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SUBJECT: Containment Properties of DCX

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Abstract

The "absolute" containment of ions in the DCX magnetic mirror field resulting from the cylindrical symmetry of the field is discussed. The regions of confinement in space and momentum are plotted for 300 Kev deuterons.

DISCLAIMER

Ions are contained in the Oak Ridge high-energy injection experiment, DCX, by means of a magnetic "mirror" field. It is well known that charged particles of sufficiently small Larmor radius are confined by magnetic mirrors due to the constraints imposed by an exact constant of motion, the kinetic energy, $p^2/2m$, together with an approximate constant, the magnetic moment. DCX orbits are much too large for this adiabatic containment criterion to apply. However, it is also well known[1] that, in addition to possible adiabatic confinement, a class among those particles encircling the magnetic axis of symmetry is absolutely contained by the action of two exact constants, the kinetic energy, again, and the canonical angular momentum, p_θ. While the ultimate mirror machine may require adiabatic confinement, in DCX the ion Larmor radius is so large that, though nonadiabatic, almost all ions are in fact subject to absolute containment. As will be pointed out, this is almost as good as adiabatic containment in this case.

The absolute containment criterion can be derived as follows. In cylindrical coordinates with the z-axis pointing in the magnetic field direction, the non-relativistic Hamiltonian for a single particle with charge e, mass m, moving in a cylindrically symmetric magnetic field arising from the vector potential, $\vec{A}$, which in this case has only one component, $A_\theta \equiv A(r,z)$, is

$$H = \frac{1}{2m}\left[p_r^2 + p_z^2 + \left(\frac{p_\theta}{r} - \frac{e}{c}A\right)^2\right]. \tag{1}$$

Here, as was mentioned above, $p_\theta = mr^2\dot{\theta} + \frac{e}{c}rA$ is a constant of the motion. Writing H, also a constant of motion, as $p^2/2m$, and multiplying through by 2m gives

$$p^2 = p_r^2 + p_z^2 + V(p_\theta, r, z) \tag{2}$$

1. A. Garren et al., Non-Adiabatic Effects in Single Particle Orbits, in Proceedings of Controlled Thermonuclear Reactions Conference, Berkeley, California, Feb. 1957, TID-7536 (Part 2), p. 170.

where

$$V(p_\theta, r, z) = \left(\frac{p_\theta}{r} - \frac{e}{c} A\right)^2 \qquad (3)$$

is, for a given p_θ, an effective potential for the r, z motion. Clearly a sufficient criterion for absolute containment is that all the energy become "potential" as a particle approaches the machine walls; that is, it is sufficient to require that

$$V \geqq p^2 \qquad (4)$$

at every point on the boundary of the machine.

Consider positive ions, $e > 0$. In our coordinate system, $A \geq 0$ inside the machine, and $A \longrightarrow 0$ as $r \longrightarrow 0$. Thus, for $p_\theta > 0$, $V = 0$ at some r for all p_θ, z, so that Eq. (4) cannot be satisfied. Such is not the case if $p_\theta < 0$, corresponding to orbits which encircle the z-axis. Then V as a function of r and z defines a bowl-shaped surface depressed in the neighborhood of $z = 0$. A particle with insufficient energy to spill over the bowl is trapped. In Fig. 1 are plotted typical curves $\sqrt{V}/p$ for 300 kev deuterons at $z = 0$, the "midplane" of DCX (mirror ratio 2:1).

Equation (4) can be written as an explicit criterion in momentum space by noting that there exists a maximum p_θ, call it $\overline{p}_\theta$, such that Eq. (4) is satisfied for all $p_\theta < \overline{p}_\theta$. ($\overline{p}_\theta$ is negative, of course.) Then an ion is <u>absolutely contained</u> if

$$p_r^2 + p_z^2 < p^2 - V(\overline{p}_\theta, r, z). \qquad (5)$$

In Fig. 2, the circles bounding the absolute containment zones in momentum space defined by Eq. (5) have been plotted for 300 kev deuterons at several radial positions in the DCX midplane. The momentum of any particle corresponds to a point within the unit circle, of course. A particle crossing the midplane

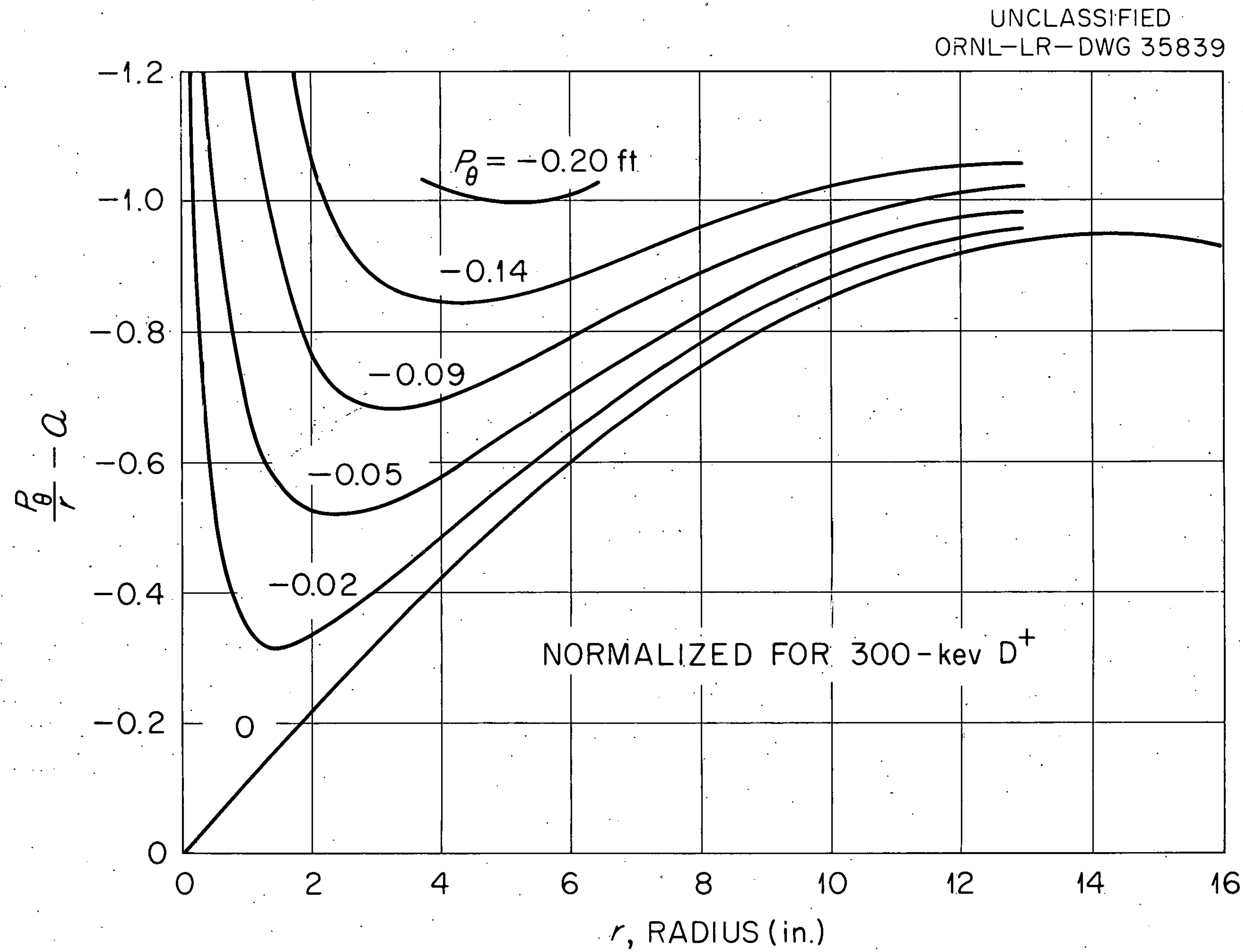

Fig. 1. Effective Potential Confining Ions in DCX

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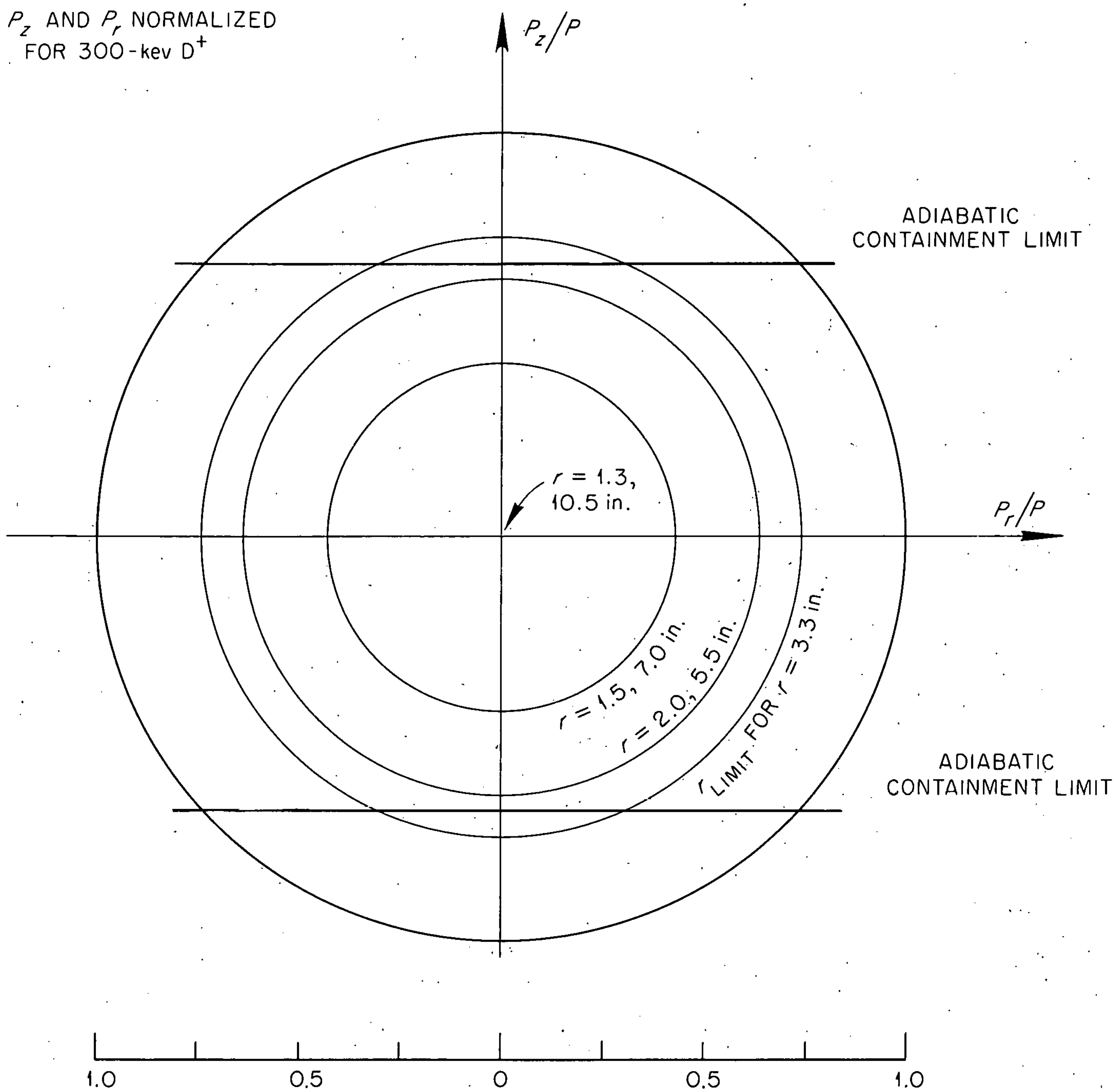

Fig. 2. Regions in Momentum Space Occupied by Anions Absolutely Contained in DCX as a Function of Its Radial Position When Crossing the Midplane

at a given radial position r is absolutely contained if its r and z momentum correspond to a point within the circle labeled by that r. There is also plotted the zone of adiabatic containment, which is bounded by straight lines rather than circles since adiabatic confinement depends only upon p_z.

Herein lies an important distinction between adiabatic and absolute containment. In terms of the angle by which a particle newly injected into DCX must be scattered in order to be lost (we inject in the midplane on the one orbit which is a perfect circle concentric to the symmetry axis), adiabatically contained particles are lost when scattered by more than a certain critical angle in the z-direction.

On the other hand, absolutely contained particles may be lost when scattered by more than another critical angle in any direction. In the case of a mirror ratio 2:1, as in DCX, the critical angles for both criteria are $\sim 45^\circ$. In that case the steady-state ion density which can be achieved with a given injection current in competition with ion losses out the mirrors by scattering is only a factor ~ 3 less with absolute containment than what it would be if adiabatic containment were applicable.

Turning now from the implication of absolute containment in momentum space to its meaning in configuration space, it is interesting to determine the spatial region of confinement. In Fig. 3 is plotted the equipotential curves $V(p_\theta, r, z) = p^2$ bounding the regions in r and z space to which particles of a given p_θ are confined. The curves are labeled by both p_θ/p and by the corresponding angle by which a newly injected ion must be scattered to have this value p_θ. Figure 4 is an enlargement of the region near the injection point.

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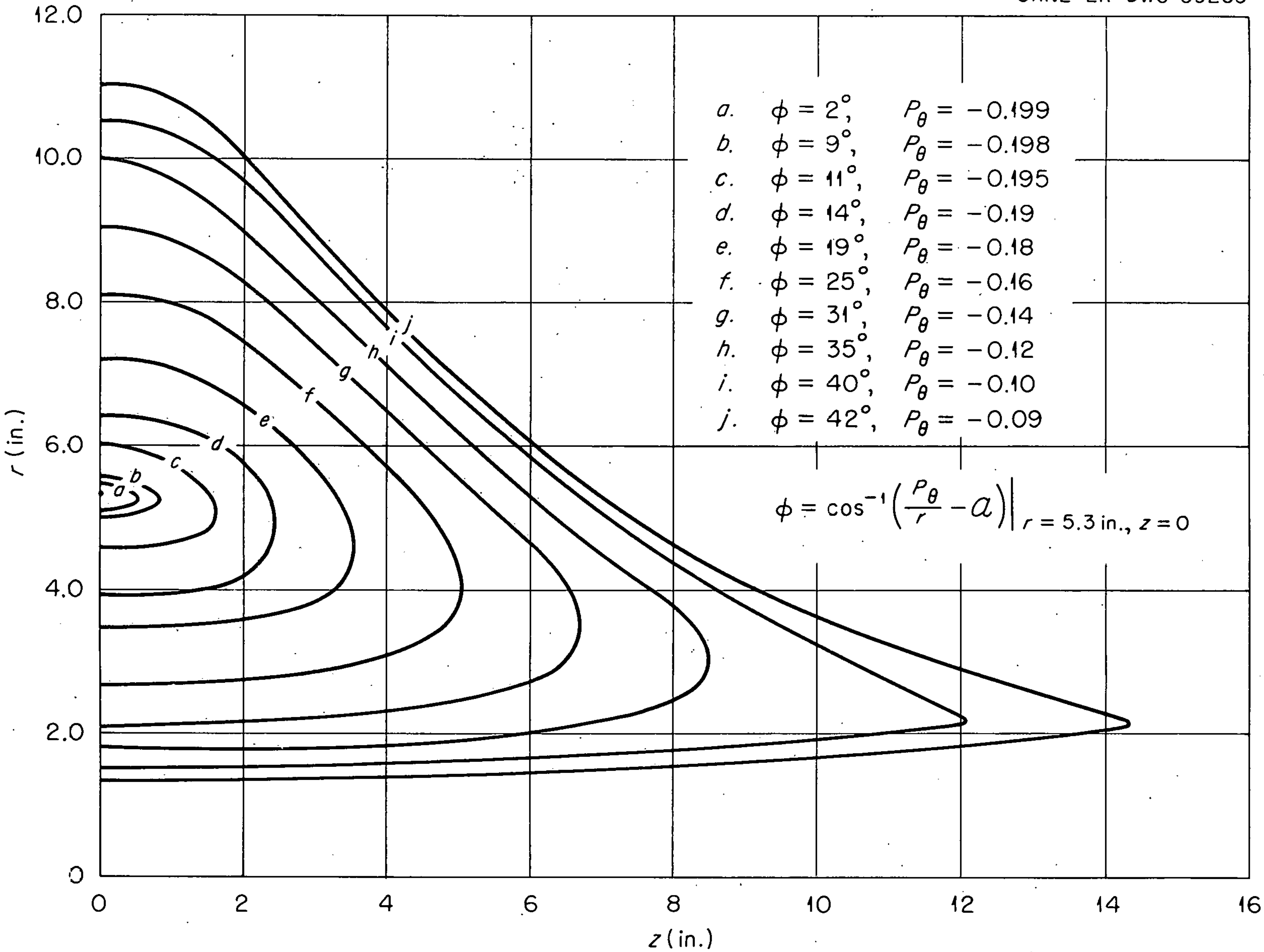

Fig. 3. Regions in Space Occupied by Anion Absolutely Contained in DCX as a Function of Its Canonical Angular Momentum

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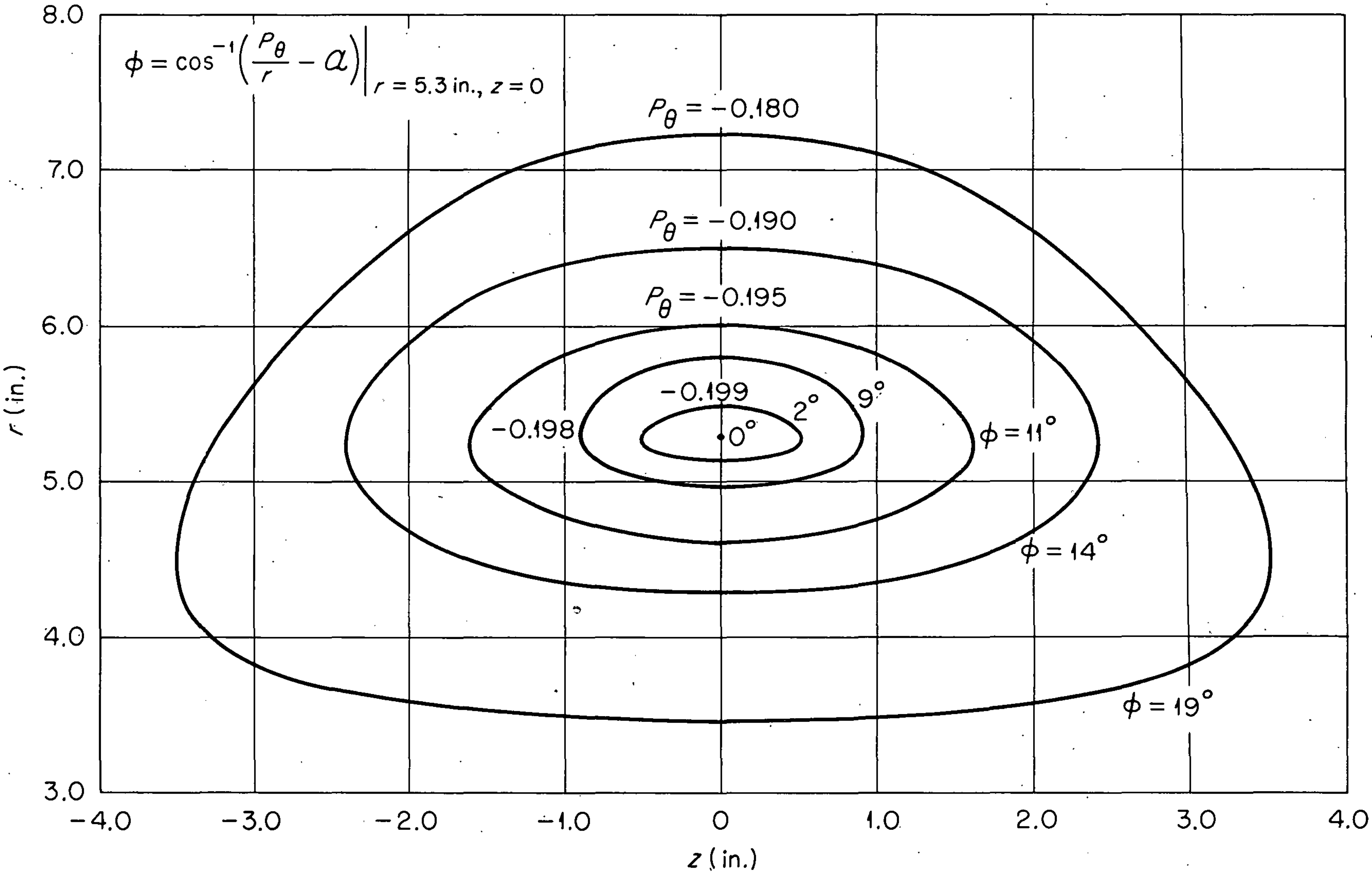

Fig. 4. Enlargement of Fig. 3 Near P_θ = -0.2

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