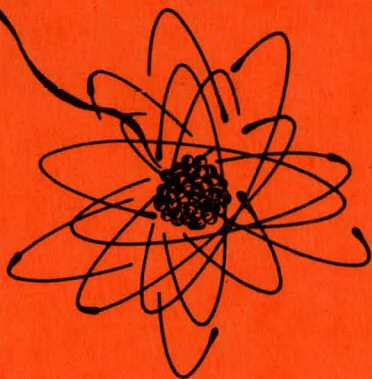YANKEE ATOMIC ELECTRIC COMPANY
RESEARCH AND DEVELOPMENT PROGRAM

MODEL STUDY OF THE PRESSURE DROP RELATIONSHIPS
IN A TYPICAL FUEL ROD ASSEMBLY

R & D SUBCONTRACT NO. I under
USAEC - YAEC CONTRACT AT (30-3)-222

FEBRUARY, 1959

WESTINGHOUSE ELECTRIC CORPORATION
ATOMIC POWER DEPARTMENT
PITTSBURGH, 30   P. O. BOX 355   PENNSYLVANIA

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Research and Development Program

MODEL STUDY OF THE PRESSURE DROP RELATIONSHIPS

IN A TYPICAL FUEL ROD ASSEMBLY

by

R. T. Berringer
A. A. Bishop
Thermal, Hydraulic and Mechanical Design Section

February, 1959

For The Yankee Atomic Electric Company
Under Research and Development Subcontract
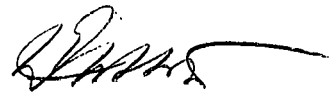No. 1 of USAEC - YAEC Contract AT(30-3)-222

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Westinghouse
ELECTRIC CORPORATION
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I. ABSTRACT

A study was made of hydraulic characteristics of Yankee type fuel rod assemblies using experimental and analytical methods. Two scale model fuel assemblies utilizing both ferrule and strap type arrangements were constructed and tested at atmospheric pressure and room temperature. Analytical methods using semi-empirical relationships are substantiated by experimental results for both the fuel assembly having strap-type spacers and the fuel assembly having cylindrical ferrule-type spacers. The experimental pressure drop across the assembly model using either straps or ferrules correlated within 5% of the value calculated by means of equations based on the equivalent diameter concept for flow inside pipes. The individual frictional drops along the rods and across the end plates and straps correlated within 15% of the predicted pressure drops. The individual pressure drops across both the staggered ferrule sections and the full ferrule section correlated to within 17% of the predicted pressure drops.

Comparison of the ferrule and the strap pressure drops indicates that the pressure drop across a level of straps was more than four times the pressure drop across a full ferruled section.

It is concluded that the analytical methods based on the equivalent diameter concept can be satisfactorily used to calculate pressure drops for flow parallel to fuel rod bundles; specifically for a fuel assembly having 0.335 inch diameter rods in a square lattice of 0.425 inch pitch. Experimental tests on this fuel rod configuration with either straps or ferrules indicated no excessive energy losses due to vortex formations.

The possibility of a sustained forced vibration of the fuel rods was examined theoretically. If the unsupported length of the fuel rod is not the length which would respond to the pump excitation frequency, no sustained forced vibration is expected.

II. INTRODUCTION

The pressure drop for a configuration similar to the fuel rod assembly shown in Figure 1 may be determined by either an experimental investigation or by a semi-empirical analysis. The usual semi-empirical apprcach to the pressure drop analysis of such a configuration utilizes the equivalent hydraulic diameter concept to adapt the actual irregular flow cross-section to the relationships determined for a circular cross section. Claiborne[1] investigated pressure drops in similar flow passages and found that for most cases the error of this procedure is less than 20%. Experiments conducted at the Bettis Plant[2] for turbulent flow outside of, and parallel to, the axis of a bundle of tubes also indicated that a theoretical analysis is suitable for most applications. On the other hand, Miller, Byrnes, Bendorado and others[3] have found that the measured frictional pressure drops are much higher than the equivalent diameter concept would predict. Their value of the friction factor varied from 50% to 100% higher for water flowing parallel to a bundle than was theoretically predicted. For any particular fuel assembly arrangement an experimental investigation is necessary to demonstrate the applicability of analytical approaches to pressure drop calculations.

The hydraulic impulses arising from the rotating pump impeller vanes may provide a source for sustaining a forced vibration in the fuel rods. If such a forced resonance existed it could possibly result in failure of the fuel elements. The various possible modes of vibration may be determined as a function of the unit weight, the moment of inertia, and the length between supports. Only if the natural frequency of the fuel rod is essentially that of the excitation frequency are serious vibrations possible.
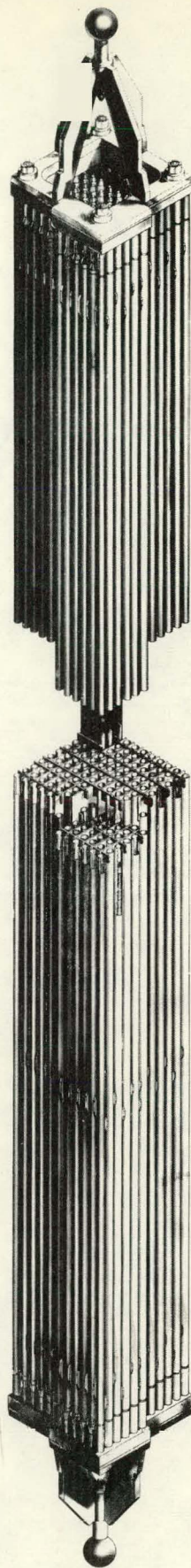
Figure 1

Full Size Strap Design Fuel Assembly Model

III. CONCLUSIONS

1. This investigation indicated that the equivalent diameter concept and related equations for calculating pressure drop inside of pipes are valid for calculating pressure drop along a parallel cluster of rods where flow is parallel to the rods. The experimental pressure drop along the fuel rods agreed with the calculated pressure drop within 15%. The pressure drops across the straps and across the end plates were also predicted within 15% using the maximum velocity based on the minimum free cross section in a region. The pressure drops across the ferrules were predicted within 17% using standard contraction and expansion relationships and including frictional losses.

   The explanation of why the correlation between the total experimental and total analytical pressure drops across the fuel assembly correlated within 5% whereas the pressure drop correlation for the straps, end plates and ferrules correlated within 17% is that there is a partial recovery of the energy loss after the individual expansion and contractions, a condition somewhat similar to the recovery downstream of a fraction of the apparent energy lost at an orifice or other similar obstruction.

   Additional experimental studies are planned, such as injecting air through the walls of selected fuel rods in the matrix to simulate two phase flow. This test will provide greater insight into the flow and pressure drop relationships in a reactor fuel assembly where local boiling can occur.

2. The pressure drop across the ferrule section with ferrules in every space was only 25% of the pressure drop across a strap array. For hydraulic considerations the ferrule method of spacing fuel rods is recommended as being preferable to the straps because the ferrules offer less resistance to the reactor flow.

3. The length between supports of fuel rods which has a fundamental frequency nearest to the 206.5 cycles per second excitation frequency of the pump is between 8.5 and 12.5 inches, probably very close to the latter. The recommended length between supports of 8 inches cannot mathematically cause resonance with the excitation frequency.

IV. NOMENCLATURE

A   - Area of flow cross-section

D   - Diameter

D_e - Equivalent diameter

E   - Modulus of elasticity

f   - Moody friction factor

g   - Acceleration of gravity

G   - Mass flow rate

I   - Moment of inertia

K   - Form factor

ℓ - Free length

L   - General length

m   - Mass

M   - Bending moment

p   - Static pressure

Δp - Pressure differential

P   - Total pressure

r   - Radius

R_e - Reynolds number

U   - Internal energy

v   - Velocity

V   - Volume

w   - Mass per unit length

W   - Total mass flow

x   - Variable

y   - Deflection

y_o - Maximum deflection

Z   - Distance along fuel assembly

ϵ - Absolute roughness

ρ - Density

θ - Slope

ω - Frequency

Subscripts

s   - Pertains to the straps

t   - Pertains to the tubes

V. OBJECT OF INVESTIGATION

The object of this report was to determine the correlation between theoretical and experimental approaches to the determination of the pressure drop and flow relationship of the fuel assembly. The flow models used were designed to obtain information on configurations proposed for the Yankee Reactor. The investigation was conducted to determine if any undesirable conditions exist in the flow and pressure drop characteristics. The accompanying effects of the straps and the ferrules on the flow were studied. The analytical methods used to predict pressure drops were to be substantiated for the range of flows which will exist in the reactor. The basic analytical methods can then be applied to the particular conditions and geometries which are proposed for the reactor.

VI. DESCRIPTION OF APPARATUS

A. Experimental Loop

The experimental phase of this investigation was conducted in the 300 gpm
unpressurized Hydraulic Test Loop which is shown in Figure 2 and is located
in the ⓦAPD High Bay Building. The cell shown in Figure 3 was used for the
experimental tests. A calibrated Meriam orifice meter was used to measure
the flow through the fuel assembly model. Flow control was achieved by ad-
justing the two valves in the pump by-pass and a three inch globe valve in
the main line. Straightening vanes located upstream from the main line orifice
helped to insure accurate measurement of flow. Straightening vanes and a surge
tank in the loop helped to provide uniform turbulent flow as the water entered
the test cell.

B. Fuel Rod Assembly

The method of arranging the fuel rods in a reactor core varies with the reactor.
The Yankee first core design uses groups of long continuous fuel rods connected
to form fuel rod subassemblies. The position of all fuel rods is maintained
by mechanically joining the ends of a selected number of fuel rods to end
plates. Additional support along the length of the fuel rods is provided by
either ferrules or straps.

C. Experimental Fuel Assembly Model, Strap Design

A drawing of the strap design fuel-rod assembly model used for the hydraulic
tests is shown in Figure 4. The basic test assembly which consisted of 93
dummy fuel rods spaced with two strap sections was contained between two end
plate configurations. The dummy fuel rods had an outside diameter of 0.335"
and were of two lengths. Sixty-six 24 inch long rods were fastened only at
the straps. Twenty-seven 25-1/2 inch rods extended to the end plates and were
retained by both the end plates and the straps. The straps were assembled in
an egg-crate configuration similar to a possible reactor design. There are
nine 1 inch x 1/32 inch straps and ten 5/8 inch x 1/32 inch straps in each array,
the overall height of one strap section was 1-1/8 inches. The two strap arrays
were 14 inches apart and symmetrical about the center of the fuel assembly model.
The strap design was constructed to permit removing individual fuel rods and
replacing them with air ejector tubes for future two-phase flow studies.

D. Experimental Fuel Assembly Model Ferrule Design

Figure 5 is a drawing of the ferrule design fuel-rod assembly model. Three
ferrule sections were fabricated using epoxy resin to secure the stainless steel
ferrules to the aluminum rods. The 1/2 inch long ferrules had a 0.265 inch O.D.
with a 20 mil wall thickness. The full ferrule section contained 74 ferrules
and the two staggered ferrule sections contained 42 and 38 ferrules. Both model
assemblies were contained within the test cell which had a dummy control rod to
better simulate reactor core configurations. The annulus between the model fuel
assembly and the test cell is 0.105 inches wide limiting the coolant flowing past
the edges of the model fuel assembly. In the prototype reactor the width of the
space around the fuel assemblies is of the same order of magnitude.

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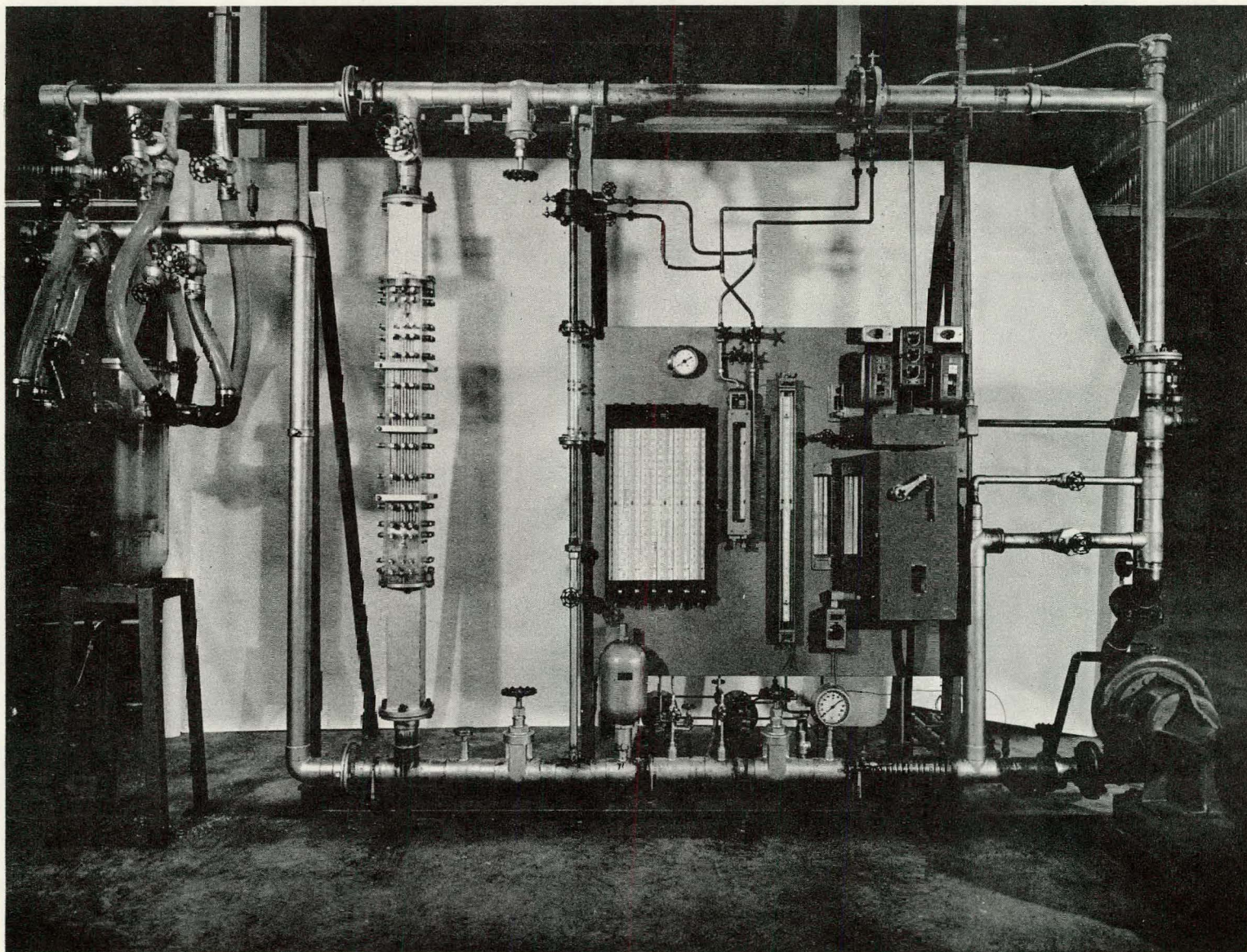
Figure 2 - Reactor Vessel and Fuel Assembly Model Hydraulic Test Loop
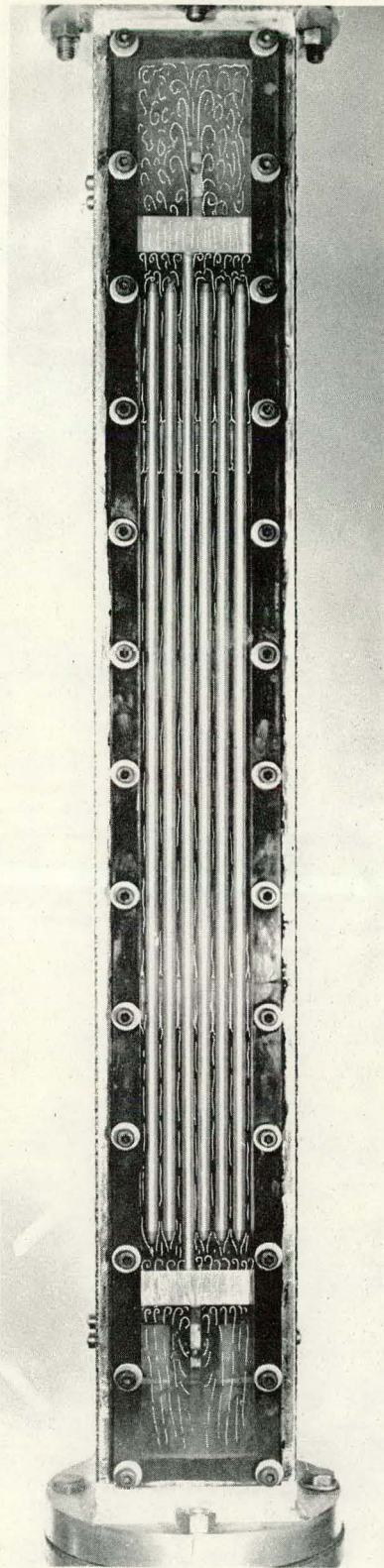
Figure 3

Fuel Rod Assembly Model Stand in Loop
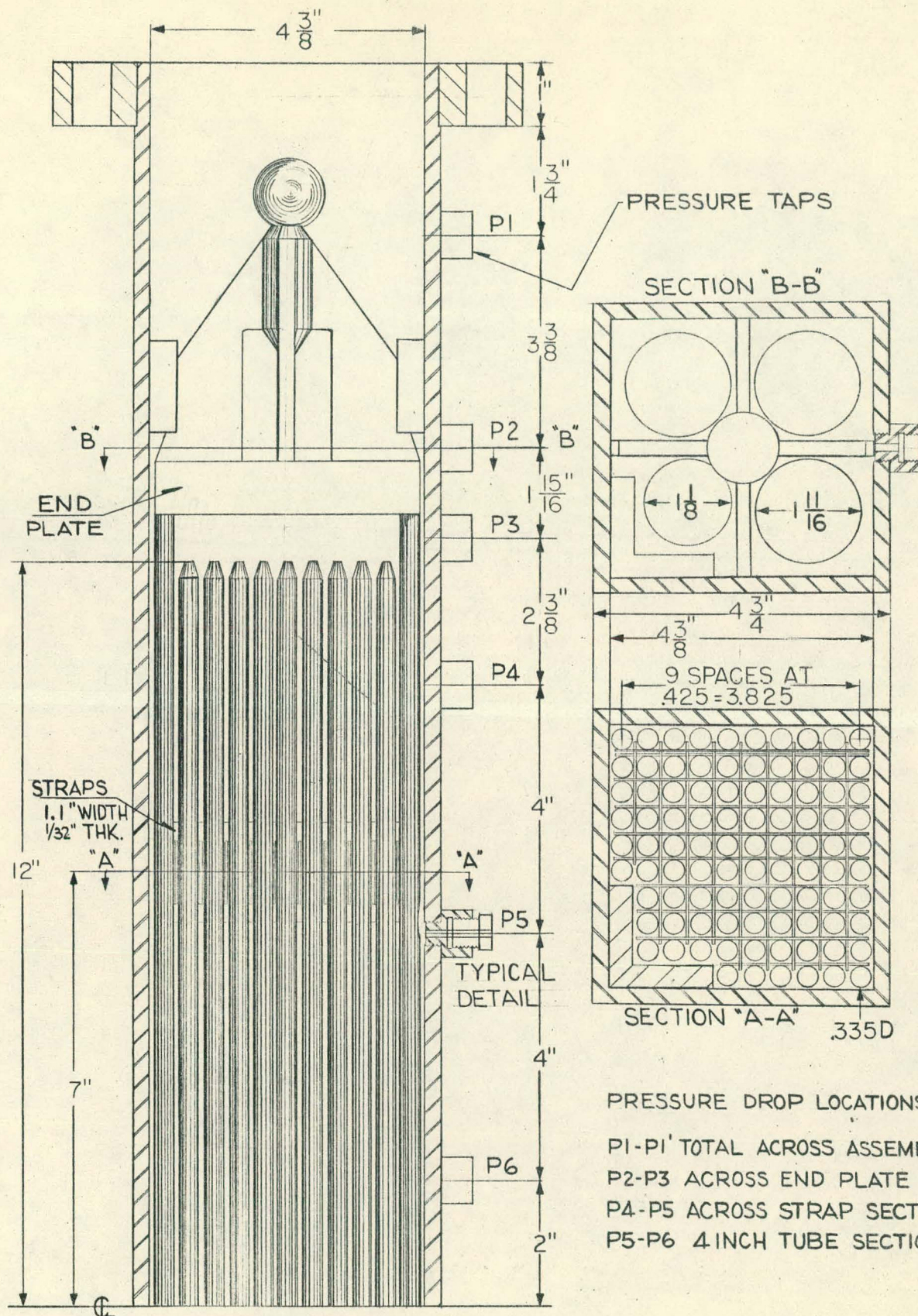
FIG 4: FUEL BUNDLE TEST ASSEMBLY
(WITH STRAPS)

PRESSURE TAPS

SECTION "B-B"

PRESSURE DROP LOCATIONS

P1-P1' TOTAL ACROSS ASSEMBLY
P2-P3 ACROSS END PLATE
P4-P5 ACROSS STRAP SECTION
P5-P6 4 INCH TUBE SECTION

SECTION "A-A"

9 SPACES AT
.425 = 3.825

STRAPS
1.1" WIDTH
1/32" THK.

END PLATE

TYPICAL DETAIL

DUMMY CONTROL ROD

SECTION-"CC"

FUEL ROD
.335" O.D.

SECTION-"BB"

FERRULE
.265 O.D.
.225 I.D.

SECTION-"AA"

"C"   "C'"   4¾
"B"   "B"   8
"A"   "A"   8
                4¾
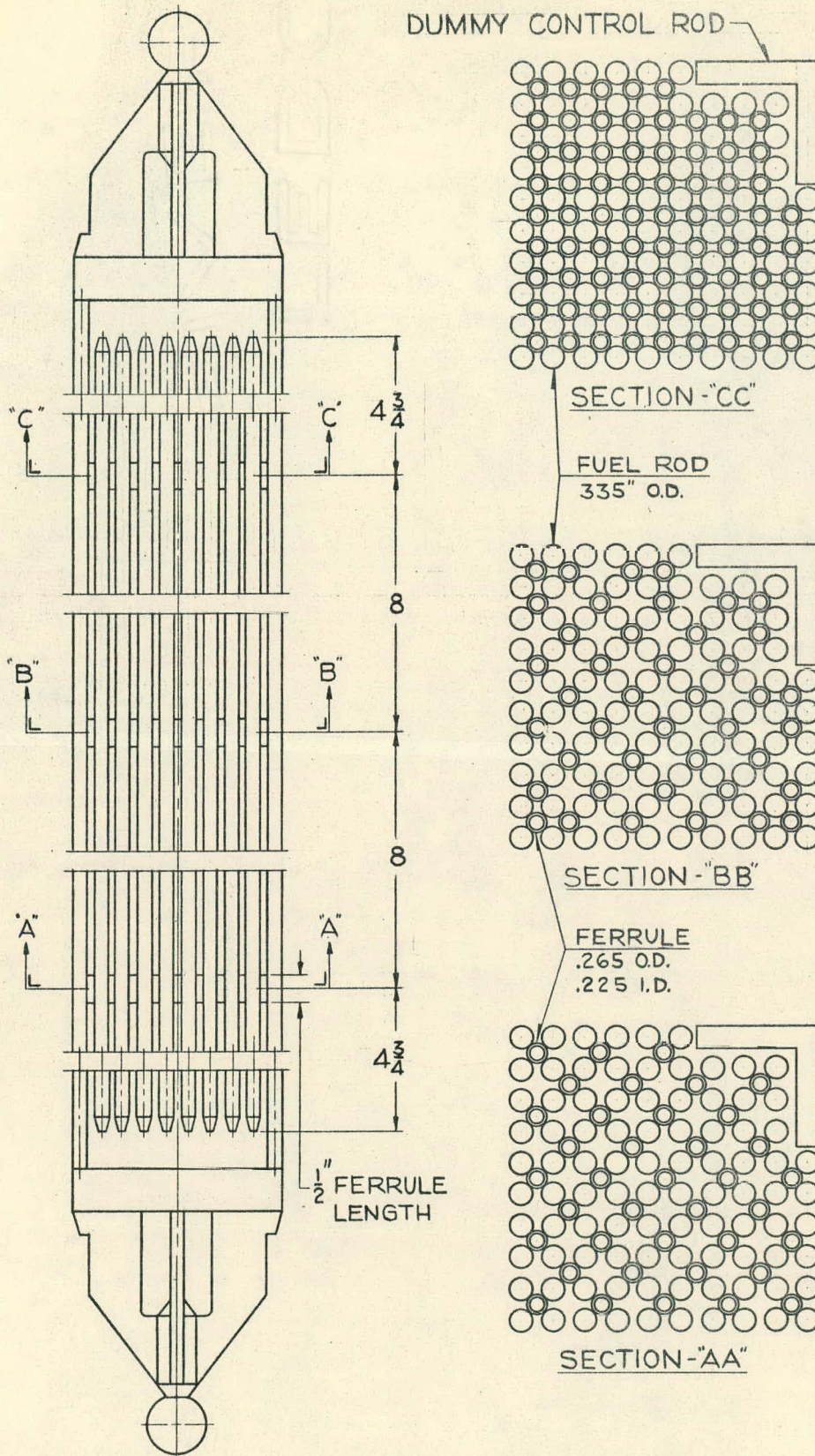
½" FERRULE
LENGTH

FUEL BUNDLE ASSEMBLY (WITH FERRULES)
FIGURE 5

VII. DISCUSSION OF PREDICTED AND MEASURED PRESSURE DROP RELATIONSHIPS

The general momentum equation for steady incompressible turbulent flow in a channel is:

$$\frac{\partial}{\partial X}\left(\frac{G^2}{\rho}\right) = -\frac{\partial P}{\partial X} - \rho g - \frac{f\,G^2}{De\,\rho}$$

The pressure drop along the test unit is obtained by integrating along the length:

$$\Delta p = P_2 - P_1 = -\left[\frac{G^2}{g}\right]_1^2 - \int_1^2 \rho\,dL - \int_1^2 \frac{f\,G\,dL}{2\rho\,g\,De}$$

The two components of the pressure loss are the friction head and the form head:

$$\text{Friction head.} \quad \cdots \cdots \cdots \quad \int_1^2 \frac{f\,G\,dL}{2\,g\,De} \approx \Sigma\,\frac{f\,G\,L}{2\,g\,De}$$

$$\text{Form head.} \quad \cdots \cdots \cdots \cdots \quad = \Sigma\,K_{form}\,\frac{G^2}{2\rho g}$$

Friction losses occur along the length of the fuel rods and along the length of the straps; form losses occur at each change in flow area in the system. Flow areas, equivalent diameters, and relative surface roughness are calculated for the various flow cross sections. The contraction and expansion form coefficients were obtained from Kays' graphs.[5] The friction factor was taken from Moody's graphs of the friction factor as a function of the surface relative roughness and the Reynolds number.[6]

The above equations do not include pressure drop losses caused by energy dissipated in vortices, the effect of the proximity of one obstruction to another, or the energy recovered after a sudden enlargement or sudden contraction.

The calculated pressure drop across the fuel assembly model containing straps based on the equivalent diameter concept was within 5% of the measured drop.

The experimental pressure drop across the ferrule type fuel assembly model agreed to within 5% of the analytical values. Sample analytical equations are developed in Appendix I and a typical conversion of experimental data is given in Appendix II. The various pressure drop versus flow relationships are graphically represented in Figures 6 and 7. Based on the mass velocity parallel to the fuel rods, a flow of 45.8 lb/sec through the test cell corresponds to the design rate of 37.8×10^6 lb/hr through the reactor core.

The Reynolds number in the test cell is 30,000 in contrast to a Reynolds number of 250,000 for the prototype reactor. The ratio which is approximately eight-to-one is small and similarity of hydrodynamic phenomena can be expected. Research on turbulence has shown that the geometry of turbulence, jet shape, and eddy formation is nearly independent of the Reynolds number at values above 5000, except in boundary layers on surfaces.[8]
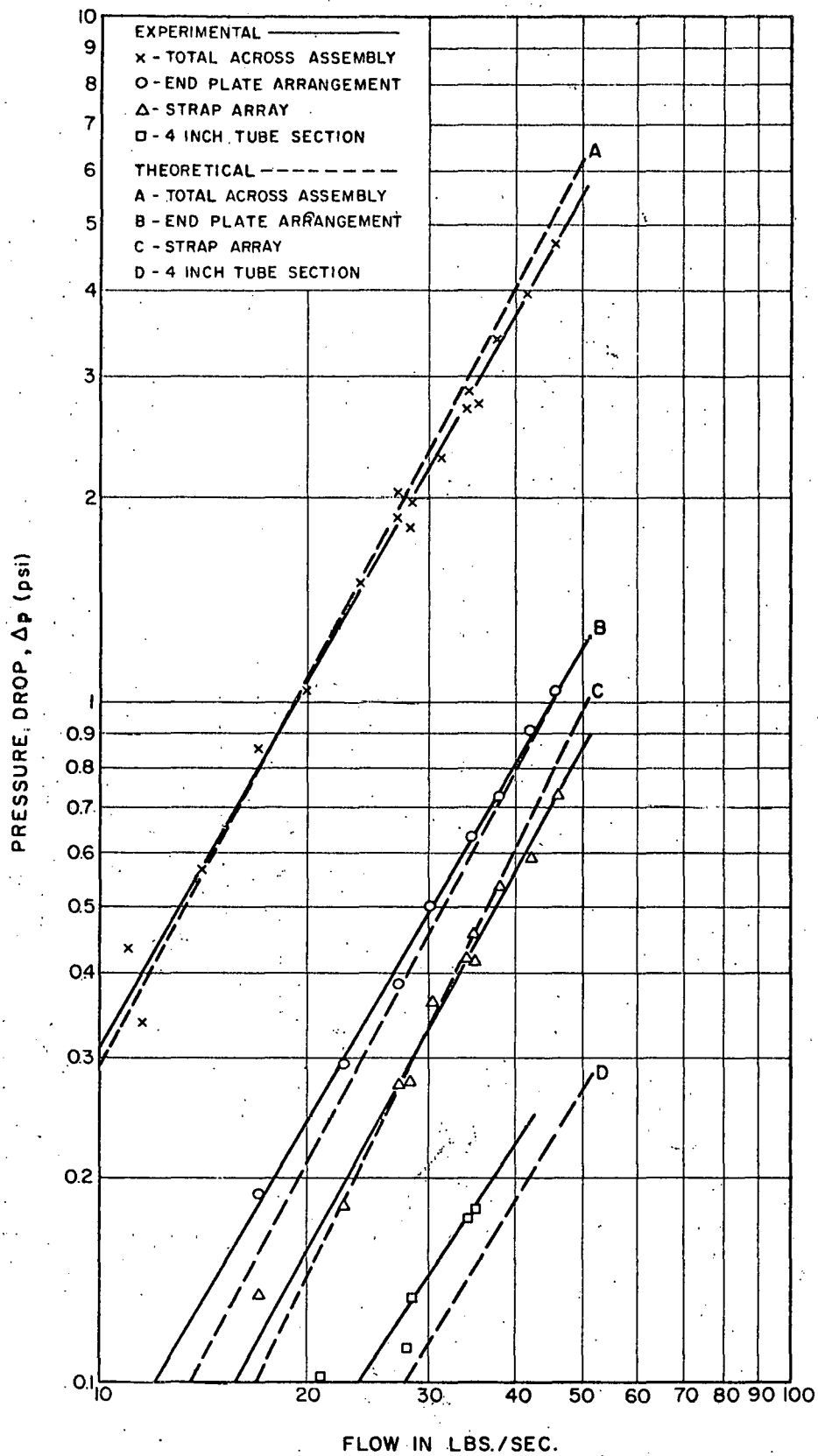
PRESSURE DROP VERSUS FLOW IN FUEL ASSEMBLY (STRAP DESIGN)

FIGURE 6
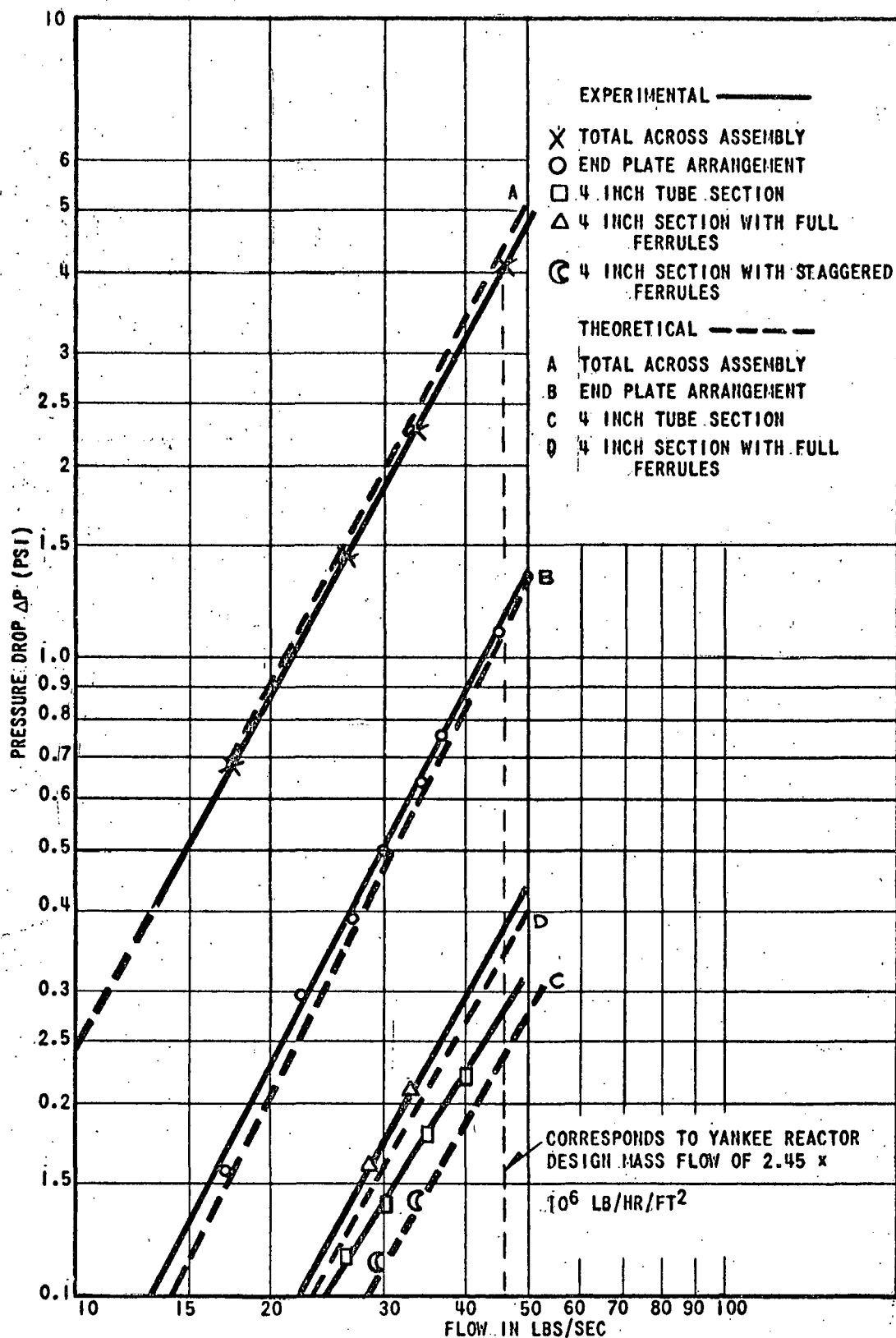
PRESSURE DROP VERSUS FLOW IN FUEL ASSEMBLY (WITH FERRULES)

FIGURE 7

520   020

VIII. COMPARISON OF STRAP AND FERRULE DESIGNS

Both straps and ferrules reduce the available flow area at a given cross section of the fuel assembly. The principal pressure losses are due to contraction and expansion losses across the section; such losses are called form losses. The extent of these form losses depends on how much the flow is restricted. The tube-side flow area of 9.67 in.^{2} is reduced to 7.50 in.^{2} by the straps and 8.54 in.^{2} by 74 ferrules. Theoretically the pressure drop due to a restriction in the flow path varies directly as the sum of the contraction and the expansion form coefficients and inversely as the square of the free flow area $[\Delta p \propto (K_c + K_e)/A^2]$. On this basis alone, the $(K_c + K_e)/A^2$ factor for the strap configuration is five times that of the section with the maximum number of ferrules. This is verified experimentally in an indirect way by subtracting the frictional pressure drop along the tubes in the test section, e.g., the friction drop along the 3.5 inch tube length in the 4 inch test section containing a ferrule array should be 7/8 of the experimental pressure drop measured along a 4 inch test section with no restrictions. Figure 8 gives these experimentally based pressure drops across only the ferrule and strap sections.

The straps and the section which had ferrules at all locations are more conservative from a pressure drop standpoint because their cross-sectional flow areas are less than the flow areas of the thinner straps and ferrules proposed for the prototype design.
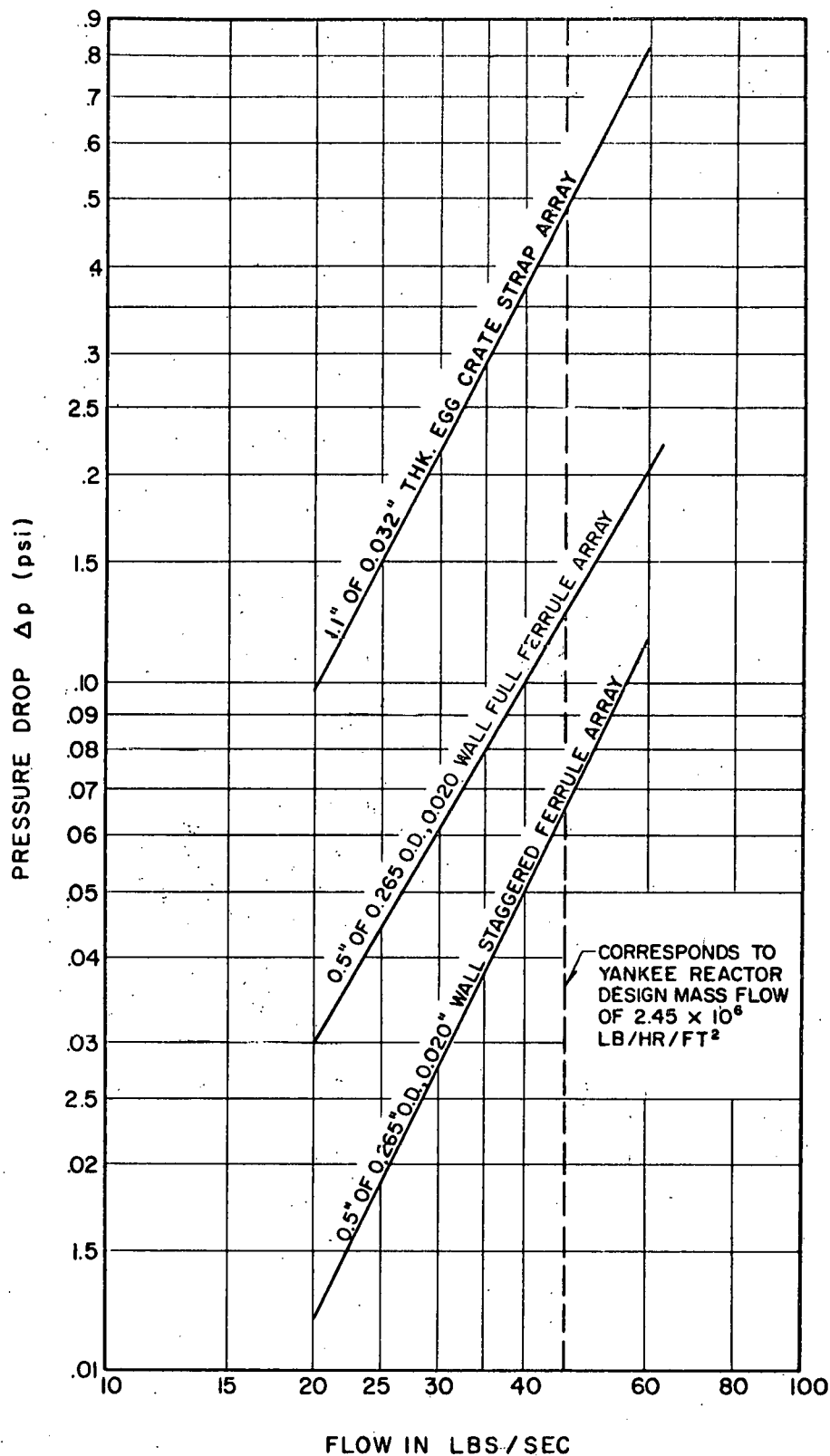
COMPARISON OF PRESSURE DROPS

THROUGH TEST SECTION FOR FERRULE AND STRAP ARRAYS

FIGURE 8

IX. DISCUSSION OF THE VIBRATION POSSIBILITIES OF THE FUEL RODS

Under specific unfavorable conditions, it is believed possible that the physical elements of the core could respond to the mechanical impulses provided by the pumps resulting in sustained forced vibrations of the fuel rods. Due to the damping forces acting on the fuel rods, a preliminary study indicated that serious vibrations would be possible only if the fundamental natural frequency of the fuel rods acting singly or in groups would resonate to the fundamental pump frequency. The fundamental pump excitation frequency is the frequency of the impeller vane impulses, 206.5 cps.

The damping forces which act on the fuel rod are forces inherent in the fuel rod and external forces exerted by the surrounding water. When a rod surrounded by a fluid vibrates, the fluid exerts a damping force on the rod proportional to the square of the velocity of the rod. The vibrational velocity of the rod is in turn proportional to the vibration frequency and the peak-to-peak amplitude. Since both the exciting force and the principal damping force are due to the motion of the fluid in the vicinity of the rod, it is felt that sustained forced vibrations are not probable. If the unsupported length of the fuel rod is such that its natural frequency is other than the excitation frequency, sustained forced vibrations are not mathematically possible.

The resonant frequencies of a system are lowered by damping in the system. Another effect of damping is to lower the magnitude of the displacements incurred during resonance. The overall effects of damping of the vibrations of the fuel rods are such that if the damping is sufficient to appreciably change the resonant frequency, the magnitude of the vibration is reduced enough that any resulting vibrations would not be serious. As a result, the natural frequency determinations of Appendix III have omitted any damping considerations.

The actual configuration of the fuel rods is such that an exacting study of the vibrational modes of the fuel rods is unfeasible. It is felt that a simplified study of a single rod or four rods acting as a bundle is sufficient to cover the possible cases for serious vibrations. Both the ferrule and the strap methods of retaining the fuel rods act as semi-fixed supports. The fuel rod will vibrate as a beam somewhere between the theoretical limits of a beam with fixed ends and of a simply supported beam. The fundamental mode of vibration is determined by the Rayleigh method of equating energies, since this method is readily applied to these basic cases where the deflection curve approximates the fundamental mode of vibration.[7] Figure 9 gives the natural frequencies of the fuel rods as a function of their free length as determined by the methods included in Appendix III.
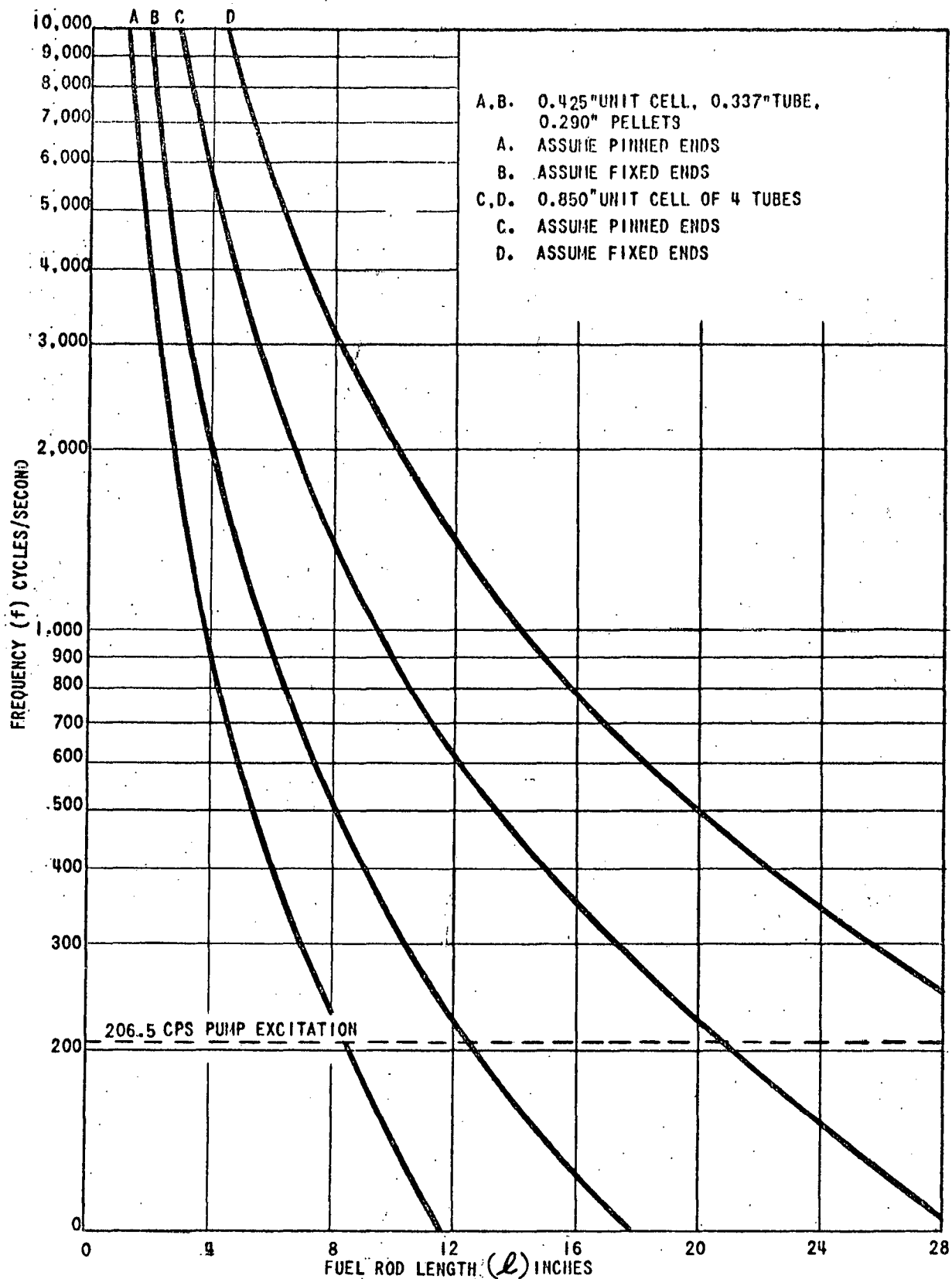
NATURAL FREQUENCY VS. LENGTH OF YANKEE FUEL RODS

Figure 9

510 024

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                              University Press, (1956).

Sample Calculations

Total pressure drop across the fuel assembly for a design mass flow rate of
682 lb/sec-ft^2.

$$\Delta p = \Sigma \frac{f \, \Delta L \, G^2}{2g \, \rho \, De} + \Sigma K/A^2 \quad \frac{G_t^2 \, A_t^2}{2g\rho}$$

$$\Delta p = \frac{f_t \, \Delta Z_t \, W^2}{2g \, \rho (De)_t A_t^2} + \frac{f_s \, \Delta Z_s \, W^2}{2g \, \rho (De)_s A_s^2} + \Sigma \frac{K}{A^2} \quad \frac{W^2}{2g\rho}$$

where:

G_t = 682 lb/sec-ft^2 W = 45.8 lb/sec ρ = 62.27 lb/ft^3

μ = 6.44 x 10^{-4} lb/ft-sec   (De)$_t$ = 0.335 in.   (De)$_s$ = 0.118 in.

A_t = 9.67 in.^{2} A_s = 7.50 in.^{2} ε = 0.000125 in. Z_t = 21.875 in.

ΔZ_s = 2.125 in.   (K/A^2)$_s$ = 0.00748/in.^{2} $\Sigma K/A^2$ = 0.04161/in.^{2}

$$Re_t = \frac{W \, De}{A \, \mu} = \frac{45.8 \text{ lb/sec x } 0.335 \text{ in. x } 12 \text{ in./ft}}{9.67 \text{ in}^2 \text{ x } 6.44 \text{ x } 10^{-4} \text{ lb/ft-sec}} = 29,500$$

$$Re_s = \frac{45.8 \text{ lb/sec x } 0.118 \text{ in. x } 12 \text{ in./ft}}{7.50 \text{ in.}^2 \text{ x } 6.44 \text{ x } 10^{-4} \text{ lb/ft-sec}} = 13,400$$

$$(\varepsilon/De)_s = \frac{0.000125}{0.118} = 0.00106 \qquad (\varepsilon/De)_t = 0.000374$$

from Moody's friction factor chart

f_s = 0.030 $\qquad$ f_t = 0.025

$$\Delta p = \frac{0.025 \text{ x } 21.875 \text{ in. x } 45.8^2 \text{ lb}^2/\text{sec}^2 \text{ x } 144 \text{ in.}^2/\text{ft}^2}{64.4 \text{ ft/sec}^2 \text{ x } 62.27 \text{ lb/ft}^3 \text{ x } 0.118 \text{ in. x } 7.50^2 \text{ in.}^4}$$

$$+ \frac{0.030 \text{ x } 2.125 \text{ in. x } 45.8^2 \text{ lb}^2/\text{sec}^2 \text{ x } 144 \text{ in.}^2/\text{ft}^2}{64.4 \text{ ft/sec}^2 \text{ x } 62.27 \text{ lb/ft}^3 \text{ x } 0.118 \text{ in. x } 7.50^2 \text{ in.}^4}$$

$$+ \frac{0.04161/in.^4 \times 45.8^2 \ lb^2/sec^2 \times 144 \ in.^2/ft^2}{64.4 \ ft/sec^2 \times 62.27 \ lb/ft^3}$$

$$\Delta p = 1.32 \ psi + 0.725 \ psi + 3.14 \ psi = 5.185 \ psi$$

Conversion of Experimental Data

Test Data

Pressure drop across Meriam Calibrated Orifice=11.3 in. Hg
Pressure drop across fuel assembly=10.3 in. Hg
Water temperature T_w = 67°F ρ = 62.3 lb/ft^3
For the calibrated orifice $Q(\text{gpm})$ = 97.5 $\sqrt{h}$ (in.Hg)

$$Q = 97.5 \sqrt{10.3} = 328 \text{ gpm}$$

$$W = Q\rho = \frac{328 \text{ gpm} \times 0.1337 \text{ ft}^3/\text{gal} \times 62.3 \text{ lb/ft}^3}{60 \text{ sec/min}} = 45.5 \text{ lb/sec}$$

Pressure drop across fuel assembly

$$\Delta p = \frac{10.3 \text{ in. Hg (on } H_2O) \times 62.3 \text{ lb/ft}^3 \times 12.6 \text{ in. } H_2O/\text{in.Hg (on } H_2O)}{1728 \text{ in.}^3/\text{ft}^3}$$

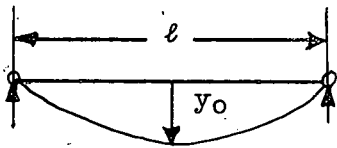$$= 4.68 \text{ psi}$$

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Natural Frequency Determination

Case I - Simply Supported Beam

Assume the deflection to be represented by a sine curve.[7]

$$y = (y_0 \sin \frac{\pi x}{\ell}) \sin wt$$

Kinetic energy $dm = wdx/g$

$$1/2 \int v^2 \, dm = 1/2 \, \omega^2 \int y^2 \, dm$$

Potential Energy

$$1/2 \int Md\theta = 1/2 \int EI \left(\frac{d^2 y}{dx^2}\right) \, dx$$

by equating energies

$$\omega^2 = \frac{\int EI \left(\frac{d^2 y}{dx^2}\right)^2 \, dx}{\int y^2 \, dm} = \frac{EI \left(\frac{\pi}{\ell}\right)^4 \int_0^\ell \sin \frac{\pi x}{\ell} \, dx}{w/g \int_0^\ell \sin^2 \frac{\pi x}{\ell}}$$

$$\omega = \pi^2 \sqrt{\frac{gEI}{w\ell^4}}$$

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If the distance between the ends of the beam is rigidly fixed, a tensile stress will be developed by the lateral deflection. This strain energy should be considered in equating energies.

additional strain energy $dU = 1/2 \ EA_t \ \varepsilon^2 dx$

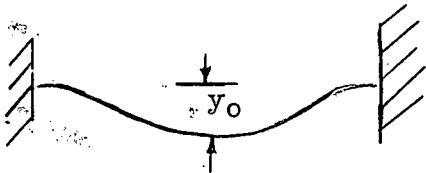where $\varepsilon = \left[\sqrt{1 + (\frac{dy}{dx})^2} - 1 \right] dx \cong 1/2 \ (\frac{dy}{dx})^2$

The corrected frequency equation then becomes

$$1/2 \ \omega^2 \int y^2 \ dm = 1/2 \int EI \ (\frac{d^2y}{dx^2})^2 \ dx + 1/2 \int \frac{EA_t}{4} \ (\frac{dy}{dx})^4 \ dx \quad (1)$$

Solving for the fundamental frequency

$$\omega = \frac{\pi^2}{\ell^2} \sqrt{\frac{EIg}{w}} + \frac{\pi^2 y_0}{\ell^2} \sqrt{\frac{3EA_t g}{16w}}$$

Case 2 — Beam with Fixed Ends
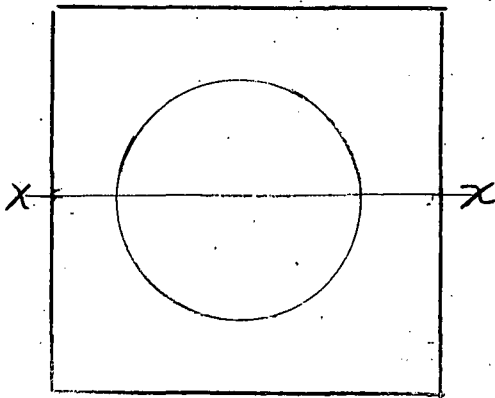

Assume a deflection curve

$$y = 1/2 \ y_0 \ (1 - \cos \ 2\pi x/\ell) \sin wt.$$

Equating the kinetic energy to the total strain energy of bending and tension the aforesaid frequency equation is obtained. (Eq. 1)

Solving for the fundamental frequency

$$\omega = \pi^2/\ell^2 \sqrt{\frac{16 \ EIg}{3w}} + \frac{\pi^2}{\ell^2} y_0 \sqrt{\frac{3EAg}{8w}} \quad (2)$$

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Consideration of a single fuel rod cell

Fuel tube O.D. = 0.337 in., I.D. = 0.295 in.

$$A = \pi/4 \ (0.337^2 - 0.295^2) = 0.021 \ \text{in.}^2$$

$$w = \rho \ V/\ell = 0.280 \ \text{lb/in.}^3 \times 0.021 \ \text{in.}^2$$

$$= 0.00585 \ \text{lb/in.}$$

$$I_{xx} = \pi/4 \ (r_o^4 - r_i^4) = 0.00025 \ \text{in.}^4$$

Fuel pellet O.D. = 0.290 in.

$$A = \pi/4 \ (0.290)^2 = 0.066 \ \text{in.}^2$$

$$w = 0.394 \ \text{lb/in}^3 \times 0.066 \ \text{in.}^2 = 0.026 \ \text{lb/in.}$$

0.425 Water Cell

$$A = 0.425^2 - \pi/4 \ (0.337)^2 = 0.0917 \ \text{in.}^2$$

$$w = 0.0249 \ \text{lb/in.}^2 \times 0.0917 \ \text{in.}^2 = 0.00228 \ \text{lb/in.}$$

Total cell weight

$$w = 0.00585 + 0.026 + 0.00228 = 0.0341 \ \text{lb/in.}$$

Natural vibration of simply supported or pinned fuel rod

$$\omega = \pi^2/\ell^2 \sqrt{\frac{gEI}{w}} + \frac{\pi^2 y_o}{\ell^2} \sqrt{\frac{3EAg}{16w}}$$

$$= \frac{\pi^2}{\ell^2} \sqrt{\frac{386 \times 29 \times 10^6 \times 0.00025}{0.0341}} + \frac{\pi^2 y_o}{\ell^2}$$

$$\sqrt{\frac{3 \times 29 \times 10^6 \times 0.021 \times 386}{16 \times 0.0341}}$$

$$= \frac{89,500}{\ell^2} + \frac{355,000 \ y_o}{\ell^2} \quad \text{radians/sec} \tag{3}$$

Since y_0 is very small for the range of lengths considered, the possibility of eliminating the second term is considered for $\ell = 15$ inches.

$$y_0 = \frac{-5w\,\ell^4}{384EI} = \frac{-5 \times 0.0341 \times 15^4}{384 \times 29 \times 10^6 \times 0.0025} = 0.0031 \text{ in.}$$

$$\frac{355,000\,y_0}{\ell^2} = \frac{1100}{\ell^2}$$

This is only 1.2 percent of the first term, corresponding to 0.78 cycles/sec and therefore may be neglected.

$$f = \frac{\omega}{2\pi} = \frac{14,260}{\ell^2} \text{ cycles/sec} \tag{4}$$

Natural vibration of fuel rod fixed at ends

$$\omega = \frac{\pi^2}{\ell^2}\sqrt{\frac{16\,EIg_t}{3w}} + \frac{\pi^2 y_0}{\ell^2}\sqrt{\frac{3EAg}{8w}}$$

$$\omega = \sqrt{\frac{16 \times 29 \times 10^6 \times 0.00025 \times 386}{3 \times 0.0341}} + \frac{y_0}{\ell^2}$$

$$\sqrt{\frac{3 \times 29 \times 10^6 \times 0.021 \times 386}{8 \times 0.0341}}$$

$$\omega = \frac{206,500}{\ell^2} + \frac{502,000\,y_0}{\ell^2} \tag{5}$$

The second term is again negligible

$$f = \frac{\omega}{2\pi} = \frac{33,000}{\ell^2} \text{ cycles/sec} \tag{6}$$

Figure 9 gives the values of the natural rod frequency as determined by Eqs. (4) and (6) for $\ell = 0$ to $\ell = 25$ inches. The pump exciting frequency is shown as 206.5 cycles/sec.

$$f_{excitation} = 1770 \text{ rev/min} \times 7 \text{ vanes} \times 1 \text{ min}/60 \text{ sec} = 206.5 \text{ cps}$$

Similarly, the natural frequencies of a 0.85 inch cell consisting of four fuel rods acting together are determined. The graphs of natural frequency versus length for both fixed and pinned ends are shown in Figure 9.