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DETERMINATION OF THE NEUTRON DIFFUSION PARAMETERS IN ROOM-TEMPERATURE BERYLLIUM

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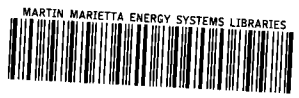

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ABSTRACT

The pulsed neutron method was used to determine the diffusion parameters of thermal neutrons in beryllium at room temperature. The following values were obtained: inverse lifetime $v\Sigma_a = 288 \pm 60$ sec^{-1}, diffusion constant $D = (1.25 \pm 0.06) \times 10^5$ cm^2/sec, diffusion cooling constant $C = 1.1 \pm 0.9$ cm^2. The experiment and the method of analyzing the data are described and the results are compared with those obtained by other groups.

INTRODUCTION

The pulsed-neutron-source technique provides a powerful tool to investigate the interaction of neutrons and matter. Basically the technique consists of observing the time behavior of the neutron density following the injection of a burst of neutrons into the medium under study. The method can be applied to study the slowing down and the diffusion of neutrons in multiplying or in non-multiplying media.[1] At the Oak Ridge National Laboratory a program is in progress to measure various reactor parameters by this method.[2,4] Part of this program was a careful determination of the diffusion parameters of neutrons in room-temperature beryllium. The results provide some insight into the problem of diffusion of neutrons in crystalline materials as well as some useful parameters for the calculation of reactors. This report describes the method by which the data was obtained and discusses the experimental results.

I. THEORY OF THE PULSED-NEUTRON-SOURCE METHOD

In the last few years the pulsed-neutron-source method has been used extensively to measure neutron-diffusion parameters in various moderators.[1] With this method a sample of moderator is irradiated by a short burst of fast neutrons (from a few microseconds to a few milliseconds), and the decay of the neutron density at the surface of the sample is observed. The fast neutrons inside the sample rapidly slow down to thermal velocities (slowing-down time in beryllium $\approx$ 100 μsec). When the neutrons approach thermal equilibrium with the moderator, the neutrons continue to leak "slowly" out of the system or are absorbed. The thermal-neutron density, n, then satisfies the following differential equation:

$$D\Delta n - v\Sigma_a n = \frac{\partial n}{\partial t}, \qquad D = \frac{\lambda_{tr}}{3} v, \tag{1}$$

where

D = diffusion coefficient,
λ_{tr} = transport mean free path,
v = velocity of the neutrons,
Σ_a = macroscopic absorption cross section,
$\partial n/\partial t$ = time derivative of the neutron density, n.

The solution of Eq. 1 which satisfies the proper boundary condition (the condition that the flux vanishes at the extrapolated boundary, $\vec{R}$, of the moderator) is

$$n(\vec{r},t) = \sum_{l,m,n} A_{lmn} S_{lmn}(\vec{r}) e^{-(v\Sigma_a + DB^2_{lmn})t}, \tag{2}$$

where the A_{lmn} are constants that depend on the source position, and S_{lmn} and B^2_{lmn} are, respectively, the eigenfunctions and eigenvalues (also called bucklings) of the equation

$$\Delta S_{lmn}(\vec{r}) = B^2_{lmn} S_{lmn}(\vec{r}), \tag{3}$$

which satisfies the given boundary condition

$$S_{lmn}(\vec{R}) = 0. \tag{3a}$$

Equation 2 shows that the neutron density $n(\vec{r},t)$ can be represented as a sum of exponential terms (modes). The term with the smallest decay constant (fundamental mode) will eventually become the dominant term. Thus, after a short time, Eq. 2 reduces to

$$n(\vec{r},t) \approx AS(r)\, e^{-\lambda t}, \tag{4}$$

$$\lambda = v\Sigma_a + DB^2,$$

where the indices l = m = n = 1 have been dropped for simplification.

Hence the measurement of the decay constant λ of the neutron population, as a function of the size or buckling B^2 of the moderating assembly, will permit the determination of the absorption cross section Σ_a and of the diffusion coefficient D.

Equation 1 describes the diffusion of monoenergetic neutrons. In reality the neutrons are not monoenergetic, but the neutron density still satisfies Eq. 1 in most cases if the parameters are properly averaged over the flux. Neutrons diffusing in an infinite medium of negligible absorption have a Maxwell-Boltzmann distribution in thermal equilibrium with the moderator. In this case the properly averaged transport mean free path is[5]

$$\lambda_{tr} = \frac{A}{L\rho} \frac{\int \frac{Ee^{-E/kT}}{\sigma_{tr}(E,T)} dE}{\int Ee^{-E/kT} dE} \tag{5}$$

where

L = Avogadro's number,

A = atomic weight of the moderator,

ρ = density of the moderator,

$\sigma_{tr}(E,T)$ = microscopic transport cross section for neutrons of energy E in the moderator at absolute temperature T,

k = Boltzmann's constant.

Neutrons diffusing through a moderator of finite size may never have a Maxwell-Boltzmann distribution in thermal equilibrium with the moderator. The neutrons that have a large diffusion coefficient (large velocity or large transport mean free path) leak out of the system first. A two-group approximation will lead to a better understanding of the diffusion process than will the monoenergetic approximation of Eq. 1. Two neutron groups, n_1 and n_2, with diffusion coefficients D_1 and D_2, respectively, will be considered. They satisfy the following pair of differential equations:

$$\left(-D_1B^2 - v\Sigma_a - \beta_{12} - \frac{\partial}{\partial t}\right) n_1 + \beta_{21}n_2 = 0, \tag{6}$$

$$\left(-D_2B^2 - v\Sigma_a - \beta_{21} - \frac{\partial}{\partial t}\right) n_2 + \beta_{12}n_1 = 0, \tag{7}$$

where the product $v\Sigma_a$ and the buckling B^2 are considered to be the same for the two groups for simplification; β_{12} and β_{21} are the probabilities per unit time for neutrons of group n_1 to be scattered into group n_2 and for neutrons of group n_2 to be scattered into group n_1. If it is assumed that $\beta_{12} = \beta_{21}$, the solution of Eqs. 6 and 7 is given by

$$n = n_1 + n_2 = e^{-(\lambda + \beta)t} (P e^{\mu t} + Q e^{-\mu t}), \tag{8}$$

where P and Q are constants and

$$\lambda = v\Sigma_a + \frac{D_1 + D_2}{2} B^2,$$

$$\mu^2 = \beta^2 + \left(\frac{D_1 - D_2}{2} \right)^2 B^4. \tag{9}$$

Equation 8 is the sum of two terms decaying with different periods, and after a certain time only the term with the smaller decay constant $(\lambda + \beta - \mu)$ will remain. Furthermore, for moderating geometries of large size (small buckling), μ^2 can be developed in series, leading to

$$N \approx P e^{-\lambda' t}, \tag{10}$$

where

$$\lambda' \approx v\Sigma_a + \bar{D}B^2 - \frac{(D_1 - D_2)^2}{8\beta} B^4,$$

$$D = \frac{(D_1 + D_2)}{2}.$$

The two-group assumption used to obtain Eq. 10 is only a very rough approximation of the true situation, but it indicates, through Eq. 10, that for large sizes of the diffusing media the plot of the decay constant λ of the neutron population vs the buckling B^2 should be a parabola:

$$\lambda = v\Sigma_a + \bar{D}B^2 - A_2 B^4, \tag{11}$$

where A_2 is a positive parameter whose magnitude depends on the spread of the diffusion coefficient D(E) over the neutron spectrum. It has become customary to write Eq. 11 in the form:

$$\lambda = v\Sigma_a + DB^2(1 - CB^2) \qquad (11a)$$

Parameter C of Eq. 11, often called the diffusion cooling constant, has been observed experimentally by many researchers.[1] A theoretical interpretation was given first by von Dardel on a somewhat different basis than the one discussed above.[6,7] The "two-group theory" given above was first developed by Antonov et al.[8] A more satisfactory interpretation of Eq. 11, due to M. Nelkin, will be discussed below in the theoretical discussion of the results.[9]

II. DESCRIPTION OF THE EXPERIMENTAL ARRANGEMENT

The equipment consists of a pulsed neutron source, a moderating assembly, a thermal neutron detector, and a time analyzer. Figure 1 is a block diagram of the experimental arrangement; the various components are described below.

a. The Pulsed Neutron Source

The pulsed neutron source consists of a 300-kev particle accelerator in which bursts of deuterons are accelerated. The deuterons, striking a target containing deuterium or tritium, produce fast neutrons by the $d(d,n)He^3$ or the $d(t,n)He^4$ reaction. A UHF radio link connects a transmitter at ground potential to a receiver at the high voltage end of the accelerator. Through this link the length and repetition rate of the deuteron pulses are controlled by means of a set of deflection plates just below the deuterium ion source.[10]

b. The Moderating Assemblies

The moderating assemblies are almost-cubical prisms of various sizes. The modal bucklings of a cube with edge a are given by

$$B^2_{lmn} = \frac{\pi^2}{\tilde{a}^2}(l^2 + m^2 + n^2) \qquad (12)$$

where $\tilde{a} = a + 1.42\,\lambda_{tr}$ is the "extrapolated size" of the side of the cube. If the neutron source is on an axis of symmetry perpendicular to two faces of the cube and the detector on the center of another face of the cube, only modes having l,m, and n odd are detected, since all other modes have a node at the detector or at the source. The lowest odd mode, after the fundamental, B^2_{113}, decays about 11/3 times as fast as the fundamental mode (in most sizes of moderators considered, the leakage DB^2 is much larger than the absorption $v\Sigma_a$). Under these conditions it is possible to separate the fundamental mode from its higher harmonics and use Eq. 4.

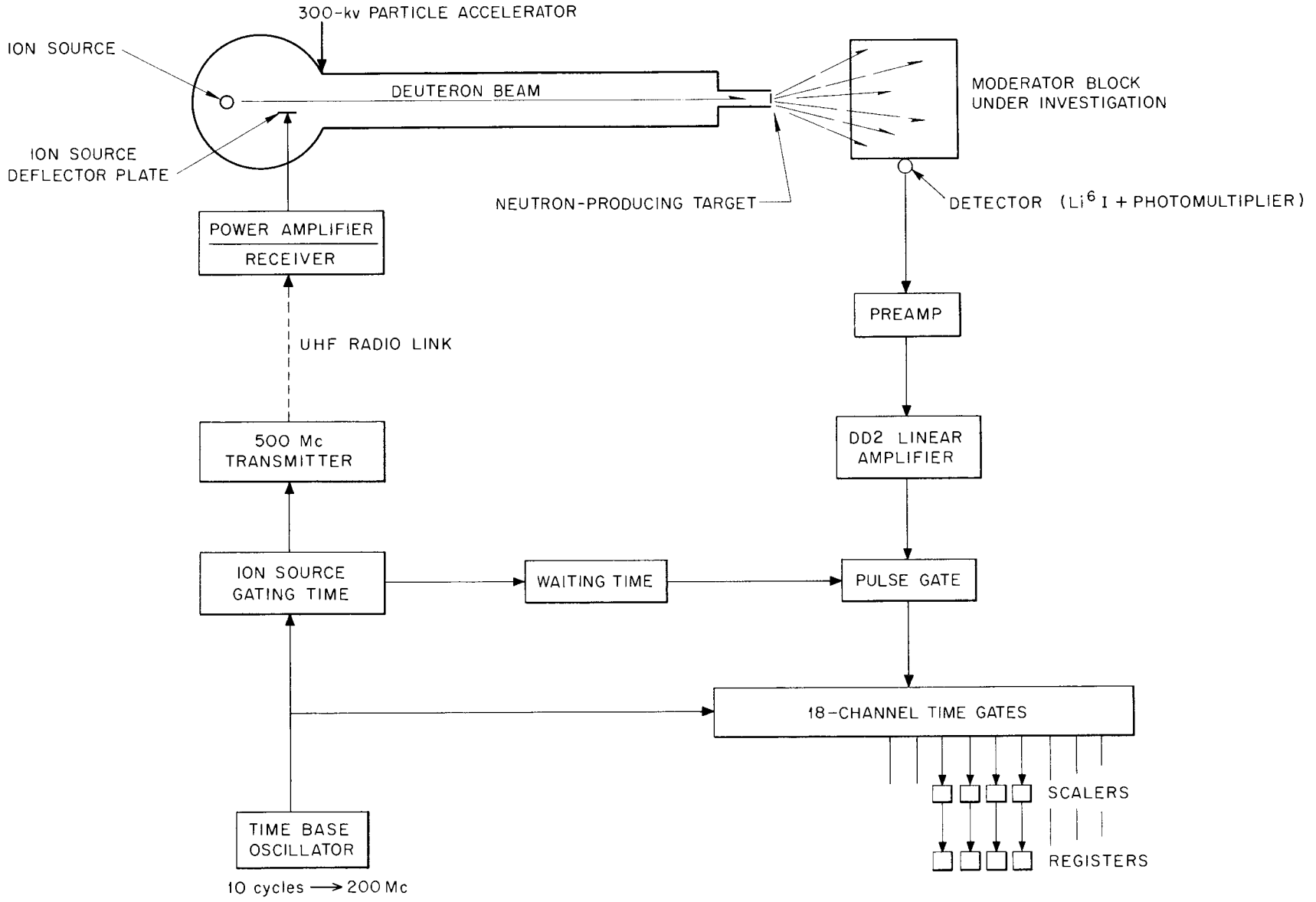

Fig. 1. Block Diagram of the Experimental Arrangement for Measurement of the Diffusion Parameters in Room Temperature Beryllium.

c. The Thermal Neutron Detector

The detector consists of a $Li^6I(Eu)$ scintillation crystal, about 2.5 cm in dia. and 0.3 cm in thickness, optically connected to an RCA 6655 2-in.-dia photomultiplier. The signals from the photomultiplier are amplified through a preamplifier and a DD2 amplifier.[11] The signals from the differential pulse-height analyzer of the amplifier are fed to the time analyzer. The Li^6I crystal is black to thermal neutrons. It responds to neutrons by means of the reaction $Li^6(n,\alpha)H^3$. The disintegration products lose their energy in the crystal and cause a large and well-defined scintillation pulse, which can be discriminated from pulses caused by gamma rays.[12]

d. The Time Analyzer[13]

The signals from the neutron detector are fed into an 18-channel time analyzer. The width of the time channels can be varied from a few microseconds to a few hundred milliseconds. The width of the fast neutron pulse and the waiting time between the end of the fast neutron pulse and the beginning of the counting can be varied as well. The time analyzer records coincidences between the neutron detector signals and a time signal provided by an oscillator running down the 18 positions of two M010 Burroughs beam switching tubes in series. The frequency of the oscillator was continuously monitored on an intervalometer and kept constant within one part in 10^4.

III. TREATMENT OF THE DATA

A precise determination of the decay constant of the various beryllium assemblies is difficult for the following main reasons: (a) a small background is always present, perhaps due in part to the fact that the neutron-producing deuterium beam cannot be completely cut off during the measurement period. (b) Faster decays are superposed on the main decay to be measured. In large blocks these faster decays usually have negative amplitude and are probably caused by higher modes. They tend to "flatten" the decay curve and decrease the apparent decay constant of the large blocks. In small blocks the faster decays have positive amplitude and are probably associated with the neutron slowing-down which occurs during times comparable to the thermal decay time of these assemblies. Here the faster decays tend to increase the apparent decay constants of the small blocks.

The following semigraphic semianalytic method was used consistently to treat the data except for the smallest assembly.*

1. The logarithm of the difference between two successive channel counts was plotted against the channel number. This process eliminates any bias due to a constant background; if the number of counts in channel k is given by

$$n_k = ae^{-k\lambda t} + b \tag{13}$$

where

a and b are constants, λ is the decay constant and t is the channel width, then,

$$\ln(n_k - n_{k+1}) = \ln[a(1 - e^{-\lambda t})] - k\lambda t \tag{14}$$

* See Appendix I for a full discussion of this case.

By plotting Eq. 14 versus k one obtains a straight line of slope λt and the bias due to the constant background has been eliminated. The first few points of the plot $\ln(n_k - n_{k+1})$ versus k may be off the straight line due to the contribution of faster decays; this permits one to determine visually from what channel on the contributions of faster decays can be neglected.

2. The first few channels corresponding to points that do not fall on a straight line in the plot of $\ln(n_k - n_{k+1})$ vs k are rejected. The remaining original data points are then analyzed in terms of a single exponential decay and a constant background, by the Cornell method.[14]

Several hundred thousand counts were collected for each determination of a decay constant. All the decay constants are determined several times, using different channel widths. Successive determinations of the same decay agreed within better than 2%, except for the smallest assembly.

Typical data on the decay of three assemblies are given in Appendix I, as well as a description of the way in which the data was treated.

IV. RESULTS

The decay rate of the neutron population has been measured in eleven cubical blocks of beryllium having fundamental-mode bucklings ranging from 0.8×10^{-2} cm^{-2} to 7.2 to 10^{-2} cm^{-2}. Smaller bucklings could not be obtained with the limited amount of beryllium available. A special effort was made to measure the decay of very small blocks (large bucklings) as this allowed a better determination of the diffusion cooling constant C of Eq. 11a. As explained above, the difficulty of measuring the decay constant in such small blocks is that the lifetime of the thermal neutrons in the blocks is of the same order as the slowing-down time of the neutrons. Figures I-1, I-2, and I-3 show the decay curves for three blocks:* one of the largest, one of medium size and the smallest. They also show the plot of the logarithm of the first differences and the line corresponding to the decay constant of the given assembly. Figure 2 shows a plot of the decay constant vs. buckling. A parabola has been fitted to the experimental points by the method of least squares. The diffusion parameters corresponding to the coefficients of this parabola are presented in Table I with other values measured by the pulsed neutron method by other groups.[8,15,16] (An iteration process was used to calculate bucklings with an extrapolation length consistent with the experimentally determined diffusion constant.) In the last line of Table I calculated values of the three diffusion parameters are also given. Table II lists derived parameters calculated from the three diffusion parameters determined by this experiment. Table III lists the sizes and bucklings of the eleven assemblies that are used, as well as the experimentally determined decay constants, and decay constants calculated with the diffusion parameters given in Table I. Table IV lists various values of the diffusion parameters obtained by various methods of analyzing the data. In the first column the parameters are calculated using all eleven assemblies and fitting the data to a three-parameter curve. In the second column only the data of the ten largest assemblies were used to determine the parameters, using the same method as in the first column. This was done because

* Figures I-1, I-2, and I-3 can be found in Appendix I.

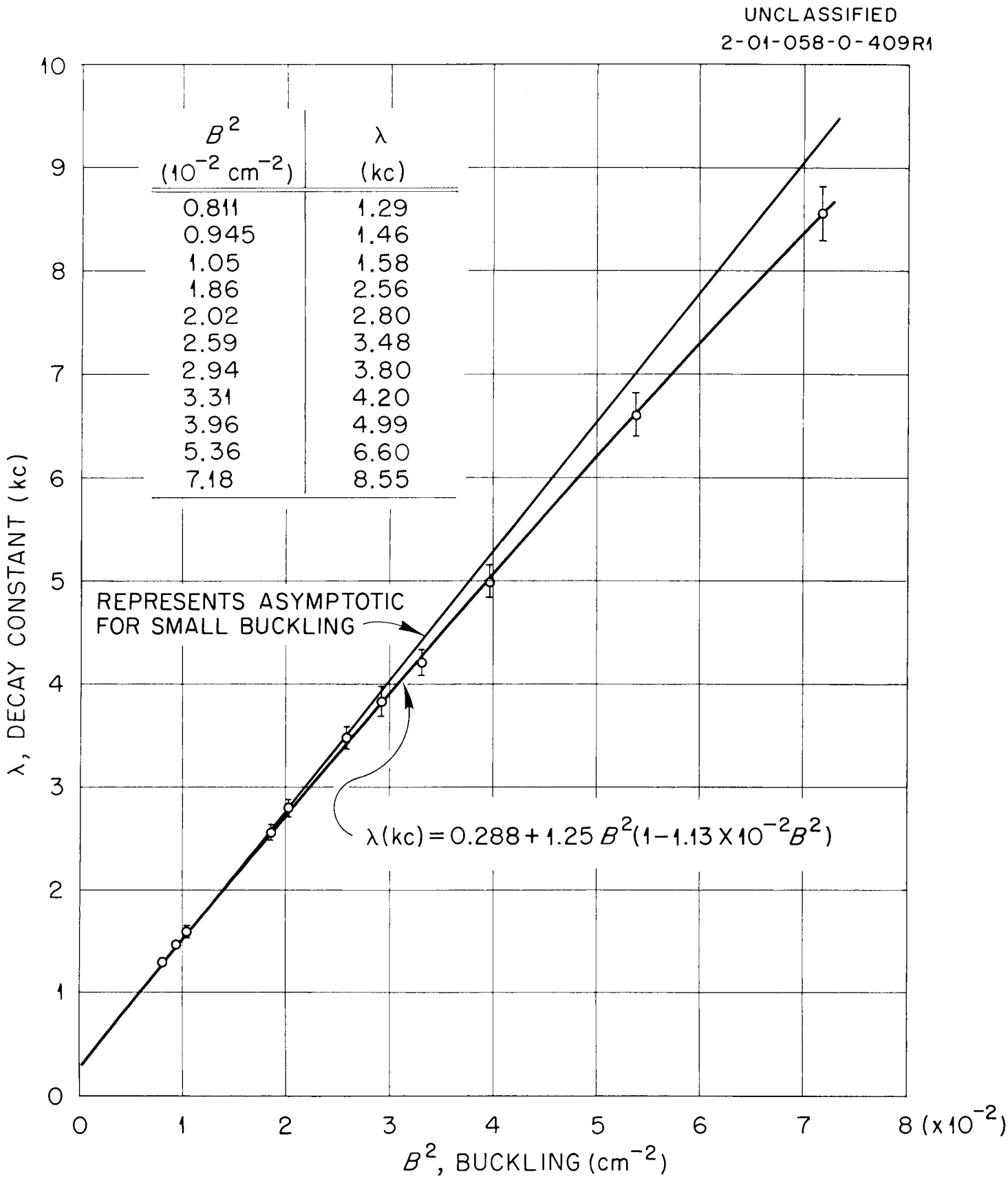

Fig. 2. Thermal Neutron Decay Constant as a Function of Buckling for Beryllium at Room Temperature.

the precision of the determination of the decay constant of the smallest assembly is not as good as for the other assemblies (see Appendix I). Finally in the last column the data obtained with the eleven assemblies have been fitted to a four-parameter curve; only the coefficient of B^6 was determined. It is seen that the value of this coefficient is small, and the precision of the measurements does not permit the conclusion that it is different from zero.

V. THEORETICAL CALCULATIONS OF THE RESULTS

In the last line of Table I some "calculated" values of the diffusion parameters are given. These values have been calculated from other measured parameters and a short discussion of these calculations will be given below.

a. $v\Sigma_a$, Calculation from σ_a

The microscopic 2200m/sec absorption cross section of beryllium is reported in BNL-325[17] as $\sigma_a = 10 \pm 1$ mb, as obtained by diffusion measurements; it is given as 10 ± 3 millibarns by Hughes,[18] as obtained by subtracting the inelastic cross section from the total cross section of cold neutrons. $v\Sigma_a$ can be calculated from this value:

$$v\Sigma_a = \frac{L\rho}{A} \cdot (v\sigma_a) = \frac{6.02 \times 10^{23} \times 1.85}{9.02} \times 2.2 \times 10^5 \times 10^{-26} = (272 \pm 30)\ \text{sec}^{-1} \quad (15)$$

where the symbols have the same meaning as in Eq. 5.

Any small amount of boron impurity in the beryllium would tend to increase $v\Sigma_a$, due to the large absorption cross section of boron. On the other hand, it is hard to see why $v\Sigma_a$ should have a value smaller than the one given in Eq. 15.

b. D, Theoretical Calculation

R. Weinstock[19] has developed a theory for calculating the elastic and inelastic transport cross sections of a crystal from the bound scattering cross section. This bound scattering cross section was measured as $7.54 \pm .07$ barns.[17] Using this value and the theory of Weinstock, Bhandari[20] and others have calculated a value of $D = 1.21 \times 10^5$ cm^2/sec for the diffusion coefficient in 300° K beryllium with density of 1.85 g/cm^3.

c. C, Theoretical Calculation

M. Nelkin[9] has presented a method of estimating the diffusion cooling

constant C. A variational expression for the decay constant is derived and evaluated from a trial solution of Maxwellian form for the neutron spectrum. The diffusion cooling coefficient is thus related to the second moment M_2 of the energy transfer. Nelkin has calculated M_2 for beryllium, with a simplified model for the crystalline binding, and obtains thus a value of the diffusion cooling constant $C = 1.1\ cm^2$. Transport theory corrections to the diffusion approximation also contribute a correction of order B^4 to Eq. 4, and there may also be correction terms of order B^6, or higher; however the magnitude of these transport theory corrections is probably small compared to the effects discussed above.

VI. COMPARISON OF THE RESULTS WITH OTHER DETERMINATIONS

Only three other measurements of the diffusion constants of beryllium by the pulsed-neutron-source method are known to us. The results obtained by the three groups are reported in Table I and will now be discussed.

a. The Measurements of Antonov et al.[8]

This Russian group used beryllium of density 1.78 g/cm^3. The measurements extended over a buckling range from .0073 to .0612 cm^{-2}. The accuracy of the measurement was not sufficient to determine the mean lifetime directly; this was calculated from the diffusion constant and the diffusion length measured by an independent experiment. There are large unexplained inconsistencies between the diffusion parameters and the decay times reported: For instance, the smallest block measured by this group is a cube of 20-cm side length, corresponding to a buckling of 6.118×10^{-2} cm^{-2}. Using their diffusion parameters, such a block should have a decay constant of 6.592×10^3 sec^{-1}, but they measure instead a decay constant of 5.405×10^3 sec^{-1}. Also the parabola λ vs B^2 given in Fig. 3 of the report does not have the same parameters as reported in their Table I and thus the interpretation of these data becomes ambiguous.

b. Measurements of E. C. Campbell and P. Stelson[15]

These measurements were made with beryllium of density $\rho = 1.85$ g/cm^3 over a range of bucklings from .5 to 3.6×10^{-1} cm^{-2}. Eight blocks were measured. The results are in good agreement with those reported in this paper, as shown in Fig. 3 where the results of various groups are compared. For large blocks (small bucklings) Campbell and Stelson find decay constants somewhat lower than those reported here. The value of $v\Sigma_a = 150$ sec^{-1} reported for the intercept at 0 buckling seems somewhat unreasonably low, corresponding to a microscopic absorption cross section of $\sigma_a = 5.5$ mb.

c. Measurements of Kloverstrom et al.[16]

The measurements were made with beryllium of density $\rho = 1.54$ gm/cm^3

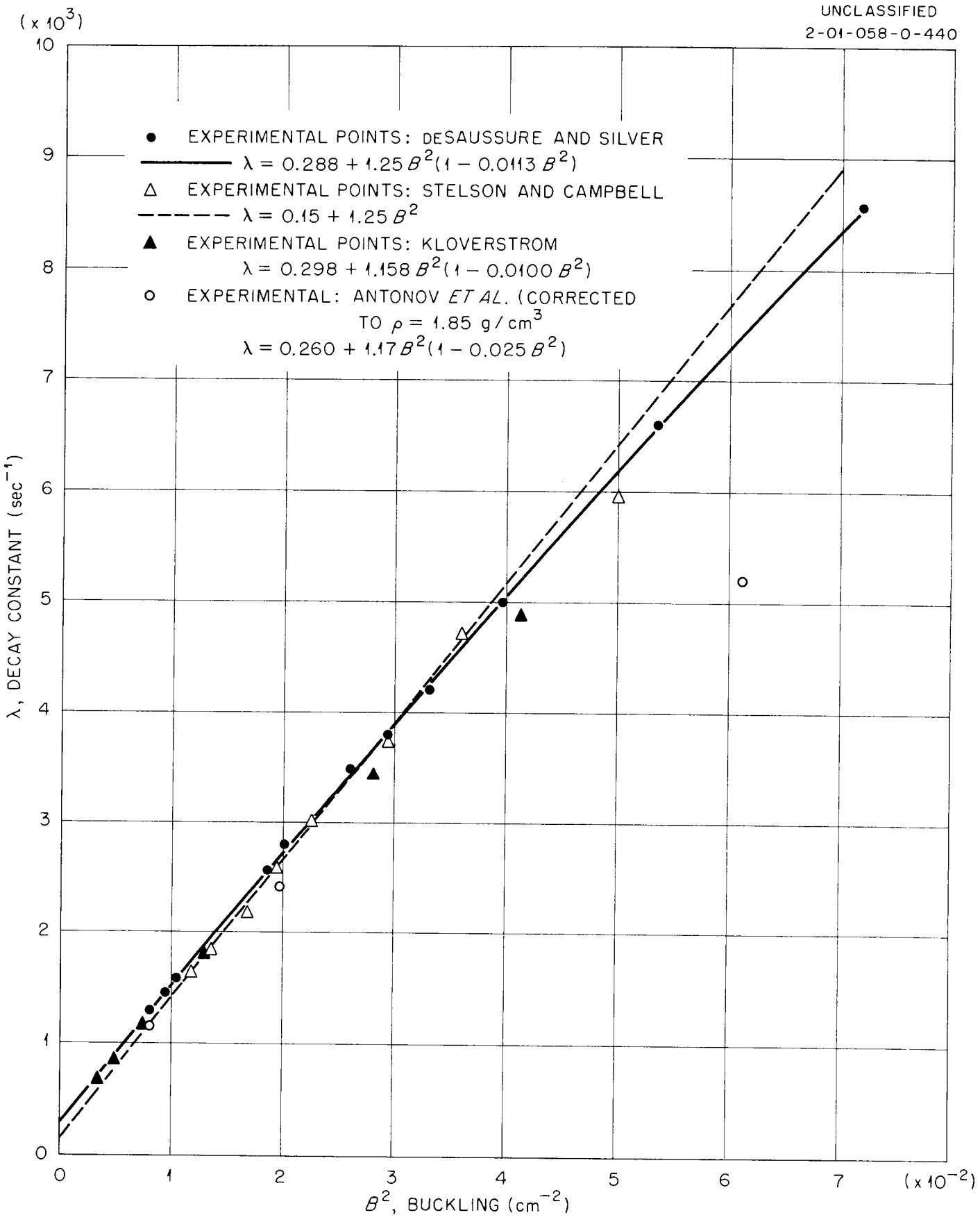

Fig. 3. Comparison of Results Obtained by Various Experimenters.

over a buckling range from .339 to 4.12 x 10^{-2} cm^{-2}. For large sizes (small bucklings) the decay constants obtained are in very good agreement with those obtained in this experiment. For their two smallest blocks Kloverstrom et al. find decay constants smaller than those found in this experiment, leading to a large value of the diffusion cooling constant.

VII. CONCLUSION

The results obtained in this experiment are in reasonably good agreement with those obtained by other workers and with calculations using other experimental constants. A special effort was made here to extend the measurements to assemblies of very small sizes (large bucklings) so as to determine with precision the diffusion cooling constants; but there still remain large discrepancies between the various values reported for this constant. Further measurements will now be made with beryllium at other than room temperature to study experimentally the variation of the diffusion constant and the diffusion cooling constant with the moderator temperature.

APPENDIX I.

DISCUSSION OF TYPICAL DECAY CURVES

Figure I-1. Decay of the Neutron Density in a Large Assembly of Beryllium

The assembly was a prism of 20-1/8 x 20-1/8 x 24 in.; $B^2 = 0.945 \times 10^{-2}$ cm^{-2}. The length of the neutron pulse was 1.5 millisec. The counting started 0.25 millisec after the end of the pulse, and the width of each of the 18 time channels was t = 0.25 millisec. The table below gives the number of counts n_k and the difference $n_k - n_{k+1}$ for each channel.

	k	1	2	3	4	5	6	7	8	9
	n_k	200,135	145,235	101,905	72,345	50,515	35,215	24,475	17,175	11,995
$n_k - n_{k+1}$		54,900	43,330	39,560	21,830	15,300	10,740	7,300	5,180	3,750
	k	10	11	12	13	14	15	16	17	18
	n_k	8,245	5,715	3,995	2,764	2,041	1,373	1,015	716	465
$n_k - n_{k+1}$		2,530	1,720	1,231	723	668	358	299	251	-

The logarithms of n_k and $n_k - n_{k+1}$ are plotted versus k in the figure. From the curve log $(n_k - n_{k+1})$ versus k it is apparent that the first few channels fall below the straight line passing through the remaining channels. This is attributed to a higher mode decay, with negative amplitude, not yet completely negligible at the beginning of the counting. The first three channels are thus discarded, and the remaining n_k's for $k > 3$ are analyzed into an exponential decay and a constant background by the Cornell method. The decay constant is given by:

$$\lambda = -\frac{1}{5t} \ln \frac{\sum_{k=9}^{13} n_k - \sum_{k=14}^{18} n_k}{\sum_{k=4}^{8} n_k - \sum_{k=9}^{13} n_k} = \frac{1}{5 \times .25} \ln \frac{32,714 - 5,610}{199,725 - 32,714} = 1.455 \text{ kc}$$

The same analysis made with n_k's for $2 < k < 18$ gives $\lambda = 1.440$ kc showing that the third channel depresses the curve. A similar analysis made with the 12 n_k's for $4 < k < 17$ gives $\lambda = 1.454$ kc, indicating that the inclusion of n_4 in the calculation does not change the value of the decay constant. Figure I-1

shows that though a straight line can be fitted graphically to the points of the plot ln n_k versus k, this line does not have a slope corresponding to the decay constant to be measured; the higher modes and the background contribute to the decay in such a way as to show a composite decay constant about 4% smaller, in this case, than the one to be measured. The value 1.46 kc given for the decay constant of this assembly in Table III is the average of many such determinations made with different values of the channel width t and of the waiting time. For this assembly the various determinations were all in agreement within 1%.

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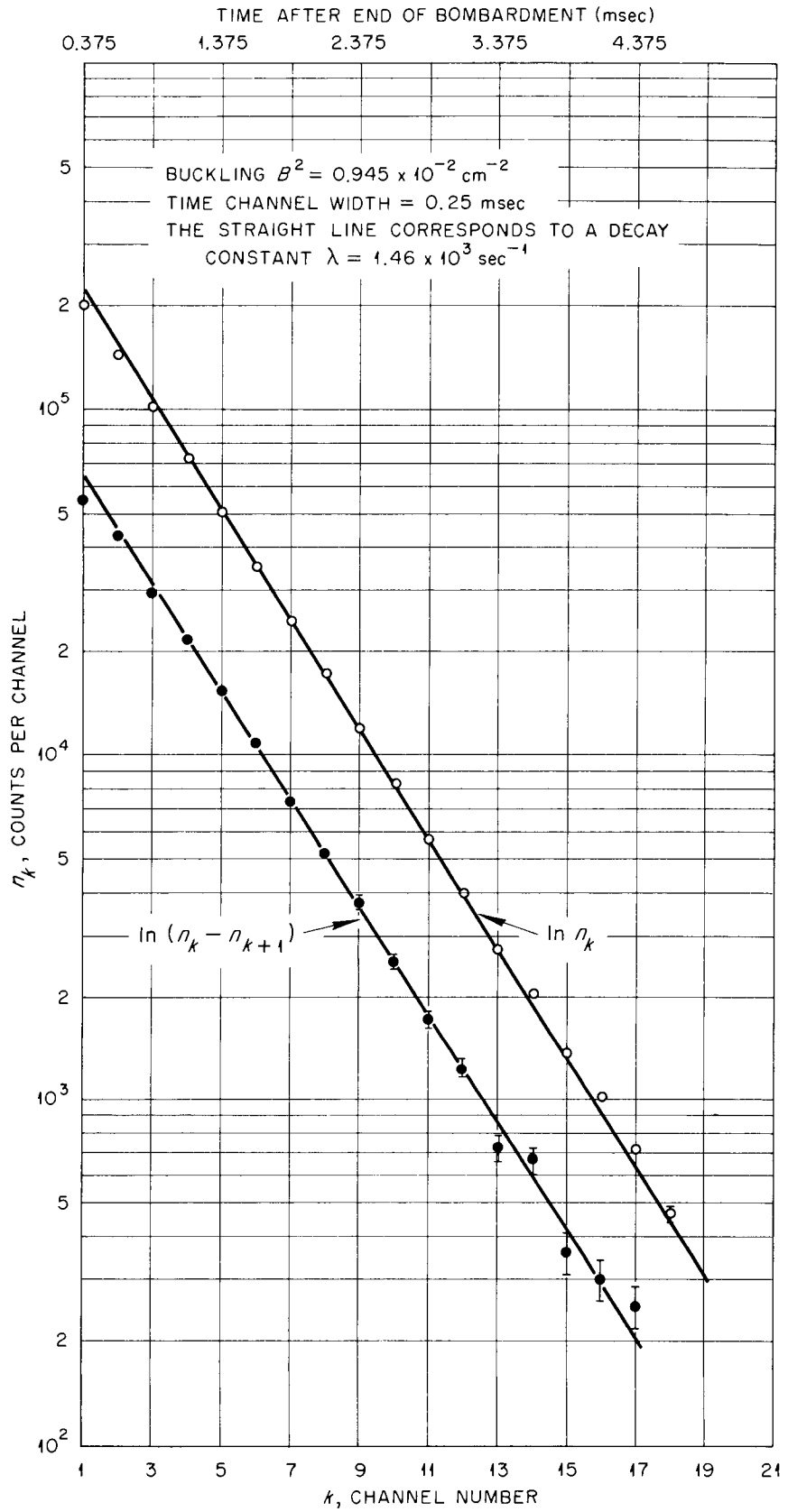

Fig. I-1. Decay of the Neutron Density in a Beryllium Assembly $20\frac{1}{8} \times 20\frac{1}{8} \times 24$ in.

Figure I-2. Decay of the Neutron Density in a Medium Sized Assembly of Beryllium

The assembly was a prism of 14-3/8 x 14-3/8 x 16 in.; $B^2 = 1.861 \times 10^{-2}$ cm^{-2}. The length of the neutron pulse was 0.6 millisec. The counting started 0.1 millisec after the end of the pulse and the width of each of the 18 time channels was t = 0.1 millisec. The table below gives the number of counts n_k and the difference $n_k - n_{k+1}$ for each channel.

k	1	2	3	4	5	6	7	8	9
n_k	100,145	78,005	60,305	46,485	36,205	28,275	21,705	16,955	13,045
$n_k - n_{k+1}$	22,140	17,700	13,820	10,280	7,930	6,570	4,755	3,910	2,960
k	10	11	12	13	14	15	16	17	18
n_k	10,085	7,835	6,165	4,782	3,780	2,915	2,249	1,752	1,395
$n_k - n_{k+1}$	2,250	1,670	1,383	1,002	865	666	497	357	-

The logarithms of n_k and $n_k - n_{k+1}$ are plotted versus k in the figure. All the points of the curve $\ln(n_k - n_{k+1})$ vs k fall on a straight line, indicating that there is no significant contribution from higher modes. The 18 n_k's are analyzed, by the Cornell method, into a single exponential decay and a background, giving a decay constant $\lambda = 2.56$ kc. If only the 15 n_k's for $1 \leq k \leq 17$ are used, one obtains the value $\lambda = 2.57$ kc. If the 15 n_k's for $2 \leq k \leq 18$ are used, the constant becomes $\lambda = 2.56$ kc again. These 0.5% variations are larger than the statistical variations expected but are probably connected with small variations in the length t of the time channels.

All the decays measured, except those of the two largest and the two smallest assemblies, had very little contamination due to higher modes, slowing down, or background, and Fig. I-2 can be considered as typical of all decay curves for the medium sized assemblies.

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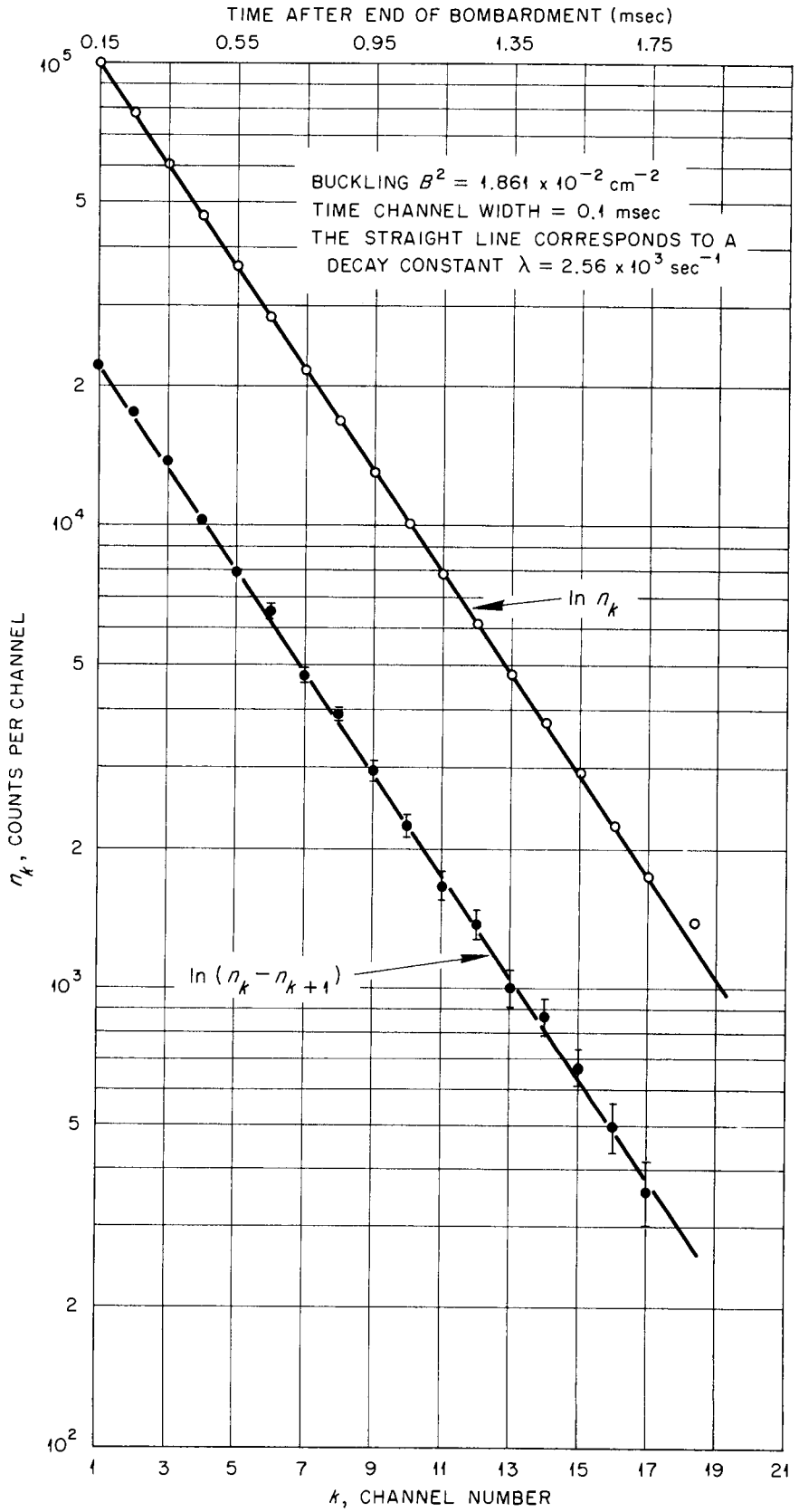

Fig. I-2. Decay of the Neutron Density in a Beryllium Assembly $14\frac{3}{8} \times 14\frac{3}{8} \times 16$ in.

Figure I-3. Decay of the Neutron Density in the Smallest Assembly of Beryllium

The assembly was a prism 5-3/4 x 8 x 8-5/8 in.; $B^2 = 7.18 \times 10^{-2}$ cm^{-2}. The length of the neutron pulse was 0.3 millisec. The counting started 0.15 millisec after the end of the pulse, and the width of each of the 18 time channels was 0.05 millisec. The table below gives the number of counts n_k and the difference $n_k - n_{k+1}$ for each channel.

k	1	2	3	4	5	6	7	8	9
n_k	401,270	260,720	166,300	108,930	71,070	46,670	31,050	21,000	14,400
$n_k - n_{k+1}$	140,550	94,420	59,370	37,860	24,400	15,620	10,050	6,600	4,640

k	10	11	12	13	14	15	16	17	18
n_k	9,760	6,840	4,900	3,705	2,819	2,287	1,862	1,559	1,452
$n_k - n_{k+1}$	2,920	1,940	1,195	886	532	425	303	107	-

The logarithms of n_k and $n_k - n_{k+1}$ are plotted versus k in the figure. An analysis of the 15 first n_k's, for $k < 16$, was made, into two exponential decays and a constant background, by the Cornell method. The resulting decay constants are: $\lambda_1 = 8.43$ kc, $\lambda_2 = 22.9$ kc. The value $\lambda_1 = 8.55$ kc given in Table III is the average of many determinations with other widths of the time channels and other values of the waiting time. These determinations all gave values of λ_1 within 3% of the value 8.55 kc quoted.

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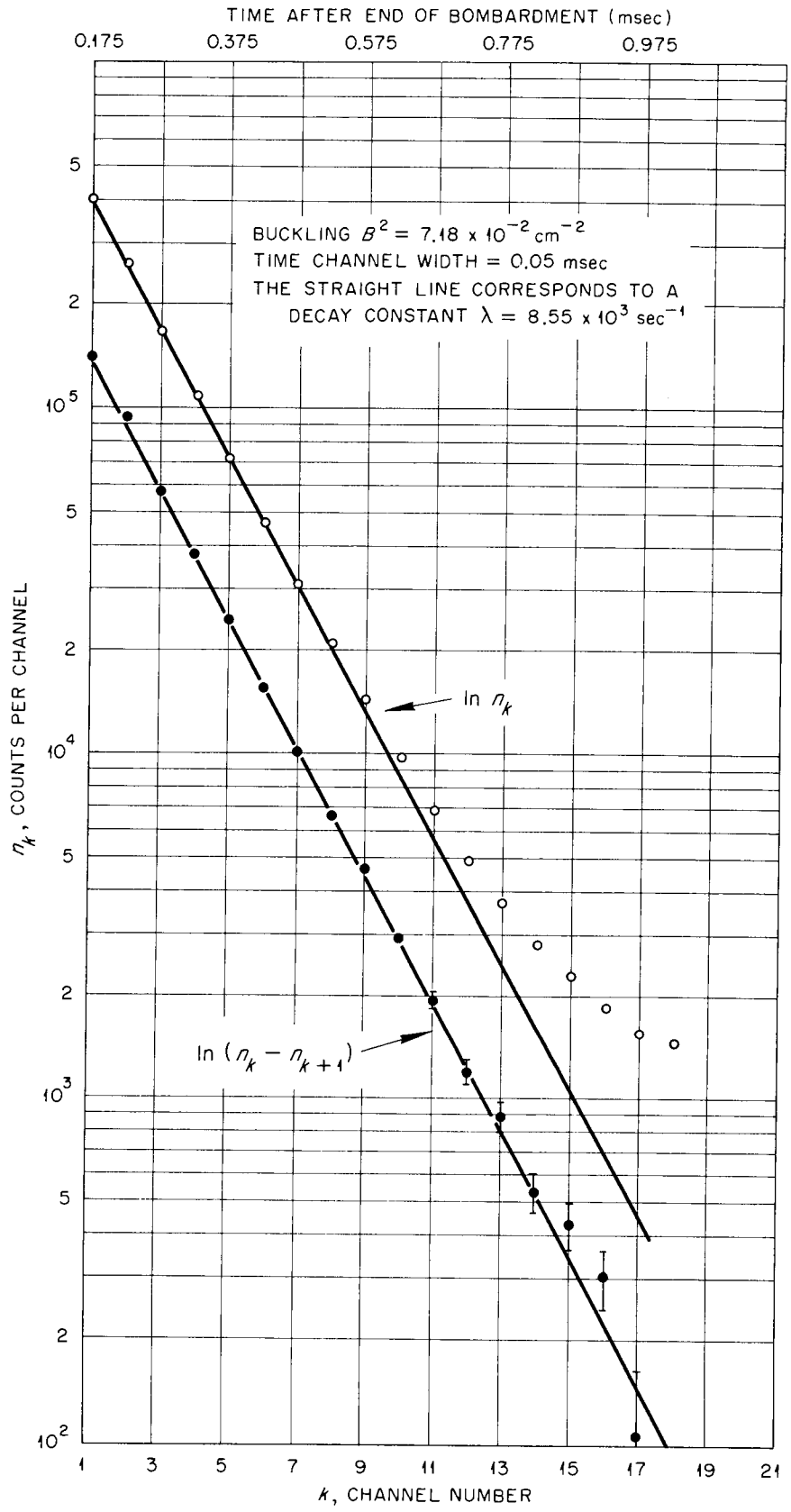

Fig. I-3. Decay of the Neutron Density in a Beryllium Assembly $5\frac{3}{4} \times 8 \times 8\frac{5}{8}$ in.

Table I. Neutron Diffusion Parameters for Beryllium at Room Temperature (Corrected to a density of 1.85 g/cc)

Authors	λ_a (sec^{-1})	D (cm^2/sec)	C (cm^2)	Range of B^2 Used (cm^{-2})
		x 10^5		
Antonov et al.[8]		1.17 ± 0.05	2.5 ± 0.9	0.008 - 0.061
Campbell et al.[15]	150	1.25	0	0.005 - 0.036
Kloverstrom et al.[16]	270 ± 19	1.24 ± 0.04	3.15 ± 0.65	0.003 - 0.041
deSaussure and Silver	288 ± 60	1.25 ± 0.06	1.1 ± 0.9	0.008 - 0.072
Calculated values	270 ± 30	1.21	1.1	

Table II. Thermal Neutron Diffusion Properties of Room-Temperature Beryllium

For density $\rho = 1.85$ gr/cm^3, derived from the constants given in the fourth row of Table I: $v\Sigma_a = 288 \pm 60$ sec^{-1}, $D = (1.25 \pm .06)\ 10^5$ cm^2 sec^{-1}.

Macroscopic thermal transport cross section	$\Sigma_{tr} = .66 \pm .03$ cm^{-1}
Transport mean free path	$\lambda_{tr} = 1.51 \pm .08$ cm
Extrapolation length	$1.42\ \lambda_{tr} = 2.14 \pm .11$ cm
Macroscopic thermal absorption cross section	$\Sigma_a = (1.31 \pm .27) \times 10^{-3}$ cm^{-1}
Microscopic thermal absorption cross section	$\sigma_a = 11 \pm 2$ millibarns
Thermal lifetime	$t_a = 3.5 \pm .7$ msec
Absorption mean free path	$\lambda_a = 763 \pm 146$ cm
Diffusion length	$L = 19.6 \pm 2.4$ cm

Table III. Decay Constant of the Fundamental Mode of the Thermal Neutron Density in Eleven Beryllium Assemblies

Assembly Size[a]	Buckling[b]	Experimental Decay Constant[c]	Estimated Error[d]	Calculated Decay Constant[e]
$in.^3$	10^{-2} cm^{-2}	10^3 sec^{-1}		10^3 sec^{-1}
20-1/8 x 24 x 25-7/8	.811	1.29	< 2%	1.29
20-1/8 x 20-1/8 x 24	.945	1.46	< 2%	1.46
20 x 20-1/8 x 20-1/8	1.048	1.585	< 2%	1.58
14-3/8 x 14-3/8 x 16	1.861	2.56	< 2%	2.565
14 x 14-3/8 x 14-3/8	2.016	2.80	< 2%	2.75
11-1/2 x 12 x 14-3/8	2.593	3.48	< 2%	3.435
11-1/2 x 11-1/2 x 12	2.937	3.80	< 2%	3.84
10 x 11-1/2 x 11-1/2	3.310	4.20	< 2%	4.27
8 x 11-1/2 x 11-1/2	3.965	4.99	< 2%	5.02
8 x 8-5/8 x 8-5/8	5.359	6.60	< 2%	6.58
5-3/4 x 8 x 8-5/8	7.180	8.55	~ 3%	8.54

a. Assembly size: length, in inches, of the three sides of the prism.

b. Buckling in 10^{-2} cm^{-2}: $B^2 = \pi^2(\frac{1}{\tilde{a}^2} + \frac{1}{\tilde{b}^2} + \frac{1}{\tilde{c}^2})$ where $\tilde{a} - a = 2.14$ cm ($\lambda_{tr} = 1.51$ cm).

c. Experimentally determined decay constant, 10^3 sec^{-1}.

d. Estimated percent error on the decay constant for all but the smallest assembly; successive measurements of the decay period, made at different times and with different channel widths of the time analyzer, all agreed within 2% of each other. The conservative error of 2% has been adopted for the ten largest assemblies to take into account possible systematic errors, and because assuming a uniform error simplifies the least square analysis of the data.

e. Calculated decay constant: $\lambda(10^3 \text{ sec}^{-1}) = .288 + 125\ B^2(1 - .011)B^2$ in 10^{-2} cm^{-2}.

Table IV. Diffusion Constants of Thermal Neutrons in Beryllium, ρ = 1.85 g/cc

	Unit	I	II	III
$A_o = v\Sigma_a$	10^3 sec^{-1}	.288 ± .06	.279 ± .05	-
$A_1 = D$	10^5 cm^2 sec^{-1}	1.252 ± .06	1.266 ± .06	(1.25 ± .06)
A_2	10^7 cm^4 sec^{-1}	-.014 ± .011	-.018 ± .012	-
A_3	10^9 cm^6 sec^{-1}	-	-	+.003 ± .007
$C = -A_2/A_1$	cm^2	1.13 ± .94	1.43 ± 1.1	-
$E = A_3/A_1$	cm^4	-	-	25 ± 57

The decay constant can be expressed as a function of buckling as:

$$\lambda = A_o + A_1 B^2 + A_2 B^4 + A_3 B^6 = A_o + A_1 B^2(1 - CB^2 + EB^4)$$

The first set of parameters given (Column I) was obtained by fitting a parabola ($A_3 = E = 0$) to the data of the eleven assemblies of Table III. The error was obtained by assuming a uniform 3% error on all the data. This is a conservative estimate as the decay constants of all but the smallest block were determined to 2% or better, but the least square analysis is much simplified if the same percentage error is assumed for all the data. The second set of parameters (Column II) was obtained by fitting a parabola to the data obtained by the ten largest assemblies in Table III. In this case a 2% error on each decay constant was assumed to calculate the error on the parameters. The value of A_3 given in Column III was obtained by fitting a four parameter curve to the data corresponding to all eleven assemblies. The other parameters were not determined and the value of E was obtained by dividing A_3 by the value of A_1 obtained in Column I.

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