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TABLES OF THE CLEBSCH-GORDAN COEFFICIENTS

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I. INTRODUCTION

This report is essentially Appendices II and III of a thesis entitled "Angular Distributions in the Elastic Scattering of Protons by Light Nuclei" submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy to the Graduate School, California Institute of Technology, in 1949. Because of recent interest in the tables presented here, it was felt that it would be worth while to issue this report at this time.

Although M. E. Rose[1] and his collaborators[2,3] at Oak Ridge National Laboratory have published extensive tables of Clebsch-Gordan and Racah coefficients, those tables suffer from the disadvantage that the coefficients are given in decimal form rather than fractional form. For calculations with large digital computing machines, the decimal form is more convenient, but for elementary calculations and for an understanding of the structure of the coefficients, it is felt that the form given here has at least some merits.

II. THE CLEBSCH–GORDAN SERIES

The Clebsch-Gordan series is the decomposition of a direct product of representations of the rotation group into a sum of representations. Since the $2J + 1$ wave functions of angular momentum J induce a representation of the rotation group, the series allows us to decompose the product of wave function into a sum of wave functions and hence becomes the mathematical basis for the vector model of the atom.[4, 5, 6]

A wave function ψ_j^m of a Hamiltonian which is invariant to rotation of coordinates can be represented in its transformation properties by the monomial

$$\frac{(2j)!(-1)^{j-m}\,\xi^{j+m}\,\eta^{j-m}}{\sqrt{(j+m)!\,(j-m)!}} \quad , \qquad \ldots(1)$$

where (ξ, η) is a spinor. This is just the coefficient of $\dfrac{a^{j-m}\,b^{j+m}}{\sqrt{(j+m)!(j-m)!}}$ in the spinor-invariant

$$(-a\eta + b\xi)^{2j} \quad , \qquad \ldots(2)$$

where (a, b) is a constant spinor. We are interested in evaluating the coefficients in the expansion

$$\psi_l^\mu \psi_s^\nu = \sum_{jm} A_{jm}^{ls\,\mu\nu}\,\psi_j^m \qquad \ldots(3)$$

or, since the transformation can be taken to be real and unitary, in the expansion

$$\psi_j^m = \sum_{\mu\nu} A_{jm}^{ls\,\mu\nu}\,\psi_l^\mu \psi_s^\nu \quad . \qquad \ldots(4)$$

We form an invariant

$$\Phi = (-AY + BX)^{2j} \quad , \qquad \ldots(5)$$

where (A, B) is a constant spinor and (X, Y) an arbitrary spinor. The terms $X^{j+m}Y^{j-m}$ of the expansion of Φ transform like the wave functions ψ_j^m.

We also write Φ in the form

$$\Phi = (-A\eta + B\xi)^{\alpha}(-A\eta' + B\xi')^{\beta}(-\xi\eta' + \eta\xi')^{\gamma} \quad . \qquad \ldots(6)$$

The spinor (ξ, η) shall be associated with the wave functions ψ_l^{μ}, and the spinor $(\xi'\eta')$ with the wave functions ψ_s^{ν}.

We must obviously have $\alpha + \beta = 2j$, since in the form (5), Φ is a homogeneous polynomial of degree $2j$ in A and B, while in the form (6) it is a homogeneous polynomial of degree $\alpha + \beta$. We shall not yet specify the values of α, β, γ any more completely.

Expanding the two expressions for Φ and equating them we have

$$\sum_{m=-j}^{j} \frac{(-1)^{j-m}(2j)!\,X^{j+m}Y^{j-m}}{(j+m)!\,(j-m)!} A^{j-m}B^{j+m}$$

$$= \sum_{p=o}^{\alpha}\sum_{q=o}^{\beta}\sum_{r=o}^{\gamma} \frac{\alpha!\,\beta!\,\gamma!\,(-1)^{p+q+r}\eta^{p+\gamma-r}\xi^{\alpha-p+r}\eta'^{q+r}\xi'^{\beta-q+\gamma-r}}{p!\,(\alpha-p)!\,q!\,(\beta-q)!\,r!\,(\gamma-r)!} A^{p+q}B^{\alpha+\beta-p-q} \quad . \qquad \ldots(7)$$

Identifying coefficients of like powers of $A^{j-m}B^{j+m}$, we have

$$p + q = j - m \qquad \alpha + \beta = 2j \, , \qquad \ldots(8)$$

and since we want to identify the monomial $\eta^{p+\gamma-r}\xi^{a-p+r}$ with the wave function ψ_l^μ, and the monomial $\eta'^{q+r}\xi'^{\beta-q+\gamma-r}$ with the wave function ψ_s^ν, we also have

$$\begin{aligned} &a + \gamma = 2l && a - \gamma - 2(p - r) = 2\mu \\ &\beta + \gamma = 2s && \beta + \gamma - 2(q + r) = 2\nu \end{aligned} \qquad \text{...(9)}$$

Since (5) and (6) must be finite polynomials, a, β, γ must be integers and l, s, j can be only integer or half integer; hence

$$\begin{aligned} &a = j + l - s && q = s - \nu + r \\ &\beta = j + s - l && m = \mu + \nu \\ &\gamma = l + s - j && p = j - \mu - s + r \end{aligned} \qquad \text{...(10)}$$

We see from this that l, s, j must each be integral, or one integral and the two others half-integral.

This gives us

$$\frac{(-1)^{j-m}(2j)!\,X^{j+m}\,Y^{j-m}}{(j + m)!\,(j - m)!}$$

$$= \sum_{\nu=l-j-r}^{s-r}\ \sum_{r=0}^{l+s-j} \frac{(j + l - s)!(j + s - l)!(l + s - j)!(-1)^{j-m+r}\,\xi^{l+\mu}\,\eta^{l-\mu}\,\xi'^{s+\nu}\,\eta'^{s-\nu}}{(j - \mu - s + r)!(l + \mu - r)!(s - \nu - r)!(j - l + \nu + r)!\,r!\,(l + s - j - r)!}$$

$$\mu + \nu = m \qquad \text{...(11)}$$

The implicit condition on the indices is that the factors in the denominator of (11) must be factorials of non-negative numbers. (We may allow the indices to run over all values if we recognize that the factorial of a negative number is infinite; hence terms containing them in the denominator vanish). This condition gives us other important relationships:

$$\begin{aligned} &j + l - s \geq 0 \\ &j + s - l \geq 0 && -s \leq \nu \leq s \\ &l + s - j \geq 0 \end{aligned} \qquad \text{...(12)}$$

If we now replace the monomials in (11) by wave functions according to (1), we have

$$\psi_j^m = C_{lsj}\sqrt{(j+m)!(j-m)!}\,\frac{(j+l-s)!(j+s-l)!(l+s-j)!}{(2l)!\,(2s)!}$$

$$\times \sum_{\nu=-s}^{s}\sum_{r}\frac{\sqrt{(l+\mu)!(l-\mu)!(s+\nu)!(s-\nu)!}\,(-1)^{j-l-s+r}\,\psi_i^{m-\nu}\,\psi_s^{\nu}}{(j-m+\nu-s+r)!(l-m-\nu-r)!(s-\nu-r)!(j-l+\nu+r)!\,r!\,(l+s-j-r)!}\,, \qquad \ldots(13)$$

where the constant C_{lsj} has been introduced because the set of functions ψ_j^m and the monomials $\dfrac{(-1)^{j-m}(2j)!\,\xi^{j+m}\,\eta^{j-m}}{\sqrt{(j+m)!(j-m)!}}$ are not equal but only have the same transformation properties. That C_{lsj} does not depend on the magnetic quantum numbers $m\mu\nu$ can be seen if the identification of monomials with wave functions is made in Eq. (7), which is a function only of lsj.

We therefore have

$$A_{jm}^{ls\mu\nu} = C_{lsj}\sum_{r}\frac{\sqrt{(j+m)!(j-m)!(l+\mu)!\,(l-\mu)!(s+\nu)!(s-\nu)!}\,(j+l-s)!(j+s-l)!(l+s-j)!(-1)^{j-l-s+r}}{(2l)!(2s)!(j-m+\nu-s+r)!(l+m-\nu-r)!(s-\nu-r)!(j-l+\nu+r)!\,r!\,(l+s-j-r)!}$$

$$\ldots(14a)$$

or

$$A_{jm}^{ls\mu\nu} = B_{lsj}\sum_{r}\frac{(-1)^r\sqrt{(j+m)!(j-m)!(l+\mu)!(l-\mu)!(s+\nu)!(s-\nu)!}}{(j-\mu-s+r)!(j+\nu-l+r)!(l+\mu-r)!(s-\nu-r)!\,r!\,(l+s-j-r)!}\,, \qquad \ldots(14b)$$

$$\mu+\nu=m$$

the summation over r being taken over all values which do not make the denominator infinite.

We must now evaluate the coefficient B_{lsj} . To do this we impose the condition that the transformation $A_{jm}^{ls\mu\nu}$ shall be unitary. Since the A's are real, this condition becomes

$$\sum_{\mu\nu} \left| A_{jm}^{ls\mu\nu} \right|^2 = 1 \qquad \text{...(15)}$$

This is most easily carried out for the case $m = j$, since then the sum (14) reduces to a single term. Since B_{lsj} is independent of m, such a choice will not affect the applicability of the solution to cases where $m \neq j$. For $m = j$ we have $\mu = j - \nu$, $r = s - \nu$, and

$$A_{jj}^{ls,j-\nu,\nu} = B_{lsj} \frac{(-1)^{s-\nu}\sqrt{(2j)!(l+j-\nu)!(l-j+\nu)!(s+\nu)!(s-\nu)!}}{(l+j-s)!(j-l+s)!(s-\nu)!(l-j+\nu)!}$$

$$= \frac{B_{lsj}\sqrt{(2j)!}\,(-1)^{s-\nu}}{(l+j-s)!(j+s-l)!} \sqrt{\frac{(l+j-\nu)!(s+\nu)!}{(l-j+\nu)!(s-\nu)!}} \ . \qquad \text{...(16)}$$

Then (15) becomes

$$\frac{B_{lsj}^2(2j)!}{[(l+j-s)!(j+s-l)!]^2} \sum_{\nu=j-l}^{s} \frac{(l+j-\nu)!(s+\nu)!}{(l-j+\nu)!(s-\nu)!} = 1 \ . \qquad \text{...(17)}$$

We write this as

$$\frac{B_{lsj}^2(2j)!}{(l+j-s)!(s+j-l)!} \sum_{\nu=l-j}^{s} \binom{l+j-\nu}{s-\nu}\binom{s+\nu}{l-j+\nu} = 1 \ , \qquad \text{...(18)}$$

where $\binom{\alpha}{\beta}$ is the coefficient of x^{β} in the expansion of $(1 + x)^{\alpha}$:

$$\binom{\alpha}{\beta} = \frac{\alpha(\alpha-1)(\alpha-2)\ldots(\alpha-\beta+1)}{\beta!} = (-1)^{\beta}\frac{(\beta-\alpha-1)(\beta-\alpha-2)\ldots(1-\alpha)(-\alpha)}{\beta!}$$

$$= (-1)^{\beta}\binom{\beta-\alpha-1}{\beta}. \qquad \ldots(19)$$

Then the sum in (18) becomes

$$\sum_{\nu=l-j}^{s}\binom{l+j-\nu}{s-\nu}\binom{s+\nu}{l-j+\nu} = \sum_{\nu=l-j}^{s}(-1)^{s+l-j}\binom{s-l-j-1}{s-\nu}\binom{l-j-s-1}{s-\nu}$$

$$= (-1)^{s+l-j}\binom{-2j-2}{l+s-j} = \binom{l+s+j+1}{l+s-j}. \qquad \ldots(20)$$

The summation of the binomial coefficients is verified by identifying the coefficient of x^{l+s-j} on each side of the identity

$$(1+x)^{s-l-j-1}(1+x)^{l-j-s-1} = (1+x)^{-2j-2} \qquad \ldots(21)$$

Solving (18) for B^2_{lsj} , we then have

$$B^2_{lsj} = \frac{(l+s-j)!(s+j-l)!(l+s-j)!(2j+1)}{(l+s+j+1)!} \quad ; \qquad \ldots(22)$$

hence, we now have from (14)

$$A_{jm}^{ls\mu\nu} = \sqrt{\frac{(l+s-j)!(l+j-s)!(s+j-l)!(2j+1)}{(l+s+j+1)!}}\ \delta_m^{\mu+\nu}$$

$$\times \sum_r \frac{(-1)^r \sqrt{(j+m)!(j-m)!(l+\mu)!(l-\mu)!(s+\nu)!(s-\nu)!}}{(j-\mu-s+r)!(j-l+\nu+r)!(l+\mu-r)!(l+\mu-r)!(s-\nu-r)!r!(l+s-j-r)!} \quad . \qquad \ldots(23)$$

The coefficients $A_{jm}^{ls\mu\nu}$ are tabulated in the following pages. Since the wave functions are representations (Darstellungen) of the rotation group, we have labeled each table with the heading $D_l \times D_s$. The symbols on the left stand for the product of any two wave functions with l and s given by the subscripts and μ and ν given by the superscripts. The symbols along the top of the table again stand for wave functions with subscript j and superscript m. The element $A_{jm}^{ls\mu\nu}$ then stands at the intersection of the row labeled $U_l^\mu V_s^\nu$ and the column labeled W_j^m. All empty intersections are zero. Each sub-matrix is then an explicit example of the general group theoretical formula

$$D_l \times D_s = D_{l+s} + D_{l+s-1} + \ldots . + D_{|l-s|} \quad . \qquad \ldots(24)$$

The coefficients A may also be thought of as a transformation of coordinate axes in function space from one set of orthogonal "base vectors" $e_{\mu\nu} = \psi_l^\mu \psi_s^\nu$ to another orthogonal set $\epsilon_{jm} = \psi_j^m$.

Algebraic tables are given for $D_{1/2} \times D_l$, $D_1 \times D_l$, and $D_{3/2} \times D_l$. A table for $D_2 \times D_l$ has been given by Condon and Shortley.[7] Falkoff *et al.*[8] have published a table for $D_3 \times D_l$, and recently Melvin and Swamy[9] have filled the gap with $D_{5/2} \times D_l$.

III. SUMMATION FORMULAE INVOLVING CLEBSCH-GORDAN COEFFICIENTS

In Section II, certain sums of matrix elements appear in the equations for the scattering cross section. Specifically, we have sums of the type

$$\sum M^{JMl}_{m\,m_j\,m_s} M^{JMl'}_{m'm_j'm_s'} \quad ,$$

with various conditions on the summed indices. The matrix elements M can be written as

$$M^{JMl}_{m\,m_j m_s} = \sum_{Tm_T} a_T A^{jsm_jm_s}_{Tm_T} A^{Tlm_Tm}_{JM}$$

The matrices A^{ls} are unitary, and hence

$$\sum_{\mu\nu} A^{ls\mu\nu}_{jm} A^{ls\mu\nu}_{j'm'} = \delta_{jj'}\delta_{mm'} \qquad \text{...(1)}$$

and

$$\sum_{jm} A^{ls\mu\nu}_{jm} A^{ls\mu'\nu'}_{jm} = \delta_{\mu\mu'}\delta_{\nu\nu'} \ , \qquad \text{...(2)}$$

which follow directly from the condition that A^{ls} is the transformation matrix from one orthonormal set of base vectors to another.

We need also the relationship*

$$\sum_{\nu m} A_{jm}^{ls\mu\nu} A_{jm}^{ls\mu'\nu} = X_{\mu\mu'} = \frac{2j+1}{2l+1}\,\delta_{\mu\mu'}. \qquad \ldots(3)$$

To prove this we make use of Schurr's lemma† that if the matrix X commutes with every matrix of an irreducible representation then X is a multiple of the unit matrix.

We first show that $A_{jm}^{ls\mu\nu}$ is a scalar (i.e., that $P_R A_{jm}^{ls\mu\nu} = A_{jm}^{ls\mu\nu}$, where P_R is the operator which subjects the coordinates of the wave function to a rotation R):

$$A_{jm}^{ls\mu\nu} = (\psi_l^{\mu}\,\psi_s^{\nu},\ \psi_j^{m});$$

hence

$$P_R A_{jm}^{ls\mu\nu} = (P_R\,\psi_l^{m}\,\psi_s^{\nu},\ P_R\,\psi_j^{m})$$

$$= \sum_{\mu'\nu'm'} D_{\mu'\mu}^{(l)*}\, D_{\nu'\nu}^{(s)*}\, (\psi_l^{\mu'}\,\psi_s^{\nu'},\ \psi_j^{m'})\, D_{m'm}^{(j)}$$

$$= \sum_{\mu'\nu'm'} \sum_{j'\sigma\sigma'} A_{j'\sigma}^{ls\mu'\nu'}\, D_{\sigma\sigma'}^{(j')*}\, A_{j'\sigma'}^{ls\mu\nu}\, A_{jm'}^{ls\mu'\nu'}\, D_{m'm}^{(j)}$$

$$= \sum_{j'm'\sigma\sigma'} \delta_{jj'}\,\delta_{m'\sigma}\, D_{\sigma\sigma'}^{(j)*}\, A_{j'\sigma'}^{ls\mu\nu}\, D_{m'm}^{(j)}$$

$$= \sum_{\sigma\sigma'} D_{\sigma\sigma'}^{(j)*}\, D_{\sigma m}^{(j)}\, A_{j\sigma'}^{ls\mu\nu} = A_{jm}^{ls\mu\nu}\ .$$

*The evaluation of this sum has been given by Breit and Darling (G. Breit and B. T. Darling, "Note on Calculation of Angular Distribution in Resonance Reactions," Phys. Rev. 71, 402 (1947).)

†See p 83 of Ref. 4 for a detailed discussion of Schurr's lemma.

Therefore,

$$P_R X_{\mu\mu'} = X_{\mu\mu'} = P_R \sum_{m\nu} (\psi_l^{\mu} \psi_s^{\nu}, \psi_j^{m})(\psi_l^{m}, \psi_l^{\mu'} \psi_s^{\nu})$$

$$= \sum_{m\nu} \sum_{\substack{\mu''\mu''' \\ \nu'\nu'' \\ m'm''}} D^{(l)*}_{\mu''\mu} D^{(s)*}_{\nu'\nu} A^{ls\mu''\nu'}_{jm'} D^{(j)}_{m'm} D^{(j)}_{m''m} A^{ls\mu'''\nu''}_{jm''} D^{(l)}_{\mu'''\mu'} D^{(s)}_{\nu''\nu}$$

$$= \sum_{m'\nu'\mu''\mu'''} D^{(l)*}_{\mu''\mu} A^{ls\mu''\nu'}_{jm'} A^{ls\mu'''\nu'}_{jm'} D^{(l)}_{\mu'''\mu'}$$

$$= \sum_{\mu''\mu'''} D^{(l)*}_{\mu''\mu} X_{\mu''\mu'''} D_{\mu'''\mu'} \ .$$

Hence $X_{\mu\mu'}$ commutes with the irreducible matrices $D^{(l)}(R)$ for any R, and therefore we can conclude that

$$X_{\mu\mu'} = X^{lsj} \delta_{\mu\mu'} \ .$$

The evaluation of X^{lsj} is now simple. We simply sum $X_{\mu\mu}$ and obtain

$$\sum_{\mu} X_{\mu\mu} = (2l + 1) X^{lsj}$$

$$= \sum_{\mu\nu m} A^{ls\mu\nu}_{jm} A^{ls\mu\nu}_{jm} = \sum_{m} 1 = 2j + 1 \ .$$

Hence $X^{lsj} = \dfrac{2j+1}{2l+1}$. This establishes Eq. (3). We now wish to sum the squares and products of matrix elements appearing in the cross-section formula.

$$\sum_{mm_jm_s} \left| M^{JMl}_{mm_jm_s} \right|^2 = \sum_{mm_jm_s} \left| \sum_{Tm_T} a_T A^{jsm_jm_s}_{Tm_T} A^{Tlm_Tm}_{JM} \right|^2$$

$$= \sum_{mm_jm_s} \sum_{TRm_Tm_R} a_T a_R A^{jsm_jm_s}_{Tm_T} A^{jsm_jm_s}_{Rm_R} A^{Tlm_Tm}_{JM} A^{Rlm_Rm}_{JM} \ ,$$

from which

$$\sum_{mm_jm_s} \left| M^{JMl}_{mm_jm_s} \right|^2 = \sum_{TR} \sum_{mm_Tm_R} a_T a_R \delta_{TR} \delta_{m_Tm_R} A^{Tlm_Tm}_{JM} A^{Rlm_Rm}_{JM}$$

$$= \sum_{T} a_T^2 \sum_{mm_T} \left| A^{Tlm_Tm}_{JM} \right|^2 = \sum_{T} a_T^2 \ . \qquad \ldots(4)$$

Another important sum is obtained by summing the same matrix elements over a different set of indices:

$$\sum_{Mm_jm_s} \left| M^{JMl}_{mm_jm_s} \right|^2 = \sum_{T} a_T^2 \sum_{Mm_T} \left| A^{Tlm_Tm}_{JM} \right|^2$$

$$= \frac{2J+1}{2l+1} \sum_{T} a_T^2 \qquad \ldots(5)$$

The third important sum is obtained by squaring the products of matrix elements:

$$\sum_{\substack{mm_jm_s \\ Mm_j'm_s'}} \left| M^{JMl}_{om_j'm_s'} M^{JMl}_{mm_jm_s} \right|^2$$

$$= \sum_{Mm} \sum_{m_j' m_s'} \left| M^{JMl}_{om_j' m_s'} \right|^2 \left[\sum_{m_j m_s} \left| M^{JMl}_{mm_j m_s} \right| \right]^2$$

$$= \sum_{Mm} \sum_{Tm_T} a_T^2 \left| A_{JM}^{Tlm_T o} \right|^2 \sum_{Rm_R} a_R^2 \left| A_{JM}^{Rlm_R m} \right|^2$$

$$= \sum_{TR} a_T^2 a_R^2 \sum_{M} \left| A_{JM}^{TlMo} A_{JM}^{Rl,M-m,m} \right|^2 \qquad \ldots(6)$$

IV. TABLES OF THE CLEBSCH–GORDAN COEFFICIENTS

TABLE I

$\underline{D_l \times D_{1/2}}$

	$W^m_{l+1/2}$	$W^m_{l-1/2}$
$U_l^{m+1/2} \; V_{1/2}^{-1/2}$	$\sqrt{\dfrac{l-m+1/2}{2l+1}}$	$\sqrt{\dfrac{l+m+1/2}{2l+1}}$
$U_l^{m-1/2} \; V_{1/2}^{1/2}$	$\sqrt{\dfrac{l+m+1/2}{2l+1}}$	$-\sqrt{\dfrac{l-m+1/2}{2l+1}}$

TABLE II

$\underline{D_l \times D_1}$

	W^m_{l+1}	W^m_{l}	W^m_{l-1}
$U_l^{m+1} \; V_1^{-1}$	$\sqrt{\dfrac{(l-m+1)(l-m)}{(2l+1)(2l+2)}}$	$\sqrt{\dfrac{(l-m)(l+m+1)}{l(2l+2)}}$	$\sqrt{\dfrac{(l+m)(l+m+1)}{(2l)(2l+1)}}$
$U_l^{m} \; V_1^{0}$	$\sqrt{\dfrac{2(l+m+1)(l-m+1)}{(2l+1)(2l+2)}}$	$\sqrt{\dfrac{m}{l(l+1)}}$	$-\sqrt{\dfrac{2(l-m)(l+m)}{(2l)(2l+1)}}$
$U_l^{m-1} \; V_1^{1}$	$\sqrt{\dfrac{(l+m+1)(l+m)}{(2l+1)(2l+2)}}$	$-\sqrt{\dfrac{(l+m)(l-m+1)}{l(2l+2)}}$	$\sqrt{\dfrac{(l-m)(l-m+1)}{(2l)(2l+1)}}$

TABLE III

$D_l \times D_{3/2}$

	$W^m_{l+3/2}$	$W^m_{l+1/2}$	$W^m_{l-1/2}$	$W^m_{l-3/2}$
$U_l^{m+3/2}\ V_{3/2}^{3/2}$	$\sqrt{\frac{(l-m-1/2)(l-m+1/2)(l-m+3/2)}{(2l+1)(2l+2)(2l+3)}}$	$\sqrt{\frac{3(l-m-1/2)(l-m+1/2)(l+m+3/2)}{(2l+3)(2l+1)(2l)}}$	$\sqrt{\frac{3(l-m+1/2)(l-m+3/2)(l-m-1/2)}{(2l-1)(2l+1)(2l+2)}}$	$\sqrt{\frac{(l+m-1/2)(l+m+1/2)(l+m+3/2)}{(2l-1)(2l)(2l+1)}}$
$U_l^{m+1/2}\ V_{3/2}^{-1/2}$	$\sqrt{\frac{3(l+m+3/2)(l-m+1/2)(l-m+3/2)}{(2l+1)(2l+2)(2l+3)}}$	$(l+3m+3/2)\sqrt{\frac{(l-m+1/2)}{(2l+3)(2l+1)(2l)}}$	$-(l-3m-1/2)\sqrt{\frac{(l+m+1/2)}{(2l-1)(2l+1)(2l+2)}}$	$-\sqrt{\frac{3(l-m-1/2)(l+m-1/2)(l+m+1/2)}{(2l-1)(2l)(2l+1)}}$
$U_l^{m-1/2}\ V_{3/2}^{1/2}$	$\sqrt{\frac{3(l+m+3/2)(l+m+1/2)(l+m+3/2)}{(2l+1)(2l+2)(2l+3)}}$	$-(l-3m+3/2)\sqrt{\frac{(l+m+1/2)}{(2l+3)(2l+1)(2l)}}$	$-(l+3m-1/2)\sqrt{\frac{(l-m+1/2)}{(2l+3)(2l+1)(2l+2)}}$	$\sqrt{\frac{3(l-m-1/2)(l-m+1/2)(l+m-1/2)}{(2l-1)(2l)(2l+1)}}$
$U_l^{m-3/2}\ V_{3/2}^{3/2}$	$\sqrt{\frac{(l+m+3/2)(l+m+1/2)(l+m-1/2)}{(2l+1)(2l+2)(2l+3)}}$	$-\sqrt{\frac{3(l+m-1/2)(l+m+1/2)(l-m+3/2)}{(2l+3)(2l+1)(2l)}}$	$\sqrt{\frac{3(l-m+1/2)(l-m+3/2)(l+m-1/2)}{(2l-2)(2l+2)(2l+2)}}$	$-\sqrt{\frac{(l-m-1/2)(l-m+1/2)(l-m+3/2)}{(2l-1)(2l)(2l+1)}}$

TABLE IV

$D_{1/2} \times D_{1/2}$

	W^{1}_{1}	W^{0}_{1}	W^{0}_{0}	W^{-1}_{1}
$U^{1/2}_{1/2} \; V^{1/2}_{1/2}$	1			
$U^{1/2}_{1/2} \; V^{-1/2}_{1/2}$		$\sqrt{\frac{1}{2}}$	$\sqrt{\frac{1}{2}}$	
$U^{-1/2}_{1/2} \; V^{1/2}_{1/2}$		$\sqrt{\frac{1}{2}}$	$-\sqrt{\frac{1}{2}}$	
$U^{-1/2}_{1/2} \; V^{-1/2}_{1/2}$				1

TABLE V

$D_{1} \times D_{1/2}$

	$W^{3/2}_{3/2}$	$W^{1/2}_{3/2}$	$W^{1/2}_{1/2}$	$W^{-1/2}_{3/2}$	$W^{-1/2}_{1/2}$	$W^{-3/2}_{3/2}$
$U^{1}_{1} \; V^{1/2}_{1/2}$	1					
$U^{1}_{1} \; V^{-1/2}_{1/2}$		$\sqrt{\frac{1}{3}}$	$\sqrt{\frac{2}{3}}$			
$U^{0}_{1} \; V^{1/2}_{1/2}$		$\sqrt{\frac{2}{3}}$	$-\sqrt{\frac{1}{3}}$			
$U^{0}_{1} \; V^{-1/2}_{1/2}$				$\sqrt{\frac{2}{3}}$	$\sqrt{\frac{1}{3}}$	
$U^{-1}_{1} \; V^{1/2}_{1/2}$				$\sqrt{\frac{1}{3}}$	$-\sqrt{\frac{2}{3}}$	
$U^{-1}_{1} \; V^{-1/2}_{1/2}$						1

TABLE VI

$D_{3/2} \times D_{1/2}$

	W_2^2	W_2^1	W_1^1	W_2^0	W_1^0	W_2^{-1}	W_1^{-1}	W_2^{-2}
$U_{3/2}^{3/2}$ $V_{1/2}^{1/2}$	1							
$U_{3/2}^{3/2}$ $V_{1/2}^{-1/2}$		$\sqrt{\frac{1}{4}}$	$\sqrt{\frac{3}{4}}$					
$U_{3/2}^{1/2}$ $V_{1/2}^{1/2}$		$\sqrt{\frac{3}{4}}$	$-\sqrt{\frac{1}{4}}$					
$U_{3/2}^{1/2}$ $V_{1/2}^{-1/2}$				$\sqrt{\frac{1}{2}}$	$\sqrt{\frac{1}{2}}$			
$U_{3/2}^{-1/2}$ $V_{1/2}^{1/2}$				$\sqrt{\frac{1}{2}}$	$-\sqrt{\frac{1}{2}}$			
$U_{3/2}^{-1/2}$ $V_{1/2}^{-1/2}$						$\sqrt{\frac{3}{4}}$	$\sqrt{\frac{1}{4}}$	
$U_{3/2}^{-3/2}$ $V_{1/2}^{1/2}$						$\sqrt{\frac{1}{4}}$	$-\sqrt{\frac{3}{4}}$	
$U_{3/2}^{-3/2}$ $V_{1/2}^{-1/2}$								1

TABLE VII

$\underline{D_1 \times D_1}$

	W_2^2	W_2^1	W_1^1	W_2^0	W_1^0	W_0^0	W_2^{-1}	W_1^{-1}	W_2^{-2}
$U_1^1 \; V_1^1$	1								
$U_1^1 \; V_1^0$		$\sqrt{\frac{1}{2}}$	$\sqrt{\frac{1}{2}}$						
$U_1^0 \; V_1^1$		$\sqrt{\frac{1}{2}}$	$-\sqrt{\frac{1}{2}}$						
$U_1^1 \; V_1^{-1}$				$\sqrt{\frac{1}{6}}$	$\sqrt{\frac{1}{2}}$	$\sqrt{\frac{1}{3}}$			
$U_1^0 \; V_1^0$				$\sqrt{\frac{2}{3}}$	0	$-\sqrt{\frac{1}{3}}$			
$U_1^{-1} \; V_1^1$				$\sqrt{\frac{1}{6}}$	$-\sqrt{\frac{1}{2}}$	$\sqrt{\frac{1}{3}}$			
$U_1^0 \; V_1^{-1}$							$\sqrt{\frac{1}{2}}$	$\sqrt{\frac{1}{2}}$	
$U_1^{-1} \; V_1^0$							$\sqrt{\frac{1}{2}}$	$-\sqrt{\frac{1}{2}}$	
$U_1^{-1} \; V_1^{-1}$									1

TABLE VIII

$D_2 \times D_{1/2}$

	$W^{5/2}_{5/2}$	$W^{3/2}_{5/2}$	$W^{3/2}_{3/2}$	$W^{1/2}_{5/2}$	$W^{1/2}_{3/2}$	$W^{-1/2}_{5/2}$	$W^{-1/2}_{3/2}$	$W^{-3/2}_{5/2}$	$W^{-3/2}_{3/2}$	$W^{-5/2}_{5/2}$
$U^{2}_{2}\ V^{1/2}_{1/2}$	1									
$U^{2}_{2}\ V^{-1/2}_{1/2}$		$\sqrt{\frac{1}{5}}$	$\sqrt{\frac{4}{5}}$							
$U^{1}_{2}\ V^{1/2}_{1/2}$		$\sqrt{\frac{4}{5}}$	$-\sqrt{\frac{1}{5}}$							
$U^{1}_{2}\ V^{-1/2}_{1/2}$				$\sqrt{\frac{2}{5}}$	$\sqrt{\frac{3}{5}}$					
$U^{0}_{2}\ V^{1/2}_{1/2}$				$\sqrt{\frac{3}{5}}$	$-\sqrt{\frac{2}{5}}$					
$U^{0}_{2}\ V^{-1/2}_{1/2}$						$\sqrt{\frac{3}{5}}$	$\sqrt{\frac{2}{5}}$			
$U^{-1}_{2}\ V^{1/2}_{1/2}$						$\sqrt{\frac{2}{5}}$	$-\sqrt{\frac{3}{5}}$			
$U^{-1}_{2}\ V^{-1/2}_{1/2}$								$\sqrt{\frac{4}{5}}$	$\sqrt{\frac{1}{5}}$	
$U^{-2}_{2}\ V^{1/2}_{1/2}$								$\sqrt{\frac{1}{5}}$	$-\sqrt{\frac{4}{5}}$	
$U^{-2}_{2}\ V^{1/2}_{1/2}$										1

TABLE IX

$D_{3/2} \times D_1$

	$W_{5/2}^{5/2}$	$W_{5/2}^{3/2}$	$W_{3/2}^{3/2}$	$W_{5/2}^{1/2}$	$W_{3/2}^{1/2}$	$W_{1/2}^{1/2}$	$W_{5/2}^{-1/2}$	$W_{3/2}^{-1/2}$	$W_{1/2}^{-1/2}$	$W_{5/2}^{-3/2}$	$W_{3/2}^{-3/2}$	$W_{5/2}^{-5/2}$
$U_{3/2}^{3/2}\ V_1^1$	1											
$U_{3/2}^{3/2}\ V_1^0$		$\sqrt{\frac{2}{5}}$	$\sqrt{\frac{3}{5}}$									
$U_{3/2}^{1/2}\ V_1^1$		$\sqrt{\frac{3}{5}}$	$-\sqrt{\frac{2}{5}}$									
$U_{3/2}^{3/2}\ V_1^{-1}$				$\sqrt{\frac{1}{10}}$	$\sqrt{\frac{2}{5}}$	$\sqrt{\frac{1}{2}}$						
$U_{3/2}^{1/2}\ V_1^0$				$\sqrt{\frac{3}{5}}$	$\sqrt{\frac{1}{15}}$	$-\sqrt{\frac{1}{3}}$						
$U_{3/2}^{-1/2}\ V_1^1$				$\sqrt{\frac{3}{10}}$	$-\sqrt{\frac{8}{15}}$	$\sqrt{\frac{1}{6}}$						
$U_{3/2}^{1/2}\ V_1^{-1}$							$\sqrt{\frac{3}{10}}$	$\sqrt{\frac{8}{15}}$	$\sqrt{\frac{1}{6}}$			
$U_{3/2}^{-1/2}\ V_1^0$							$\sqrt{\frac{3}{5}}$	$-\sqrt{\frac{1}{15}}$	$-\sqrt{\frac{1}{3}}$			
$U_{3/2}^{-3/2}\ V_1^1$							$\sqrt{\frac{1}{10}}$	$-\sqrt{\frac{2}{5}}$	$\sqrt{\frac{1}{2}}$			
$U_{3/2}^{-1/2}\ V_1^{-1}$										$\sqrt{\frac{3}{5}}$	$\sqrt{\frac{2}{5}}$	
$U_{3/2}^{-3/2}\ V_1^0$										$\sqrt{\frac{2}{5}}$	$-\sqrt{\frac{3}{5}}$	
$U_{3/2}^{-3/2}\ V_1^{-1}$												1

TABLE X

$D_{5/2} \times D_{1/2}$

	W_3^3	W_3^2	W_2^2	W_3^1	W_2^1	W_3^0	W_2^0	W_3^{-1}	W_2^{-1}	W_3^{-2}	W_2^{-2}	W_3^{-3}
$U_{5/2}^{5/2} \; V_{1/2}^{1/2}$	1											
$U_{5/2}^{5/2} \; V_{1/2}^{-1/2}$		$\sqrt{\frac{1}{6}}$	$\sqrt{\frac{5}{6}}$									
$U_{5/2}^{3/2} \; V_{1/2}^{1/2}$		$\sqrt{\frac{5}{6}}$	$-\sqrt{\frac{1}{6}}$									
$U_{5/2}^{3/2} \; V_{1/2}^{-1/2}$				$\sqrt{\frac{1}{3}}$	$\sqrt{\frac{2}{3}}$							
$U_{5/2}^{1/2} \; V_{1/2}^{1/2}$				$\sqrt{\frac{2}{3}}$	$-\sqrt{\frac{1}{3}}$							
$U_{5/2}^{1/2} \; V_{1/2}^{-1/2}$						$\sqrt{\frac{1}{2}}$	$\sqrt{\frac{1}{2}}$					
$U_{5/2}^{-1/2} \; V_{1/2}^{1/2}$						$\sqrt{\frac{1}{2}}$	$-\sqrt{\frac{1}{2}}$					
$U_{5/2}^{-1/2} \; V_{1/2}^{-1/2}$								$\sqrt{\frac{2}{3}}$	$\sqrt{\frac{1}{3}}$			
$U_{5/2}^{-3/2} \; V_{1/2}^{1/2}$								$\sqrt{\frac{1}{3}}$	$-\sqrt{\frac{2}{3}}$			
$U_{5/2}^{-3/2} \; V_{1/2}^{-1/2}$										$\sqrt{\frac{5}{6}}$	$\sqrt{\frac{1}{6}}$	
$U_{5/2}^{-5/2} \; V_{1/2}^{1/2}$										$\sqrt{\frac{1}{6}}$	$-\sqrt{\frac{5}{6}}$	
$U_{5/2}^{-5/2} \; V_{1/2}^{-1/2}$												1

TABLE XI

$D_2 \times D_1$

	W_3^3	W_3^2	W_2^2	W_3^1	W_2^1	W_1^1	W_3^0	W_2^0	W_1^0	W_3^{-1}	W_2^{-1}	W_1^{-1}	W_3^{-2}	W_2^{-2}	W_3^{-3}
$U_2^2\ V_1^1$	1														
$U_2^2\ V_1^0$		$\sqrt{\frac{1}{3}}$	$\sqrt{\frac{2}{3}}$												
$U_2^1\ V_1^1$		$\sqrt{\frac{2}{3}}$	$-\sqrt{\frac{1}{3}}$												
$U_2^2\ V_1^{-1}$				$\sqrt{\frac{1}{15}}$	$\sqrt{\frac{1}{3}}$	$\sqrt{\frac{3}{5}}$									
$U_2^1\ V_1^0$				$\sqrt{\frac{8}{15}}$	$\sqrt{\frac{1}{6}}$	$-\sqrt{\frac{3}{10}}$									
$U_2^0\ V_1^1$				$\sqrt{\frac{6}{15}}$	$-\sqrt{\frac{1}{2}}$	$\sqrt{\frac{1}{10}}$									
$U_2^1\ V_1^{-1}$							$\sqrt{\frac{1}{5}}$	$\sqrt{\frac{1}{2}}$	$\sqrt{\frac{3}{10}}$						
$U_2^0\ V_1^0$							$\sqrt{\frac{3}{5}}$	0	$-\sqrt{\frac{2}{5}}$						
$U_2^{-1}\ V_1^1$							$\sqrt{\frac{1}{5}}$	$-\sqrt{\frac{1}{2}}$	$\sqrt{\frac{3}{10}}$						
$U_2^0\ V_1^{-1}$										$\sqrt{\frac{6}{15}}$	$\sqrt{\frac{1}{2}}$	$\sqrt{\frac{1}{10}}$			
$U_2^{-1}\ V_1^0$										$\sqrt{\frac{8}{15}}$	$-\sqrt{\frac{1}{6}}$	$-\sqrt{\frac{3}{10}}$			
$U_2^{-2}\ V_1^1$										$\sqrt{\frac{1}{15}}$	$-\sqrt{\frac{1}{3}}$	$\sqrt{\frac{3}{5}}$			
$U_2^{-1}\ V_1^{-1}$													$\sqrt{\frac{2}{3}}$	$\sqrt{\frac{1}{3}}$	
$U_2^{-2}\ V_1^0$													$\sqrt{\frac{1}{3}}$	$-\sqrt{\frac{2}{3}}$	
$U_2^{-2}\ V_1^{-1}$															1

TABLE XII

$D_{3/2} \times D_{3/2}$

	W_3^3	W_3^2	W_2^2	W_3^1	W_2^1	W_1^1	W_3^0	W_2^0	W_1^0	W_0^0	W_3^{-1}	W_2^{-1}	W_1^{-1}	W_3^{-2}	W_2^{-2}	W_3^{-3}
$U^{3/2}_{3/2}\ V^{3/2}_{3/2}$	1															
$U^{3/2}_{3/2}\ V^{1/2}_{3/2}$		$\sqrt{\frac{1}{2}}$	$\sqrt{\frac{1}{2}}$													
$U^{1/2}_{3/2}\ V^{3/2}_{3/2}$		$\sqrt{\frac{1}{2}}$	$-\sqrt{\frac{1}{2}}$													
$U^{3/2}_{3/2}\ V^{-1/2}_{3/2}$				$\sqrt{\frac{1}{5}}$	$\sqrt{\frac{1}{2}}$	$\sqrt{\frac{3}{10}}$										
$U^{1/2}_{3/2}\ V^{1/2}_{3/2}$				$\sqrt{\frac{3}{5}}$	0	$-\sqrt{\frac{2}{5}}$										
$U^{-1/2}_{3/2}\ V^{3/2}_{3/2}$				$\sqrt{\frac{1}{5}}$	$-\sqrt{\frac{1}{2}}$	$\sqrt{\frac{3}{10}}$										
$U^{3/2}_{3/2}\ V^{-3/2}_{3/2}$							$\sqrt{\frac{1}{20}}$	$\sqrt{\frac{1}{4}}$	$\sqrt{\frac{9}{20}}$	$\sqrt{\frac{1}{4}}$						
$U^{1/2}_{3/2}\ V^{-1/2}_{3/2}$							$\sqrt{\frac{9}{20}}$	$\sqrt{\frac{1}{4}}$	$-\sqrt{\frac{1}{20}}$	$-\sqrt{\frac{1}{4}}$						
$U^{-1/2}_{3/2}\ V^{1/2}_{3/2}$							$\sqrt{\frac{9}{20}}$	$\sqrt{\frac{1}{4}}$	$-\sqrt{\frac{1}{20}}$	$\sqrt{\frac{1}{4}}$						
$U^{-3/2}_{3/2}\ V^{3/2}_{3/2}$							$\sqrt{\frac{1}{20}}$	$-\sqrt{\frac{1}{4}}$	$\sqrt{\frac{9}{20}}$	$-\sqrt{\frac{1}{4}}$						
$U^{1/2}_{3/2}\ V^{-3/2}_{3/2}$											$\sqrt{\frac{1}{5}}$	$\sqrt{\frac{1}{2}}$	$\sqrt{\frac{3}{10}}$			
$U^{-1/2}_{3/2}\ V^{-1/2}_{3/2}$											$\sqrt{\frac{3}{5}}$	0	$-\sqrt{\frac{2}{5}}$			
$U^{-3/2}_{3/2}\ V^{1/2}_{3/2}$											$\sqrt{\frac{1}{5}}$	$-\sqrt{\frac{1}{2}}$	$\sqrt{\frac{3}{10}}$			
$U^{-1/2}_{3/2}\ V^{-3/2}_{3/2}$														$\sqrt{\frac{1}{2}}$	$\sqrt{\frac{1}{2}}$	
$U^{-3/2}_{3/2}\ V^{-1/2}_{3/2}$														$\sqrt{\frac{1}{2}}$	$-\sqrt{\frac{1}{2}}$	
$U^{-3/2}_{3/2}\ V^{-3/2}_{3/2}$																1

TABLE XIII

$\underline{D}_3 \times \underline{D}_{1/2}$

	$W_{7/2}^{7/2}$	$W_{7/2}^{5/2}$	$W_{5/2}^{5/2}$	$W_{7/2}^{3/2}$	$W_{5/2}^{3/2}$	$W_{7/2}^{1/2}$	$W_{5/2}^{1/2}$	$W_{7/2}^{-1/2}$	$W_{5/2}^{-1/2}$	$W_{7/2}^{-3/2}$	$W_{5/2}^{-3/2}$	$W_{7/2}^{-5/2}$	$W_{5/2}^{-5/2}$	$W_{7/2}^{-7/2}$
$U_3^3 \; V_{1/2}^{1/2}$	1													
$U_3^3 \; V_{1/2}^{-1/2}$		$\sqrt{\frac{1}{7}}$	$\sqrt{\frac{6}{7}}$											
$U_3^2 \; V_{1/2}^{1/2}$		$\sqrt{\frac{6}{7}}$	$-\sqrt{\frac{1}{7}}$											
$U_3^2 \; V_{1/2}^{-1/2}$				$\sqrt{\frac{2}{7}}$	$\sqrt{\frac{5}{7}}$									
$U_3^1 \; V_{1/2}^{1/2}$				$\sqrt{\frac{5}{7}}$	$-\sqrt{\frac{2}{7}}$									
$U_3^1 \; V_{1/2}^{-1/2}$						$\sqrt{\frac{3}{7}}$	$\sqrt{\frac{4}{7}}$							
$U_3^0 \; V_{1/2}^{1/2}$						$\sqrt{\frac{4}{7}}$	$-\sqrt{\frac{3}{7}}$							
$U_3^0 \; V_{1/2}^{-1/2}$								$\sqrt{\frac{4}{7}}$	$\sqrt{\frac{3}{7}}$					
$U_3^{-1} \; V_{1/2}^{1/2}$								$\sqrt{\frac{3}{7}}$	$-\sqrt{\frac{4}{7}}$					
$U_3^{-1} \; V_{1/2}^{-1/2}$										$\sqrt{\frac{5}{7}}$	$\sqrt{\frac{2}{7}}$			
$U_3^{-2} \; V_{1/2}^{1/2}$										$\sqrt{\frac{2}{7}}$	$-\sqrt{\frac{5}{7}}$			
$U_3^{-2} \; V_{1/2}^{-1/2}$												$\sqrt{\frac{6}{7}}$	$\sqrt{\frac{1}{7}}$	
$U_3^{-3} \; V_{1/2}^{1/2}$												$\sqrt{\frac{1}{7}}$	$-\sqrt{\frac{6}{7}}$	
$U_3^{3} \; V_{1/2}^{-1/2}$														1

TABLE XIV

$D_{5/2} \times D_1$

	$W^{7/2}_{7/2}$	$W^{5/2}_{7/2}$	$W^{5/2}_{5/2}$	$W^{3/2}_{7/2}$	$W^{3/2}_{5/2}$	$W^{3/2}_{3/2}$	$W^{1/2}_{7/2}$	$W^{1/2}_{5/2}$	$W^{1/2}_{3/2}$	$W^{-1/2}_{7/2}$	$W^{-1/2}_{5/2}$	$W^{-1/2}_{3/2}$	$W^{-3/2}_{7/2}$	$W^{-3/2}_{5/2}$	$W^{-3/2}_{3/2}$	$W^{-5/2}_{7/2}$	$W^{-5/2}_{5/2}$	$W^{-7/2}_{7/2}$
$U^{5/2}_{5/2}\ V^1_1$	1																	
$U^{5/2}_{5/2}\ V^1_1$		$\sqrt{\frac{2}{7}}$	$\sqrt{\frac{5}{7}}$															
$U^{3/2}_{5/2}\ V^1_1$		$\sqrt{\frac{5}{7}}$	$\sqrt{\frac{2}{7}}$															
$U^{3/2}_{5/2}\ V^{-1}_1$				$\sqrt{\frac{1}{21}}$	$\sqrt{\frac{2}{7}}$	$\sqrt{\frac{2}{3}}$												
$U^{3/2}_{5/2}\ V^0_1$				$\sqrt{\frac{10}{21}}$	$\sqrt{\frac{9}{35}}$	$-\sqrt{\frac{4}{15}}$												
$U^{1/2}_{5/2}\ V^1_1$				$\sqrt{\frac{10}{21}}$	$-\sqrt{\frac{16}{35}}$	$\sqrt{\frac{1}{15}}$												
$U^{3/2}_{5/2}\ V^{-1}_1$							$\sqrt{\frac{1}{7}}$	$\sqrt{\frac{16}{35}}$	$\sqrt{\frac{2}{5}}$									
$U^{1/2}_{5/2}\ V^0_1$							$\sqrt{\frac{4}{7}}$	$\sqrt{\frac{1}{35}}$	$-\sqrt{\frac{2}{5}}$									
$U^{-1/2}_{5/2}\ V^1_1$							$\sqrt{\frac{2}{7}}$	$-\sqrt{\frac{18}{35}}$	$\sqrt{\frac{1}{5}}$									
$U^{1/2}_{5/2}\ V^1_1$										$\sqrt{\frac{2}{7}}$	$\sqrt{\frac{18}{35}}$	$\sqrt{\frac{1}{5}}$						
$U^{-1/2}_{5/2}\ V^0_1$										$\sqrt{\frac{4}{7}}$	$-\sqrt{\frac{1}{35}}$	$-\sqrt{\frac{2}{5}}$						
$U^{-3/2}_{5/2}\ V^1_1$										$\sqrt{\frac{1}{7}}$	$-\sqrt{\frac{16}{35}}$	$\sqrt{\frac{2}{5}}$						
$U^{-1/2}_{5/2}\ V^{-1}_1$													$\sqrt{\frac{10}{21}}$	$\sqrt{\frac{16}{35}}$	$\sqrt{\frac{1}{15}}$			
$U^{-3/2}_{5/2}\ V^0_1$													$\sqrt{\frac{10}{21}}$	$-\sqrt{\frac{9}{35}}$	$-\sqrt{\frac{4}{15}}$			
$U^{-3/2}_{5/2}\ V^1_1$													$\sqrt{\frac{1}{21}}$	$-\sqrt{\frac{2}{7}}$	$\sqrt{\frac{2}{3}}$			
$U^{-3/2}_{5/2}\ V^{-1}_1$																$\sqrt{\frac{5}{7}}$	$\sqrt{\frac{2}{7}}$	
$U^{-5/2}_{5/2}\ V^0_1$																$\sqrt{\frac{2}{7}}$	$-\sqrt{\frac{5}{7}}$	
$U^{-5/2}_{5/2}\ V^{-1}_1$																		1

TABLE XV

$D_2 \times D_{3/2}$

	$W^{7/2}_{7/2}$	$W^{5/2}_{7/2}$	$W^{5/2}_{5/2}$	$W^{3/2}_{7/2}$	$W^{3/2}_{5/2}$	$W^{3/2}_{3/2}$	$W^{1/2}_{7/2}$	$W^{1/2}_{5/2}$	$W^{1/2}_{3/2}$	$W^{1/2}_{1/2}$	$W^{-1/2}_{7/2}$	$W^{-1/2}_{5/2}$	$W^{-1/2}_{3/2}$	$W^{-1/2}_{1/2}$	$W^{-3/2}_{7/2}$	$W^{-3/2}_{5/2}$	$W^{-3/2}_{3/2}$	$W^{-5/2}_{7/2}$	$W^{-5/2}_{5/2}$	$W^{-7/2}_{7/2}$
$U_2^2\ V^{3/2}_{3/2}$	1																			
$U_2^2\ V^{1/2}_{3/2}$		$\sqrt{\frac{3}{7}}$	$\sqrt{\frac{4}{7}}$																	
$U_2^1\ V^{3/2}_{3/2}$		$\sqrt{\frac{4}{7}}$	$-\sqrt{\frac{3}{7}}$																	
$U_2^2\ V^{-1/2}_{3/2}$				$\sqrt{\frac{1}{7}}$	$\sqrt{\frac{16}{35}}$	$\sqrt{\frac{2}{5}}$														
$U_2^1\ V^{1/2}_{3/2}$				$\sqrt{\frac{4}{7}}$	$\sqrt{\frac{18}{35}}$	$-\sqrt{\frac{2}{5}}$														
$U_2^0\ V^{3/2}_{3/2}$				$\sqrt{\frac{2}{7}}$	$-\sqrt{\frac{18}{35}}$	$\sqrt{\frac{1}{5}}$														
$U_2^2\ V^{-3/2}_{3/2}$							$\sqrt{\frac{1}{35}}$	$\sqrt{\frac{6}{35}}$	$\sqrt{\frac{2}{5}}$	$\sqrt{\frac{2}{5}}$										
$U_2^1\ V^{-1/2}_{3/2}$							$\sqrt{\frac{12}{35}}$	$\sqrt{\frac{5}{14}}$	0	$-\sqrt{\frac{3}{10}}$										
$U_2^0\ V^{1/2}_{3/2}$							$\sqrt{\frac{18}{35}}$	$\sqrt{\frac{3}{35}}$	$-\sqrt{\frac{1}{5}}$	$\sqrt{\frac{1}{5}}$										
$U_2^{-1}\ V^{3/2}_{3/2}$							$\sqrt{\frac{4}{35}}$	$-\sqrt{\frac{27}{70}}$	$\sqrt{\frac{2}{5}}$	$-\sqrt{\frac{1}{10}}$										
$U_2^1\ V^{-3/2}_{3/2}$											$\sqrt{\frac{4}{35}}$	$\sqrt{\frac{27}{70}}$	$\sqrt{\frac{2}{5}}$	$\sqrt{\frac{1}{10}}$						
$U_2^0\ V^{-1/2}_{3/2}$											$\sqrt{\frac{18}{35}}$	$\sqrt{\frac{3}{35}}$	$-\sqrt{\frac{1}{5}}$	$-\sqrt{\frac{1}{5}}$						
$U_2^{-1}\ V^{1/2}_{3/2}$											$\sqrt{\frac{12}{35}}$	$-\sqrt{\frac{5}{14}}$	0	$\sqrt{\frac{3}{10}}$						
$U_2^{-2}\ V^{3/2}_{3/2}$											$\sqrt{\frac{1}{35}}$	$-\sqrt{\frac{6}{35}}$	$\sqrt{\frac{2}{5}}$	$-\sqrt{\frac{2}{5}}$						
$U_2^0\ V^{-3/2}_{3/2}$															$\sqrt{\frac{2}{7}}$	$\sqrt{\frac{18}{35}}$	$\sqrt{\frac{1}{5}}$			
$U_2^{-1}\ V^{-1/2}_{3/2}$															$\sqrt{\frac{4}{7}}$	$-\sqrt{\frac{1}{35}}$	$-\sqrt{\frac{2}{5}}$			
$U_2^{-2}\ V^{1/2}_{3/2}$															$\sqrt{\frac{1}{7}}$	$-\sqrt{\frac{16}{35}}$	$\sqrt{\frac{2}{5}}$			
$U_2^{-1}\ V^{-3/2}_{3/2}$																		$\sqrt{\frac{4}{7}}$	$\sqrt{\frac{3}{7}}$	
$U_2^{-2}\ V^{-1/2}_{3/2}$																		$\sqrt{\frac{3}{7}}$	$-\sqrt{\frac{4}{7}}$	
$U_2^{-2}\ V^{-3/2}_{3/2}$																				1

TABLE XVI

$\underline{D_{7/2} \times D_{1/2}}$

	W_4^4	W_4^3	W_3^3	W_4^2	W_3^2	W_4^1	W_3^1	W_4^0	W_3^0	W_4^{-1}	W_3^{-1}	W_4^{-2}	W_3^{-2}	W_4^{-3}	W_3^{-3}	W_4^{-4}
$U_{7/2}^{7/2} V_{1/2}^{1/2}$	1															
$U_{7/2}^{7/2} V_{1/2}^{-1/2}$		$\sqrt{\frac{1}{8}}$	$\sqrt{\frac{7}{8}}$													
$U_{7/2}^{5/2} V_{1/2}^{1/2}$		$\sqrt{\frac{7}{8}}$	$-\sqrt{\frac{1}{8}}$													
$U_{7/2}^{5/2} V_{1/2}^{-1/2}$				$\sqrt{\frac{1}{4}}$	$\sqrt{\frac{3}{4}}$											
$U_{7/2}^{3/2} V_{1/2}^{1/2}$				$\sqrt{\frac{3}{4}}$	$-\sqrt{\frac{1}{4}}$											
$U_{7/2}^{3/2} V_{1/2}^{-1/2}$						$\sqrt{\frac{3}{8}}$	$\sqrt{\frac{5}{8}}$									
$U_{7/2}^{1/2} V_{1/2}^{1/2}$						$\sqrt{\frac{5}{8}}$	$-\sqrt{\frac{3}{8}}$									
$U_{7/2}^{1/2} V_{1/2}^{-1/2}$								$\sqrt{\frac{1}{2}}$	$\sqrt{\frac{1}{2}}$							
$U_{7/2}^{-1/2} V_{1/2}^{1/2}$								$\sqrt{\frac{1}{2}}$	$-\sqrt{\frac{1}{2}}$							
$U_{7/2}^{-1/2} V_{1/2}^{-1/2}$										$\sqrt{\frac{5}{8}}$	$\sqrt{\frac{3}{8}}$					
$U_{7/2}^{-3/2} V_{1/2}^{1/2}$										$\sqrt{\frac{3}{8}}$	$-\sqrt{\frac{5}{8}}$					
$U_{7/2}^{-3/2} V_{1/2}^{-1/2}$												$\sqrt{\frac{3}{4}}$	$\sqrt{\frac{1}{4}}$			
$U_{7/2}^{-5/2} V_{1/2}^{1/2}$												$\sqrt{\frac{1}{4}}$	$-\sqrt{\frac{3}{4}}$			
$U_{7/2}^{-5/2} V_{1/2}^{-1/2}$														$\sqrt{\frac{7}{8}}$	$\sqrt{\frac{1}{8}}$.	
$U_{7/2}^{-7/2} V_{1/2}^{1/2}$														$\sqrt{\frac{1}{8}}$	$-\sqrt{\frac{7}{8}}$	
$U_{7/2}^{7/2} V_{1/2}^{-1/2}$																1

TABLE XVII

$\underline{D}_3 \times \underline{D}_1$

	W_4^4	W_4^3	W_3^3	W_4^2	W_3^2	W_2^2	W_4^1	W_3^1	W_2^2	W_4^0	W_3^0	W_2^0	W_4^{-1}	W_3^{-1}	W_2^{-1}	W_4^{-2}	W_3^{-2}	W_2^{-2}	W_4^{-3}	W_3^{-3}	W_4^{-4}
$U_3^3\ V_1^1$	1																				
$U_3^3\ V_1^0$		$\sqrt{\frac{1}{4}}$	$\sqrt{\frac{3}{4}}$																		
$U_3^2\ V_1^1$		$\sqrt{\frac{3}{4}}$	$-\sqrt{\frac{1}{4}}$																		
$U_3^3\ V_1^{-1}$				$\sqrt{\frac{1}{28}}$	$\sqrt{\frac{1}{4}}$	$\sqrt{\frac{5}{7}}$															
$U_3^2\ V_1^0$				$\sqrt{\frac{3}{7}}$	$\sqrt{\frac{1}{3}}$	$-\sqrt{\frac{5}{21}}$															
$U_3^1\ V_1^1$				$\sqrt{\frac{15}{28}}$	$-\sqrt{\frac{5}{12}}$	$\sqrt{\frac{1}{21}}$															
$U_3^2\ V_1^{-1}$							$\sqrt{\frac{3}{28}}$	$\sqrt{\frac{5}{12}}$	$\sqrt{\frac{10}{21}}$												
$U_3^1\ V_1^0$							$\sqrt{\frac{15}{28}}$	$\sqrt{\frac{1}{12}}$	$-\sqrt{\frac{8}{21}}$												
$U_3^0\ V_1^1$							$\sqrt{\frac{10}{28}}$	$-\sqrt{\frac{6}{12}}$	$\sqrt{\frac{3}{21}}$												
$U_3^1\ V_1^{-1}$										$\sqrt{\frac{3}{14}}$	$\sqrt{\frac{1}{2}}$	$\sqrt{\frac{2}{7}}$									
$U_3^0\ V_1^0$										$\sqrt{\frac{8}{14}}$	0	$-\sqrt{\frac{3}{7}}$									
$U_3^{-1}\ V_1^1$										$\sqrt{\frac{3}{14}}$	$-\sqrt{\frac{1}{2}}$	$\sqrt{\frac{2}{7}}$									
$U_3^0\ V_1^{-1}$													$\sqrt{\frac{10}{28}}$	$\sqrt{\frac{6}{12}}$	$\sqrt{\frac{3}{21}}$						
$U_3^{-1}\ V_1^0$													$\sqrt{\frac{15}{28}}$	$-\sqrt{\frac{1}{12}}$	$-\sqrt{\frac{8}{21}}$						
$U_3^{-2}\ V_1^1$													$\sqrt{\frac{3}{28}}$	$-\sqrt{\frac{5}{12}}$	$\sqrt{\frac{10}{21}}$						
$U_3^{-1}\ V_1^{-1}$																$\sqrt{\frac{15}{28}}$	$\sqrt{\frac{5}{12}}$	$\sqrt{\frac{1}{21}}$			
$U_3^{-2}\ V_1^0$																$\sqrt{\frac{3}{7}}$	$-\sqrt{\frac{1}{3}}$	$-\sqrt{\frac{5}{21}}$			
$U_3^{-3}\ V_1^1$																$\sqrt{\frac{1}{28}}$	$-\sqrt{\frac{1}{4}}$	$\sqrt{\frac{5}{7}}$			
$U_3^{-2}\ V_1^{-1}$																			$\sqrt{\frac{3}{4}}$	$\sqrt{\frac{1}{4}}$	
$U_3^{-3}\ V_1^0$																			$\sqrt{\frac{1}{4}}$	$-\sqrt{\frac{3}{4}}$	
$U_3^{-3}\ V_1^{-1}$																					1

TABLE XVIII

$D_{5/2} \times D_{3/2}$

	W_4^4	W_4^3	W_3^3	W_4^2	W_3^2	W_2^2	W_4^1	W_3^1	W_2^1	W_1^1	W_4^0	W_3^0	W_2^0	W_1^0	W_4^{-1}	W_3^{-1}	W_2^{-1}	W_1^{-1}	W_4^{-2}	W_3^{-2}	W_2^{-2}	W_4^{-3}	W_3^{-3}	W_4^{-4}
$U_{5/2}^{5/2} V_{3/2}^{3/2}$	1																							
$U_{5/2}^{5/2} V_{3/2}^{1/2}$		$\sqrt{\frac{3}{8}}$	$\sqrt{\frac{5}{8}}$																					
$U_{5/2}^{3/2} V_{3/2}^{3/2}$		$\sqrt{\frac{5}{8}}$	$-\sqrt{\frac{3}{8}}$																					
$U_{5/2}^{5/2} V_{3/2}^{-1/2}$				$\sqrt{\frac{3}{28}}$	$\sqrt{\frac{5}{12}}$	$\sqrt{\frac{10}{21}}$																		
$U_{5/2}^{3/2} V_{3/2}^{1/2}$				$\sqrt{\frac{15}{28}}$	$\sqrt{\frac{1}{12}}$	$-\sqrt{\frac{8}{21}}$																		
$U_{5/2}^{1/2} V_{3/2}^{3/2}$				$\sqrt{\frac{10}{28}}$	$-\sqrt{\frac{6}{12}}$	$\sqrt{\frac{3}{21}}$																		
$U_{5/2}^{5/2} V_{3/2}^{-3/2}$							$\sqrt{\frac{1}{56}}$	$\sqrt{\frac{1}{8}}$	$\sqrt{\frac{5}{14}}$	$\sqrt{\frac{1}{2}}$														
$U_{5/2}^{3/2} V_{3/2}^{-1/2}$							$\sqrt{\frac{15}{56}}$	$\sqrt{\frac{49}{120}}$	$\sqrt{\frac{1}{42}}$	$-\sqrt{\frac{3}{10}}$														
$U_{5/2}^{1/2} V_{3/2}^{1/2}$							$\sqrt{\frac{30}{56}}$	$-\sqrt{\frac{1}{60}}$	$-\sqrt{\frac{25}{84}}$	$\sqrt{\frac{3}{20}}$														
$U_{5/2}^{-1/2} V_{3/2}^{3/2}$							$\sqrt{\frac{10}{56}}$	$-\sqrt{\frac{9}{20}}$	$\sqrt{\frac{27}{84}}$	$-\sqrt{\frac{1}{20}}$														
$U_{5/2}^{3/2} V_{3/2}^{-3/2}$											$\sqrt{\frac{1}{14}}$	$\sqrt{\frac{3}{10}}$	$\sqrt{\frac{3}{7}}$	$\sqrt{\frac{1}{5}}$										
$U_{5/2}^{1/2} V_{3/2}^{-1/2}$											$\sqrt{\frac{3}{7}}$	$\sqrt{\frac{1}{5}}$	$-\sqrt{\frac{1}{14}}$	$-\sqrt{\frac{3}{10}}$										
$U_{5/2}^{-1/2} V_{3/2}^{1/2}$											$\sqrt{\frac{3}{7}}$	$-\sqrt{\frac{1}{5}}$	$-\sqrt{\frac{1}{14}}$	$\sqrt{\frac{3}{10}}$										
$U_{5/2}^{-3/2} V_{3/2}^{3/2}$											$\sqrt{\frac{1}{14}}$	$-\sqrt{\frac{3}{10}}$	$\sqrt{\frac{3}{7}}$	$-\sqrt{\frac{1}{5}}$										
$U_{5/2}^{1/2} V_{3/2}^{-3/2}$															$\sqrt{\frac{10}{56}}$	$\sqrt{\frac{9}{20}}$	$\sqrt{\frac{9}{28}}$	$\sqrt{\frac{1}{20}}$						
$U_{5/2}^{-1/2} V_{3/2}^{-1/2}$															$\sqrt{\frac{30}{56}}$	$\sqrt{\frac{1}{60}}$	$-\sqrt{\frac{25}{84}}$	$-\sqrt{\frac{3}{20}}$						
$U_{5/2}^{-3/2} V_{3/2}^{1/2}$															$\sqrt{\frac{15}{56}}$	$-\sqrt{\frac{49}{120}}$	$\sqrt{\frac{1}{42}}$	$\sqrt{\frac{3}{10}}$						
$U_{5/2}^{-5/2} V_{3/2}^{3/2}$															$\sqrt{\frac{1}{56}}$	$-\sqrt{\frac{1}{8}}$	$\sqrt{\frac{5}{14}}$	$-\sqrt{\frac{1}{2}}$						
$U_{5/2}^{-1/2} V_{3/2}^{-3/2}$																			$\sqrt{\frac{10}{28}}$	$\sqrt{\frac{6}{12}}$	$\sqrt{\frac{1}{7}}$			
$U_{5/2}^{-3/2} V_{3/2}^{-1/2}$																			$\sqrt{\frac{15}{28}}$	$-\sqrt{\frac{1}{12}}$	$-\sqrt{\frac{8}{21}}$			
$U_{5/2}^{-5/2} V_{3/2}^{1/2}$																			$\sqrt{\frac{3}{28}}$	$-\sqrt{\frac{5}{12}}$	$\sqrt{\frac{10}{21}}$			
$U_{5/2}^{-3/2} V_{3/2}^{-3/2}$																						$\sqrt{\frac{5}{8}}$	$\sqrt{\frac{3}{8}}$	
$U_{5/2}^{-5/2} V_{3/2}^{-1/2}$																						$\sqrt{\frac{3}{8}}$	$-\sqrt{\frac{5}{8}}$	
$U_{5/2}^{-5/2} V_{3/2}^{-3/2}$																								1

<u>TABLE XIX</u>

$\underline{D_2 \times D_2}$

	W^4_4	W^3_4	W^3_3	W^2_4	W^2_3	W^2_2	W^1_4	W^1_3	W^1_2	W^1_1	W^0_4	W^0_3	W^0_2	W^0_1	W^0_0	W^{-1}_4	W^{-1}_3	W^{-1}_2	W^{-1}_1	W^{-2}_4	W^{-2}_3	W^{-2}_2	W^{-3}_4	W^{-3}_3	W^{-4}_4
$U^2_2\ V^2_2$	1																								
$U^2_2\ V^1_2$		$\sqrt{\frac{1}{2}}$	$\sqrt{\frac{1}{2}}$																						
$U^1_2\ V^2_2$		$\sqrt{\frac{1}{2}}$	$-\sqrt{\frac{1}{2}}$																						
$U^2_2\ V^0_2$				$\sqrt{\frac{3}{14}}$	$\sqrt{\frac{1}{2}}$	$\sqrt{\frac{2}{7}}$																			
$U^1_2\ V^1_2$				$\sqrt{\frac{8}{14}}$	0	$\sqrt{\frac{3}{7}}$																			
$U^0_2\ V^2_2$				$\sqrt{\frac{3}{14}}$	$-\sqrt{\frac{1}{2}}$	$\sqrt{\frac{2}{7}}$																			
$U^2_2\ V^{-1}_2$							$\sqrt{\frac{1}{14}}$	$\sqrt{\frac{3}{10}}$	$\sqrt{\frac{6}{14}}$	$\sqrt{\frac{2}{10}}$															
$U^1_2\ V^0_2$							$\sqrt{\frac{6}{14}}$	$\sqrt{\frac{2}{10}}$	$-\sqrt{\frac{1}{14}}$	$-\sqrt{\frac{3}{10}}$															
$U^0_2\ V^1_2$							$\sqrt{\frac{6}{14}}$	$-\sqrt{\frac{2}{10}}$	$-\sqrt{\frac{1}{14}}$	$\sqrt{\frac{3}{10}}$															
$U^{-1}_2\ V^2_2$							$\sqrt{\frac{1}{14}}$	$-\sqrt{\frac{3}{10}}$	$\sqrt{\frac{6}{14}}$	$-\sqrt{\frac{2}{10}}$															
$U^2_2\ V^{-2}_2$											$\sqrt{\frac{1}{70}}$	$\sqrt{\frac{1}{10}}$	$\sqrt{\frac{4}{14}}$	$\sqrt{\frac{4}{10}}$	$\sqrt{\frac{1}{5}}$										
$U^1_2\ V^{-1}_2$											$\sqrt{\frac{16}{70}}$	$\sqrt{\frac{4}{10}}$	$\sqrt{\frac{1}{14}}$	$-\sqrt{\frac{1}{10}}$	$-\sqrt{\frac{1}{5}}$										
$U^0_2\ V^0_2$											$\sqrt{\frac{36}{70}}$	0	$-\sqrt{\frac{4}{14}}$	0	$\sqrt{\frac{1}{5}}$										
$U^{-1}_2\ V^1_2$											$\sqrt{\frac{16}{70}}$	$-\sqrt{\frac{4}{10}}$	$\sqrt{\frac{1}{14}}$	$\sqrt{\frac{1}{10}}$	$-\sqrt{\frac{1}{5}}$										
$U^{-2}_2\ V^2_2$											$\sqrt{\frac{1}{70}}$	$-\sqrt{\frac{1}{10}}$	$\sqrt{\frac{4}{14}}$	$-\sqrt{\frac{4}{10}}$	$\sqrt{\frac{1}{5}}$										
$U^1_2\ V^{-2}_2$																$\sqrt{\frac{1}{14}}$	$\sqrt{\frac{3}{10}}$	$\sqrt{\frac{6}{14}}$	$\sqrt{\frac{2}{10}}$						
$U^0_2\ V^{-1}_2$																$\sqrt{\frac{6}{14}}$	$\sqrt{\frac{2}{10}}$	$-\sqrt{\frac{1}{14}}$	$-\sqrt{\frac{3}{10}}$						
$U^{-1}_2\ V^0_2$																$\sqrt{\frac{6}{14}}$	$-\sqrt{\frac{2}{10}}$	$-\sqrt{\frac{1}{14}}$	$\sqrt{\frac{3}{10}}$						
$U^{-2}_2\ V^{-2}_2$																$\sqrt{\frac{1}{14}}$	$\sqrt{\frac{3}{10}}$	$\sqrt{\frac{6}{14}}$	$-\sqrt{\frac{2}{10}}$						
$U^0_2\ V^{-2}_2$																				$\sqrt{\frac{3}{14}}$	$\sqrt{\frac{1}{2}}$	$\sqrt{\frac{2}{7}}$			
$U^{-1}_2\ V^{-1}_2$																				$\sqrt{\frac{8}{14}}$	0	$-\sqrt{\frac{3}{7}}$			
$U^{-2}_2\ V^0_2$																				$\sqrt{\frac{3}{14}}$	$-\sqrt{\frac{1}{2}}$	$\sqrt{\frac{2}{7}}$			
$U^{-1}_2\ V^{-2}_2$																							$\sqrt{\frac{1}{2}}$	$\sqrt{\frac{1}{2}}$	
$U^{-2}_2\ V^{-1}_2$																							$\sqrt{\frac{1}{2}}$	$-\sqrt{\frac{1}{2}}$	
$U^{-2}_2\ V^{-2}_2$																									1

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