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FLIP – AN IBM-704 CODE TO SOLVE THE P_L AND DOUBLE-P_L EQUATIONS IN SLAB GEOMETRY

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B. L. Anderson • J. A. Davis • E. M. Gelbard • P. H. Jarvis

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CONTENTS

A method of obtaining the few-group form of the P_L and double-P_L equations is given for slab geometry. Anisotropic scattering is allowed within specified limitations. The difference equations and associated recursion relations are discussed; the features and restrictions of FLIP are explained; and a detailed discussion of the input and output is presented. Operating instructions are given, and a sample problem is included.

FLIP—AN IBM-704 CODE TO SOLVE THE P_L AND DOUBLE-P_L EQUATIONS IN SLAB GEOMETRY

B. L. Anderson, J. A. Davis, E. M. Gelbard, and P. H. Jarvis

INTRODUCTION

FLIP, a reactor design code for the IBM-704 computer, uses the P_L and double-P_L approximations to solve the one-velocity, one-dimensional transport equation in slab geometry. The following approximations are available in FLIP: P_3, P_5, P_7, double-P_1, double-P_2, and double-P_3. For all approximations, flux components are computed over a mesh of, at most, 500 intervals, distributed through a slab containing no more than 50 homogeneous regions. The source function is assumed to be isotropic, but it may vary from mesh point to mesh point. Anisotropic scattering is permitted within limitations which will be clarified in the following sections.

DEVELOPMENT OF THE METHOD

Conventional P_L Approximations

Essentially, FLIP is an extension of the IBM-704 SIMPL codes (Ref 1). The method used to derive the P_3 SIMPL equations may be applied directly to the higher P_L equations as well. The resulting second order differential equations can be written in the form (Ref 2)

$$-\frac{\partial}{\partial x} D_i \frac{\partial}{\partial x} \varphi_i + \Sigma_{Ti}\varphi_i = A_i S + \sum_{\substack{j=1 \\ (j\neq i)}}^{\frac{1}{2}(L+1)} \alpha_{ij}\varphi_j \tag{1}$$

where

$$D_i = \frac{1}{(4i - 1)\Sigma_i} \tag{2}$$

and

$$\Sigma_i = \Sigma_a + \Sigma_{s0} - \Sigma_{si} \tag{3}$$

Further,

$$
\left.\begin{aligned}
\Sigma_{T1} &= \Sigma_a \\
\Sigma_{T2} &= \frac{4}{9}\Sigma_a + \frac{5}{9}\Sigma_2 \\
\Sigma_{T3} &= \frac{64}{225}\Sigma_a + \frac{16}{45}\Sigma_2 + \frac{9}{25}\Sigma_4 \\
\Sigma_{T4} &= \frac{256}{1225}\Sigma_a + \frac{64}{225}\Sigma_2 + \frac{324}{1225}\Sigma_4 + \frac{13}{49}\Sigma_6
\end{aligned}\right\} \qquad (4)
$$

The coupling parameters α_{ij}, symmetric in i and j, are given by the expressions

$$
\left.\begin{aligned}
\alpha_{12} &= \alpha_{21} = \frac{2}{3}\Sigma_a \\
\alpha_{13} &= \alpha_{31} = -\frac{8}{15}\Sigma_a \\
\alpha_{14} &= \alpha_{41} = \frac{16}{35}\Sigma_a \\
\alpha_{23} &= \alpha_{32} = \frac{16}{45}\Sigma_a + \frac{4}{9}\Sigma_2 \\
\alpha_{24} &= \alpha_{42} = -\frac{32}{105}\Sigma_a - \frac{8}{21}\Sigma_2 \\
\alpha_{34} &= \alpha_{43} = \frac{128}{525}\Sigma_a + \frac{32}{105}\Sigma_2 + \frac{54}{175}\Sigma_4
\end{aligned}\right\} \qquad (5)
$$

while the A_i, which are independent of all physical constants, are given by

$$
\left.\begin{aligned}
A_1 &= 1 \\
A_2 &= -\frac{2}{3} \\
A_3 &= \frac{8}{15} \\
A_4 &= -\frac{16}{35}
\end{aligned}\right\} \qquad (6)
$$

All the F_L (the Legendre coefficients of the flux) may be expressed in terms of φ_i:

$$
\left.\begin{aligned}
F_0 &= \varphi_1 - \frac{2}{3}\varphi_2 + \frac{8}{15}\varphi_3 - \frac{16}{35}\varphi_4 \\
F_2 &= \frac{1}{3}\varphi_2 - \frac{4}{15}\varphi_3 + \frac{8}{35}\varphi_4 \\
F_4 &= \frac{1}{5}\varphi_3 - \frac{6}{35}\varphi_4 \\
F_6 &= \frac{1}{7}\varphi_4 \\
F_1 &= -D_1\varphi_1' \\
F_3 &= -D_2\varphi_2' \\
F_5 &= -D_3\varphi_3' \\
F_7 &= -D_4\varphi_4'
\end{aligned}\right\} \qquad (7)
$$

The source, S, must be isotropic but may vary from point to point.

All of the previous equations have been written explicitly for a P_7 approximation. Corresponding relations for a P_5 approximation may be obtained simply by deleting all fourth group* parameters. P_3 relations remain when both the third and fourth group parameters are suppressed. It should be noted that D_i and φ_i are so defined as to guarantee the continuity of

$$D_i \frac{\partial \varphi_i}{\partial x}$$

across interfaces.

FLIP solves Eq (1) by a Gauss-Seidel iterative process. Assume that the k^{th} iteration has been completed, and that k^{th} iterates, $\varphi^{(k)}$, of φ are available in all groups. These are inserted into the right-hand side of Eq (1), and a $(k+1)^{th}$ iterate of φ_1 is computed. Now $\varphi_1^{(k+1)}$ and the k^{th} iterates of all other φ's are inserted into the right-hand side of Eq (1), and $\varphi_2^{(k+1)}$ is computed, etc. After each iteration, the function $F_0^{(k)}$ is computed at every mesh point n from the relation

$$F_{On}^{(k)} = \varphi_{1n}^{(k)} - \frac{2}{3}\varphi_{2n}^{(k-1)} + \frac{8}{15}\varphi_{3n}^{(k-1)} - \frac{48}{105}\varphi_{4n}^{(k-1)} , \tag{8}$$

and divided by $F_{On}^{(k-1)}$, yielding

$$\lambda_n^{(k)} \equiv \frac{F_{On}^{(k)}}{F_{On}^{(k-1)}} . \tag{9}$$

The iterative process terminates when

$$\left| \left[\lambda_n^{(k)} \right]_{max} - 1 \right| < \epsilon$$

and

$$\left| \left[\lambda_n^{(k)} \right]_{min} - 1 \right| < \epsilon$$

for all mesh points, and for an ϵ whose value is supplied as input.

It has been found convenient to start the iterative process from a zero guess, $\varphi_i^{(0)} = 0$, in all groups. The first iterate of the scalar flux, $F_0^{(1)}$, is then identical with the P_1 solution. With an ϵ of 0.00005, which is more than adequate in practice, convergence requires about four to ten iterations in all P_L approximations.

Extrapolation of the iterative process is available in FLIP, as in the WANDA code (Ref 3), but because of the rapidity of convergence, its use has never been necessary.

Double-P_L Approximations

The derivation of the FLIP double-P_L equations is not completely straightforward and will, therefore, be discussed in some detail. Anisotropic double-P_L equations have been developed by Mertens (Ref 4). These will be written, first, for a double-P_3 approximation. Equations for lower approximations will be obtained by deleting terms from the double-P_3 equations.

In double-P_3, then,

$$\frac{1}{2}\frac{\partial \psi_1}{\partial x} + \frac{1}{2}\frac{\partial \psi_0}{\partial x} + \Sigma_T \psi_0 = \frac{1}{2} S_0 + \frac{1}{2}\Sigma_{s0}F_0 + \frac{3}{4}\Sigma_{s1}F_1 - \frac{7}{16}\Sigma_{s3}F_3 + \frac{11}{32}\Sigma_{s5}F_5 - \frac{75}{256}\Sigma_{s7}F_7 , \tag{10}$$

$$\frac{1}{2}\frac{\partial \chi_1}{\partial x} - \frac{1}{2}\frac{\partial \chi_0}{\partial x} + \Sigma_T \chi_0 = \frac{1}{2} S_0 + \frac{1}{2}\Sigma_{s0}F_0 - \frac{3}{4}\Sigma_{s1}F_1 + \frac{7}{16}\Sigma_{s3}F_3 - \frac{11}{32}\Sigma_{s5}F_5 + \frac{75}{256}\Sigma_{s7}F_7 , \tag{11}$$

* The word "group" is used in this report in an artificial sense.

$$\frac{1}{3}\frac{\partial\psi_2}{\partial x}+\frac{1}{2}\frac{\partial\psi_1}{\partial x}+\frac{1}{6}\frac{\partial\psi_0}{\partial x}+\Sigma_T\psi_1=\frac{1}{4}\Sigma_{s1}F_1-\frac{5}{8}\Sigma_{s2}F_2+\frac{7}{16}\Sigma_{s3}F_3$$

$$+\frac{3}{16}\Sigma_{s4}F_4-\frac{11}{32}\Sigma_{s5}F_5-\frac{13}{128}\Sigma_{s6}F_6+\frac{75}{256}\Sigma_{s7}F_7 \quad , \qquad (12)$$

$$\frac{1}{3}\frac{\partial\chi_2}{\partial x}-\frac{1}{2}\frac{\partial\chi_1}{\partial x}+\frac{1}{6}\frac{\partial\chi_0}{\partial x}+\Sigma_T\chi_1=\frac{1}{4}\Sigma_{s1}F_1-\frac{5}{8}\Sigma_{s2}F_2+\frac{7}{16}\Sigma_{s3}F_3$$

$$+\frac{3}{16}\Sigma_{s4}F_4-\frac{11}{32}\Sigma_{s5}F_5-\frac{13}{128}\Sigma_{s6}F_6+\frac{75}{256}\Sigma_{s7}F_7 \quad , \qquad (13)$$

$$\frac{3}{10}\frac{\partial\psi_3}{\partial x}+\frac{1}{2}\frac{\partial\psi_2}{\partial x}+\frac{1}{5}\frac{\partial\psi_1}{\partial x}+\Sigma_T\psi_2=\frac{1}{8}\Sigma_{s2}F_2+\frac{7}{16}\Sigma_{s3}F_3+\frac{9}{16}\Sigma_{s4}F_4$$

$$+\frac{11}{64}\Sigma_{s5}F_5-\frac{39}{128}\Sigma_{s6}F_6-\frac{57}{256}\Sigma_{s7}F_7 \quad , \qquad (14)$$

$$\frac{3}{10}\frac{\partial\chi_3}{\partial x}-\frac{1}{2}\frac{\partial\chi_2}{\partial x}+\frac{1}{5}\frac{\partial\chi_1}{\partial x}+\Sigma_T\chi_2=\frac{1}{8}\Sigma_{s2}F_2-\frac{7}{16}\Sigma_{s3}F_3+\frac{9}{16}\Sigma_{s4}F_4$$

$$-\frac{11}{64}\Sigma_{s5}F_5-\frac{39}{128}\Sigma_{s6}F_6+\frac{57}{256}\Sigma_{s7}F_7 \quad , \qquad (15)$$

$$\frac{1}{2}\frac{\partial\psi_3}{\partial x}+\frac{3}{14}\frac{\partial\psi_2}{\partial x}+\Sigma_T\psi_3=\frac{1}{16}\Sigma_{s3}F_3+\frac{9}{32}\Sigma_{s4}F_4+\frac{33}{64}\Sigma_{s5}F_5+\frac{13}{32}\Sigma_{s6}F_6-\frac{15}{256}\Sigma_{s7}F_7 \quad , \qquad (16)$$

and

$$-\frac{1}{2}\frac{\partial\chi_3}{\partial x}+\frac{3}{14}\frac{\partial\chi_2}{\partial x}+\Sigma_T\chi_3=\frac{1}{16}\Sigma_{s3}F_3-\frac{9}{32}\Sigma_{s4}F_4+\frac{33}{64}\Sigma_{s5}F_5-\frac{13}{32}\Sigma_{s6}F_6-\frac{15}{256}\Sigma_{s7}F_7 \quad . \qquad (17)$$

Here, the scattering cross section is truncated at Σ_{s7}. The ψ's and χ's are the usual half-range moments of the flux,

$$\left.\begin{aligned}\psi_L(x) &= \int_0^1 F(\mu,x)P_L(2\mu-1)\,d\mu\\ \chi_L(x) &= \int_{-1}^0 F(\mu,x)P_L(2\mu+1)\,d\mu\end{aligned}\right\} , \qquad (18)$$

while F_L and f_L are double-P_3 expressions for the full-range moments:

$$F_0=\psi_0+\chi_0 \quad , \qquad (19)$$

$$F_1=\frac{1}{2}(\psi_1+\chi_1)+\frac{1}{2}(\psi_0-\chi_0) \quad , \qquad (20)$$

$$F_2=\frac{1}{4}(\psi_2+\chi_2)+\frac{3}{4}(\psi_1-\chi_1) \quad , \qquad (21)$$

$$F_3=\frac{1}{8}(\psi_3+\chi_3)+\frac{5}{8}(\psi_2-\chi_2)+\frac{3}{8}(\psi_1+\chi_1)-\frac{1}{8}(\psi_0-\chi_0) \quad , \qquad (22)$$

$$f_4=\frac{7}{16}(\psi_3-\chi_3)+\frac{5}{8}(\psi_2+\chi_2)-\frac{1}{8}(\psi_1-\chi_1) \quad , \qquad (23)$$

$$f_5=\frac{21}{32}(\psi_3+\chi_3)+\frac{5}{32}(\psi_2-\chi_2)-\frac{3}{16}(\psi_1+\chi_1)+\frac{1}{16}(\psi_0-\chi_0) \quad , \qquad (24)$$

$$f_6=\frac{7}{16}(\psi_3-\chi_3)-\frac{15}{64}(\psi_2+\chi_2)+\frac{3}{64}(\psi_1-\chi_1) \quad , \qquad (25)$$

and

$$f_7 = -\frac{7}{128}(\psi_3 + \chi_3) - \frac{19}{128}(\psi_2 - \chi_2) + \frac{15}{128}(\psi_1 + \chi_1) - \frac{5}{128}(\psi_0 - \chi_0) \quad . \tag{26}$$

The symbol f_L denotes a full-range moment which cannot be represented exactly in a given half-range approximation.

The double-P_1 equations may be obtained by deleting all terms containing scattering components beyond Σ_{s3} and moments beyond ψ_1 and χ_1 from Eqs (10) through (17) and from Eqs (19) through (26). These may be written in terms of the approximate full-range moments:

$$F_1' + (\Sigma_T - \Sigma_{s0})\, F_0 = S \quad , \tag{27}$$

$$\frac{2}{3} f_2' + \frac{1}{3} F_0' + (\Sigma_T - \Sigma_{s1})\, F_1 = 0 \quad , \tag{28}$$

$$\frac{1}{2} f_3' + \frac{3}{8} F_1' + (\Sigma_T - \frac{15}{16}\Sigma_{s2})\, f_2 = 0 \quad , \tag{29}$$

and

$$\frac{1}{6} f_2' + (\Sigma_T - \frac{7}{16}\Sigma_{s3})\, f_3 = 0 \quad , \tag{30}$$

where

$$F_0 = \psi_0 + \chi_0 \quad , \tag{31}$$

$$F_1 = \frac{1}{2}(\psi_1 + \chi_1) + \frac{1}{2}(\psi_0 - \chi_0) \quad , \tag{32}$$

$$f_2 = \frac{3}{4}(\psi_1 - \chi_1) \quad , \tag{33}$$

and

$$f_3 = \frac{3}{8}(\psi_1 + \chi_1) - \frac{1}{8}(\psi_0 - \chi_0) . \tag{34}$$

When the odd full-range moments are eliminated, the result is two, coupled, second-order equations of the form of Eq (1), where

$$\Sigma_{T1} = \Sigma_a \quad , \quad \Sigma_{T2} = \frac{9}{16}\Sigma_a + \frac{3}{4}\Sigma_2 \quad ; \tag{35}$$

$$D_1 = \frac{1}{3\Sigma_1} \quad , \quad D_2 = \frac{1}{16\Sigma_3} \quad ; \tag{36}$$

$$\alpha_{12} = \frac{3}{4}\Sigma_a \quad , \quad \alpha_{21} = \frac{3}{4}\Sigma_a \quad ; \tag{37}$$

$$A_1 = 1 \quad , \quad A_2 = -\frac{3}{4} \quad ; \tag{38}$$

and

$$\left.\begin{array}{l} F_0 = \varphi_1 - \frac{3}{4}\varphi_2 \quad , \quad f_2 = \frac{3}{8}\varphi_2 \\ F_1 = -D_1\varphi_1' \quad , \quad f_3 = -D_2\varphi_2' \end{array}\right\} . \tag{39}$$

Second order double-P_2 equations may, of course, be derived in a number of ways. For practical reasons, however, it is convenient to work with equations which are as simple as possible, and which are formally similar to the few-group equations. For these reasons, two conditions are imposed on the second-order equations. First, it is required that they have the form of Eq (1) and second, the quantities

$$- D_i \frac{\partial \varphi_i}{\partial x}$$

are required to be continuous across interfaces. To meet these conditions, it is necessary to neglect all scattering components beyond Σ_{s2}. This has been done in both the double-P_2 and double-P_3 equations, with the result that the scattering cross section is represented with the same degree of precision as the RDR 4 (Ref 5). It should be noted that no such difficulties arise in the conventional P_L approximation for any value of L.

Having neglected higher scattering components, one may write the double-P_2 equations in the form

$$F_1' + \Sigma_a F_0 = S \quad , \tag{40}$$

$$\frac{2}{3} F_2' + \frac{1}{3} F_0' + (\Sigma_T - \Sigma_{s1}) F_1 = 0 \quad , \tag{41}$$

$$\frac{3}{5} f_3' + \frac{2}{5} F_1' + (\Sigma_T - \Sigma_{s2}) F_2 = 0 \quad , \tag{42}$$

$$\frac{17}{32} f_4' + \frac{27}{64} F_2' + \Sigma_T f_3 = 0 \quad , \tag{43}$$

$$\frac{16}{45} f_5' + \frac{31}{90} f_3' + \Sigma_T f_4 = 0 \quad , \tag{44}$$

and

$$\frac{5}{128} f_4' - \frac{9}{256} F_2' + \Sigma_T f_5 = 0 \quad . \tag{45}$$

Here the F's and f's are defined by Eqs (19) through (24) with appropriate deletions. From the definitions

$$F_0 \equiv \varphi_1 - \frac{5}{144} \varphi_2 + \frac{17}{144} \varphi_3 \quad , \tag{46}$$

$$F_2 \equiv \frac{5}{288} \varphi_2 - \frac{17}{288} \varphi_3 \quad , \tag{47}$$

and

$$f_4 \equiv \frac{1}{64} \varphi_2 + \frac{3}{64} \varphi_3 \quad , \tag{48}$$

it follows that

$$F_1 = - D_1 \varphi_1' \quad , \quad f_3 = - D_2 \varphi_2' \quad , \quad f_5 = - D_3 \varphi_3' \tag{49}$$

with

$$D_1 = \frac{1}{3 (\Sigma_{s0} + \Sigma_a - \Sigma_{s1})} \quad , \tag{50}$$

$$D_2 = \frac{1}{64 (\Sigma_{s0} + \Sigma_a)} \quad , \tag{51}$$

and

$$D_3 = \frac{1}{256 (\Sigma_{s0} + \Sigma_a)} \tag{52}$$

When the odd F_L's and f_L's are eliminated from Eqs (40) through (45), three, coupled, second-order equations in φ result. In the notation of Eq (1),

$$\Sigma_{T1} = \Sigma_a \quad ; \tag{53}$$

$$\Sigma_{T2} = \frac{5}{96}\Sigma_a + \frac{25}{864}(\Sigma_{s0} - \Sigma_{s2}) \quad ; \tag{54}$$

$$\Sigma_{T3} = \frac{233}{768}\Sigma_a + \frac{785}{3456}(\Sigma_{s0} - \frac{527}{1256}\Sigma_{s2}) \quad ; \tag{55}$$

$$A_1 = 1 \quad , \quad A_2 = -\frac{2}{3} \quad , \quad A_3 = \frac{31}{48} \quad ; \tag{56}$$

$$\alpha_{21} = \frac{2}{3}\Sigma_a \quad , \quad \alpha_{12} = \frac{5}{144}\Sigma_a \quad ; \tag{57}$$

$$\alpha_{31} = -\frac{31}{48}\Sigma_a \quad , \quad \alpha_{13} = -\frac{17}{144}\Sigma_a \quad ; \tag{58}$$

and

$$\alpha_{32} = \frac{5}{768}\Sigma_a - \frac{55}{3456}(\Sigma_{s0} + \frac{155}{88}\Sigma_{s2}) \quad , \quad \alpha_{23} = \frac{17}{96}\Sigma_a + \frac{85}{864}(\Sigma_{s0} - \Sigma_{s2}) \tag{59}$$

The D_i's have already been defined by Eqs (50) through (52).

The procedure used to derive the double-P_1 and double-P_2 FLIP equations may be summarized as follows: one writes the double-P_L equations in terms of the full-range moments and uses the derivative terms in alternate equations to define group variables. For instance, from Eqs (41), (43), and (45),

$$F_0 + 2F_2 = \varphi_1 \quad , \tag{60}$$

$$27\, F_2 + 34\, f_4 = \varphi_2 \quad , \tag{61}$$

and

$$10\, f_4 - 9\, F_2 = \varphi_3 \quad \text{in double-}P_2. \tag{62}$$

Next, D's are defined in such a way as to yield analogues to Fick's law (see Eq (49) for example), and the odd full-range moments are eliminated.

In terms of the full-range moments, the double-P_3 equations take the form

$$F_1' + \Sigma_a F_0 = S \quad , \tag{63}$$

$$\frac{2}{3}F_2' + \frac{1}{3}F_0' + \Sigma_1 F_1 = 0 \quad , \tag{64}$$

$$\frac{3}{5}F_3' + \frac{2}{5}F_1' + \Sigma_2 F_2 = 0 \quad , \tag{65}$$

$$\frac{4}{7}f_4' + \frac{3}{7}F_2' + \Sigma_T F_3 = 0 \quad , \tag{66}$$

$$\frac{83}{156}f_5' + \frac{85}{192}F_3' + \Sigma_T f_4 = \frac{5}{156}f_7' \quad , \tag{67}$$

$$\frac{34}{77}f_6' + \frac{32}{77}f_4' + \Sigma_T f_5 = 0 \quad , \tag{68}$$

$$\frac{1}{4}f_7' + \frac{1}{4}f_5' + \Sigma_T f_6 = \frac{1}{64}f_3' \quad , \tag{69}$$

and

$$-\frac{5}{77} f_6' + \Sigma_T f_7 = \frac{153}{1540} f_4' \quad . \tag{70}$$

If second-order equations are derived from Eqs (63) through (70), the resulting Σ_{T3} will be negative. It can be shown, in consequence, that the Gauss-Seidel process will not converge for certain types of problems. This difficulty may be circumvented by reformulating the equations in terms of new independent variables:

$$H_4 = \frac{77}{85} F_4 \quad , \tag{71}$$

$$H_5 = \frac{1}{39} (83\ F_5 - 5\ F_7) \quad , \tag{72}$$

$$H_6 = \frac{3}{85} F_4 + F_6 \quad , \tag{73}$$

and

$$G_7 = F_5 + F_7 \quad . \tag{74}$$

One then finds that

$$F_1' + \Sigma_a F_0 = S \quad , \tag{75}$$

$$\frac{2}{3} F_2' + \frac{1}{3} F_0' + \Sigma_1 F_1 = 0 \quad , \tag{76}$$

$$\frac{3}{5} F_3' + \frac{2}{5} F_1' + \Sigma_2 F_2 = 0 \quad , \tag{77}$$

$$\frac{340}{539} H_4' + \frac{3}{7} F_2' + \Sigma_T F_3 = 0 \quad , \tag{78}$$

$$\frac{77}{340} H_5' + \frac{77}{192} F_3' + \Sigma_T H_4 = 0 \quad , \tag{79}$$

$$\frac{876}{924} H_6' + \frac{881}{924} H_4' + \Sigma_T H_5 = 0 \quad , \tag{80}$$

$$\frac{85}{340} G_7' + \frac{3}{340} H_5' + \Sigma_T H_6 = 0 \quad , \tag{81}$$

and

$$\frac{348}{924} H_6' + \frac{309}{924} H_4' + \Sigma_T G_7 = 0 \quad . \tag{82}$$

Of course, substitutions, Eqs (71) through (74), are somewhat arbitrary. They are, however, designed to bring the double-P_3 equations into a form similar to that of the P_7 equations, since the P_7 version of FLIP is rapidly convergent.

In Eqs (75) through (82) one unsatisfactory feature remains. It is implied in Eq (80) that

$$\varphi_3 \propto 876\ H_6 + 881\ H_4 \quad , \tag{83}$$

while, from Eq (82),

$$\varphi_4 \propto 348\ H_6 + 309\ H_4 \quad . \tag{84}$$

So defined, φ_3 and φ_4 are nearly proportional to each other. Perhaps for this reason, convergence of a double-P_3 FLIP, based on Eqs (75) through (82), is rather slow. One additional transformation eliminates convergence problems.

$$H_7 = \frac{881}{309} G_7 - H_5 \tag{85}$$

The double-P_3 equations now take the final form

$$F_1' + \Sigma_a F_0 = S \quad , \tag{86}$$

$$\frac{2}{3} F_2' + \frac{1}{3} F_0' + \Sigma_1 F_1 = 0 \quad , \tag{87}$$

$$\frac{3}{5} F_3' + \frac{2}{5} F_1' + \Sigma_2 F_2 = 0 \quad , \tag{88}$$

$$\frac{340}{539} H_4' + \frac{3}{7} F_2' + \Sigma_T F_3 = 0 \quad , \tag{89}$$

$$\frac{77}{340} H_5' + \frac{77}{192} F_3' + \Sigma_T H_4 = 0 \quad , \tag{90}$$

$$\frac{876}{924} H_6' + \frac{881}{924} H_4' + \Sigma_T H_5 = 0 \quad , \tag{91}$$

$$\frac{309}{3524} H_7' + \frac{7227}{74885} H_5' + \Sigma_T H_6 = 0 \quad , \tag{92}$$

and

$$\frac{272}{2163} H_6' + \Sigma_T H_7 = 0 \quad . \tag{93}$$

The resulting second-order equations have the form of Eq (1) with

$$\Sigma_{T1} = \Sigma_a \quad ; \tag{94}$$

$$\Sigma_{T2} = \frac{4}{9} \Sigma_a + \frac{5}{9} \Sigma_2 \quad ; \tag{95}$$

$$\Sigma_{T3} = \frac{145}{1463616} \left[85\ (4\ \Sigma_a + 5\ \Sigma_2) + 432\ \Sigma_T \right] \quad ; \tag{96}$$

$$\Sigma_{T4} = \frac{1}{162624} \left[73\ (4\ \Sigma_a + 5\ \Sigma_2) + \frac{91003}{73} \Sigma_T \right] \quad ; \tag{97}$$

$$\left. \begin{array}{ll} D_1 = \dfrac{1}{3\ \Sigma_1} \quad , & D_2 = \dfrac{1}{7\ \Sigma_3} \\ D_3 = \dfrac{25549}{2764608\ \Sigma_T} \quad , & D_4 = \dfrac{881}{14839440\ \Sigma_T} \end{array} \right\} \quad ; \tag{98}$$

$$A_1 = 1 \quad , \quad A_2 = -\frac{2}{3} \quad , \quad A_3 = \frac{85}{72} \quad , \quad A_4 = -\frac{1}{24} \quad ; \tag{99}$$

and

$$\left. \begin{array}{llll} & \alpha_{12} = \frac{2}{3} \Sigma_a \ , & \alpha_{13} = -\frac{145}{5082} \Sigma_a \ , & \alpha_{14} = \frac{73}{1694} \Sigma_a \\ \alpha_{21} = \frac{2}{3} \Sigma_a \ , & \alpha_{23} = \frac{145}{30492} (4\ \Sigma_a + 5\ \Sigma_2) \ , & \alpha_{24} = -\frac{73}{10164} (4\ \Sigma_a + 5\ \Sigma_2) & \\ \alpha_{31} = -\frac{85}{72} \Sigma_a \ , & \alpha_{32} = \frac{85}{432} (4\ \Sigma_a + 5\ \Sigma_2) \ , & \alpha_{34} = \frac{73}{487872} \left[85\ (4\ \Sigma_a + 5\ \Sigma_2) + 432\ \Sigma_T \right] & \\ \alpha_{41} = \frac{1}{24} \Sigma_a \ , & \alpha_{42} = -\frac{1}{144} (4\ \Sigma_a + 5\ \Sigma_2) \ , & \alpha_{43} = \frac{29}{8293824} \left[85\ (4\ \Sigma_a + 5\ \Sigma_2) + 432\ \Sigma_T \right] & \end{array} \right\} \quad ; \tag{100}$$

where

$$\Sigma_i = \Sigma_T - \Sigma_{si} \quad (1 \leq i \leq 2)$$

and

$$\Sigma_i = \Sigma_T \qquad (3 \leq i \leq 7) \quad .$$

Also,

$$\left.\begin{aligned}
F_0 &= \varphi_1 - \frac{2}{3}\varphi_2 + \frac{145}{5082}\varphi_3 - \frac{73}{1694}\varphi_4 \\
F_2 &= \frac{1}{3}\varphi_2 - \frac{145}{10164}\varphi_3 + \frac{73}{3388}\varphi_4 \\
f_4 &= \frac{145}{13552}\varphi_3 - \frac{219}{13552}\varphi_4 \\
f_6 &= -\frac{87}{230384}\varphi_3 + \frac{829}{54208}\varphi_4 \\
F_1 &= -D_1\varphi_1' \\
F_3 &= -D_2\varphi_2' \\
f_5 &= -\frac{10608}{22906}D_3\varphi_3' - \frac{14235}{22906}D_4\varphi_4' \\
H_5 &= -D_3\varphi_3' \\
f_7 &= \frac{2574}{22906}D_3\varphi_3' - \frac{236301}{22906}D_4\varphi_4' \\
H_7 &= -\frac{3212}{103}D_4\varphi_4'
\end{aligned}\right\} \qquad (101)$$

Convergence of the double-P_3 version of FLIP, based on Eqs (86) through (101), is rapid. In this final form the double-P_3 FLIP is operational as a production code.

Gray Boundary Conditions

The code described in the preceding sections is now referred to as FLIP 1. In FLIP 1, either the directional flux or the current must vanish at each boundary. FLIP 2 is a modified form of FLIP 1, with gray boundary conditions applicable at left- and/or right-hand boundaries.

Only the double-P_1 approximation is available in FLIP 2. In this approximation, one may require that

$$\left.\begin{aligned}
\psi_0 &= a_{1L}\chi_0 + b_{1L}\chi_1 \\
\psi_1 &= a_{2L}\chi_0 + b_{2L}\chi_1
\end{aligned}\right\} \qquad (102)$$

and/or that

$$\left.\begin{aligned}
\chi_0 &= a_{1R}\psi_0 + b_{1R}\psi_1 \\
\chi_1 &= a_{2R}\psi_0 + b_{2R}\psi_1
\end{aligned}\right\} \qquad (103)$$

Here the subscripts L and R denote the left- and right-hand boundaries, respectively. The a's and b's, which must be supplied as input to FLIP 2, may be derived from the blackness theory (Ref 6). At a black boundary all the a's and b's are zero. At a reflecting boundary, $a_1 = 1$, $b_2 = -1$, and $a_2 = b_1 = 0$.

NUMERICAL SOLUTION

Difference Equations and Associated Recursion Relations

All the approximations discussed previously lead to equations of the form

$$\left\{ -\nabla\cdot\left[D^i(x)\,\nabla\varphi^i(x)\right] + \Sigma_T^i(x)\varphi^i(x) = A^iS(x) + \sum_{\substack{j=1\\(j\neq i)}}^{k} \alpha_{ij}(x)\varphi^j(x) \right\}_{i=1}^{k}$$

where k takes on values as follows:

k = 2 (P_3, double-P_1)
k = 3 (P_5, double-P_2)
k = 4 (P_7, double-P_3)

These may be expressed as difference equations and solved by well-known methods (Ref 7). In brief, one finds for points interior to a region that

$$\varphi_n^i = \frac{\varphi_{n+1}^i + \beta_n^i}{\alpha_n^i} \quad , \tag{104}$$

$$\delta_n^i = \frac{\delta_{n-1}^i}{1 + \delta_{n-1}^i} + \frac{h^2}{D_n^i}\,\Sigma_{T,n}^i = \alpha_n^i - 1 \quad , \tag{105}$$

$$\beta_n^i = \frac{\beta_{n-1}^i}{1 + \delta_{n-1}^i} + c_n^i \quad , \tag{106}$$

and

$$c_n^i = \frac{h^2}{D_n^i}\left[A^iS_n + \sum_{\substack{j=1\\(j\neq i)}}^{k} \alpha_{ij,n}\varphi_n^j\right] \quad . \tag{107}$$

Introduction of the variable δ_n^i in place of α_n^i is a device which has long been used in the WANDA code (Ref 3) to minimize roundoff error.

At interfaces,

$$\delta_B^i = \frac{1}{\gamma^i}\left\{\frac{\delta_{B-1}^i}{1 + \delta_{B-1}^i} + \frac{1}{2}\left[\frac{h^2}{D_B^i}\,\Sigma_{T,B}^i + \gamma^i\,\frac{h'^2}{D_B'^i}\,\Sigma_{T,B}'^i\right]\right\} \quad , \tag{108}$$

$$\beta_B^i = \frac{1}{\gamma^i}\left[\frac{\beta_{B-1}^i}{1 + \delta_{B-1}^i} + \frac{1}{2}\left(c_B^i + \gamma^i c_B'^i\right)\right] \quad , \tag{109}$$

$$\gamma^i = \frac{D_B'^i h}{D_B^i h'} \quad , \tag{110}$$

$$c_B^i = \frac{h^2}{D_B^i}\left[A^iS_B + \sum_{\substack{j=1\\(j\neq i)}}^{k} \alpha_{ij,B}\varphi_B^j\right] \quad , \tag{111}$$

and

$$c'^{i}_{B} = \frac{h'^{2}}{D'^{i}_{B}} \left[A^{i} S'_{B} + \sum_{\substack{j=1 \\ (j \neq i)}}^{k} \alpha'_{ij,B} \varphi^{j}_{B} \right] , \tag{112}$$

where the primed variables represent the right-hand region.

At the left- and right-hand boundaries, either the flux or the gradient of its even moments may be forced to zero. The related equations are summarized below. At the origin (n = 0):

1) Zero Flux

$$\beta^{i}_{1} = c^{i}_{1} \tag{113}$$

and

$$\delta^{i}_{1} = 1 + \frac{h^{2}}{D^{i}_{1}} \Sigma^{i}_{T,1} \tag{114}$$

2) Zero Gradient

$$\beta^{i}_{0} = \frac{h^{2}}{2D^{i}_{0}} \left[A^{i} S_{0} + \sum_{\substack{j=1 \\ (j \neq i)}}^{k} \alpha_{ij,0} \varphi^{j}_{0} \right] = \frac{c^{i}_{0}}{2} \tag{115}$$

and

$$\delta^{i}_{0} = \frac{h^{2}}{2D^{i}_{0}} \Sigma^{i}_{T,0} . \tag{116}$$

At the outer boundary (n = N):

1) Zero Flux

$$\varphi^{i}_{N} = 0 . \tag{117}$$

2) Zero Gradient

$$\varphi^{i}_{N} = \frac{\dfrac{c^{i}_{N}}{2} + \dfrac{\beta^{i}_{N-1}}{1 + \delta^{i}_{N-1}}}{\dfrac{h^{2}}{2D^{i}_{N}} \Sigma^{i}_{T,N} + \dfrac{\delta^{i}_{N-1}}{1 + \delta^{i}_{N-1}}} , \tag{118}$$

where

$$c^{i}_{N} = \frac{h^{2}}{D^{i}_{N}} \left[A^{i} S_{N} + \sum_{\substack{j=1 \\ (j \neq i)}}^{k} \alpha_{ij,N} \varphi^{j}_{N} \right] . \tag{119}$$

Gray Boundary Conditions

The relationship of the a's and b's of Eqs (102) and (103) to the basic equations will now be shown. Consider a slab with gray (or black) conditions applied at the left-hand boundary. The use of Eq (102) to substitute for ψ_0 and ψ_1 in Eqs (31) and (33), respectively, gives two simultaneous linear equations in χ_0 and χ_1. When these are solved it is found that

$$\chi_0 = \frac{(b_{2L} - 1) F_0 - \frac{4}{3} b_{1L} f_2}{(a_{1L} + 1)(b_{2L} - 1) - a_{2L} b_{1L}} \tag{120}$$

and

$$\chi_1 = \frac{\frac{4}{3}(a_{1L} + 1) f_2 - a_{2L} F_0}{(a_{1L} + 1)(b_{2L} - 1) - a_{2L} b_{1L}} \quad . \tag{121}$$

Let these be written as

$$\chi_0 = A_{1L} F_0 + B_{1L} f_2 \tag{122}$$

and

$$\chi_1 = A_{2L} F_0 + B_{2L} f_2 \quad , \tag{123}$$

where the definitions of A_{1L}, B_{1L}, A_{2L}, B_{2L} are evident from Eqs (120) and (121). Substitution of Eq (102) into Eqs (32) and (34) gives

$$F_1 = \frac{1}{2}\left[(a_{1L} + a_{2L} - 1)\chi_0 + (b_{1L} + b_{2L} + 1)\chi_1\right]$$

and

$$f_3 = \frac{1}{8}\left[(1 - a_{1L} + 3a_{2L})\chi_0 + (3 - b_{1L} + 3b_{2L})\chi_1\right]$$

which, by means of Eqs (122) and (123), can be written

$$F_1 = \frac{1}{2}\left\{\left[A_{1L}(a_{1L} + a_{2L} - 1) + A_{2L}(b_{1L} + b_{2L} + 1)\right] F_0 + \left[B_{1L}(a_{1L} + a_{2L} - 1) + B_{2L}(b_{1L} + b_{2L} + 1)\right] f_2\right\} \tag{124}$$

and

$$f_3 = \frac{1}{8}\left\{\left[A_{1L}(1 - a_{1L} + 3a_{2L}) + A_{2L}(3 - b_{1L} + 3b_{2L})\right] F_0 + \left[B_{1L}(1 - a_{1L} + 3a_{2L}) + B_{2L}(3 - b_{1L} + 3b_{2L})\right] f_2\right\} \quad . \tag{125}$$

If Eq (39) is used in Eqs (124) and (125) to remove F_0 and f_2, the result is

$$F_1 = \frac{1}{2}\left[A_{1L}(a_{1L} + a_{2L} - 1) + A_{2L}(b_{1L} + b_{2L} + 1)\right]\varphi_1 + \frac{3}{16}\left[(B_{1L} - 2A_{1L})(a_{1L} + a_{2L} - 1) + (B_{2L} - 2A_{2L})(b_{1L} + b_{2L} + 1)\right]\varphi_2 \tag{126}$$

and

$$f_3 = \frac{1}{8}\left[A_{1L}(1 - a_{1L} + 3a_{2L}) + A_{2L}(3 - b_{1L} + 3b_{2L})\right]\varphi_1 + \frac{3}{64}\left[(B_{1L} - 2A_{1L})(1 - a_{1L} + 3a_{2L}) + (B_{2L} - 2A_{2L})(3 - b_{1L} + 3b_{2L})\right]\varphi_2 \quad . \tag{127}$$

These equations can be written, with the aid of Eq (39), as

$$- D_1 \varphi_1' = \alpha_{L1}\varphi_1 + T_{L1}\varphi_2 \tag{128}$$

and

$$- D_2 \varphi_2' = \alpha_{L2}\varphi_2 + T_{L2}\varphi_1 \quad , \tag{129}$$

where the definitions of the α_L's and T_L's are obvious from Eqs (126) and (127). If gray (or black) conditions are applied at the right-hand boundary, a similar technique is used to find the corresponding α_{R1}, T_{R1}, α_{R2}, and T_{R2}. Consequently, it can be shown that

$$\left.\begin{aligned} \alpha_{R1} &= -\alpha_{L1} \\ T_{R1} &= -T_{L1} \\ \alpha_{R2} &= -\alpha_{L2} \\ T_{R2} &= -T_{L2} \end{aligned}\right\} \tag{130}$$

It now remains to show how the parameters in Eq (130) modify the flux calculations. Let Eqs (128) and (129) be written as

$$-D^i \frac{d\varphi^i}{dx} = \alpha_L^i \varphi^i + \sum_{\substack{j=1 \\ (j\neq i)}}^{k} T_L^i \varphi^j \quad \left(\begin{array}{l} i = 1, 2 \\ k = 2 \end{array}\right) . \tag{131}$$

Consider the left-hand boundary. At x = 0, Eq (131) can be written in difference equation form as

$$-D_0^i \left[\frac{\varphi_1^i - \varphi_{-1}^i}{2h}\right] = \alpha_L^i \varphi_0^i + \sum_{\substack{j=1 \\ (j\neq i)}}^{k} T_L^i \varphi_0^j . \tag{132}$$

A three-point difference equation in φ, obtainable by methods cited in Ref 6, is used to eliminate φ_{-1}^i from Eq (132). Suitable algebraic manipulation, together with the use of Eq (107), allows the resulting expression to be written as

$$\varphi_0^i = \frac{\varphi_1^i + \beta_0^{*i}}{1 + \delta_0^{*i}} , \tag{133}$$

where

$$\beta_0^{*i} = \frac{h^2}{2D_0^i}\left[A^i S_0 + \sum_{\substack{j=1 \\ (j\neq i)}}^{k} (\alpha_{ij,0} + t_L^i)\, \varphi_0^j\right] = \frac{c_0^{*i}}{2} , \tag{134}$$

$$\delta_0^{*i} = \frac{h^2}{2D_0^i}\left[\Sigma_{T,0}^i - \frac{2}{h}\alpha_L^i\right] \tag{135}$$

and

$$t_L^i = \frac{2}{h} T_L^i . \tag{136}$$

At the right-hand boundary, where $x = x_N$, Eq (131) is written in difference equation form as

$$-D_N^i \left[\frac{\varphi_{N+1}^i - \varphi_{N-1}^i}{2h}\right] = \alpha_R^i \varphi_N^i + \sum_{\substack{j=1 \\ (j\neq i)}}^{k} T_R^i \varphi_N^j . \tag{137}$$

Manipulation of Eq (137) in a manner similar to that used with Eq (132) permits it to be written as

$$\varphi_N^i = \frac{\dfrac{c_N^{*i}}{2} + \dfrac{\beta_{N-1}^i}{1 + \delta_{N-1}^i}}{\dfrac{h^2}{2D_N^i}\Sigma_{T,N}^i + \dfrac{\delta_{N-1}^i}{1 + \delta_{N-1}^i} + \dfrac{h}{D_N^i}\alpha_R^i} \tag{138}$$

where

$$c_N^{*i} = \frac{h^2}{D_N^i}\left[A^i S_N + \sum_{\substack{j=1 \\ (j\neq i)}}^{k} (\alpha_{ij,N} - t_R^i)\,\varphi_N^j\right] \tag{139}$$

and

$$t_R^i = \frac{2}{h} T_R^i \quad . \tag{140}$$

At reflecting boundaries (or axes of symmetry), α_L^i, t_L^i and/or α_R^i, t_R^i are zero. Under these conditions, Eqs (134) and (135) reduce to Eqs (115) and (116), and/or Eq (138) reduces to Eq (118), respectively.

A SUMMARY OF FLIP CHARACTERISTICS

Features

Flexible Input

For most problems, the usual procedure is to have the code automatically calculate all the coefficients of Eq (104). The input required for this is called "regular" input. However, for special applications it is possible to specify these coefficients directly. Hence, this kind of input is called "direct" input. Either regionwise or pointwise external sources can be used with each kind of input, which makes a total of four possible methods of input presentation.

Choice of Output

For each input method, a total of four output edits is available. With regular input, these edits consist of various combinations of the scalar flux, its even moments, and the α_{ij}. With direct input, the appropriate group fluxes are printed out instead of the scalar flux and the even moments, and the convergence criterion is based directly on φ_{1n} instead of F_{0n}.

Stopping after One Iteration

Any problem can be set to stop automatically after one iteration. Because of the manner in which iterations are counted, stopping after the first one will give the P_1 solution.

Scaling of External Sources

In certain rare cases, a given FLIP 1 problem, usually a double-P_3 approximation, will fail to converge because the values being computed for the β_n^i are creating an underflow condition. Such a condition is caused by the computer's attempting to handle numbers smaller than 10^{-38} (except for zero). It has been found that underflow may occur in a problem where the external source is constant in a region several mean free paths in width. If such an underflow occurs, FLIP 1 automatically scales the external sources up by a factor of 10^4 and repeats the calculation. If the problem converges after sufficient scaling, the print-out will indicate how many times the external sources (and consequently, the fluxes) have been scaled. If, however, a total of three such scalings is insufficient to remedy the situation, the problem is automatically stopped, and the region causing the

the difficulty is indicated on the print-out. If such problems are to be run successfully, one should remove excessive mesh points in those parts of cells where the flux remains at an asymptotic value for several mean free paths.

Extrapolation

An extrapolation technique, using the same extrapolation factor on all fission sources, can be specified. Suppose that for the k^{th} iteration $\varphi_i^{(k)}$ $(i > 1)$ has been calculated. Instead of using this value of $\varphi_i^{(k)}$ in the fission term of Eq (104) for the next iteration, it is modified according to the relation

$$\left[\varphi_i^{(k)}\right]_{new} = \varphi_i^{(k)} + \theta \left[\varphi_i^{(k)} - \varphi_i^{(k-1)}\right] \quad (0 \leq \theta \leq 1) \quad .$$

If extrapolation is specified, FLIP will wait until the third iteration to begin extrapolating.

Problem Running Time

The actual running time for most FLIP problems, with a convergence criterion in the order of 0.00005 and without extrapolation, ranges from less than one-half minute for the smaller P_3 and double-P_1 problems to about nine or ten minutes for the largest P_7 and double-P_3 problems. The number of iterations usually increases with the higher approximations, but it should seldom exceed fifteen; sometimes, for a P_3 or double-P_1 approximation the number of iterations is as low as four or five.

Restrictions

Geometric Restrictions

A maximum of 500 mesh intervals and 50 homogeneous regions is allowed with FLIP 1. For FLIP 2 this becomes 250 mesh intervals and 25 homogeneous regions. Only slab geometry can be used.

Mathematical Restrictions

FLIP 1 handles the P_3, P_5, P_7, double-P_1, double-P_2, and double-P_3 approximations. The boundary conditions are either zero current or zero flux. FLIP 2 allows any degree of grayness, from reflecting to blackness, for its boundary conditions. Grayness may be specified at either one or both boundaries. When it is specified at only one boundary, the conditions at the other boundary are specified as in FLIP 1. Only the double-P_1 approximation is available in FLIP 2.

Computer Restrictions

The minimum computer requirements are a 32,768 word core storage unit, one tape unit, an on-line card reader, an on-line printer, and an off-line printer. No drums are used.

PREPARATION OF INPUT

FLIP 1 Input Data

Title Card

Columns	
02 - 06	FLIP 1
07 - 72	Problem number and any other information the requestor wishes to use for identification purposes

Control Card No. 1

Columns	
01 - 04	Card number (0001)
05 - 08	Type of approximation (0003, 0005, 0007, 0021, 0022, 0023)

09 - 12	Number of regions ≤ 50
13 - 16	Number of points ≤ 501. This number, which includes the origin as point No. 1, must be odd.
17 - 20	Left boundary: symmetric (0001), nonsymmetric (0000)
21 - 24	Right boundary: symmetric (0001), nonsymmetric (0000)
25 - 28	Type of input: regular (0000), direct (0001)
29 - 32	Type of output: (0000): F_0 at all points (0001): F_0 at all points, α_{ij} for all regions (0002): F_0, F_2, (F_4), (F_6) at all points (0003): F_0, F_2, (F_4), (F_6) at all points, α_{ij} for all regions
33 - 36	Type of source: regionwise (0000), pointwise (0001)
37, 38 - 43, 44, 45	Sign, convergence criterion (floating point), sign, exponent
46, 47 - 52, 53, 54	Sign, extrapolation factor (floating point), sign, exponent

Note: Floating point numbers are written to the base 10. For example, a convergence criterion of 0.00005 should be written as + 500000 - 4.

Control Card No. 2

This card controls the option to specify fuel regions. If none are to be specified, place zeros in columns 5 - 6.

Columns	
01 - 04	Card number (0002)
05 - 06	Total number of fuel regions
07 - 08 09 - 10 11 - 12 etc.	Region number(s) of the fuel regions

Geometry

Columns	
01 - 04	Card number
05 - 08 09 - 12 etc.	Interface numbers and the outer boundary. These numbers must be odd, counting the origin as point number one. Use as many cards as necessary.

Mesh Spacing

Columns	
01 - 04	Card number
05, 06 - 13, 14, 15	Sign, mesh width (floating point), sign, exponent
16, 17 - 24, 25, 26	Sign, mesh width (floating point), sign, exponent

27, 28 - 35, 36, 37	Sign, mesh width (floating point), sign, exponent
38, 39 - 46, 47, 48	Sign, mesh width (floating point), sign, exponent
49, 50 - 57, 58, 59	Sign, mesh width (floating point), sign, exponent
60, 61, 68, 69, 70	Sign, mesh width (floating point), sign, exponent

Use as many cards as necessary.

Source

Sources are listed, using the same format as the mesh spacing. If a regionwise source is specified, there must be a source for each region.

If a pointwise source is specified, there must be a source for each point and an interface source for each interface as well as one at the outer boundary. To list the interface sources, start a new card after all pointwise sources have been listed. Since a double-valued source is not used at the outer boundary, a zero must be used at this point.

Sigmas

List the sigmas in the same format as the mesh spacing.

a) P_3 and double-P_1 require one card for each region, containing
   Card number, Σ_a, Σ_{s_0}, Σ_{s_1}, Σ_{s_2}, Σ_{s_3}

b) Double-P_2 and double-P_3 require one card for each region, containing
   Card number, Σ_a, Σ_{s_0}, Σ_{s_1}, Σ_{s_2}

c) P_5 requires two cards for each region, containing
   Card number, Σ_a, Σ_{s_0}, Σ_{s_1}, Σ_{s_2}, Σ_{s_3}
   Card number, Σ_{s_4}, Σ_{s_5}

d) P_7 requires two cards for each region, containing
   Card number, Σ_a, Σ_{s_0}, Σ_{s_1}, Σ_{s_2}, Σ_{s_3}
   Card number, Σ_{s_4}, Σ_{s_5}, Σ_{s_6}, Σ_{s_7}

List all the sigmas for the first region, followed by all the sigmas for the second region, etc.

Direct Input

List the data through the sources in the manner described above. However, instead of specifying values for the sigmas, the requestor must supply values for D_i, Σ_{Ti}, A_i, and α_{ij}.

a) D_i: There will be one card for each region, containing
   Card number, D_1, D_2, (D_3), (D_4)

If any of the D_i do not apply to the approximation, their space on the card may be left blank.

b) Σ_{Ti}: Use one card for each region, containing
   Card number, Σ_{T_1}, Σ_{T_2}, (Σ_{T_3}), (Σ_{T_4})

c) A_i: One card, containing
   Card number, A_1, A_2, (A_3), (A_4)

d) α_{ij}: Use either one or two cards per region, depending upon the approximation, containing
   Card number, α_{12}, α_{21}, α_{13}, α_{31}, α_{23}, α_{32}
   Card number, α_{14}, α_{41}, α_{24}, α_{42}, α_{34}, α_{43}

One Iteration

One-iteration problems may be done by setting the convergence criterion to zero. First group fluxes will be calculated and printed.

FLIP 2 Input Data

The input instructions are identical with those of FLIP 1 except for the following differences:

Title Card Changes

Columns	
02 - 06	FLIP 2
07 - 72	Problem number and any other identification

Control Card No. 1 Changes

Columns	
05 - 08	Type of approximation (0021)
09 - 12	Number of regions ≤ 25
13 - 16	Number of points ≤ 251. This number, which includes the origin as point No. 1, must be odd.
17 - 20	Left boundary: symmetric (0001), nonsymmetric (0000), grayness (0001)
21 - 24	Right boundary: symmetric (0001), nonsymmetric (0000), grayness (0001)

All other necessary information regarding gray boundary conditions appears on additional cards which must be put at the end of the deck. The numbering of these cards must be consecutive with the rest of the deck. The order and description of the additional cards are given below.

Gray Boundary Control Card

Columns	
01 - 04	Card number
05 - 06	Gray boundary conditions: no grayness at either boundary (00), grayness at right boundary only (01), grayness at left boundary only (10), grayness at both boundaries (11)

Gray Boundary Input Parameters

If no grayness is specified the gray boundary control card is the last card in the deck. If grayness is specified at only one boundary, one additional card containing a_1, b_1, a_2, b_2, is necessary. If grayness is specified at both boundaries, two more cards are necessary. The first must contain a_1, b_1, a_2, b_2 for the right boundary, while the second must contain these four parameters for the left boundary.

Columns	
01 - 04	Card number
05, 06 - 13, 14, 15	Sign, a_1 (floating point), sign, exponent
16, 17 - 24, 25, 26	Sign, b_1 (floating point), sign, exponent

27, 28 - 35, 36, 37 Sign, a_2 (floating point), sign, exponent

38, 39 - 46, 47, 48 Sign, b_2 (floating point), sign, exponent

OUTPUT

Output information will be printed in the following order:

1) A listing is made of the data deck.

2) The calculated group parameters, D^i, Σ_T^i, and A^i, are listed for all the regions for the first group, followed by all the regions for the second group, etc.

   The α_{ij} will be printed if requested. The values are read as if they were the columns of a matrix: α_{11}, α_{21}, α_{31}, α_{41}, α_{12}, α_{22}, etc., and there will be a set of these for each region.

3) Values of λ_{max} and λ_{min} will be printed after each iteration except the first. These values must approach unity in order for convergence to take place.

   If one of the F_{on}'s, which appear in the denominator of the ratio λ_n, is very small or zero (except at a nonsymmetric boundary), the ratio cannot be computed, and a test for convergence cannot be made. However, a test will be attempted after the next iteration. Hence, there may be certain iterations for which no values of λ_{max} and λ_{min} are printed.

   If the underflow condition occurs, the sources will be scaled by a factor of 10^4, and the calculation which caused the underflow will be repeated. Each time the sources are scaled, the scaling factor will be printed. After scaling has occurred, the iteration counter will not be reset, and values of λ_{max} and λ_{min} will continue to be calculated and printed. If, after scaling the sources three times, the underflow condition persists, the problem stops and prints out an indication of the region which causes the difficulty.

4) The point number, radius, and fluxes will be printed after convergence has taken place. For regular input, the F_n's will be printed; for direct input, the φ_n's will be printed. If it was necessary to scale the sources, the fluxes will have been scaled by the same factor.

5) The width of each region is listed.

6) The following integrals, calculated over each region by means of Simpson's Rule, will be printed:

   a) $\int \Sigma_a F_0 dx$

   b) $\int F_0 dx$

7) If the requestor wishes, the above integrals will be summed over fuel and non-fuel regions. This option is under control of input card No. 2. The sums of the integrals will be printed in the following order:

   a) $\sum \int F_0 dx$ for fuel regions

   b) $\sum \int \Sigma_a F_0 dx$ for fuel regions

   c) $\sum \int F_0 dx$ for non-fuel regions

   d) $\sum \int \Sigma_a F_0 dx$ for non-fuel regions

8) Balance checks are calculated for each region and for each group.

Let the boundaries of a region R be at n and N, as shown in Fig. 1.

The basic equations solved by the code may be written as

$$- D^i \nabla^2 \varphi^i = S_T^i - \Sigma_T^i \varphi^i \tag{141}$$

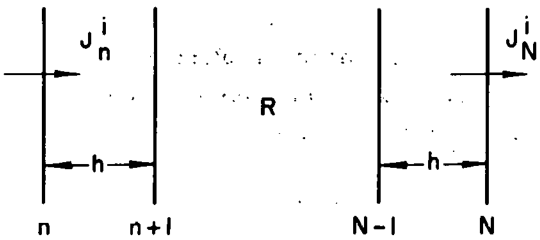

Figure 1

where S_T^i is the total source (fission + slowing down + external). Integrating the right-hand side of Eq (141) across the region R, one may write

$$\int_n^N (S_T^i - \Sigma_T^i \varphi^i)\, dx = J_N^i - J_n^i \quad . \tag{142}$$

Integrating the left-hand side of Eq (141) in a similar manner, one may write

$$- D^i \frac{\partial \varphi^i}{\partial x}\Bigg|_{x=n}^{x=N} = j_N^i - j_n^i \quad , \tag{143}$$

where

$$j_N^i = - D^i \left[\frac{\varphi_N^i - \varphi_{N-1}^i}{h}\right] + \frac{h}{2} (S_T^i)_N - \frac{h}{2} \Sigma_T^i \varphi_N^i \tag{144}$$

and

$$j_n^i = - D^i \left[\frac{\varphi_{n+1}^i - \varphi_n^i}{h}\right] - \frac{h}{2} (S_T^i)_n + \frac{h}{2} \Sigma_T^i \varphi_n^i \quad . \tag{145}$$

The balance checks are values of $(J_N^i - J_n^i)$, which the code calculates by Simpson's Rule, and of $(j_N^i - j_n^i)$, which it calculates with Eqs (144) and (145). A comparison between $(J_N^i - J_n^i)$ and $(j_N^i - j_n^i)$ should provide some estimate of the adequacy of the mesh spacing for a particular problem. For each group, $(J_N^i - J_n^i)$ is printed out for all the regions and followed by $(j_N^i - j_n^i)$ for all the regions.

It will be seen, from Eq (49) for example, that the j^i's are simply related to the odd moments of the directional flux. The first moment is the current.

If at least one of the boundaries is symmetric, the current at each interface may be obtained from the balance checks by performing a simple hand computation. From Eq (142), it is evident that if one starts at a symmetric boundary, the current at the nearest interface can be found. Proceeding in this manner, it is possible to construct, in order, the currents for all interfaces.

OPERATING INSTRUCTIONS

General

FLIP may be run either from tape or cards, but tape is preferable. The routine WB CTB 2 is used to prepare the program tape from absolute binary and correction/transfer cards.

FLIP 1

Printer Board–SHARE No. 2

Tapes–Logical No. 1: Program tape (this is not used if the code is being run from binary cards).
Logical No. 2: Blank

Sense Switches–All normal

Note: FLIP 1 contains a special device, which is under the control of SSW No. 5, that may be used for on-line monitoring of the progress of any problem. With SSW No. 5 depressed, values for λ_{max} and λ_{min} are printed out on-line after each iteration. If the problem is converging, each of these values must be approaching unity. This monitor is not meant to be used as a general procedure, but rather only if a problem seems to be running a long time.

Console–CLEAR, LOAD TAPE (or LOAD CARDS)

Card Reader–START

Note: Any number of problem decks may be loaded into the reader. A blank card must be inserted after each problem deck, and two blank cards should follow the last problem deck. Note that FLIP 1 does not read the blank card until after the problem is finished.

Last Problem Ends–With a SELECT on the card reader.

Pressing START on the card reader causes the following:

(1) An end-of-file is written on the output tape.

(2) The end-of-program stop $(14776)_8$ appears on the console.

Print Output Tape–Program Control

Program Stops

All numbers are octal.

7543	Machine error
7572	Probably requestor error in specifying interface numbers. Possibly machine error. Push START on console to read in next problem.
7575	Cards are incorrectly numbered or out of order. If it stops on reading the blank card, there are not enough cards in the deck.
11512	Machine error
11514	Machine error
11516	Machine error
11520	Machine error
11522	Machine error
11524	Machine error
11526	Machine error
11530	Machine error
11532	Machine error
11534	Machine error
11536	Machine error
11540	Machine error
11542	Machine error
11544	Machine error
11545	Machine error
11550	Machine error
11552	Machine error
11554	Machine error
12757	Inappropriate character encountered in a data field in reading cards
13002	Inappropriate character encountered in a data field in reading cards
13022	Inappropriate character encountered in a data field in reading cards
13036	Inappropriate character encountered in a data field in reading cards
13301	Non-Hollerith character encountered in reading card
14667	Echo check in printing. Press RESET and START to repeat line and continue.
14776	End of file in reading cards End of program

FLIP 2

The operating instructions for FLIP 1 apply, with the following exceptions:

Sense Switches–(a) All normal for off-line output

(b) SSW No. 5 down for on-line output (logical tape No. 2 is not needed)

Note: FLIP 2 contains no provision for monitoring the convergence of a problem. It will write the entire output for a problem either off-line or on-line, but it will not do both.

Last Problem Ends–Same as FLIP 1, except that the end-of-program stop is $(14153)_8$.

Program Stops

All numbers are octal.

7271	Machine error
7330	Probably requestor error in specifying interface numbers. Possibly machine error. Push START on console to read in next problem.
7334	Cards are incorrectly numbered or out of order. If it stops on reading the blank card, there are not enough cards in the deck.
13003	Machine error
13005	Machine error
13007	Machine error
13011	Machine error
13013	Machine error
13015	Machine error
13017	Machine error
13021	Machine error
13023	Machine error
13025	Machine error
13027	Machine error
13031	Machine error
13033	Machine error
13035	Machine error
13037	Machine error
13041	Machine error
13043	Machine error
13045	Machine error
13047	Machine error
13051	Machine error
13053	Machine error
13055	Machine error
13057	Machine error
13061	Machine error
13063	Machine error
13065	Machine error
13067	Machine error
13071	Machine error
13073	Machine error
13075	Machine error
13077	Machine error
13101	Machine error
13103	Machine error
13105	Machine error
13107	Machine error
13111	Machine error

13113	Machine error
13115	Machine error.
14153	End of file in reading cards
	End of program
14241	Inappropriate character encountered in a data field in reading cards
14274	Inappropriate character encountered in a data field in reading cards
14314	Inappropriate character encountered in a data field in reading cards
14330	Inappropriate character encountered in a data field in reading cards
14573	Non-Hollerith character encountered in reading a card
16161	Echo check in printing; press RESET and START to repeat line and continue

```
 FLIP1 SAMPLE PROBLEM FOR INCLUSION IN WAPD-TM-134
00010023000200230001000100000030000+500000-4+000000+0
000200
00030011 0023
0004+12700000+0+10583333+0
0005+00000000+0+10000000+1
0006+10000000+1+30000000+0+00000000+0+00000000+0+00000000+0+00000000+0
0007+20000000-1+31200000+1+00000000+0+00000000+0+00000000+0+00000000+0
```

```
FLIP1 SAMPLE PROBLEM FOR INCLUSION IN WAPD-TM-134

 1  23   2  23   1   1   0   3   0  0.00004999  0.
 2 0-0-0-0-0-0-0-0-0-0-0-0-0-0-0-0-0-0-0-0-0-0-0-0-0-0-0-0-0-0-0-0-0
 3  11  23  -0  -0  -0  -0  -0  -0  -0  -0  -0  -0  -0  -0  -0  -0  -0
 4     0.12700000    0.10583333   -0.          -0.          -0.          -0.
 5     0.            1.00000000   -0.          -0.          -0.          -0.
 6     1.00000000    0.30000000    0.           0.           0.
 7     0.02000000    3.11999997    0.           0.           0.
```

GROUP PARAMETERS

D

0.256410E-00	0.106157E-00
0.109890E-00	0.454959E-01
0.710881E-02	0.294314E-02
0.456683E-04	0.189073E-04

SIGMA TOTAL

0.100000E 01	0.200000E-01
0.116667E 01	0.175333E 01
0.144057E-00	0.267268E-00
0.146737E-01	0.311536E-01

A

0.100000E 01	-0.666667E 00	0.118056E 01	-0.416667E-01

ALPHA(I,J)

0.	0.666667E 00	-0.118056E 01	0.416667E-01	0.666667E 00	0.	0.206597E 01	-0.729167E-01
-0.285321E-01	0.499311E-01	0.	0.508437E-02	0.430933E-01	-0.754132E-01	0.217576E-00	0.
0.	0.133333E-01	-0.236111E-01	0.533333E-03	0.133333E-01	0.	0.310486E 01	-0.109583E-00
-0.570641E-03	0.750394E-01	0.	0.543300E-02	0.861365E-03	-0.113335E-00	0.403667E-00	0.

```
ITERATION    2  MAX.LAMDA= 1.18599050  MIN.LAMDA= 0.91486744
ITERATION    3  MAX.LAMDA= 1.01316142  MIN.LAMDA= 0.93454427
ITERATION    4  MAX.LAMDA= 1.00449115  MIN.LAMDA= 0.98420712
ITERATION    5  MAX.LAMDA= 1.00131807  MIN.LAMDA= 0.99431303
ITERATION    6  MAX.LAMDA= 1.00024803  MIN.LAMDA= 0.99897350
ITERATION    7  MAX.LAMDA= 1.00018603  MIN.LAMDA= 0.99991928
ITERATION    8  MAX.LAMDA= 1.00029315  MIN.LAMDA= 0.99991538
ITERATION    9  MAX.LAMDA= 1.00018854  MIN.LAMDA= 0.99994268
ITERATION   10  MAX.LAMDA= 1.00009735  MIN.LAMDA= 0.99996679
ITERATION   11  MAX.LAMDA= 1.00004533  MIN.LAMDA= 0.99998210
```

POINT NUMBER	RADIUS	F0	F2	F4	F6
1	0.	0.40992952E-00	0.14727695E-00	0.52112377E-02	-0.41895828E-02
2	0.12700000E-00	0.41887802E-00	0.14924917E-00	0.45688239E-02	-0.41762618E-02
3	0.25399999E-00	0.44631242E-00	0.15515279E-00	0.25892777E-02	-0.40944359E-02
4	0.38099999E-00	0.49407287E-00	0.16493060E-00	-0.88603934E-03	-0.38063686E-02
5	0.50799999E 00	0.56548587E 00	0.17842150E-00	-0.61272082E-02	-0.30391946E-02
6	0.63499998E 00	0.66581014E 00	0.19524218E-00	-0.13520667E-01	-0.13103072E-02
7	0.76199998E 00	0.80301062E 00	0.21456551E-00	-0.23562659E-01	0.21913470E-02
8	0.88899998E 00	0.98922962E 00	0.23463550E-00	-0.36755811E-01	0.87287277E-02
9	0.10159999E 01	0.12448151E 01	0.25113507E-00	-0.52821651E-01	0.19679661E-01
10	0.11429999E 01	0.16171027E 01	0.24843492E-00	-0.64978283E-01	0.32051197E-01
11	0.12699999E 01	0.23006754E 01	0.14095263E-00	-0.18360290E-01	0.24726263E-02
12	0.13758332E 01	0.36557463E 01	-0.17311437E-01	0.28315678E-01	-0.16009320E-01
13	0.14816666E 01	0.46886316E 01	-0.63380176E-01	0.24294705E-01	-0.67821943E-02
14	0.15874999E 01	0.55758391E 01	-0.84418596E-01	0.17765163E-01	-0.22331306E-02
15	0.16933332E 01	0.63478401E 01	-0.94725899E-01	0.12447877E-01	-0.44308313E-03
16	0.17991665E 01	0.70172437E 01	-0.99792038E-01	0.85077415E-02	0.17392222E-03
17	0.19049998E 01	0.75906590E 01	-0.10221548E-00	0.57133918E-02	0.32895885E-03
18	0.20108332E 01	0.80718863E 01	-0.10329124E-00	0.37857959E-02	0.31955788E-03
19	0.21166665E 01	0.84632798E 01	-0.10368863E-00	0.24863559E-02	0.26481593E-03
20	0.22224998E 01	0.87663814E 01	-0.10376567E-00	0.16351038E-02	0.20914961E-03
21	0.23283331E 01	0.89822233E 01	-0.10371871E-00	0.11063237E-02	0.16699158E-03
22	0.24341664E 01	0.91114756E 01	-0.10365503E-00	0.81393493E-03	0.14170130E-03
23	0.25399993E 01	0.91545185E 01	-0.10362836E-00	0.72793411E-03	0.13334854E-03

```
REGION WIDTHS
  0.127000E 01  0.127000E 01

INTEGRAL OF (SIGMA A)( FLUX )

  0.108247E 01  0.178079E-00

INTEGRAL OF FLUX ACROSS EACH REGION

  0.108247E 01  0.890393E 01

BALANCE CHECKS

GROUP 1

 -0.108247E 01  0.109192E 01
 -0.109220E 01  0.109220E 01

GROUP 2

  0.184677E-00 -0.183818E-00
  0.195668E-00 -0.195668E-00

GROUP 3

 -0.190577E-00  0.183899E-00
 -0.215559E-00  0.215559E-00

GROUP 4

  0.407038E-02 -0.370959E-02
  0.528856E-02 -0.528866E-02
```

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