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Accelerator Neutrino Physics, Present and Future--

A Review for Theorists and Experimentalists

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CONTENTS

1) KINEMATICS AND REVIEW OF "STANDARD" V-A THEORY[1]

We will be discussing throughout reactions of the form

$$\ell_1(q_1) + N(p) \rightarrow \ell_2(q_2) + \Gamma$$

$\ell_{1,2}$ leptons

N nucleon; mass M_N; at rest in lab

Γ hadron or hadrons (we will consider both exclusive and inclusive processes)

Metric (1, -1, -1, -1) $\qquad p^2 = M_N^2$

Two important variables:

$q = q_1 - q_2$ = leptonic momentum transfer.

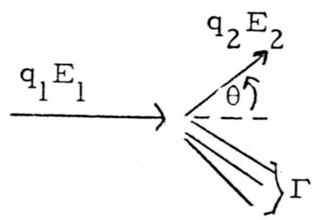
q^2 = leptonic momentum transfer squared.

$\nu = M_N q_0^{lab} = M_N(E_1 - E_2)$ = leptonic energy transfer.

Obviously $\nu = q \cdot p$ is corresponding invariant.

$q^2 = 4E_1E_2 \sin^2\frac{\theta}{2}$, θ = lab leptonic scattering angle.

Lab picture is:



The "standard" V-A theory of weak interactions is obtained from a current-current effective Lagrangian:

$$\mathcal{L}_{eff} = \frac{G}{\sqrt{2}} J_\lambda^+ J^\lambda \qquad G \approx 10^{-5}/M_N^2 = \text{Fermi constant}$$

$$J^\lambda = J_\ell^\lambda + J_h^\lambda$$

$$J_\ell^\lambda = \bar{\nu}_\mu \gamma^\lambda (1-\gamma_5)\mu + \bar{\nu}_e \gamma^\lambda (1-\gamma_5) e$$

$$J_h^\lambda = (V^\lambda_{1+i2} - A^\lambda_{1+i2}) \cos\theta_C + (V^\lambda_{4+i5} - A^\lambda_{4+i5}) \sin\theta_C .$$

$$\theta_C \approx 15^0 = \text{Cabibbo angle} .$$

Comments:

(i) Scale of V, A fixed by current algebra:

$$[V_k^0(\vec{x}, t), V_\ell^0(\vec{y}, t)] = i\,\delta(\vec{x}-\vec{y}) f_{k\ell m} V_m^0(\vec{x}, t) \text{ etc.}$$

(ii) The Lagrangian has charged currents only; possibility of neutral currents will be discussed extensively below.

(iii) $\left.\begin{array}{l} V^\lambda_{1+i2} \text{ has G parity } +1 \text{ (like } \rho) \\ A^\lambda_{1+i2} \text{ has G parity } -1 \text{ (like } \pi) \end{array}\right\}$ i.e. $\mathcal{L}_{eff}$ has first class currents only

Experimental bounds on a possible second class current [V(ΔS=0) with G = -1, A(ΔS=0) with G = 1] are not very good--such a current, with strength comparable to the usual beta decay current, is still not excluded.

2) LEPTON CONSERVATION RESULTS FROM NEUTRINO EXPERIMENTS

First major result from neutrino reactions was $\nu_\mu \neq \nu_e$. This is incorporated into $\mathcal{L}_{eff}$ above in the form of two additive leptonic quantum numbers:

$$N_\mu + N_{\nu_\mu} = \text{CONST}$$

$$N_e + N_{\nu_e} = \text{CONST}$$

Possibility of multiplicative law.[3] $\left.\begin{array}{l} P_{\mu,\nu_\mu} = -1 \\ P_{e,\nu_e} = +1 \end{array}\right\}$ muon parity; Anti-particle parities opposite in sign to particle parities.

In this scheme the product of muon parities is conserved, as well as $N_\mu + N_{\nu_\mu} + N_e + N_{\nu_e}$. Multiplicative law allows $\mu^+ \to e^+ + \bar{\nu}_e + \nu_\mu$ as well as $\mu^+ \to e^+ + \nu_e + \bar{\nu}_\mu$. So have

$$r = \frac{\mu^+ \to e^+ + \bar{\nu}_e + \nu_\mu}{\text{all } \mu^+ \text{ modes}} = \begin{cases} 0 & \text{additive law} \\ .5 & \text{multiplicative law} \end{cases} .$$

In a ν beam (obtained by sign selection) should have ν_e but negligible $\bar{\nu}_e$ if $r = 0$. (There will be a small residual $\bar{\nu}_e$ component from K^0_L decays.) That is

$$\frac{\text{flux } \bar{\nu}_e}{\text{flux } \nu_e} \quad \text{sensitively measures } r .$$

From $e^\pm$ determinations CERN Gargamelle group finds[4] $r < .25$ at 90% confidence. So additive law is favored. This is important in constructing Lagrangian models of the weak interactions.

3) EXCLUSIVE REACTIONS

(A) Quasielastic

Have $\Gamma = N$: single nucleon.

Matrix element of hadronic current for $\nu_\mu + n \to \mu^- + p$ is

$$\langle p(p_2) | J_h^\lambda | n(p_1) \rangle = \cos\theta_C \, \bar{u}(p_2)\, \Gamma^\lambda u(p_1)$$

$$\Gamma^\lambda = \gamma^\lambda F_V^1(q^2) + \frac{i\sigma^{\lambda\nu} q_\nu}{2M_N} F_V^2(q^2) + \frac{q^\lambda}{M_N} F_V^3(q^2)$$

$$-\gamma^\lambda \gamma_5 g_A(q^2) - q^\lambda \gamma_5 h_A(q^2) + \frac{(p_1 + p_2)_\lambda \gamma_5}{M_N} F_A^3(q^2)$$

No second class currents $\Rightarrow F_V^3 = F_A^3 = 0.$

CVC also $\Rightarrow \begin{cases} F_V^3 = 0. \\ F_V^{1,2} \text{ given by electron scattering data} \end{cases}$

$q^\lambda \quad < \mu^- | J_{\ell\lambda} | \nu_\mu > \propto m_\mu$, so h_A (induced pseudoscalar) term is strongly suppressed. (h_A probably well described by pion pole dominance.)

$g_A(0) \approx 1.24$. So the only thing not known is q^2 - dependence of g_A. We parametrize this in the form

$$g_A(q^2) = 1.24/\left(1 - \frac{q^2}{M_A^2}\right)^2 .$$

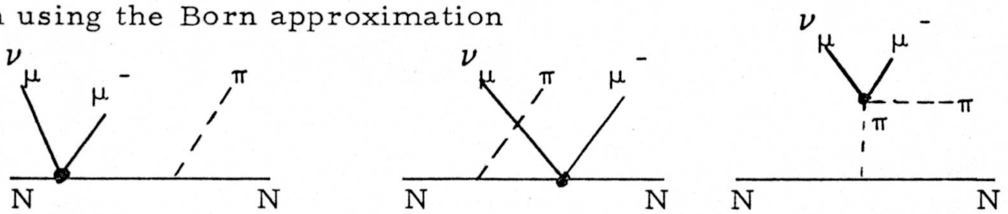
Most recent result[5] from Argonne bubble chamber filled with deuterium: $M_A = 0.95 \pm 0.12$ GeV/c^2. (Older CERN experiments gave a somewhat lower value).

In satisfactory agreement with a determination of $g_A(q^2)$ from a pion electroproduction low energy theorem.[5]

(B) Δ (1236) production (3, 3 resonance)

$$\Gamma = \Delta\,(1236) \to N + \pi$$

Can make a relativistic version[6] of the static model for this process, obtained from using the Born approximation



for non-resonant partial waves + unitarized Born approx (using experimental resonant πN amplitude) for resonant multipoles. Gives essentially unique predictions in terms of elastic weak form factors $F_V^{1,2}$, g_A, h_A, F_π. This model works well for pion electroproduction for moderate q^2 [$q^2 \lesssim 1 \left(\frac{\text{GeV}}{c}\right)^2$] ; breaks down for larger q^2.

In weak prod., is in satisfactory agreement[7] with recent Argonne experiment in D_2 (change from older CERN data in propane). Relativistic quark model (Feynman, Kislinger and Ravndal[8]) gives similar results. Ability to successfully model Δ(1236) production by charged weak currents is of importance in discussing neutral current tests involving Δ(1236).

(C) Forward lepton theorem:[9] PCAC and CVC tests

Consider any inelastic exclusive reaction ($\Gamma \neq N$) with the final lepton in the forward direction ($\theta = 0$)

$$\xrightarrow{\ \ell_1\ } \quad \xrightarrow{\ \ell_2\ }$$

Can show $\langle \Gamma | J_h^\lambda | N \rangle \langle \ell_2 | J_{\ell\lambda} | \ell_1 \rangle \propto \langle \Gamma | \partial_\lambda J_h^\lambda | N \rangle$ up to lepton mass corrections. So can measure the divergence of the hadronic current in the forward configuration.

How forward is forward? Need[10] $q^2 \lesssim 0.04\ (\text{GeV}/c)^2 \sim 2\, M_\pi^2$ to avoid appreciable interference with transverse parts of J_h^λ. (Extension of this region possible but model dependent.)

Applications in strangeness-conserving reactions:

(i) CVC $\Rightarrow \partial_\lambda V^\lambda_{1+i2} = 0$

$\Rightarrow$ V-A interference, and hence parity violating effects, vanish in forward configuration. Get a CVC test. Will return to this idea when we discuss properties of neutral currents.

(ii) Assuming CVC, only the axial-vector current remains.

According to the partially-conserved axial-vector current hypothesis,

$$\partial_\lambda A^\lambda_{1+i2} = c\, \underbrace{\phi_{\pi^+}}_{\text{pion field}}$$

so we get a proportionality between forward lepton cross sections and cross sections for pion-induced reactions,

$$\left.\frac{d^2\sigma\left(\frac{\nu}{\bar\nu} + N \rightarrow \frac{\mu^-}{\mu^+} + \Gamma\right)}{d\,q^2\; d\,M_\Gamma}\right|_{\theta=0} = \begin{bmatrix} \text{KNOWN} \\ \text{CONSTANTS} \end{bmatrix} \times \sigma\,(\pi^\pm + N \rightarrow \Gamma)$$

Remark: Most current algebra applications involve only PCAC sandwiched between single particle on shell or low mass composite states. Thus, it is still possible PCAC could fail badly for matrix elements involving off shell states or composite systems of high mass--this is the possibility of so-called "weak" PCAC discussed by Drell, Brandt, and Preparata. Since for large $E_2 - E_1$ we get large M_Γ, above relation will serve as a test to distinguish between "strong" and "weak" PCAC:[11]

"strong" PCAC $\Rightarrow$ relation holds for all M_Γ

"weak" PCAC $\Rightarrow$ relation holds for small M_Γ (say, in the resonance region) but is violated by $\sim 30\%$ in region of large M_Γ (say $M_\Gamma \gtrsim 2.5\ \text{GeV}/c^2$)

4) INCLUSIVE REACTIONS: SUM OVER ALL Γ FOR FIXED M_Γ

Both because of their experimental accessibility, and their connection with scaling and the light cone, inclusive reactions occupy a central position in accelerator neutrino physics.

Squaring the current-current form we find

$$\sigma^{\nu,\bar{\nu}} \propto \ell^{\alpha\beta}_{\nu,\bar{\nu}} H^{\nu,\bar{\nu}}_{\alpha\beta}$$

$$H^{\nu}_{\alpha\beta} = \frac{1}{2}\sum_{\Gamma} \overline{\sum_{N\,\mathrm{spin}}} \langle N|J_{h\alpha}|\Gamma\rangle\langle\Gamma|J^{\dagger}_{h\beta}|N\rangle (2\pi)^3 \delta^4(q_1+p-q_2-p_\Gamma)$$

$H^{\bar{\nu}}_{\alpha\beta}$ obtained by $J_h \longleftrightarrow J^{\dagger}_h$

General structure of $H^{\nu}_{\alpha\beta}$ is[12]

$$H^{\nu}_{\alpha\beta} = -g_{\alpha\beta} W^{\nu}_1 + \frac{p_\alpha p_\beta}{M_N^2} W^{\nu}_2 - i\,\frac{\varepsilon_{\alpha\beta\sigma\lambda} p^\sigma q^\lambda}{2M_N^2} W^{\nu}_3 + \frac{q_\alpha q_\beta}{M_N^2} W^{\nu}_4$$

$$+ \frac{(p_\alpha q_\beta + p_\beta q_\alpha)}{2M_N^2} W^{\nu}_5 + i\,\frac{(p_\alpha q_\beta - p_\beta q_\alpha)}{2M_N^2} W^{\nu}_6 \, .$$

When we contract with the leptonic tensor, terms with q_α, q_β are proportional to the lepton mass. So we find

$$\frac{d^2\sigma^{\nu,\bar{\nu}}}{d|q^2|\,d\nu} = \frac{G^2}{2\pi M_N^2}\frac{E_2}{E_1}\left(\cos^2\frac{\theta}{2} W^{\nu,\bar{\nu}}_2 + 2\sin^2\frac{\theta}{2} W^{\nu,\bar{\nu}}_1 \mp \frac{E_1+E_2}{M_N}\sin^2\frac{\theta}{2} W^{\nu,\bar{\nu}}_3\right)$$

$$+ O(m_\ell^2)$$

(A) Before turning to scaling, we consider tests of the Gell-Mann local current algebra[13] in high energy neutrino reactions. We form the commutator

$$\left[\int d^3x\, J_h^0(\vec{x},0)\, e^{i\vec{q}\cdot\vec{x}},\ \int d^3y\, J_h^0(\vec{y},0)^\dagger e^{-i\vec{q}\cdot\vec{y}} \right]$$

$$= 4\, I_3 \cos^2\theta_C + (3Y+2I_3)\sin^2\theta_C + \ldots \begin{cases} \text{pseudoscalar} \\ \text{or } \Delta S \neq 0;\ \text{one nucleon} \\ \text{spin-averaged matrix} \\ \text{element vanishes} \end{cases}$$

Taking the one-nucleon to one-nucleon spin-averaged matrix element and using the $P \to \infty$ method, we get the Adler sum rule[14] $(q^2 = -\vec{q}^{\,2})$

$$\frac{1}{M_N^2} \int_0^\infty d\nu \left[W_2^{\bar{\nu}}(\nu, q^2) - W_2^{\nu}(\nu, q^2) \right] = \langle 4 \cos^2\theta_C I_3 + (3Y+2I_3)\sin^2\theta_C \rangle_N$$

$$= 2\cos^2\theta_C + 4\sin^2\theta_C \quad \text{proton target}$$

$$- 2\cos^2\theta_C + 2\sin^2\theta_C \quad \text{neutron target}$$

$$= \text{constant, independent of } q^2.$$

Equivalently, this can be written as

$$\lim_{E_1 \to \infty} \left[\frac{d\sigma^{\bar{\nu}p}}{d|q^2|} - \frac{d\sigma^{\nu p}}{d|q^2|} \right] = \frac{G^2}{\pi} (\cos^2\theta_C + 2\sin^2\theta_C)$$

for <u>all</u> q^2

The q^2-independence of the right-hand side tests the <u>local</u> current algebra. The precise <u>value</u> of the constant tests the construction of the hadronic current from pieces which individually obey current algebra. Adding further terms to the current would change the constant--what such terms might be will be discussed later on, when we consider "charm."

(B) Scaling variables and scaling assumption[12]

Let us introduce new variables and dimensionless structure functions as follows:

$$W_1(\nu, q^2) = G_1(\omega, |q^2|/M_N^2)$$

$$\frac{\nu W_2(\nu, q^2)}{M_N^2} = G_2(\omega, |q^2|/M_N^2)$$

$$\frac{\nu W_3(\nu, q^2)}{M_N^2} = G_3(\omega, |q^2|/M_N^2)$$

$$\omega = \frac{2q\cdot p}{-q^2} = \frac{2\nu}{-q^2} \qquad 1 \leq \omega < \infty \text{ is allowed kinematic range}$$

$$\left.\begin{aligned} y &= \frac{\nu}{M_N E_1} = 1 - \frac{E_2}{E_1} \\ x &= \frac{1}{\omega} \end{aligned}\right\} \qquad 0 \leq x, y \leq 1 \ .$$

In terms of the G's the doubly differential cross section takes the form

$$\frac{d^2\sigma^{\nu,\bar{\nu}}}{dx\,dy} = \frac{G^2 M_N E_1}{\pi}\left[\left(1-y-\frac{1}{2}xy\frac{M_N}{E_1}\right)G_2^{\nu,\bar{\nu}} + x y^2 G_1^{\nu,\bar{\nu}} \mp xy\left(1-\frac{1}{2}y\right)G_3^{\nu,\bar{\nu}}\right]$$

Predictive content of this rewriting comes through making the Bjorken scaling[15] assumption:

$$\lim_{\substack{|q^2|\to\infty \\ x=\omega^{-1}\text{ fixed}}} G_i(\omega, |q^2|/M_N^2) = F_i(x) \qquad \text{exists}$$

Can attain large $|q^2|$ only for large neutrino energy E_1; dropping the M_N/E_1 term we get the scaling regime expression

$$\frac{d^2\sigma^{\nu,\bar\nu}}{dx\,dy} = \frac{G^2 M_N E_1}{\pi}\left[xy^2 F_1^{\nu,\bar\nu}(x) + (1-y)F_2^{\nu,\bar\nu}(x) \mp xy(1-\tfrac{1}{2}y)F_3^{\nu,\bar\nu}(x)\right] .$$

Since the hadronic squared tensor $H_{\alpha\beta}$ is a positive semidefinite form, we have $\varepsilon^{\alpha}\varepsilon^{\beta *} H_{\alpha\beta} \geq 0$ for arbitrary polarization vector ε. Thinking ahead to the intermediate boson exchange picture and taking ε to correspond to absorption of scalar, left-handed and right-handed boson polarization components, we get the positivity conditions

$$0 \leq \sigma_S = \frac{\pi}{\nu+q^2/2}\left[W_2\left(\frac{\nu^2}{-q^2 M_N^2}+1\right) - W_1\right] \approx \frac{\pi}{\nu(1-x)}\left[\frac{F_2}{2x} - F_1\right]$$

$$0 \leq \sigma_R = \frac{\pi}{\nu+q^2/2}\left[W_1 + \frac{1}{2}\sqrt{\frac{\nu^2}{M_N^4} - \frac{q^2}{M_N^2}}\,W_3\right] \approx \frac{\pi}{\nu(1-x)}\left[F_1 + \frac{1}{2}F_3\right]$$

$$0 \leq \sigma_L = \frac{\pi}{\nu+q^2/2}\left[W_1 - \frac{1}{2}\sqrt{\frac{\nu^2}{M_N^4} - \frac{q^2}{M_N^2}}\,W_3\right] \approx \frac{\pi}{\nu(1-x)}\left[F_1 - \frac{1}{2}F_3\right] ,$$

i.e. in the scaling limit we have

$$F_2(x) \geq 2x\,F_1(x)$$

$$F_1(x) \geq \frac{1}{2}|F_3(x)|$$

When y is integrated out we get

$$\frac{d\sigma^{\nu,\bar{\nu}}}{dx} = \frac{G^2 M_N E_1}{\pi} \left[\frac{1}{3} x\, F_1^{\nu,\bar{\nu}}(x) + \frac{1}{2} F_2^{\nu,\bar{\nu}}(x) \mp x \frac{1}{3} F_3^{\nu,\bar{\nu}}(x) \right],$$

or rearranging the $\nu, \bar{\nu}$ cases separately to exploit the positivity conditions,

$$\frac{d\sigma^{\bar{\nu}}}{dx} = \frac{G^2 M_N E_1}{\pi} \left[a_S^{\bar{\nu}} + \frac{1}{3} x\, a_L^{\bar{\nu}} + x\, a_R^{\bar{\nu}} \right]$$

$$\frac{d\sigma^{\nu}}{dx} = \frac{G^2 M_N E_1}{\pi} \left[a_S^{\nu} + x\, a_L^{\nu} + \frac{1}{3} x\, a_R^{\nu} \right]$$

$$a_S^{\nu,\bar{\nu}} = \frac{1}{2} F_2^{\nu,\bar{\nu}} - x\, F_1^{\nu,\bar{\nu}} \geq 0 \qquad a_L^{\nu,\bar{\nu}} = F_1^{\nu,\bar{\nu}} - \frac{1}{2} F_3^{\nu,\bar{\nu}} \geq 0$$

$$a_R^{\nu,\bar{\nu}} = F_1^{\nu,\bar{\nu}} + \frac{1}{2} F_3^{\nu,\bar{\nu}} \geq 0$$

(C) Regge asymptotics

Let us briefly consider what happens when we combine Regge asymptotics with the scaling limit. <u>Before</u> going to the scaling limit a standard Regge analysis gives for the asymptotic behavior of $W_i(\nu, q^2)$

$$W_1 \underset{\nu \to \infty}{\rightarrow} \beta_1(q^2)\, \nu^{\alpha_1(0)}$$

$$W_2 \underset{\nu \to \infty}{\rightarrow} \beta_2(q^2)\, \nu^{\alpha_2(0)-2}$$

$$W_3 \underset{\nu \to \infty}{\rightarrow} \beta_3(q^2)\, \nu^{\alpha_3(0)-1},$$

with each α the $t = 0$ intercept of the appropriate leading trajectory. Since the Pomeron can contribute to $W_{1,2}$ we have $\alpha_{1,2}(0) = 1$; the leading trajectories for W_3 (which comes from the negative G-parity V-A interference) have $\alpha_3(0) = \frac{1}{2}$. Now let us suppose that <u>we can take the Regge and scaling limits simultaneously</u>. This assumption uniquely specifies the large-q^2 form of $\beta_i(q^2)$ to be power-behaved, and we get

$$F_1 \underset{\omega\to\infty}{\to} \beta_1 \omega^{\alpha_1(0)}$$

$$F_2 \underset{\omega\to\infty}{\to} \beta_2 \omega^{\alpha_2(0)-1}$$

$$F_3 \underset{\omega\to\infty}{\to} \beta_3 \omega^{\alpha_3(0)} \quad .$$

Thus, Regge ideas combined with scaling suggest that $F_2(\omega)$ will behave as $\beta_2\omega^{1-1}$ = CONST as $\omega\to\infty$, which appears to be the observed behavior. Of course, if we take a linear combination such as $F_2^{\nu p} - F_2^{\bar{\nu} p}$ from which the Pomeron decouples, we expect the dominant trajectory to be the ρ [with $\alpha_\rho(0) \approx \frac{1}{2}$] , and the asymptotic behavior becomes $\omega^{-\frac{1}{2}}$ as $\omega\to\infty$. This fact guarantees convergence of the scaling form of the local current algebra sum rule [see immediately below] .

(D) Applications of the scaling formalism

(i) First rewrite the local current algebra sum rule in scaling form:

$$\int_1^\infty \frac{d\omega}{\omega} [F_2^{\bar{\nu}} - F_2^{\nu}] = \int_0^1 \frac{dx}{x} [F_2^{\bar{\nu}} - F_2^{\nu}]$$

$$= \langle 4 \cos^2\theta_C I_3 + (3Y+2I_3)\sin^2\theta_C \rangle_N$$

Scaling makes the q^2-independence of the integral <u>automatic</u>; the key question becomes the value of ω at which the sum rule saturates and the constant thus produced.

(ii) Next we consider the total cross section.[15a] Integrating on x and y we get

$\sigma^{\nu,\bar{\nu}} = C^{\nu,\bar{\nu}} E_1$: cross sections rise linearly with lab neutrino energy. This rise is seen from CERN[16] energies up to and beyond E_1=150 GeV at NAL[17]. (Added note: Possible deviations from linearity in σ^ν were reported by Mann at this Conference.)

Experimentally,[16,17] $\frac{C^{\bar{\nu}}}{C^{\nu}} \approx \frac{1}{3}$ on targets with roughly equal nos. of protons and neutrons. To interpret theoretically, we consider isoscalar target ($Z = \frac{1}{2} A$) and neglect strangeness-changing contribution to structure functions ($\sin^2\theta_C / \cos^2\theta_C << 1$) Then charge symmetry ($V_{1+i2}$, V_{1-i2} in <u>same</u> isospin multiplet and likewise for V→A) implies that

$$F_i^{\nu n} = F_i^{\bar{\nu} p};\ F_i^{\nu p} = F_i^{\bar{\nu} n}$$

and hence

$$\frac{1}{2}(F_i^{\nu p} + F_i^{\nu n}) = \frac{1}{2}(F_i^{\bar{\nu} p} + F_i^{\bar{\nu} n}) \equiv F_i,$$

so we can drop superscripts $\nu, \bar{\nu}$ when discussing an average nucleon target under the above-stated assumptions. Hence

$$\frac{C^{\bar{\nu}}}{C^{\nu}} = \frac{\int_0^1 dx\, a_S + \frac{1}{3}\int_0^1 dx\, x\, a_L + \int_0^1 dx\, x\, a_R}{\int_0^1 dx\, a_S + \int_0^1 dx\, x\, a_L + \frac{1}{3}\int_0^1 dx\, x\, a_R}$$

$$\Rightarrow \qquad \frac{1}{3} \leq \frac{C^{\bar{\nu}}}{C^{\nu}} \leq 3$$

Experiment gives $\approx$ extremal value of $\frac{1}{3} \Rightarrow$

$$\int_0^1 dx\, a_S \approx 0$$

$$\int_0^1 dx\, x\, a_R \approx 0$$

Since $a_S \geq 0$ and $a_R \geq 0$ for all x, we learn

$a_S \approx 0$ i.e. $F_2(x) \approx 2x\, F_1(x)$ (Callan-Gross[17] relation for spin - 1/2 constituent)

$a_R \approx 0$, i.e. $F_3(x) \approx -2\, F_1(x)$ (V-A interference is maximal)*

Since there is only <u>one</u> independent structure function now, we find for <u>the y distribution on an isoscalar target</u>

*This relation holds for all x except very near $x = 0$, where Regge asymptotics (see p. 13) requires $F_3 \propto x^{\frac{1}{2}}\, F_1$ as $x \to 0$.

$$\left.\begin{aligned}\frac{d\sigma^{\nu}}{dx\,dy} &= \frac{G^2 M_N E_1}{\pi} F_2(x) \\ \frac{d\sigma^{\bar{\nu}}}{dx\,dy} &= \frac{G^2 M_N E_1}{\pi} F_2(x)(1-y)^2\end{aligned}\right\} \text{ Remarkably simple forms!}$$

Caltech-NAL experiment[16] for ν: (a) consistent with flat y distribution

(b) finds $F_2^{\nu N}(x)$ which agrees with

$\frac{5}{18} F_2^{eN}(x)$ measured in electron scattering

(here $N = \frac{1}{2}(n+p)$ = average nucleon target)

Mean muon (secondary lepton) energy:

$$\nu \qquad \langle E_2/E_1\rangle = \langle 1-y\rangle = \frac{\int_0^1 dy\,(1-y)}{\int_0^1 dy} = \frac{1}{2}$$

$$\bar{\nu} \qquad \langle 1-y\rangle = \frac{\int_0^1 dy\,(1-y)^3}{\int_0^1 dy\,(1-y)^2} = \frac{3}{4}$$

(iii) Another useful scaling variable:[18] $v = xy = \frac{|q^2|}{2M_N E_1}$

$= 2\left(\frac{E_2}{M_N}\right)\sin^2\frac{\theta}{2}$ <u>independent of initial neutrino energy</u>.

Combining with simplified neutrino $\frac{d\sigma^{\nu}}{dx\,dy}$ above:

$$\frac{\frac{d\sigma^{\nu}}{dv}}{\sigma^{\nu}} = \frac{\int_v^1 \frac{dx}{x} F_2(x)}{\int_0^1 dx\, F_2(x)}$$

independent of neutrino energy E_1 and of the neutrino flux.

So use of the v variable allows scaling tests, and extraction of F_2, even if initial neutrino energy and flux cannot be determined. The NAL experiments actually do have information on E_1 (from calorimetry) and on the neutrino flux, so this trick is not essential.

(iv) Suppose there is an intermediate boson (or scaling violation through a form factor). Then the formula $\frac{d\sigma}{dx\,dy} = E_1\,\Phi(x,y)$ gets replaced by $\frac{d\sigma}{dx\,dy} = \frac{E_1\,\Phi(x,y)}{\left(1-\frac{q^2}{M_W^2}\right)^2}$

To calculate the large-E_1 behavior of the total cross section:

$$-q^2 = 2M_N E_1 xy$$

$$\sigma = \int_0^1\int_0^1 dx\,dy\,\frac{E_1\Phi(x,y)}{\left(1+\frac{2M_N E_1 xy}{M_W^2}\right)^2} \underset{E_1\to\infty}{\approx} \Phi(0,0)\int_0^1\int_0^1 dx\,dy\,\frac{E_1}{\left(1+\frac{2M_N E_1 xy}{M_W^2}\right)^2}$$

$$= \Phi(0,0)\,\frac{M_W^2}{2M_N}\,\ell n\left(1+\frac{2M_N E_1}{M_W^2}\right)$$

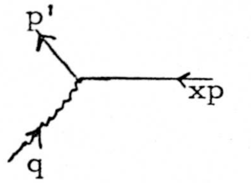
linear rise turns over into a logarithmic rise

5) QUARK PARTON MODEL[12]

A linearly rising cross section is suggestive of the asymptotic behavior of neutrino scattering from a free nucleon. This is the motivation of the <u>quark parton model</u> - the nucleon is regarded as an assemblage of almost free <u>partons</u> (and antipartons) of light mass, which carry quark (antiquark) quantum numbers. When interacting with an energetic neutrino (or electron) the partons scatter incoherently, so the total scattering cross section is a sum on cross sections for the individual partons. The picture is supposed to apply in frames in which the target nucleon has very large momentum $\vec{p}$, so that the target four-momentum p can be regarded as essentially lightlike, $p^2 \approx 0$ (i.e., we are approaching the infinite-momentum frame)

Have quarks	p	n	λ	antiquarks	$\bar{p}$	$\bar{n}$	$\bar{\lambda}$
Q	$\frac{2}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$		$-\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{3}$
B	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$		$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$
S	0	0	-1		0	0	1

Each parton of type i is assumed to have a density distribution $u_i(x)$ for carrying fraction x of the total proton four-momentum p, $0 \leq x \leq 1$. Now consider scattering of an individual parton



Since partons are quasi-free, the final parton must be on the mass shell for the process to be kinematically allowed, i.e. we must have

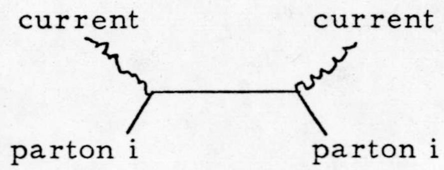
$$0 \approx m^2_{\text{parton}} = p'^2 = (q+xp)^2 = q^2 + 2x\, q\cdot p + x^2 \underbrace{p^2}_{\approx 0}$$

$$\Rightarrow x = -\frac{q^2}{2q\cdot p}:$$ just the scaling variable introduced before. Get scaling, of course, from approximation of neglecting masses.

<u>So for a given q^2 and ν, deep inelastic lepton scattering "sees" only that part of the parton distribution with (longitudinal) momentum fraction</u> $x = -\frac{q^2}{2q\cdot p}$. Get the total deep inelastic structure function by summing over contributions from the different types of partons,

$$H_{\alpha\beta} = \sum_{i=1}^{6} u_i(x)\, h^i_{\alpha\beta}$$

↑ Structure function calculated in Born approx:



Now in terms of basic quark couplings, J^λ_h is $\cos\theta_C \bar{p}\gamma^\lambda(1-\gamma_5)n + \ldots$, i.e.,

it has pure V-A character. For $h_{\alpha\beta}$ we find by a simple calculation

$$h_{\alpha\beta} = -g_{\alpha\beta} + 2x\frac{p_\alpha p_\beta}{p\cdot q} \underbrace{\pm}_{\substack{+\ \text{parton} \\ -\ \text{antiparton}}} i\,\frac{\varepsilon_{\alpha\beta\sigma\lambda}p^\sigma q^\lambda}{p\cdot q} + \ldots$$

$$= -g_{\alpha\beta}w_1 + \frac{p_\alpha p_\beta}{M_N^2}w_2 - i\,\frac{\varepsilon_{\alpha\beta\sigma\lambda}p^\sigma q^\lambda}{2M_N^2}w_3 + \underbrace{\ldots}_{w_4 \text{ to } w_6 \text{ terms}}$$

Hence we identify

$$f_1 = w_1 = 1$$

$$f_2 = \frac{p\cdot q}{M_N^2}\, w_2 = 2x$$

$$f_3 = \frac{p\cdot q}{M_N^2}\, w_3 = \mp 2$$

$f_2 = 2xf_1 \Rightarrow F_2 = 2x\, F_1$ when we sum over partons: Callan-Gross relation

$f_3 = -2f_1$ partons

$ = 2f_1$ antipartons

Experimentally see $F_3(x) \approx -2F_1(x) \Rightarrow$ antiparton content of nucleon is small:* $u_{\bar{p}} \approx 0$, $u_{\bar{n}} \approx 0$, $u_{\bar{\lambda}} \approx 0$

Because the basic parton couplings have pure V-A form, the VV and AA contributions to $F_2(x)$ are equal in the quark parton model:

$$F_2^{VV}(x) = F_2^{AA}(x)\,.$$

Evaluating the sums over partons (and keeping antipartons in) one gets linear relations for the structure functions

---

* These relations cannot hold near x=0, where Pomeron dominance tells us that the antiparton and parton content of the nucleon become equal. See note on p. 14.

$$F_j^R(x) = \sum_{i=1}^{6} C_{ji}^R\, u_i(x) f_j, \quad C_{ji}^R \text{ constants determined by the quark parton quantum numbers}$$

$j = 1, 2, 3$ (3 structure functions)

$R = \nu p \to, \bar{\nu} p \to, \nu n \to, \bar{\nu} n \to, ep \to, en \to$ (6 reactions of interest)

From manipulating these linear relations one gets:[19]

(i) Equalities - in certain cases one can take linear combinations which eliminate the u's altogether, e.g.

$$12(F_1^{ep} - F_1^{en}) = F_3^{\nu p} - F_3^{\nu n} \qquad \text{(not tested)}$$

(ii) Sum rules - Integrals over appropriate combinations of the u_i must give the target quantum numbers:

$$S = \int_0^1 dx\, [u_\lambda(x) - u_{\bar{\lambda}}(x)]$$

$$I_3 = \int_0^1 dx\, [\tfrac{1}{2}(u_p(x) - u_{\bar{p}}(x)) - \tfrac{1}{2}(u_n(x) - u_{\bar{n}}(x))]$$

$$B = \int_0^1 dx\, \tfrac{1}{3}[u_p(x) + u_n(x) + u_\lambda(x) - u_{\bar{p}}(x) - u_{\bar{n}}(x) - u_{\bar{\lambda}}(x)]$$

From these we get the current algebra sum rule given previously, and in addition the Gross-Llewellyn-Smith[20] sum rule

$$-\int_1^\infty \frac{d\omega}{\omega^2}(F_3^{\bar{\nu}} + F_3^{\nu}) = \langle 4B + Y(2 - 3\sin^2\theta_C) + 2I_3 \sin^2\theta_C \rangle_N$$

(Current algebra sum rule sometimes called the I_3 sum rule, Gross-Llewellyn-Smith relation the B or Y sum rule since this is what they involve when $\sin^2\theta_C = 0$.)

(iii) Inequalities. The u_i's are all densities and therefore are positive semidefinite, $u_i \geq 0$. This gives many inequalities on the weak and electroproduction structure functions. Some of the most important are[12, 19]

(a) $$F_2^{ep} + F_2^{en} - \frac{5}{18}(F_2^{\nu p} + F_2^{\nu n})_{\theta_C = 0} = \text{positive}. \; [u_{\bar{\lambda}}^{(p+n)} + u_{\lambda}^{(p+n)}] \geq 0$$

Experimentally can extract $\int_0^1 dx(F_2^{\nu p} + F_2^{\nu n})$ directly from neutrino total cross section data on an average nucleon target. Find that[16]

$$\int_0^1 dx(F_2^{ep} + F_2^{en}) \approx \frac{5}{18} \int_0^1 dx\, (F_2^{\nu p} + F_2^{\nu n})$$

$\Rightarrow u_\lambda \approx u_{\bar{\lambda}} \approx 0$ i.e. strange quark densities in nucleon are small

$\Rightarrow F_2^{eN}(x) \approx \frac{5}{18} F_2^{\nu N}(x)$ consistent with Caltech result, as mentioned above

$N = \frac{1}{2}(n+p)$

(b)[21] $\frac{1}{4} \leq r_1(x) \leq 4$ with $r_1(x) = \dfrac{F_2^{en}(x)}{F_2^{ep}(x)}$

$$0 \leq r_2(x) \leq \frac{8}{5} \qquad \frac{r_1(x) - 1/4}{1 - r_1(x)} \qquad \frac{1}{4} \leq r_1 \leq \frac{2}{3}$$

$$\leq 2 \qquad \frac{2}{3} \leq r_1 \leq 1$$

with $r_2(x) = \dfrac{F_2^{\nu p}(x)}{F_2^{\nu n}(x)}$

This latter pair of inequalities tells us that if $r_1(x) \to \frac{1}{4}$, $r_2(x)$ must <u>vanish</u>! Experimentally, one finds that for $x \to 1$, r_1 gets very close to $1/4$. Hence for small and moderate ω, $F_2^{\nu p}(x)$ becomes negligible relative to $F_2^{\nu n}(x)$. Now setting $\theta_C = 0$ and using charge symmetry, the current algebra sum rule becomes

$$\int_1^\infty \frac{d\omega}{\omega}(F_2^{\bar{\nu} p} - F_2^{\nu p}) = \int_1^\infty \frac{d\omega}{\omega}(F_2^{\nu n} - F_2^{\nu p}) = 2,$$

and evidently $F_2^{\nu n} - F_2^{\nu p} > 0$ is what is needed to make sum rule work! Estimates based on quark-parton models for the structure functions plus Regge asymptotics, and preliminary experimental evidence, suggest that very large ω is needed to actually saturate the sum rule - perhaps ω as large as 400 for 90% saturation.[22]

Many more detailed inequalities for the structure functions and their moments can be found in papers of Nachtmann.[21]

Light-cone algebra[23]

The hadronic tensor $H_{\alpha\beta}$ is the absorptive part of a forward current-hadron scattering amplitude, and therefore can be written as the Fourier transform of the commutator of the weak current with its adjoint,

$$H_{\alpha\beta} = \int d^4x\, e^{iq\cdot x} \langle p | [J_{h\alpha}(\tfrac{x}{2}),\ J^{\dagger}_{h\beta}(-\tfrac{x}{2})] | p \rangle$$

An analysis of the Bjorken limit of $H_{\alpha\beta}$ ($|q^2|$, $q\cdot p \rightarrow \infty$ with ω fixed) shows that the dominant contribution comes from the light-cone region $x^2 \approx 0$ (but $x \neq 0$!) of the integrand; hence the statement that "the scaling limit studies the light-cone".

Light-cone algebra assumes that the leading light-cone singularity structure of $[J, J^{\dagger}]$ is the same as in a free quark field theory, where it is represented as a sum of bilocal operators of the form

$$\sum \bar{\psi}(\tfrac{x}{2})\ \Gamma\ \ \lambda\ \ \psi(-\tfrac{x}{2})$$

γ-matrices λ-matrix (internal symmetry matrix)

The linear relations thus obtained (together with the positivity of absorptive parts) give exactly those constraints of the quark parton model which follow for general $u_i(x)$. So the free light-cone algebra gives an equivalent, field-theoretic way of deriving the parton model predictions.

What happens in an interacting field theory ? This bring us to our next topic:

6) SCALING BREAKDOWN

The possibility of scaling breakdown has been brought to the fore by the SPEAR experiment, which shows that (confirming CEA)

$$\sigma_{e^+e^- \to \text{hadron}}(s) \sim \text{CONST} \quad \text{to } s = 25\ (\text{GeV})^2$$

whereas the quark parton model predicts

$$\sigma_{e^+e^- \to \text{hadron}}(s) \sim \text{CONST}/s\ .$$

SPEAR II, which will run about a year from now and will extend the measurements to $s = 81\ (\text{GeV})^2$, should indicate whether the constant behavior continues, or is just a pre-asymptotic effect. If effect persists in SPII experiment, all versions of the parton model are in serious trouble. Chanowitz and Drell[24] have speculated that scaling breakdown occurs on the basis of three pieces of evidence:

(i) The SPEAR (and similar, earlier CEA) results

(ii) Deviations of the nucleon electromagnetic form factor from a pure dipole form, which indicate a mass scale $\sim 10\ \text{GeV}/c^2$.

(iii) Systematic trends in the SLAC data, which can be made to scale by use of the Bloom-Gilman variable $[\,\omega' = \omega + M_N^2/|q^2|\,]$ but can also be interpreted as indicating a scaling breakdown on a mass scale of $\sim 10\ \text{GeV}/c^2$.

They suggest that there will be scaling breakdown characterized by a form factor

$$\frac{\nu W_2(\nu, q^2)}{M_N^2} = G_2(\omega, |q^2|/M_N^2) \approx F_2(x)\left[1 - \frac{2(-q^2)}{\Lambda^2}\right]$$

$$\Lambda \sim 10\ \text{GeV}/c,$$

and interpret it as an indication of parton structure effects which are becoming visible.

While these speculations give a reasonable estimate of the magnitude of a possible scaling breakdown, the pure form factor structure is probably too naive. A more realistic form for scaling breakdown is obtained by returning to the light-cone analysis of $H_{\alpha\beta}$. We consider the product of currents appearing in the forward Compton amplitude of which $H_{\alpha\beta}$ is the absorptive part, and write its Wilson operator product expansion[25]

$$J_{h\alpha}\left(\frac{x}{2}\right) J^{\dagger}_{h\beta}\left(-\frac{x}{2}\right)$$

$$= \sum_{n=0}^{\infty} C^{(n)}(x^2)\, \underbrace{O_{\alpha\beta\mu_1\ldots\mu_n}(0)\, x^{\mu_1}\ldots x^{\mu_n}}$$

term which contributes to W_2 structure function

+ terms which contribute to W_1, W_3

+ terms of higher twist [subdominant by full powers of $\frac{1}{|q^2|}$ in the large $|q^2|$, ν limit]
(twist > 2)

$O_{\alpha\beta\mu_1\ldots\mu_n}(0)$ is a local operator with

- spin = n+2 (traceless and symmetric)
- canonical dimension = n+4 (powers of [mass])
- twist = dimension-spin = 2

The $C^{(n)}$ are c-number functions of their argument. Taking hadronic matrix element of O and spin averaging one finds

$$\langle p|O_{\alpha\beta\mu_1\ldots\mu_n}|p\rangle_{\text{spin av.}} = \text{CONST} \times p_\alpha p_\beta p_{\mu_1}\cdots p_{\mu_n},$$

verifying that it gives a contribution to W_2 (the coefficient of $p_\alpha p_\beta$ in $H_{\alpha\beta}$). Note that the p-dependence of the nth term of $\langle p|JJ^{\dagger}|p\rangle$ is completely explicit: it contains exactly n+2 factors p. Comparing with the dispersion relation for the $p_\alpha p_\beta$ part of the forward current-hadron amplitude,

$$p_\alpha p_\beta \int d\nu' \frac{W_2(\nu', q^2)}{\nu'-\nu} = p_\alpha p_\beta \sum_{n=0}^{\infty} \underbrace{\nu^n}_{\substack{(p\cdot q)^n \\ \text{exactly } n \text{ factors } p}} \int \underbrace{\frac{d\nu'}{(\nu')^{n+2}}}_{\frac{d\nu'}{(\nu')^{n+2}} \propto \frac{d\omega'}{(\omega')^{n+2}} \propto dx'(x')^n} [\nu' W_2(\nu', q^2)]$$

So by equating powers of p, we find that the nth moment of νW_2 with respect to x is uniquely related to the Fourier transform of the nth term (spin n+2) in the operator product expansion. Keeping track of explicit powers of q^2 we get[26]

$$\int_0^1 dx\, x^n \left[\frac{\nu W_2(\nu, q^2)}{M_N^2}\right] = \tilde{C}^{(n)}(q^2) \times \text{CONST}$$

$$\tilde{C}^{(n)}(q^2) = (q^2)^{n+1}\left(\frac{\partial}{\partial q^2}\right)^n \int d^4x\, e^{iq\cdot x} C^{(n)}(x^2) = \text{Fourier transform of operator product expansion coefficient}$$

Application of this apparatus to discuss scaling (and its breakdown) in field theory:

In free quark model: $O_{\nu_1 \ldots \nu_{n+2}}(x) = \underset{\text{spin-index}}{\text{symmetrized}} [\bar{\psi}(x)\gamma_{\nu_1} \overleftrightarrow{\partial}_{\nu_2} \ldots \overleftrightarrow{\partial}_{\nu_{n+2}} \lambda(1\pm\gamma_5)\psi(x)]$

$\tilde{C}^{(n)}(q^2) = \text{CONST independent of } q^2$

So all moments of νW_2 scale $\Rightarrow$ νW_2 scales

In interacting model: Have O's involving gluon as well as Fermion fields.

For set of $O^{(n)}$'s of twist 2 and common spin n+2, we must do a finite matrix diagonalization to get a basis $O^{(n)\,i}$ whose coefficients $C^{(n)\,i}$ have independent large-q^2 behavior. Renormalization group arguments then $\Rightarrow$

$$\tilde{C}^{(n)\,i}(q^2) \underset{-q^2 \to \infty}{\sim} (-q^2)^{-\frac{1}{2}\gamma_{(n)i}(g^*)}$$

where $\begin{cases} \gamma_{(n)i} \text{ are power series in } g^*;\ \gamma_{(n)i} = 0 \text{ at } g^* = 0 \\ g^* = \text{coupling constant fixed point of theory} \end{cases}$

[root of the Gell-Mann - Low equation or of the Callan-Symanzik function β, i. e. $\beta(g^*) = 0$] which governs asymptotic behavior .

The $\gamma_{(n)i}$ is the <u>anomalous dimension</u> of the operator $O^{(n)i}$. Positivity of $\nu W_2 \Rightarrow$ (for i=1 or for tower of smallest γ' s if i > 1)

$\gamma_{(n)}$ increasing monotonically with n

$\gamma_{(n)}$ convex downward $\Rightarrow$ <u>if any two γ_n are zero, all are zero</u>

Now consider the energy-momentum tensor $\theta_{\mu\nu}$: $\begin{matrix}\text{dimension } 4 \\ \text{spin } 2\end{matrix} \Rightarrow$ twist 2 . From exact conservation of $\theta_{\mu\nu}$, can show that it has anomalous dimension <u>zero</u>.

Moment $x^n \longleftrightarrow$ spin n+2

$\Rightarrow \qquad x^0 \longleftrightarrow$ spin 2

$\Rightarrow \qquad \int_0^1 dx \frac{\nu W_2}{M_N^2}$ = CONST if $\theta_{\mu\nu}$ is <u>only</u> dimension 4, spin 2 operator (as in φ^4 theory);

= CONST + CONST' $(-q^2)^{-\frac{1}{2}\gamma_{(2)2}}$ if there are <u>two</u> dimension 4, spin 2 operators (as in vector gluon theory);

etc.

Experimental implication: area $\int_0^1 dx \frac{\nu W_2}{M_N^2}$ has a component which is nonvanishing as $-q^2 \to \infty$ coming from the energy-momentum tensor, in all field theory models. Since the energy momentum tensor is an isotopic singlet this piece will contribute equally to νp, νn and hence (by charge symmetry, when $\theta_C = 0$) equally to νp, $\bar{\nu} p$.

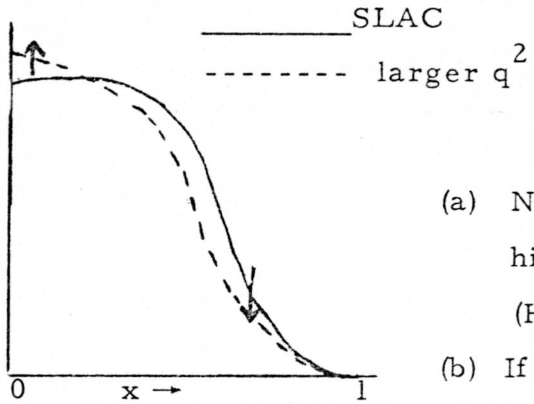
Now have two cases:

(A) $g^* \neq 0$ (non-asymptotically free theories--all field theory models for the strong interactions except a pure non-Abelian gauge theory based on a semisimple Lie group)

Then in general $\gamma_{(n)}(g^*) \neq 0$; moments show $(q^2)^{-\frac{1}{2}\gamma_{(n)}}$ deviations from scaling.

Behavior of[26, 27] $\dfrac{\nu W_2(\nu, q^2)}{M_N^2}$:

$\dfrac{\nu W_2(\nu, q^2)}{M_N^2}$



(a) Near $x \approx 1$ decreases to make higher moments decrease with q^2 (High n moments "see" $x \approx 1$ region)

(b) If $\gamma_{(2)\,i}$ all small, then area $\approx$ CONST

(c) Near $x \approx 0$, must increase to keep area approximately constant [also Regge argument[27] for rise near $x \approx 0$]

(B) $g^* = 0$ (asymptotically free theories - field theory models for the strong interactions based on a semisimple non-Abelian Lie group)

$\gamma_{(n)\,i} \equiv 0$

However, because the "effective coupling" $g(q^2)$ turns off only logarithmically in the asymptotic region, $g \sim \dfrac{\text{CONST}}{\ell n(-q^2)}$, one does not get exact Bjorken scaling, but instead there are logarithmic corrections

$$\tilde{C}^{n(i)}(q^2) \underset{-q^2 \to \infty}{\sim} (\ell n\text{-}q^2)^{-\frac{1}{2}\alpha_{(n)i}}$$

$\alpha_{(n)i}$ are numbers, computable in low order perturbation theory, which depend on structure of Lie group .

Comments on the $g^* = 0$ case:

(i) Moments now $(\ell n)^{\text{power}}$ behaved.

(ii) Qualitative picture for $\nu W_2 / M_N^2$ as before.*

(iii) Since all $\alpha_{(n)i}$ are known (for a given model), one can give an extrapolation procedure to go from given ω, q^2 to same ω, larger q^2[28].

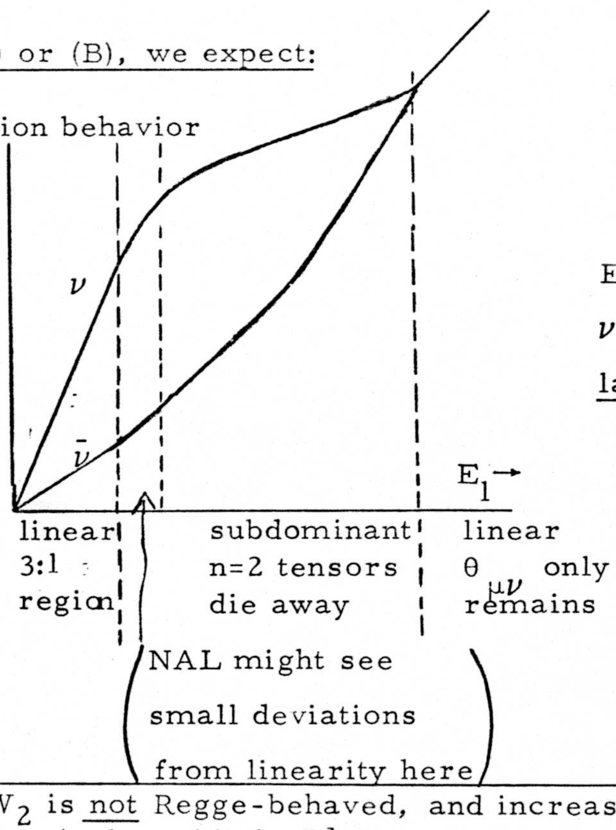
(iv) Asymptotic freedom cannot explain precocious onset of scaling.

(v) Asymptotic freedom predicts $\sigma_{e^+e^- \to \text{hadron}}(s) \sim \frac{\text{CONST}}{s}\left(1 + \frac{c}{\ell n\, s}\right)$ $c > 0$

<u>If scaling breaks down according to either mechanism (A) or (B), we expect:</u>

(i) Cross section behavior

Total cross section σ ↑



Energy scale for $\nu = \bar{\nu}$ is <u>very, very large</u> !

Note: The conventional wisdom outlined above says that ν must drop below its low energy straight line to meet $\bar{\nu}$. However, a new result of Treiman, Wilczek and Zee (to be published) shows that in any theory containing vectors (Abelian or non-Abelian), ν can rise <u>above</u> the low-energy line and still meet $\bar{\nu}$, which rises faster.

*Except that νW_2 is <u>not</u> Regge-behaved, and increases to infinity as $x \to 0$. [Treiman, Wilczek and Zee (to be published)] .

(ii) Current algebra sum rule still valid (but q^2-independence of right-hand side is not automatic in the region of scaling breakdown.)

(iii) Gross-Llewellyn-Smith sum rule <u>fails</u> if $g^* \neq 0$; <u>holds</u> in asymptotically free theories but is approached <u>logarithmically</u>

(i.e. corrections vanish as $\frac{1}{\ell n\text{-}q^2}$.)

<u>If exact scaling remains valid, all known field theory models of the strong interactions are in trouble!</u>

7) PROBLEMS WITH HIGH ENERGY AND HIGHER ORDER WEAK INTERACTIONS - MOTIVATIONS FOR RENORMALIZABLE THEORIES

(A) Unitarity troubles in traditional weak interaction theory[12]

(i) Local current-current theory: Consider $\nu_e e$ scattering

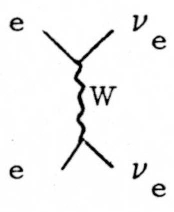
$$\sigma = \frac{G^2}{\pi} 2m_e E_1 = \frac{G^2}{\pi} W^2 \qquad W = \text{center of mass energy}$$

familiar point particle
linearly rising cross section

But amplitude is pure S-wave $\Rightarrow \sigma \leq \frac{8\pi}{W^2}$ by unitarity

So for $W \geq 2\left(\frac{\pi}{\sqrt{2}G}\right)^{1/2} = 900$ GeV, the local current-current theory violates unitarity

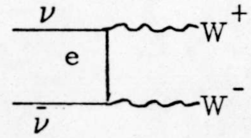
(ii) Naive intermediate boson theory:



S-wave amplitude is $\frac{GM_W^2}{W} \ell n \left(\frac{W^2}{M_W^2}\right)$ - unitarity violated only at astronomical energy where

$$G\, M_W^2 \ell n\left(\frac{W^2}{M_W^2}\right) \sim 1$$

But consider $\nu + \bar{\nu} \to W^+ + W^-$



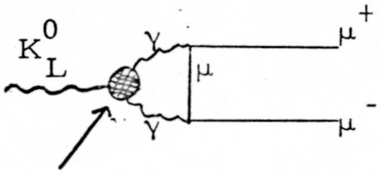
Here get unitarity breakdown at $W \geq \left(\frac{24\pi}{G}\right)^{1/2} \sim 2700$ GeV.

Same breakdown scale as for local current-current theory

(B) Smallness of $\Delta S \neq 0$ neutral hadronic transitions[12]

Suppose we use the local Fermi theory to calculate higher order weak interaction effects. Consider (mainly for pedagogical purposes) $K_L^0 \to \mu^+\mu^-$

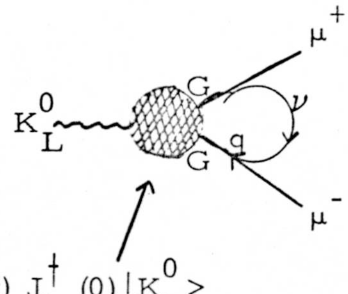
From



order $G\alpha$

get a unitarity lower bound $\dfrac{\Gamma(K_L^0 \to \mu^+\mu^-)}{\Gamma(K_L^0 \to \text{all})} \sim 6.10^{-9}$

But there is also an order G^2 process which contributes:



$$\underbrace{\int d^4x\, e^{-iq\cdot x} < 0 \,|\, J_h(x)\, J_h^\dagger(0) \,|\, K_L^0 >}$$

Dominant piece as $q \to \infty$ can be estimated from current algebra using the Bjorken-Johnson-Low limit.

Find $\dfrac{\Gamma(K_L^0 \to \mu^+\mu^- \ \ \text{order } G^2)}{\Gamma(K_L^0 \to \text{all})} \sim 2.5 \left(\dfrac{G\Lambda^2}{2\pi^2}\right)^2$ Λ = cutoff

$\sim$ unitarity bound $\Rightarrow \Lambda \lesssim 10$ GeV/c^2

So modifications must appear to current-current theory at a relatively low mass!

Other $\Delta S \neq 0$ neutral hadronic processes give similar estimates .

Discussion:

One natural way to deal with these problems is to construct a renormalizable field theory of weak interactions. Such a theory will be

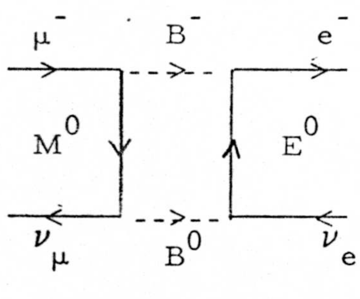
(i) unitarity - solves unitarity difficulties

(ii) finite and calculable - no cutoffs appear in evaluating higher order processes. However, to keep $\Delta S \neq 0$ neutral hadronic transitions as small as they are experimentally, we will be forced to introduce a new hadronic quantum number "charm" and new "charmed" hadrons with masses $\lesssim 10\ \mathrm{GeV}/c^2$.

Two types of renormalizable field theories of the weak interactions:

(C) Theories without fundamental vectors. For example, the models of Kummer and Segré[29], elaborated on by Shabalin[30] and Christ.[30] These theories treat the observed weak interactions as fourth order effects mediated by spin-0 boson exchange:

A 1-loop amplitude →



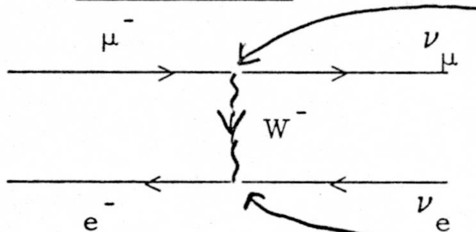
M^0, E^0 are heavy neutral leptons with μ, e leptonic quantum numbers; $B^{\pm,0}$ are heavy spin-0 bosons

This theory is renormalizable, and at energies much lower than M_B it simulates the usual V-A effective coupling. These theories fell out of favor after Christ showed that imposing all known physical conditions (smallness or absence of neutral currents, $\Delta S \neq 0$ hadronic transitions strongly suppressed, etc.) required introducing many new particles.

(D) Theories with fundamental vectors--ie. --intermediate vector boson theories. Of great current interest is the Weinberg-Salam[32] class of intermediate vector boson theories--the so called gauge theories of the weak and electromagnetic interactions. Characteristics of these models:

(i) They unify weak and electromagnetic interactions. The fundamental weak vector boson coupling is of order e = electric charge, and the basic weak process is the second order tree (no loop) amplitude

coupling $g \sim e$



propagator $\dfrac{1}{M_W^2 - q^2} \sim \dfrac{1}{M_W^2}$

coupling $g \sim e$

$$G \sim \frac{g^2}{M_W^2} \sim \frac{e^2}{M_W^2} \Rightarrow M_W \sim \sqrt{\frac{e^2}{G}}$$ typically in range 40-100 GeV/c^2

(ii) They are based on Lagrangians with non-Abelian (and possibly additional Abelian) gauge symmetry groups. Reason the gauge symmetry is needed: the propagator for a massive intermediate vector boson is

$$\frac{-g_{\mu\nu} + \dfrac{q_\mu q_\nu}{M_W^2}}{M_W^2 - q^2}$$

$g_{\mu\nu}$ term $\sim \dfrac{1}{q^2}$ as $q \to \infty$ renormalizable

$\dfrac{1}{M_W^2 - q^2} \dfrac{q_\mu q_\nu}{M_W^2}$ term ~ 1 as $q \to \infty$ spoils renormalizability

If propagator only couples to a conserved current, as is true in a gauge-invariant theory, we have $q_\nu J^\nu = 0$ and the offending term drops out. This is the argument in the Abelian case; in the non-Abelian case the Ward identities (current conservation relations) are more complicated, but they still guarantee renormalizability. Unfortunately, the Ward identities are <u>exact</u> only when all particles are <u>massless</u>. When masses are put in the Lagrangian in the conventional way, the Ward identities are broken and renormalizability is destroyed.

(iii) The Weinberg-Salam theories solved the problem of getting renormalizability in a realistic theory with masses by generating the masses by <u>spontaneous symmetry breaking</u>--essentially a way of gently breaking the gauge symmetry so that masses appear, but the high energy behavior is still that of the gauge-symmetric theory and therefore is renormalizable. Technically, this is accomplished by introducing scalar fields ϕ (<u>Higgs scalars</u>) which couple to the vectors and which develop a non-vanishing vacuum expectation $\langle \phi \rangle_0$ to supply masses. So gauge models have <u>scalar exchange</u> as well as vector exchange graphs; coupling of the Higgs scalars to leptons can be made <u>very weak</u> and therefore is negligible in most applications.

(iv) <u>Tree unitarity</u>

Spontaneously broken gauge theories may be characterized as follows: they are the (essentially) unique vector theories of the weak interactions which are <u>tree unitary</u>[33]

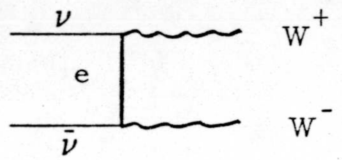
tree graphs: graphs with no loops

T_N = invariant amplitude for N-point tree graph .

Have tree unitarity if and only if T_N is bounded by E^{4-N} when all invariants $p_i \cdot p_j$ approach infinity as a characteristic squared energy E^2. (This tree bound holds for all garden-variety renormalizable field theories.) Significance of tree unitarity: "bad" high energy behavior of one tree graph is cancelled by one or more other tree graphs. Consider example

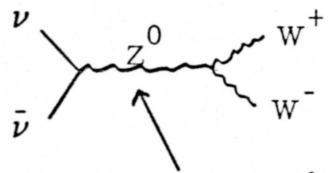
$$\nu + \bar{\nu} \to W^+ W^-$$

Have graph



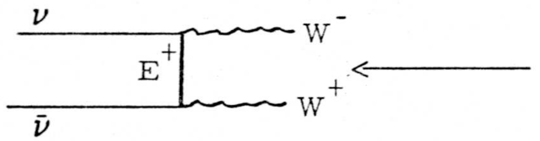
Gauge theories save unitarity by cancelling this with one (or both) of following:

(a)



neutral intermediate boson - neutral current alternative

(b)



heavy lepton with same lepton no. as electron but opposite electric charge--heavy lepton alternative

Because of their tree unitary nature, gauge theories involve either

(a) neutral currents

(b) heavy leptons

So searches for these in neutrino experiments are of great importance. From now on we will concentrate our attention on gauge theories. But first some cautionary remarks:

(i) Gauge theories may, like the scalar exchange theories, need many new particles (see "charm" discussion below).

(ii) Can get effective V-A without fundamental V, A couplings. An experimental case for fundamental vector mediation of the weak interaction must be made.

8) WEINBERG-SALAM MODEL FOR LEPTONS AND HADRONS[34]

Although there are many variants of gauge models of the weak and electromagnetic interactions, we will concentrate for sake of definiteness on the simplest, the original model of Weinberg and Salam. This model is based on SU(2) × U(1) gauge symmetry, which is the smallest gauge group incorporating the known leptons and the known leptonic weak and electromagnetic interactions. To see this we consider first just the electron and its neutrino (will incorporate the muon and its neutrino, and hadrons, later on)

Define a leptonic left-handed doublet by

$$L = \frac{1}{2}(1-\gamma_5)\begin{pmatrix}\nu_e \\ e\end{pmatrix}$$

Weak currents are

$$\left.\begin{array}{l} \bar{e}\gamma_\sigma \frac{1}{2}(1-\gamma_5)\nu_e = \bar{L}\gamma_\sigma \tau_- L \\ \bar{\nu}_e\gamma_\sigma \frac{1}{2}(1-\gamma_5)e = \bar{L}\gamma_\sigma \tau_+ L \end{array}\right\} \begin{array}{ll} \text{associated} & \int d^3x L^\dagger \tau_- L \\ \text{charges:} & \int d^3x L^\dagger \tau_+ L \end{array}$$

Charges form a closed SU(2) algebra if we adjoin the additional charge $\int d^3x\, L^\dagger\, \tau_3 L$ associated with the current

$$\bar{L}\gamma_\sigma \tau_3 L = \bar{\nu}_e\gamma_\sigma \frac{1}{2}(1-\gamma_5)\nu_e - \bar{e}\,\gamma_\sigma \frac{1}{2}(1-\gamma_5)e = \text{neutral} .$$

The presence of this current to complete the weak interaction algebra implies that we will find weak neutral current effects.

To include electromagnetism we define a right-handed singlet

$$R = \frac{1}{2}(1+\gamma_5)\, e$$

Electromagnetic current is

$$\begin{aligned}\bar{e}\gamma_\sigma e &= \bar{e}\gamma_\sigma \frac{1}{2}(1-\gamma_5)e + \bar{e}\gamma_\sigma \frac{1}{2}(1+\gamma_5)e \\ &= \bar{L}\gamma_\sigma \frac{1}{2}(1-\tau_3)L + \bar{R}\gamma_\sigma R \\ &= -\frac{1}{2}\underset{\substack{\uparrow \\ \text{neutral member} \\ \text{of } SU(2)}}{\bar{L}\gamma_\sigma \tau_3 L} + \underbrace{\bar{R}\gamma_\sigma R + \frac{1}{2}\bar{L}\gamma_\sigma L}\end{aligned}$$

associated charge $\int R^\dagger R + \frac{1}{2}L^\dagger L$ <u>commutes</u> with all SU(2) charges; generates independent U(1) group.

So have $\bar{L}\gamma_\sigma \vec{\tau} L$; associated charges generate SU(2)

$\bar{e}\gamma_\sigma e + \frac{1}{2}\bar{L}\gamma_\sigma \tau_3 L = \bar{R}\gamma_\sigma R + \frac{1}{2}\bar{L}\gamma_\sigma L$ = singlet under the SU(2); associated charge generates U(1).

So the minimal leptonic group is SU(2) $\times$ U(1).

SU(2) $\longleftrightarrow$ triplet $\vec{A}_\sigma$ of gauge fields.

U(1) $\longleftrightarrow$ singlet B_σ of gauge fields.

Coupling term in Lagrangian is

$$\mathcal{L}_{int} = \frac{1}{2} g\, \bar{L}\gamma^\sigma \vec{\tau} L \cdot \vec{A}_\sigma + \frac{1}{2} g'(\bar{L}\gamma^\sigma \tau_3 L + 2\bar{e}\gamma^\sigma e) B_\sigma$$

Note that B_σ is <u>not</u> the photon field: it couples to $2\bar{e}\gamma_\sigma e + \bar{L}\gamma_\sigma \tau_3 L$. To identify the photon field A_σ, we must find the linear combination of A^3_σ and B_σ which couples to $\bar{e}\gamma_\sigma e$ alone. The orthogonal linear combination Z_σ will be an intermediate weak boson. Also, we must put in Higgs mechanism to give the weak bosons a large mass (while keeping the photon massless.) We get the following physical fields:

(i) $$W_\sigma = \frac{1}{\sqrt{2}}(A^1_\sigma + i\,A^2_\sigma),\quad W^\dagger_\sigma = \frac{1}{\sqrt{2}}(A^1_\sigma - i\,A^2_\sigma)$$

Two charged vector bosons, mass $M_W^2 = \frac{1}{4}\lambda^2 g^2$,

$$\lambda = \langle \phi \rangle_0 \; .$$

(ii) $$Z_\sigma = \frac{(g\,A^3_\sigma + g'\,B_\sigma)}{\sqrt{g^2 + g'^2}}$$

A neutral vector boson, mass $M_Z^2 = \frac{1}{4}\lambda^2 (g^2 + g'^2)$.

(iii) $$A_\sigma = \frac{(-g'\,A^3_\sigma + g\,B_\sigma)}{\sqrt{g^2 + g'^2}}$$

Photon, mass $M_A^2 = 0$.

Electric charge $e = \frac{gg'}{\sqrt{g^2 + g'^2}}$.

Expressing A^3_σ, B_σ in terms of A_σ, Z_σ we can rewrite the coupling term given above in terms of the physical fields:

$$\mathcal{L}_{int} = W_\sigma(\ldots) + W^\dagger_\sigma(\ldots) + A_\sigma(\ldots) + Z_\sigma(\ldots) \; .$$

From the charged vector boson exchange piece, we identify Fermi constant:

$$\frac{G}{\sqrt{2}} = \frac{g^2}{8M_W^2}$$

Writing for convenience $$\frac{e^2}{g^2} = \sin^2\theta_W$$ Weinberg angle

we get the mass relations

$$M_W = \sqrt{\frac{\sqrt{2}\,e^2}{8G}}\,\frac{1}{\sin\theta_W} = \frac{37\ \mathrm{GeV}/c^2}{\sin\theta_W} \geq 37\ \mathrm{GeV}/c^2$$

$$M_Z = \frac{37\ \mathrm{GeV}/c^2}{\sin\theta_W \cos\theta_W} \geq 74\ \mathrm{GeV}/c^2 \; .$$

From the neutral vector boson exchange piece get neutral current leptonic effects. Leptonic sector predictions will be summarized below.

<u>To incorporate muons:</u> Take $L_{(\mu)} = \frac{1}{2}(1-\gamma_5)\binom{\nu_\mu}{\mu}$ as left-handed doublet.

Add coupling

$$\mathcal{L}_{int} = \tfrac{1}{2} g \, \bar{L}_{(\mu)} \gamma^\sigma \vec{\tau} L_{(\mu)} \cdot \vec{A}_\sigma + \tfrac{1}{2} g' (\bar{L}_{(\mu)} \gamma^\sigma \tau_3 L_{(\mu)} + 2\bar{\mu}\gamma^\sigma \mu) B_\sigma$$

<u>To incorporate hadrons:</u>[35]

(A) Ignore strange particles. Take $L_{(h)} = \frac{1}{2}(1-\gamma_5) N$ as left-handed doublet, with $N = \binom{p}{n}$. So add coupling

$$\mathcal{L}_{int} = \tfrac{1}{2} g \, \bar{L}_{(h)} \gamma^\sigma \vec{\tau} L_{(h)} \cdot \vec{A}_\sigma + \tfrac{1}{2} g' (\bar{L}_{(h)} \gamma^\sigma \tau_3 L_{(h)} - 2\bar{p}\gamma^\sigma p) B_\sigma$$

[Note: by analogy with above

$$\bar{L}_{(h)} \gamma^\sigma \tau_3 L_{(h)} + 2\bar{n}\gamma^\sigma n = \text{singlet}$$

$$-2\bar{n}\gamma^\sigma n - 2\bar{p}\gamma^\sigma p = \text{obviously singlet}$$

So $\bar{L}_{(h)} \gamma^\sigma \tau_3 L_{(h)} - 2\bar{p}\gamma^\sigma p$ = singlet; this form couples photon to p rather than n, as required]

The charged boson thus couples to the current

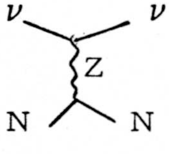
$$\bar{N}\gamma^\sigma \tfrac{1}{2}(\tau_1 + i\tau_2)(1-\gamma_5) N = V^\sigma_{1+i2} - A^\sigma_{1+i2} \equiv \mathcal{J}^\sigma_W$$

as expected. Expressing A^3_σ and B_σ in terms of A_σ and Z_σ we find that the neutral boson Z_σ couples to the <u>hadronic neutral current</u>

$$\bar{N}\gamma^\sigma \tfrac{1}{2}\tau_3 (1-\gamma_5) N - 2\sin^2\theta_W \bar{N}\gamma^\sigma \tfrac{1}{2}(1+\tau_3) N$$

$$= V^\sigma_3 - A^\sigma_3 - 2\sin^2\theta_W J^\sigma_{em} \equiv \mathcal{J}^\sigma_Z$$

Working out the effective coupling coming from the tree graph



one gets at low energies the effective neutral current Lagrangian

$$\mathcal{L}_{eff} = \frac{G}{\sqrt{2}} \bar{\nu}\gamma_\sigma (1-\gamma_5) \nu \mathcal{J}^\sigma_Z$$

(B) With strange particles. The analog of the usual Cabibbo trick would be to take

$$L_{(h)} = \tfrac{1}{2}(1-\gamma_5) N, \text{ with } N = \begin{pmatrix} p \\ n \cos\theta_C + \lambda \sin\theta_C \end{pmatrix}$$

as the left-handed doublet, and to proceed as before. We would get

$$\mathcal{J}^\sigma_W = \bar{N}\gamma^\sigma \tfrac{1}{2}(\tau_1 + i\tau_2)(1-\gamma_5) N$$

$$= \cos\theta_C \underbrace{\bar{p}\gamma^\sigma (1-\gamma_5) n}_{\Delta S = 0 \text{ charged current}} + \sin\theta_C \underbrace{\bar{p}\gamma^\sigma (1-\gamma_5) \lambda}_{\Delta S \neq 0 \text{ charged current}} \qquad \text{which is OK}$$

But the neutral current is now

$$\mathcal{J}^\sigma_Z = \underbrace{\bar{N}\gamma^\sigma \tfrac{1}{2}\tau_3 (1-\gamma_5) N}_{\tfrac{1}{2}\bar{p}\gamma^\sigma(1-\gamma_5)p - \tfrac{1}{2}(\bar{n}\cos\theta_C + \bar{\lambda}\sin\theta_C)\gamma^\sigma(1-\gamma_5)(n\cos\theta_C + \lambda\sin\theta_C)} - 2\sin^2\theta_W \overbrace{\bar{N}\gamma^\sigma \tfrac{1}{2}(1+\tau_3) N}^{J^\sigma_{em} \text{ as before: this term OK}}$$

$$\tfrac{1}{2}\bar{p}\gamma^\sigma (1-\gamma_5) p - \tfrac{1}{2}\underbrace{(\bar{n}\cos\theta_C + \bar{\lambda}\sin\theta_C)\gamma^\sigma (1-\gamma_5)(n\cos\theta_C + \lambda\sin\theta_C)}_{\text{Contains } \sin\theta_C \cos\theta_C [\bar{n}\gamma^\sigma(1-\gamma_5)\lambda + \bar{\lambda}\gamma^\sigma(1-\gamma_5) n]}$$

which is a neutral $\Delta S \neq 0$ weak current.

So we get neutral, $\Delta S \neq 0$ effects at order G; experimentally, they are much suppressed, appearing only at order G^2 or order $G\alpha$.

Simplest solution to this problem: GIM[36] (Glashow, Maiani, Iliopoulos) mechanism. Introduce a new additively conserved quantum number of the strong interactions called "charm".[37] Assume <u>two</u> fundamental left handed doublets

$$L_{(h)} = \tfrac{1}{2}(1-\gamma_5) N \qquad N = \begin{pmatrix} p \\ n\cos\theta_C + \lambda\sin\theta_C \end{pmatrix}$$

$$L'_{(h)} = \tfrac{1}{2}(1-\gamma_5) N' \qquad N' = \begin{pmatrix} p' \\ -n\sin\theta_C + \lambda\cos\theta_C \end{pmatrix}.$$

Here p' is a "charmed" quark with electric charge +1. The two doublets couple identically to the gauge vector mesons,

$$\mathcal{L}_{int} = \tfrac{1}{2} g (\bar{L}_{(h)} \gamma^\sigma \vec{\tau} L_{(h)} + \bar{L}'_{(h)} \gamma^\sigma \vec{\tau} L'_{(h)}) \cdot \vec{A}_\sigma$$

$$+ \tfrac{1}{2} g' (\bar{L}_{(h)} \gamma^\sigma \tau_3 L_{(h)} + \bar{L}'_{(h)} \gamma^\sigma \tau_3 L'_{(h)} - 2\bar{p}\gamma^\sigma p - 2\bar{p}'\gamma^\sigma p') B_\sigma \, .$$

Now we get

$$\mathcal{J}^\sigma_W = \bar{N} \gamma^\sigma \tfrac{1}{2} (\tau_1 + i\tau_2)(1-\gamma_5) N + \bar{N}' \gamma^\sigma \tfrac{1}{2} (\tau_1 + i\tau_2)(1-\gamma_5) N'$$

$$= \cos\theta_C \, \bar{p} \gamma^\sigma (1-\gamma_5) n + \sin\theta_C \, \bar{p} \gamma^\sigma (1-\gamma_5) \lambda$$

Usual charged current as written in quark model

$$- \sin\theta_C \, \bar{p}' \gamma^\sigma (1-\gamma_5) n + \cos\theta_C \, \bar{p}' \gamma^\sigma (1-\gamma_5) \lambda$$

Charged "charm-changing" current: causes semi-leptonic decay of charmed hadrons into uncharmed hadrons

while the neutral current becomes

$$\mathcal{J}^\sigma_Z = \underbrace{\bar{N} \gamma^\sigma \tfrac{1}{2} \tau_3 (1-\gamma_5) N + \bar{N}' \gamma^\sigma \tfrac{1}{2} \tau_3 (1-\gamma_5) N'}_{} - 2 \sin^2\theta_W J^\sigma_{em}$$

$$\tfrac{1}{2} \bar{p} \gamma^\sigma (1-\gamma_5) p - \tfrac{1}{2} (\bar{n} \cos\theta_C + \bar{\lambda} \sin\theta_C) \gamma^\sigma (1-\gamma_5)(n \cos\theta_C + \lambda \sin\theta_C)$$

$$+ \tfrac{1}{2} \bar{p}' \gamma^\sigma (1-\gamma_5) p' \underbrace{- \tfrac{1}{2} (-\bar{n} \sin\theta_C + \bar{\lambda} \cos\theta_C) \gamma^\sigma (1-\gamma_5)(-n \sin\theta_C + \lambda \cos\theta_C)}_{}$$

$$\tfrac{1}{2} \bar{n} \gamma^\sigma (1-\gamma_5) n + \tfrac{1}{2} \bar{\lambda} \gamma^\sigma (1-\gamma_5) \lambda$$

$\Delta S \neq 0$ pieces cancel by construction!

That is,

$$\mathcal{J}^\sigma_Z = V^\sigma_3 - A^\sigma_3 - 2 \sin^2\theta_W J^\sigma_{em} + \underbrace{\tfrac{1}{2} J^\sigma_C - \tfrac{1}{2} J^\sigma_S}_{\text{pure isoscalar}}$$

with

$$V^\sigma_3 - A^\sigma_3 = \tfrac{1}{2} \bar{p} \gamma^\sigma (1-\gamma_5) p - \tfrac{1}{2} \bar{n} \gamma^\sigma (1-\gamma_5) n$$

$$J^\sigma_C = \bar{p}' \gamma^\sigma (1-\gamma_5) p' = \text{V-A "charm" current}$$

$$J^\sigma_S = \bar{\lambda} \gamma^\sigma (1-\gamma_5) \lambda = \text{V-A strangeness current}$$

So: introducing "charm" eliminates $\Delta S \neq 0$ neutral effects of order G. Must still worry about <u>induced</u> $\Delta S \neq 0$ neutral hadronic transitions due to intermediate boson radiative corrections. Since the fundamental boson couplings are g, $g' \sim e$, radiative corrections can induce effects of order $G\alpha$ (α = fine structure constant.) So must worry about strongly suppressed processes like $K_L^0 \to \mu^+\mu^-$, $K_L - K_S$ mass difference, etc. Gaillard and Lee[38] have analyzed rare K decay modes in great detail in the GIM-modified Weinberg-Salam model. Their conclusions (which, they argue, are valid in many other popular gauge models as well):

(i) $K_L^0 \to \mu^+\mu^-$ suppressed by fortuitous cancellation.

(ii) To explain non-suppression of $K_L^0 \to \gamma\gamma$ along with small $K_L^0 - K_S^0$ mass differences need

$$\frac{m_{p'}}{m_p} >> 1 \qquad \begin{array}{l} m_p = \text{p quark mass} \\ m_{p'} = \text{p' ("charm") quark mass} \end{array}$$

but $\frac{m_{p'}}{M_W} << 1$ --in fact $m_{p'} \lesssim 5\ \text{GeV}/c^2$.

(iii) Phenomenological arguments indicate that average mass of "charmed" pseudoscalar states $< 10\ \text{GeV}/c^2$.

(iv) $K^+ \to \pi^+ e^+ e^-$ should occur with a branching ratio $\sim 10^{-6}$: comparable to the presently available experimental upper bound. <u>Should push on this decay mode</u>.

Conclusion: Hadrons can be successfully incorporated in gauge models, but a new strong interaction quantum number "charm" is probably needed, with charmed states light enough so that they will be produced in the NAL neutrino beam.

9) TESTS OF GAUGE THEORIES IN NEUTRINO REACTIONS

(A) Existence of W-boson

(i) Single most crucial test of gauge theories would be to produce and detect W-bosons. Unfortunately, in most gauge models the W's are very heavy. Have

$$E_1^{thresh} \sim \frac{M_W^2}{2M_N} \sim \frac{700 \text{ GeV}}{\sin^2\theta_W} \text{ for } M_W = (37 \text{ GeV}/c^2)/\sin\theta_W$$

and to have an appreciable cross section one would want

$$E_1 \sim 2\, E_1^{thresh} \sim \frac{1400 \text{ GeV}}{\sin^2\theta_W}$$

So W's will not be seen directly for a long time.

(ii) Alternative way to see W's is through effect of their propagator on semileptonic reactions. If Bjorken scaling were exact the effect of a W, as we have noted, would be to replace

$$\frac{d\sigma}{dx\,dy} = E_1 \Phi(x, y)$$

by

$$\frac{d\sigma}{dx\,dy} = \frac{E_1\Phi(x,y)}{\left(1 - \frac{q^2}{M_W^2}\right)^2} \approx E_1\Phi(x,y)\left(1 + \frac{2q^2}{M_W^2}\right)$$

in the deep inelastic region. Roughly, $\frac{\langle -q^2\rangle \text{ in } (\text{GeV}/c)^2}{E_1 \text{ in GeV}} \simeq .25$, so for $E_1 = 200$ GeV we have $\langle -q^2\rangle = 50\ (\text{GeV}/c)^2$, and $\frac{2\langle -q^2\rangle}{M_W^2} = \frac{2\cdot 50}{(37)^2}\sin^2\theta_W \sim .07\sin^2\theta_W$.
Would need very good statistics and control over systematics to see this. If scaling is not exact, and breaks down on a mass scale

$\Lambda \sim 10\text{-}20$ GeV/c, this method fails.

(iii) Finally, Sehgal[39] (generalizing work of Terazawa[40]) has derived some nice relations satisfied by leptonic cross sections in any intermediate vector boson theory with μ-e symmetry (valid in gauge theories since scalar boson couplings $\propto$ lepton mass and therefore are negligible in lowest order):

$$\frac{1}{3} \leq \frac{\sigma(\bar{\nu}_e e)}{\sigma(\nu_e e)}, \frac{\sigma(\bar{\nu}_\mu e)}{\sigma(\nu_\mu e)} \leq 3$$

$$\sigma(\bar{\nu}_e e) - \sigma(\bar{\nu}_\mu e) = \frac{1}{3}[\sigma(\nu_e e) - \sigma(\nu_\mu e)]$$

$$[\sigma(\nu_e e) - \frac{1}{3}\sigma(\bar{\nu}_e e)]^{\frac{1}{2}} - [\sigma(\nu_\mu e) - \frac{1}{3}\sigma(\bar{\nu}_\mu e)]^{\frac{1}{2}} = \frac{4}{3},$$

with the cross sections in the last relation measured in units of $G^2 m_e E_1/\pi$.

(B) Search for heavy leptons

The Weinberg-Salam model discussed above uses only the presently known leptons. Other gauge models with neutral currents, and all models without neutral currents, have heavy leptons.

Let M^+, M^0 be heavy leptons with the same lepton number as the μ^-. They will be produced in the reactions

$\nu_\mu + N \rightarrow M^+ + \text{hadrons}$

$\quad\hookrightarrow \left.\begin{array}{l} \nu_\mu + \nu_\mu + \mu^+ \\ \nu_\mu + \nu_e + e^+ \end{array}\right\}$ wrong sign lepton: good signature

$\quad\nu_\mu + \text{hadrons}$

$\nu_\mu + N \rightarrow M^0 + \text{hadrons}$ (This is a neutral current reaction)

$\quad\hookrightarrow \left.\begin{array}{l} \nu_\mu + \mu^+ + \mu^- \\ \nu_e + e^+ + \mu^- \end{array}\right\}$ two leptons: good signature

$\quad\mu^- + \text{hadrons}$

Bjorken and Llewellyn-Smith[41] have estimated cross sections. Conclude

(i) NAL should be able to set a mass limit in the 4-10 GeV/c^2 range,

(ii) Branching ratio into leptons $\sim$ 50%.

(iii)

In $\left\{\begin{array}{l} \nu_\mu + N \rightarrow M^+ + \text{hadrons} \rightarrow \mu^+ + (\nu_\mu + \nu_\mu) + \text{hadrons} \\ \nu_\mu + N \rightarrow M^0 + \text{hadrons} \rightarrow \mu^- + \text{hadrons} \end{array}\right.$ one would see apparent violations of scaling and lepton locality, and so could distinguish from the direct

$$\text{reactions} \begin{cases} \bar{\nu}_\mu + N \to \mu^+ + \text{hadrons} \\ \nu_\mu + N \to \mu^- + \text{hadrons} \end{cases} .$$

(C) Search for neutral current. Here, as we have heard, there is accumulating evidence for an effect. First priority is obviously confirmation of existence of neutral currents. Bearing in mind the necessary cautions about the existence of many other models with neutral currents, both gauge and non-gauge, let us systematically discuss neutral current effects within the framework of the Weinberg-Salam model.

(i) Leptonic channel[42]

Have (E_2 = lab energy of final electron; E_1 = incident neutrino energy)

$$\frac{d\sigma}{dE_2} = \frac{G^2 m_e}{\pi} \left[(g_V + g_A)^2 + (g_V - g_A)^2 \left(1 - \frac{E_2}{E_1}\right)^2 + \frac{m_e E_2}{E_1^2} (g_A^2 - g_V^2) \right] .$$

g_V and g_A are given by the following table:

Reaction	Weinberg-Salam		"Standard" V-A Theory	
	g_V	g_A	g_V	g_A
$\nu_e + e^- \to e^- + \nu_e$	$\frac{1}{2} + 2\sin^2\theta_W$	$\frac{1}{2}$	1	1
$\bar{\nu}_e + e^- \to e^- + \bar{\nu}_e$	$\frac{1}{2} + 2\sin^2\theta_W$	$-\frac{1}{2}$	1	-1
$\nu_\mu + e^- \to e^- + \nu_\mu$	$-\frac{1}{2} + 2\sin^2\theta_W$	$-\frac{1}{2}$	0	0
$\bar{\nu}_\mu + e^- \to e^- + \bar{\nu}_\mu$	$-\frac{1}{2} + 2\sin^2\theta_W$	$\frac{1}{2}$	0	0

Announced results:

(a) Write $\sigma(\bar{\nu}_e + e^- \to e^- + \bar{\nu}_e) = C \cdot 10^{-41} \text{cm}^2 \left(\frac{E_1}{\text{GeV}}\right)$

$C = 0.54$ in "standard" V-A theory

$= 0.136 - 2.86$ in Weinberg-Salam model

Gurr, Reines and Sobel[43] Savannah River reactor - find $\sigma \leq 3\sigma_{V-A}$ at 90% confidence level $\Rightarrow \sin^2\theta_W \leq 0.33$ at 90% confidence level

(b) $\left.\begin{array}{l}\sigma(\nu_\mu + e^- \rightarrow e^- + \nu_\mu) \\ \sigma(\bar{\nu}_\mu + e^- \rightarrow e^- + \bar{\nu}_\mu)\end{array}\right\}$ zero in "standard" V-A theory; nonzero in Weinberg-Salam model

CERN	Gargamelle[44]	375,000	ν_μ pictures
		360,000	$\bar{\nu}_\mu$ pictures

	Weinberg-Salam predictions min.	max.	estimated background	observed
ν_μ	0.6	6.0	0.3 ± 0.2	0
$\bar{\nu}_\mu$	0.4	8.0	0.03 ± 0.02	1

$0.1 < \sin^2\theta_W < 0.6$ as 90% confidence limits

(ii) Hadronic channel

(a) Inclusive reactions

Define $R_\nu = \dfrac{\sigma(\nu_\mu + N \rightarrow \nu_\mu + \Gamma)}{\sigma(\nu_\mu + N \rightarrow \mu^- + \Gamma')}$ where we deal with inclusive reactions, so that all allowed hadron final states are included in Γ, Γ'.

Also define $R_{\bar{\nu}} = \dfrac{\sigma(\bar{\nu}_\mu + N \rightarrow \bar{\nu}_\mu + \Gamma)}{\sigma(\bar{\nu}_\mu + N \rightarrow \mu^+ + \Gamma'')}$.

$N = \frac{1}{2}(n+p)$ = average nucleon target

Pais and Treiman[45], Paschos and Wolfenstein[46] derive the following bounds in the Weinberg-Salam model (with GIM extension):

(1) Assuming scaling in deep inelastic electroproduction (but not in weak production) one finds

$$R_\nu \geq \tfrac{1}{2}\{1 - 2\sin^2\theta_W\, t^{\frac{1}{2}}\}^2,$$

$$t = \frac{\frac{G^2}{\pi}\frac{4}{3}M_N E_1 \int_0^1 dx\, F_2^{eN}(x)}{\sigma(\nu_\mu + N \rightarrow \mu^- + \Gamma')}$$

Using $\int_0^1 dx\, F_2^{eN}(x) \approx 0.14$

$$\sigma(\nu_\mu + N \to \mu^- + \Gamma') \approx \frac{G^2}{\pi} M_N E_1 \cdot 0.52$$

one gets $t \approx 0.36.$

Hence for $\sin^2\theta_W \leq 0.33$ one gets $R_\nu \geq 0.18$.

(2) Assuming scaling in weak production as well as in electroproduction, the bound is improved to

$$R_\nu \geq \tfrac{1}{2}\left[\frac{2}{3} + \frac{1}{3}x - (1-x^2)t\right], \qquad x = 1-2\sin^2\theta_W$$

(not to be confused with the scaling variable x used above!)

For $\sin^2\theta_W \leq 0.33$ get now $R_\nu \geq 0.23$.

(3) Taking $t \approx 1/3$ (very close to experiment) this bound becomes

$$R_\nu \geq \frac{1}{6}(1 + x + x^2) \geq 0.24 \text{ for } \sin^2\theta_W \leq 0.33 \ .$$

Similarly, using $t \approx 1/3$ and $\sigma^{\bar\nu}/\sigma^{\nu} \approx 1/3$, we get

$$R_\nu \geq \tfrac{1}{2}(1-x + x^2) \geq 0.39 \text{ for } \sin^2\theta_W \leq 0.33.$$

Announced results:

CERN Gargamelle[47]

$R_\nu = 0.23 \pm 0.04$ Consistent with

$R_{\bar\nu} = 0.43 \pm 0.12$ $\sin^2\theta_W \sim 0.3$ to 0.4 .

NAL[48] $0.63\, R_\nu + 0.37\, R_{\bar\nu} = 0.20 \pm 0.05$.

The corresponding CERN result for this $\mu/\bar\mu$ mix is 0.30 ± 0.05.

So NAL and CERN are roughly consistent, within errors. (CERN is not strictly in the deep inelastic region, and so need not precisely agree with NAL.)

Weinberg-Salam lower bounds, with simplifications of (3) above, for NAL $\mu/\bar\mu$ mix:

$$0.63\, R_\nu + 0.37\, R_{\bar\nu} \geq 0.63\, \frac{1}{6}\,(1+x+x^2) + 0.37\, \tfrac{1}{2}\,(1-x+x^2)$$

$$= 0.29\,(1+x^2) - 0.08x \geq 0.28 \quad . \qquad \left[\begin{array}{c}\text{Minimized by } x=0.14 \\ \text{i.e. } \sin^2\theta_W = 0.43\end{array}\right]$$

Hence the Weinberg-Salam model is being pushed a little, but it is too soon to say anything definitive.

(b) Exclusive reactions

(1) The quasielastic reaction $\nu_\mu + p \to \nu_\mu + p$ is hard to detect experimentally, because the proton tends to recoil with low momentum. In the Weinberg-Salam model, one finds the bounds[35]

$$0.15 \leq \frac{\sigma(\nu_\mu + p \to \nu_\mu + p)}{\sigma(\nu_\mu + n \to \mu^- + p)} \leq 0.25 \quad \text{for} \quad \sin^2\theta_W \leq 0.5 \ .$$

Experiment gives 0.12 ± 0.06 for this ratio.

(2) Weak π production

Consider
$$\hat{R} = \frac{\sigma(\nu_\mu + p \to \nu_\mu + p + \pi^0) + \sigma(\nu_\mu + n \to \nu_\mu + n + \pi^0)}{2\sigma(\nu_\mu + n \to \mu^- + p + \pi^0)} \ .$$

In $\Delta(1236)$ - dominance approximation, one finds[49]

$$\hat{R} \gtrsim 0.4 - 0.5 \quad \text{for} \quad \sin^2\theta_W \leq 0.33 \ .$$

Two corrections to this result are needed[16]

1. $I = \frac{1}{2}$ final states are not negligible --this reduces the theoretical prediction.

2. When experiments are done in nuclear targets (say ${}_6C^{12}$ or ${}_{13}Al^{27}$), charge exchange effects further reduce the theoretical prediction. Charged currents copiously produce $\pi^\pm$; when these charge exchange into π^0, they increase the denominator of $\hat{R}$, and hence <u>reduce</u> the $\hat{R}$ measured on a nucleus.

Theoretical estimates of[50] 1. (via relativistic generalization of static model used to discuss Argonne $\nu + p \rightarrow \mu^- + p + \pi^+$) and of[51] 2. (via detailed model for nuclear charge exchange) gives (for ${}_{13}Al^{27}$; neutrino energy E_1 =1GeV)

	Δ(1236) only	Δ(1236)+I=1/2	Δ(1236)+I=1/2 + Charge exchange corrections for ${}_{13}Al^{27}$	$\sin^2\theta_W$
$\hat{R}$	0.56	0.40	0.23	0.3
	0.46	0.33	0.18	0.4

Uncertainty is perhaps ~30%. [Could test the charge exchange model by measuring $\pi^{\pm 0}$ electroproduction on nuclear targets] .

Announced results:

W. Lee[52] (old Columbia spark chamber experiment on ${}_{13}Al^{27}$).

$\hat{R} < 0.14$ at 90% confidence level (no candidates).

At Argonne will be able to look for π production without having to worry about nuclear charge-exchange corrections. Predictions of production model[50] averaged over the Argonne neutrino spectrum are:

$\frac{\sigma(\nu_\mu + p \rightarrow \nu_\mu + p + \pi^0)}{\sigma(\nu_\mu + p \rightarrow \mu^- + p + \pi^+)}$	$\frac{\sigma(\nu_\mu + p \rightarrow \nu_\mu + n + \pi^+)}{\sigma(\nu_\mu + p \rightarrow \mu^- + p + \pi^+)}$	SUM	$\sin^2\theta_W$
0.12	0.09	0.21	0.3
0.10	0.08	0.18	0.4

Conclusion: There seems to be evidence for neutral currents. All experiments to date are consistent (but in a number of cases just barely so) with Weinberg-Salam phenomenology with $\sin^2\theta_W \sim 0.3$-0.4. To determine the phenomenology will obviously need many experiments in many different channels.

(iii) Low energy nuclear search possibilities

A number of authors have discussed possible nuclear effects arising from the presence of neutral weak currents. We recall again

$$\mathcal{J}_Z^\sigma = V_3^\sigma - A_3^\sigma - 2\sin^2\theta_W \underbrace{J_{em}^\sigma}_{V_3^\sigma + \frac{1}{\sqrt{3}} V_8^\sigma} + \underbrace{\tfrac{1}{2} J_C^\sigma - \tfrac{1}{2} J_S^\sigma}$$

Isoscalar electromagnetic current

(Isoscalar) "charm" and strangeness currents; presumably will have very small nuclear matrix elements at low energy

(a) Nuclear Gamow-Teller transitions

The axial-vector part of $\mathcal{J}_Z^\sigma$ is $-A_3^\sigma$ and is independent of θ_W. Stanford group (Donnelly et. al.[53]) have discussed reactions of the form

$$\bar{\nu}_e + {}^A T \rightarrow \bar{\nu}_e + {}^A T^* \qquad \text{via allowed Gamow-Teller transition,}$$

initiated by reactor $\bar{\nu}_e$. One would detect ${}^A T^*$ by its γ-ray decay. Some cases allow an additional delayed coincidence, from a further decay after γ-emission, which increases the signal to noise ratio at the expense of counting rate. Some typical reactions are:

$${}^7\mathrm{Li}(\tfrac{3}{2}^- \tfrac{1}{2}) \rightarrow {}^7\mathrm{Li}^*(\tfrac{1}{2}^- \tfrac{1}{2}, 0.478 \text{ MeV})$$

$${}^{19}\mathrm{F}(\tfrac{1}{2}^+ \tfrac{1}{2}) \rightarrow {}^{19}\mathrm{F}^*(\tfrac{3}{2}^+ \tfrac{1}{2}, 1.554 \text{ MeV}).$$

Counting rates at Savannah River for reasonable assemblies are ~1/day; in the case of ${}^{19}F$, a decay chain involving two γ's would permit signal/noise ratio ~ 1:1 .

(b) Giant dipole resonance excitation

Here a vector-current,* isovector transition is involved, so the relevant part of the neutral current is $(1-2\sin^2\theta_W)\, V_3^\sigma$. Bilenky and Dadajan[54] estimate the cross section as

*Actually, axial contributions may not be negligible. More detailed calculations are desirable.

$$\sigma^{\nu+\bar{\nu}}_{T\to T^*} \geq (1-2\sin^2\theta_W)^2\sigma_0$$

E_1 (MeV) $\to$	30	50	100	
Ta^{181}	$1.7\cdot 10^{-41}$	$2.3\cdot 10^{-40}$	$5.5\cdot 10^{-39}$	σ_0 in cm^2
V^{51}	$1.4\cdot 10^{-42}$	$2.5\cdot 10^{-41}$	$7.1\cdot 10^{-40}$	

so this might be a suitable experiment for neutrino beans at meson factories. (They do not discuss experimental problems connected with detection of the excited state T*.)

(c) Coherent nuclear scattering

Freedman[55] has pointed out that neutral currents will lead to coherent neutrino-nucleus scattering $\nu T \to \nu T$. At very low energies the matrix element is proportional to $< I_3 - 2\sin^2\theta_W Q >_T$, with I_3 and Q respectively the operators for the 3rd component of isospin and the charge. For higher energies there will also be a momentum-transfer-dependent form factor. For heavy nuclei, coherent processes will show a rate enhancement factor $\gtrsim$ A compared to incoherent neutrino induced processes. Since the coherent cross section is almost energy independent, meson factory energies of order 100 MeV might be more suitable than higher energies; experimental observation would require detection of the recoil nucleus T.

Possible astrophysical implication of this process: In stellar collapse to form a supernova, coherent ν F_e scattering could lead to an <u>enhanced neutrino radiation pressure</u> which could give observed blowing off of the outer layers.

(iv) Neutral current phenomenology

We have discussed neutral current searches within the framework of the Weinberg-Salam phenomenology, but the above experiments are of interest regardless of the underlying theory, and will help to pin down the structure of the neutral current. In a more general vein, Pais and Treiman[56] have examined how one might test for various structural properties of the neutral current in accelerator neutrino experiments. For example, one important question is whether the vector part of $\mathcal{J}^{\sigma}_{Z}$ is conserved. Again, the forward lepton theorem discussed above can be used. We consider

$$\nu + N \rightarrow \nu + \Gamma .$$

Presence of parity violating effects at $q^2 = 0$ (i.e., when the final neutrino emerges precisely in the forward direction) $\Rightarrow$ vector part of $\mathcal{J}^{\sigma}_{Z}$ is not conserved. Presence of parity violating effects for $q^2 \neq 0$, which vanish always when $q^2 \rightarrow 0$, would suggest that the vector part of $\mathcal{J}^{\sigma}_{Z}$ is conserved.
One final comment: Even talking about a neutral "current" reflects a theoretical bias that the effective Fermi interaction involved is V, A and not S, T or P. Since we are dealing with a new phenomenon this assumption will in time have to be subjected to experimental test.[57]

(D) Search for "charm" (any additive quantum number of hadrons beyond I_3 and Y).

We have seen that new "charmed" hadrons are probably needed to incorporate hadrons into gauge models of the weak and electromagnetic interactions, and that the masses of such "charmed" states are likely to be $\lesssim 10\ \text{GeV}/c^2$. So the search for "charm" becomes relevant at NAL energies.

(i) Detection via production and decay[58]

"Charmed" particles with masses $\gtrsim$ a few GeV/c^2 will only go a fraction of a cm. before decaying, even when produced at NAL energies, so will not see tracks. Reasonable to assume about 10-50% decay into leptons, as a first guess.

Produce in "charm" baryon

$$\nu_\mu + N \to \mu^- + B_C + \text{hadrons}$$

"charm" meson

$$\mu^- + M_C + \text{hadrons}$$

Leptonic B_C or M_C decay will then produce a <u>two lepton</u> signature $\mu^- e^+, \mu^- \mu^+$.

If leptonic decays are strongly suppressed, detection via production and decay will be very difficult. In the GIM model, we have heard in Gaillard's talk that the leptonic breaking ratio of charmed particles may well be suppressed down to a level of order 3%.

(ii) Changes in the saturation values of the Adler and Gross-Llewellyn-Smith sum rules.[59]

We recall that the local current algebra sum rule measures $[J_h^0(\vec{x},0), J_h^0(\vec{y},0)^\dagger]$. When additional terms are present in the weak charged current, the value of this commutator is changed, giving

$$\frac{1}{M_N^2}\int_0^\infty d\nu[W_2^{\bar{\nu}} - W_2^{\nu}] = A \neq \langle 4\cos^2\theta_C I_3 + (3Y+2I_3)\sin^2\theta_C\rangle_N$$

A = a number computable from structure of "charmed" part of the weak current. Similarly, the Gross-Llewellyn-Smith sum rule is modified to read

$$\lim_{q^2 \to -\infty} \int_{-q^2/2}^{\infty} d\nu \left(\frac{q^2}{2M_N^2 \nu}\right)\left[W_3^{\bar{\nu}} + W_3^{\nu}\right] = B \neq \langle 4B+Y(2-3 \sin^2\theta_C)+2I_3\sin^2\theta_C \rangle_N$$

B = a second structure-dependent number

Obviously, to see the deviation of the sum rules from their standard values we must integrate the experimental data <u>to</u> ν <u>values well above charm production threshold.</u>

Remark: Standard current algebra low energy theorems are not altered by the presence of "charm".

(iii) Charge symmetry violations[60]

Let us neglect θ_C. When "charmed" particles are not present, the charged weak current is

$$\mathcal{J}_W^\sigma \approx \bar{p}\,\gamma^\sigma (1-\gamma_5) n = V_{1+i2}^\sigma - A_{1+i2}^\sigma,$$

which satisfies the <u>charge symmetry relation</u>

$$e^{-i\pi I_2}\, \mathcal{J}_W^\sigma\, e^{i\pi I_2} = -(J_W^\sigma)^\dagger,$$

with I_2 the second component of the isotopic spin. In other words, $\mathcal{J}_W^\sigma$ and $(\mathcal{J}_W^\sigma)^\dagger$ transform as members of the same I = 1 multiplet. When "charmed" particles are included, $\mathcal{J}_W^\sigma$ is augmented by a piece

$$\Delta\, \mathcal{J}_W^\sigma = \bar{p}'\, \gamma^\sigma (1-\gamma_5)\lambda,$$

which is an isotopic scalar and therefore satisfies

$$e^{-i\pi I_2}\, \Delta \mathcal{J}_W^\sigma\, e^{i\pi I_2} = \Delta \mathcal{J}_W^\sigma \neq -(\Delta \mathcal{J}_W^\sigma)^\dagger .$$

Thus, above "charm" threshold there will be strong charge symmetry violations. In particular, the relations (valid when $\theta_C = 0$ if charge symmetry is respected)

$$W_i^{\nu N} = W_i^{\bar{\nu} N}, \qquad N = \frac{1}{2}(n+p)$$

would be strongly violated. Many tests for charge symmetry violation can be based on this fact.

(iv) Temporary scaling breakdown associated with "charm" threshold.[60]

The appearance of a fundamental new threshold might lead to scaling breakdown in deep inelastic neutrino reactions when this threshold is surpassed. Assuming that scaling is a fundamental asymptotic property, scaling behavior would reappear at energies sufficiently far beyond "charm" threshold. However, one can skeptically ask what is special about the "charm" threshold--why don't similar (unobserved) scaling violations appear as other thresholds, say for antibaryon production are passed?

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