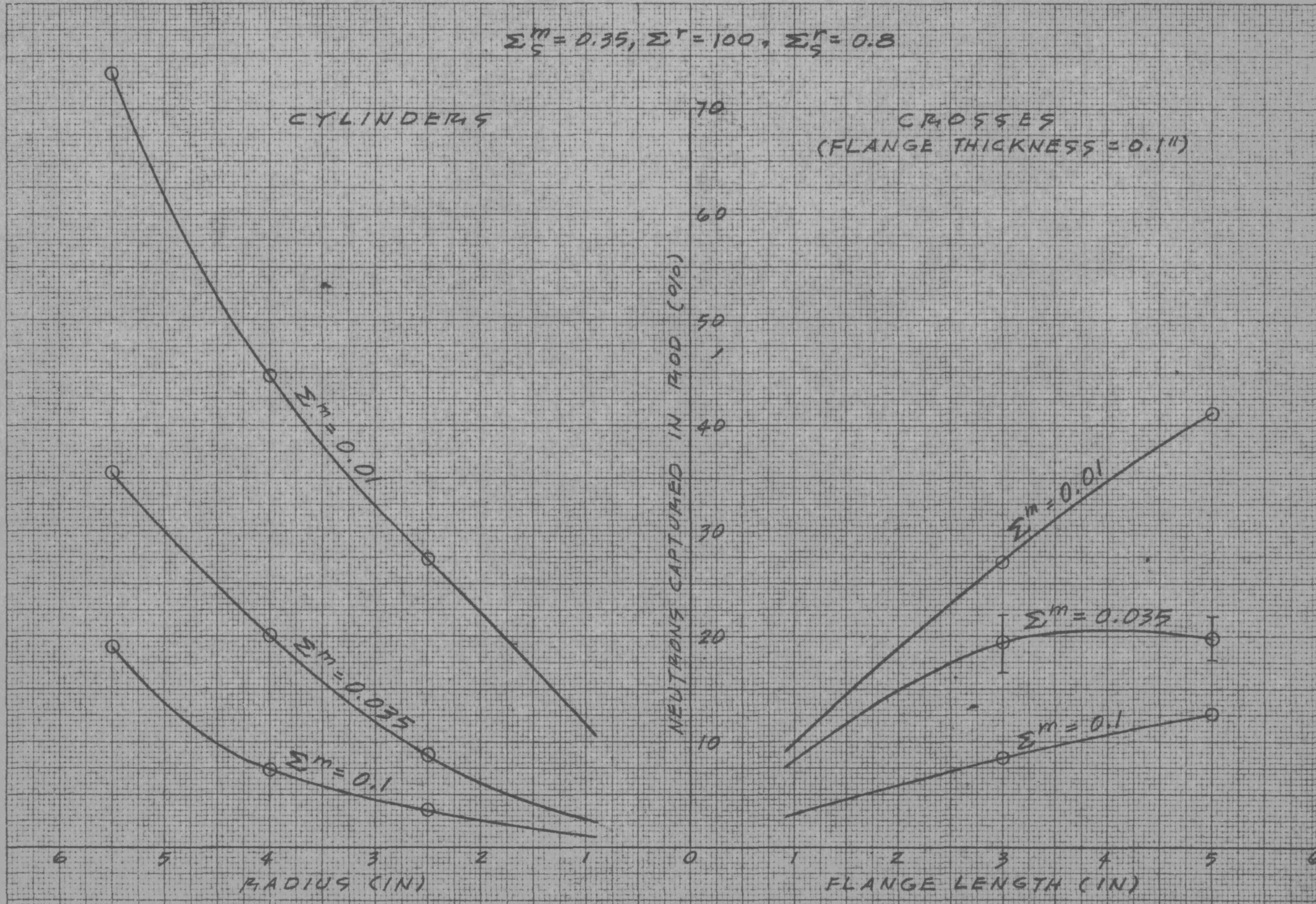

FIG. 3 PERCENTAGE OF NEUTRONS CAPTURED IN A ROD VS. RADIUS OF ROD & FLANGE LENGTH

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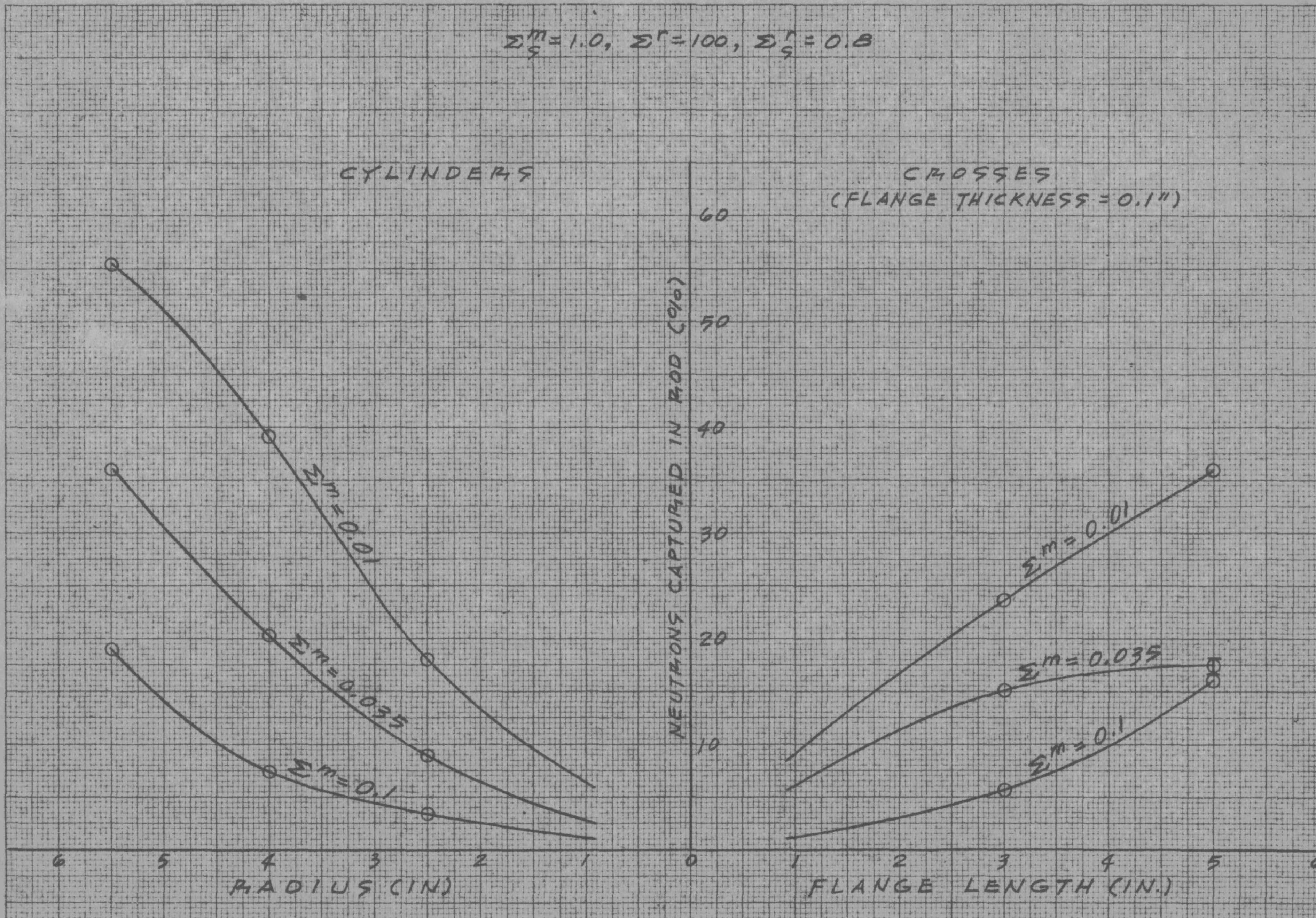

FIG. 4 PERCENTAGE OF NEUTRONS CAPTURED IN A ROD VS. RADIUS OF ROD & FLANGE LENGTH

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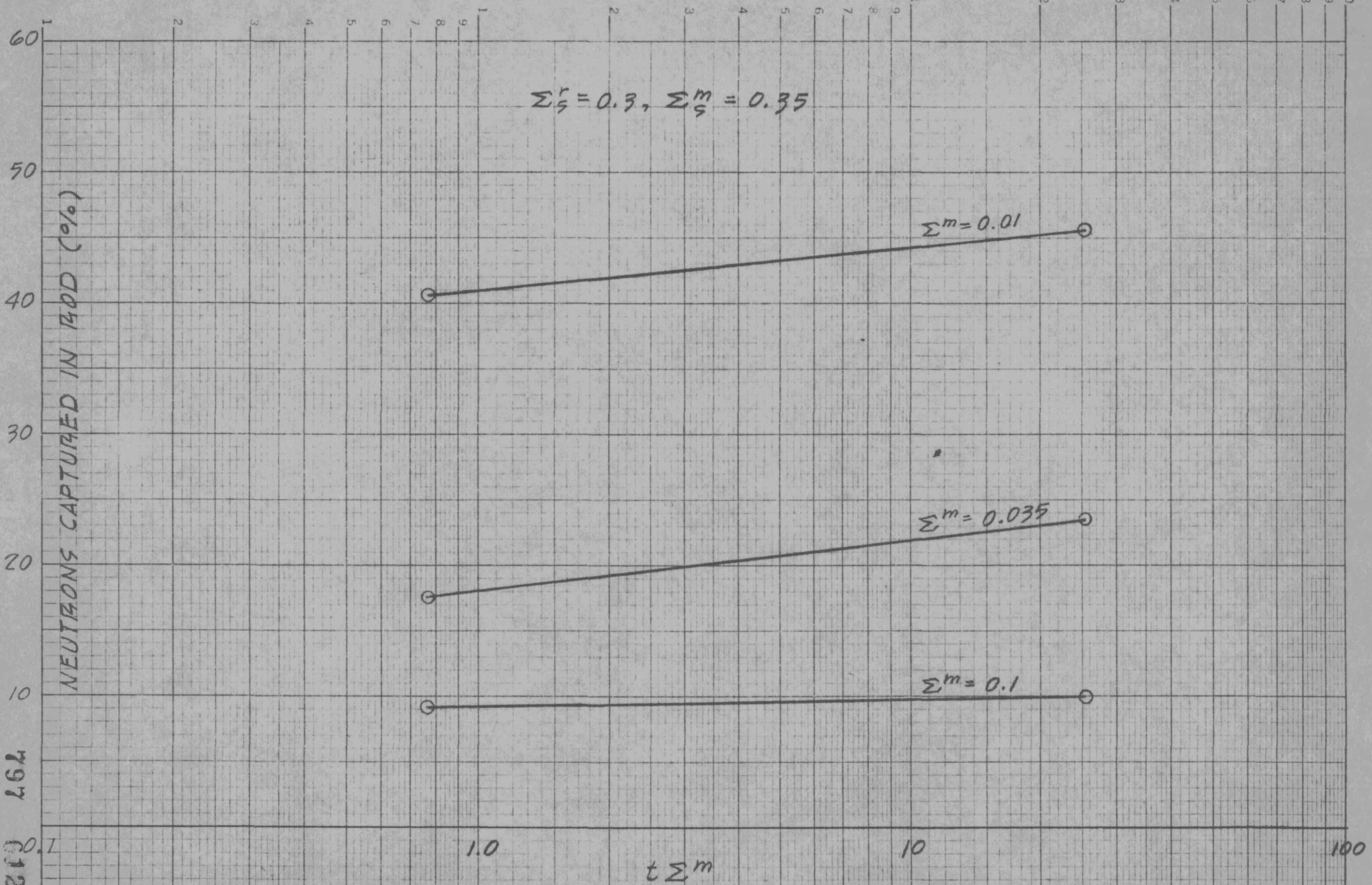

FIG. 5 PERCENTAGE OF NEUTRONS CAPTURED IN A ROD VS. $t\Sigma^m$ OF CROSS ROD

It is therefore concluded from this investigation that Monte Carlo would be more profitably employed in this type of problem not as a survey over the ranges of the variables but rather as a check against the several methods of control rod analysis in current use in order to define the range over which the simpler calculations are valid and to determine, in fact, which approximations employed in them are justified. This type of investigation, if conducted on an appropriate scale, could result in a definitive treatment of control rods and yield short-cut methods that have been thoroughly verified by Monte Carlo calculations and experiments.

APPENDIX -- ANALYSIS OF VARIANCE

It is usually advisable to print out intermediate results of the Monte Carlo calculations so that the progress can be observed and checked for unreasonable answers. In addition, these auxiliary data provide a basis for evaluating the statistical significance of the final result. While the number of interactions experienced by the 'neutrons' in particular regions is accumulated by the code, the computations between successive printouts are actually independent attempts to solve the problem. These incremental results are, therefore, subject to the laws of statistics which may be used to advantage. Since a different number of neutrons is usually followed in the independent trials, the weighted mean fraction captured in the rod is the statistic giving the best estimate of the capture fraction. Similarly, the weighted variance is used to estimate the precision of the final results.[5]

For problem number 111111, the number of interactions in the rod (I_r) and the medium (I_m) plus the number of neutrons (n_i) involved in the i^{th} trial are given in the first three columns of Table II.

TABLE II. DATA FOR PROBLEM NO. 111111

Column No.	(1)	(2)	(3)	(4)	(5)	(6)
Trial	I_r	I_m	n	R_r	R_m	f
1	1.685	147.8	9	1.680	4.109	0.2902
2	0.924	58.39	3	0.9212	1.623	0.3621
3	6.385	323.7	17	6.366	8.998	0.4143
4	1.721	266.5	5	1.716	7.410	0.1880
5	2.568	61.91	4	2.560	1.721	0.5980
6	5.436	184.5	10	5.420	5.128	0.4174
7	3.748	112.7	10	3.737	3.132	0.5440
8	0.00480	5.857	2	0.004786	0.1628	0.0286
9	0.3214	9.192	8	0.3204	0.2555	0.5563
10	0.0653	7.075	2	0.0651	0.1967	0.2487

The fourth and fifth columns give the number of neutrons removed (R) from the two regions, in general, non-integral numbers because they include the tally

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of expectation calculations.[2] These values are obtained from the number of interactions by multiplying by the ratio of removal cross section to total cross section, in this instance amounting to 0.997 and 0.0278 for the rod and medium respectively. Finally in column six is given the fraction of removals occurring in the rod. The weighted mean capture fraction is therefore

$$\bar{f} = \frac{\sum_{i=1}^{10} f_i \, n_i}{\sum_{i=1}^{10} n_i} = 0.410 \qquad (1)$$

The variance of the mean is found to be

$$s^2 = \left(\frac{1}{9}\right) \frac{\sum_{i=1}^{10} n_i \, (f_i - \bar{f})^2}{\sum_{i=1}^{10} n_i} = 0.002165$$

and the probable error, equal to 0.6745s, is 0.0314. Thus, the probability is 0.50 that the true mean lies in the range 41.0 ± 3.1%.

The capture fraction obtained from a single trial should asymtotically approach the true value as the number of neutrons is increased. Considering all the above trials as a single computation yields 38.0% as the estimate. Similar data for a particular group of problems is presented in Table III.

TABLE III. ESTIMATES OF CAPTURE FRACTIONS AND VARIANCE

Problem No.	Neutrons	Capture Fractions (%)		Probable Error (%)
		Accumulative Estimate	Weighted Mean	
111112	73	16.5	20.6	± 4.1
111121	78	10.5	13.1	± 2.1
111122	77	13.0	14.8	± 2.2
111123	66	6.25	2.74	± 4.4
111131	58	5.78	10.1	± 2.9
111132	48	2.76	2.44	± 0.6
111133	69	3.56	4.50	± 4.4
111211	123	27.2	29.1	± 1.8
111212	115	23.6	23.7	± 3.1
111221	91	19.5	20.6	± 2.6
111222	54	15.1	12.4	± 2.4
111223	68	0.979	1.68	± 3.8
111231	73	8.33	9.21	± 6.1
111232	82	5.54	10.3	± 3.0
111233	87	0.615	1.51	± 3.0

Statistical variations, generally about 3%, are seen in Table III to approach acceptable levels for strongly absorbing rods but are too gross for weak rods. In most instances, however, the cumulative estimates are in fair agreement with the weighted means. Because the calculation of weighted means is rather tedious and the variance is so great, the cumulative estimates are used in the text for the capture fraction.

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FOOTNOTES AND REFERENCES

1. These theories treat cylindrical rods by a one or two-group model for a variety of geometrical arrangements. For example, see

a) Murray, R. L. and Niestlie, J. W., "Reactor Control-Rod Theories", Nucleonics, Vol. 13, No. 2, 1955.

b) Scalettar, R. and Nordheim, L. W., "Theory of Pile Control Rods", MDDC 42, 1946.

c) Hurwitz, H. and Roe, G. M., "Absorption of Neutrons by Black Control Rods", KAPL-1336, March 1955.

d) Smith, J. H. and Stewart, J. C., "One-Group Criticality Conditions for a Black Eccentric Control Rod", KAPL-141, October 1955.

e) Avery, R., "Two-Group Diffusion Theory for a Ring of Cylindrical Rods", Nuclear Science and Engineering, Vol. 3, p. 504, 1958, also ANL-5729, June 1957.

f) Garabedian, H. L., "Control Rod Theory for a Cylindrical Reactor", AECD-3666, August 1950.

g) Pearlstein, S., Ruane, T. F., and Storm, M. L., "The Evaluation of Control Rods by the Area Absorption Method", Paper presented at ANS Meeting June 2-5, 1958.

While these methods are still valuable to investigators without access to high speed computing equipment, they are being supplanted at the more progressive installations by others having a different heritage. In this connection, see

h) Case, K. M., De Hoffmann, F. and Placzek, G., "Introduction to the Theory of Neutron Diffusion", Vol. I, Los Alamos Scientific Laboratory, June 1953.

i) Schiff, D. and Stein, S., "Escape Probability and Capture Fraction for Gray Slabs", WAPD-149, June 1956.

j) Voorhis, A. D. and Ryan, T. M., "Investigation of Absorbing Membranes in Slab Reactors", Paper presented at AEC Control Rod Meeting, March 6-8, 1957.

k) Wachspress, E. L., "Thin Regions in Diffusion Theory Calculations", Nuclear Science and Engineering, Vol. 3, 186, 1958.

l) Ruane, T. F. and Storm, M. L., "Epithermal Parameters for the Calculation of Control Rod Worth in Thermal Reactors", Paper presented at ANS Meeting June 2-5, 1958.

These last mentioned references culminate in representing cruciform rods explicitly in two dimensional diffusion calculations in which fictitious properties of the absorbing material are determined from boundary conditions.

In addition to methods based upon perturbation theory such as

m) Nowak, M. J., "Effectiveness of Reactor Control Absorbers Sensitive to Neutrons of All Energies", ASAE-5, February 1957, and

n) Wolfe, B. and Fischer, D. L., "Perturbation Theory of Control Elements", Part I, GEAP 2039 and GEAP 2059.

promising developments along other lines are taking place through the efforts of Stuart, G. W. and Woodruff, R. W., "Method of Successive Generations", Nuclear Science and Engineering, Vol. 3, 339, 1958.

2. Leshan, E. J., "A General Purpose Monte Carlo Program for the IBM 704", Part I: METHOD ASAE-2, September 1956.

3. Dayton, R. W., "The Effectiveness of Control Rod Materials", BMI-1196, June 1957.

4. Henry, A. F., "Review of Methods Used in Control Rod Analysis for Reactor Design at Bettis Plant", WAPD-BT-4, Bettis Technical Review, Vol. 1, No. 4, October 1957.

5. Dixon, W. J. and Massey, F. J., "Introduction to Statistical Analysis", McGraw-Hill, p. 109, 1951.

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