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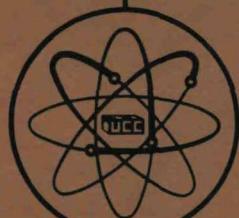
**MASTER**

**Y-1248**

Criticality Studies

**CRITICAL ASSEMBLIES OF URANIUM METAL**

R. GWIN  
W. T. MEE



**Y-12 Plant**

**UNION CARBIDE NUCLEAR COMPANY**  
DIVISION OF UNION CARBIDE CORPORATION

**Oak Ridge, Tennessee**

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Oak Ridge, Tennessee

March 26, 1959

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ABSTRACT

Data on critical assemblies of fissionable uranium metals are summarized. Empirical studies are made for specific reflectors and geometries to determine the feasibility of extrapolating these data for conditions concerning nuclear safety problems. Also included are the influences on critical systems by various reflectors, U-235 isotopic enrichment, density, and small metal pieces homogeneously distributed in water.

## INTRODUCTION

In order to facilitate the treatment of criticality hazards problems, a study of the available data on critical assemblies of uranium metal was undertaken. The purpose of this study is to present these data in graphic and composite form which will aid in the interpretation and evaluation of such problems.

Fortunately, sufficient data are presently available to enable the analysis of most problems without resorting to long extrapolations of the data. However, since problems will arise where experimental data are not directly available, a study of this type should simplify intelligent handling of such problems.

All of the data used in this report were taken by the W-2 Group of the Los Alamos Scientific Laboratory, Los Alamos, New Mexico. In reporting the data, it was stated that general uncertainties in specifying the core and reflector materials (density and U-235 concentration) introduced errors of approximately  $\pm 1\%$  in the reported critical mass values.(1) Coupled with this is the uncertainty introduced by calculating the critical mass from the multiplication of sub-critical assemblies. The probable error introduced in this manner is reported to be approximately  $\pm 1\%$ , 2%, 5%, and 10% for corresponding multiplications of 100, 50, 20, and 10. For hydogeneous reflectors, the probable errors are somewhat larger.

Uranium metal, as used in this report, has a U-235 isotopic enrichment range of 93.4% to 94% U-235 and a density of approximately  $18.5 \text{ gm/cm}^3$ .

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(1) Paxton, Hugh C., "Summary of Delayed Critical Mass Data," W-2-342, Los Alamos Scientific Laboratory, March 8, 1952. (classified)

### UNREFLECTED ASSEMBLIES OF URANIUM

The data on unreflected assemblies of uranium metal were found to be closely approximated by the elementary pile theory relation for the geometrical buckling<sup>(1)</sup> when appropriate values for the extrapolation length and the material buckling were used.

The experimental data and the corresponding empirical values are given in Table I.<sup>(2)</sup>

TABLE I  
CRITICAL ASSEMBLIES OF UNREFLECTED URANIUM  
Cylindrical Geometry

Diameter cm	Critical Height, cm		Critical Mass kg (exp) U	Maximum Multiplication
	Experimental*	Empirical		
12.07	> 46	$\infty$	> 100	13.3
13.97	24.59	24.58	70.6	96
16.18	15.42	15.34	58.8	85
17.78	12.85	12.87	59.2	76
19.05	11.62	11.64	61.9	54

#### Spherical Geometry

Diameter, cm		Critical Mass, kg U		Maximum Multiplication
Experimental	Empirical	Experimental	Empirical	
17.72	17.92	53.9	56.34	180

\* The experimental cylinder heights were determined using the given h/d ratio and the cylinder diameter.

The empirical values for the heights were determined using the relation

$$\frac{9.87}{(h + 2\epsilon)^2} + \frac{23.136}{(d + 2\epsilon)^2} = B^2 \quad \text{Eq. (1)}$$

where

$$\begin{aligned} h &= \text{cylinder height in cm} & \epsilon &= \text{extrapolation length} = 2.745 \text{ cm} \\ d &= \text{cylinder diameter in cm} & B^2 &= \text{constant} = 0.07201 \end{aligned}$$

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- (1) Soodak, H., Campbell, E. C., "Elementary Pile Theory," AECD-2201, Oak Ridge National Laboratory, August 4, 1948.
- (2) Paxton, Hugh C., "Summary of Delayed Critical Mass Data," W-2-342, Los Alamos Scientific Laboratory, March 8, 1952, Table 1. (classified)

The extrapolation length and  $B^2$  are assumed to be independent of geometry.

The variation of the critical height and critical mass with the cylinder diameter, as determined by Eq. 1, is presented graphically in Figure 1.

### CRITICAL ASSEMBLIES OF WATER-REFLECTED URANIUM

The water-reflected data was found to be approximated by the relation<sup>(1)</sup>

$$(d-a)(h-b) = c \quad \text{Eq. (2)}$$

where

$h$  = cylinder height  
 $d$  = cylinder diameter  
 $a$  = infinite cylinder diameter  
 $b$  = infinite slab thickness  
 $c$  = empirical constant

The data used are given in Table II along with the empirical values.<sup>(2)</sup>

TABLE II  
 CRITICAL ASSEMBLIES OF WATER-REFLECTED URANIUM  
 Cylindrical Geometry

Diameter cm	Height, cm		Critical Mass, kg U Experimental	Maximum Multiplication
	Experimental*	Empirical		
10.11	19.21	19.17	28.5	200
12.06	11.82	12.01	25.3	101
13.97	9.22	9.08	26.0	200
16.19	7.45	7.26	27.6	150
17.78	6.49	6.44	29.5	108
19.05	5.72	5.94	30.9	53

### Spherical Geometry

Sphere Diameter, cm	Critical Mass, kg U	Maximum Multiplication
13.69	24.9	49

\* The heights were determined from the cylinder diameter and the given  $h/d$  ratio. U-235 isotopic enrichment ranged from 93.7% to 94%.

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(1) Schuske, C. L., Morfitt, J. W., "An Empirical Study of Some Critical Mass Data," Y-533, Carbide and Carbon Chemicals Corporation, December 6, 1949. (classified)

(2) Paxton, Hugh C., "Summary of Delayed Critical Mass Data," W-2-342, Los Alamos Scientific Laboratory, March 8, 1952, Table 5. (classified)

The graph of the critical height and mass as a function of the cylinder diameter, obtained using the constants  $a = 7.34$  cm,  $b = 1.84$  cm, and  $c = 48$  in Equation (2) is given in Figure 2.

### CRITICAL ASSEMBLIES OF GRAPHITE-REFLECTED URANIUM

The graphite-reflected assemblies of uranium were treated in the same manner as the water-reflected assemblies. The thickness of the graphite was approximately 17 inches. The data and empirical results are shown in Table III.(1)

TABLE III

### CRITICAL ASSEMBLIES OF GRAPHITE-REFLECTED URANIUM

#### Cylindrical Geometry

Diameter cm	Height, cm		Critical Mass kg U	Maximum Multiplication
	Experimental*	Empirical		
8.26	24.37	24.55	24.0	17
10.11	13.14	13.10	19.5	109
12.065	9.83	8.92	18.7	82
13.97	6.92	6.89	19.7	78
16.19	5.59	5.50	21.3	107
19.05	4.48	4.42	24.1	150

#### Spherical Geometry

Diameter, cm	Critical Mass, kg U	Maximum Multiplication
12.49	18.1	48

\* The cylinder heights were calculated from cylinder diameter and the given h/d ratio. The U-235 isotopic enrichment was between 93.7% and 94%.

The constants which were used in Equation (2) are:

$a = 6.25$  cm, infinite cylinder diameter

$b = 0.67$  cm, infinite slab thickness

$c = 48$ , empirical constant

The graph of the critical height and mass as a function of cylinder diameter, using the above constants in Equation (2), is shown in Figure 3.

(1) Paxton, Hugh C., "Summary of Delayed Critical Mass Data," W-2-342, Los Alamos Scientific Laboratory, March 8, 1952, Table 5. (classified)

### COMPARISON OF VARIOUS REFLECTORS

The critical mass of a spherical core of uranium is plotted in Figure 4 as a function of reflector thickness for various reflector materials which may be encountered in plant operations.<sup>(1)</sup>

The effect of a composite reflector was investigated by H. C. Paxton in a series of experiments to determine safe casting limits in crucibles of various sizes.<sup>(2)</sup> It was found that the presence of the MgO crucible and the graphite heater ring resulted in a lower critical mass when water-reflected than the corresponding system which had only water-reflection.

### THE EFFECT OF U-235 ISOTOPIC ENRICHMENT

Figure 5 shows the variation of the critical mass of U-235, in spherical geometry as a function of the U-235 isotopic enrichment for a tuballoy-reflected system.<sup>(3)</sup> The curve plotted on a log-log graph indicates that the critical mass of uranium is inversely proportional to the 0.75 power of the isotopic enrichment ( $M \propto 1/E^{.75}$ ). No corrections have been attempted for slight variations in the density of the core and in the thickness of the reflector.

Figure 6 is a plot of the variation of the critical mass of unreflected U-235 in spherical geometry as a function of the U-235 isotopic enrichment. The cylindrical data was converted to spherical geometry by elementary pile theory, making the assumption that the extrapolation length found for uranium will give good results for lower enrichment material. The curve plotted on a log-log graph indicates that the critical mass of uranium is inversely proportional to the 0.74 power of the isotopic enrichment ( $M \propto 1/E^{.74}$ ).

### CRITICAL MASS OF SMALL URANIUM PIECES IN WATER

Experiments were performed to gain information on the critical conditions of small pieces of uranium when flooded.<sup>(4)</sup> The experiments were performed by

- (1) The data were taken from "Summary of Delayed Critical Mass Data," W-2-342, Table 2, 3, 4, and 5, Los Alamos Scientific Laboratory, March 8, 1952, compiled by H. C. Paxton. (classified)
- (2) Paxton, H. C., "Summary of Critical Mass Data," W-2-468 and W-2-469, Los Alamos Scientific Laboratory, April 29, 1953. (classified)
- (3) Paxton, H. C., "Summary of Delayed Critical Mass Data," W-2-432, Table 2, Los Alamos Scientific Laboratory, February 6, 1953. (classified)
- (4) Paxton, H. C., "Summary of Delayed Critical Mass Data," W-2-436, Supplement I, Los Alamos Scientific Laboratory, February 6, 1953. (classified)

stacking 1/2-inch cubes of uranium metal in a simple lattice at various separations and determining the corresponding critical mass when the system was flooded. The array contained lucite blocks for spacing the uranium cubes.

The data are presented in graphical form in Figure 7.

#### EFFECT OF DENSITY ON CRITICAL MASS OF URANIUM

The available data on the variation of the critical mass of uranium with the density of the uranium indicates that the relation

$$M = C \rho^{-1.2}$$

where

M = critical mass of uranium

$\rho$  = density of uranium

C = constant

holds over the range of experimental densities (9.35 gm/cm<sup>3</sup> to 18.7 gm/cm<sup>3</sup>). (1)  
The graph of this data is shown in Figure 8. (2)

#### CONCLUSION

This material has been assembled for the purpose of aiding in the analysis of nuclear safety problems in plant operations. It would be unwise to depend entirely on the values obtained by empirical techniques for cases which are well outside the range of the experimental data; however, the results should be sufficiently reliable to permit treatment of most plant problems.

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(1) Paxton, H. C., "Summary of Delayed Critical Mass Data," W-2-342, Table 2, Los Alamos Scientific Laboratory, March 8, 1952. (classified)

(2) No correction has been made for slight changes in reflector density or thickness.

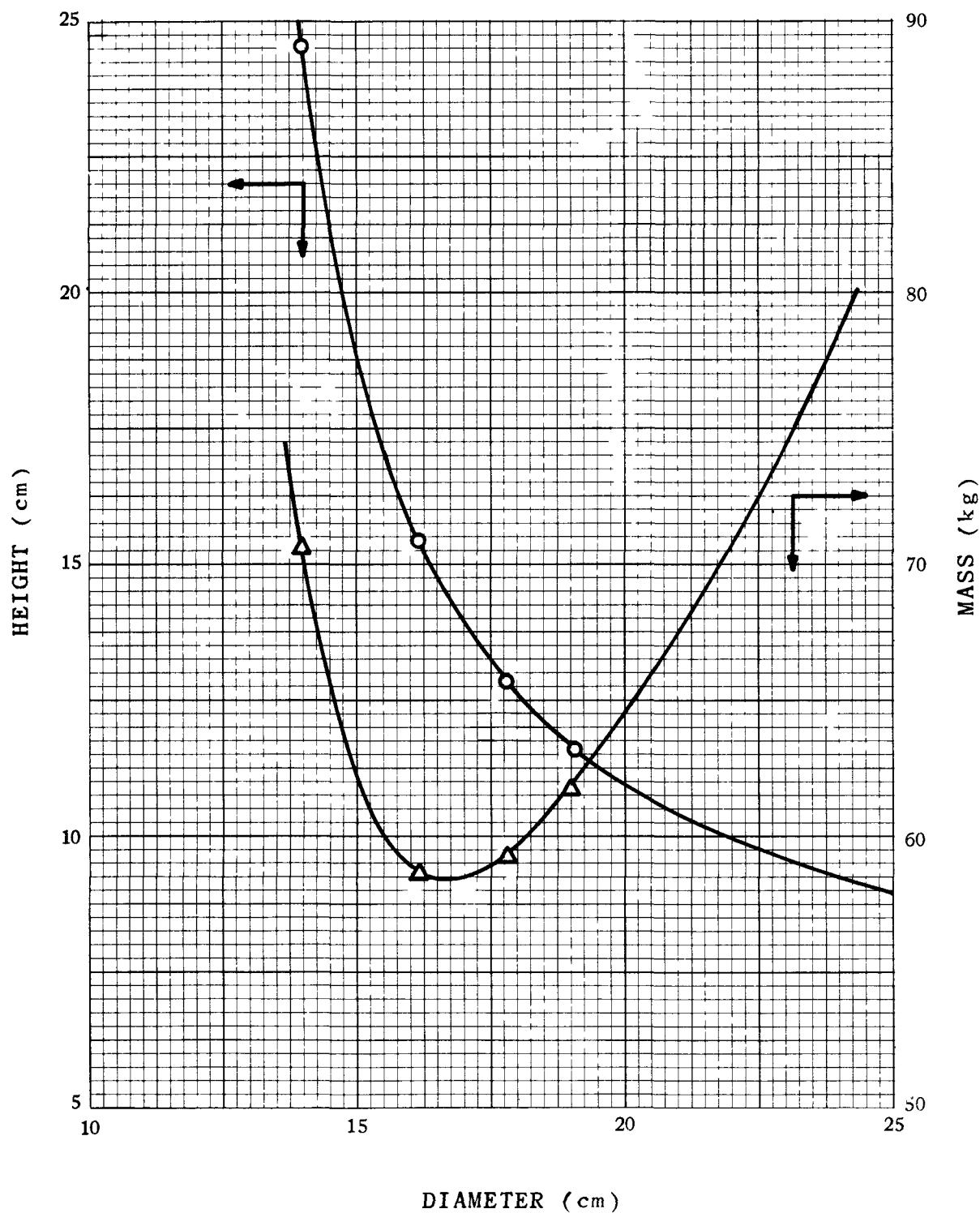


FIGURE 1. CRITICAL MASS AND HEIGHT OF UNREFLECTED URANIUM METAL CYLINDERS vs CYLINDER DIAMETER

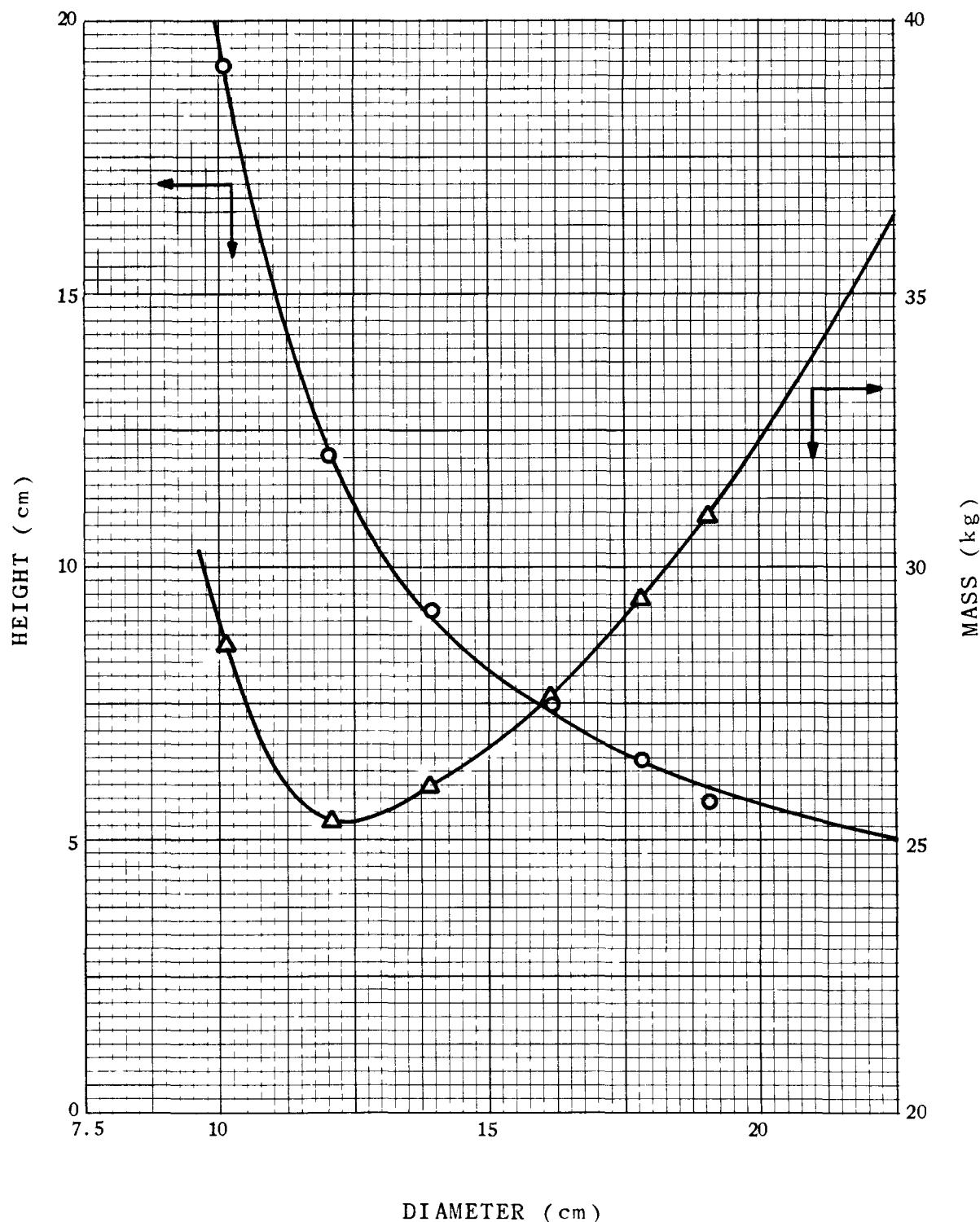


FIGURE 2. CRITICAL MASS AND HEIGHT OF WATER REFLECTED URANIUM METAL CYLINDERS vs CYLINDER DIAMETER

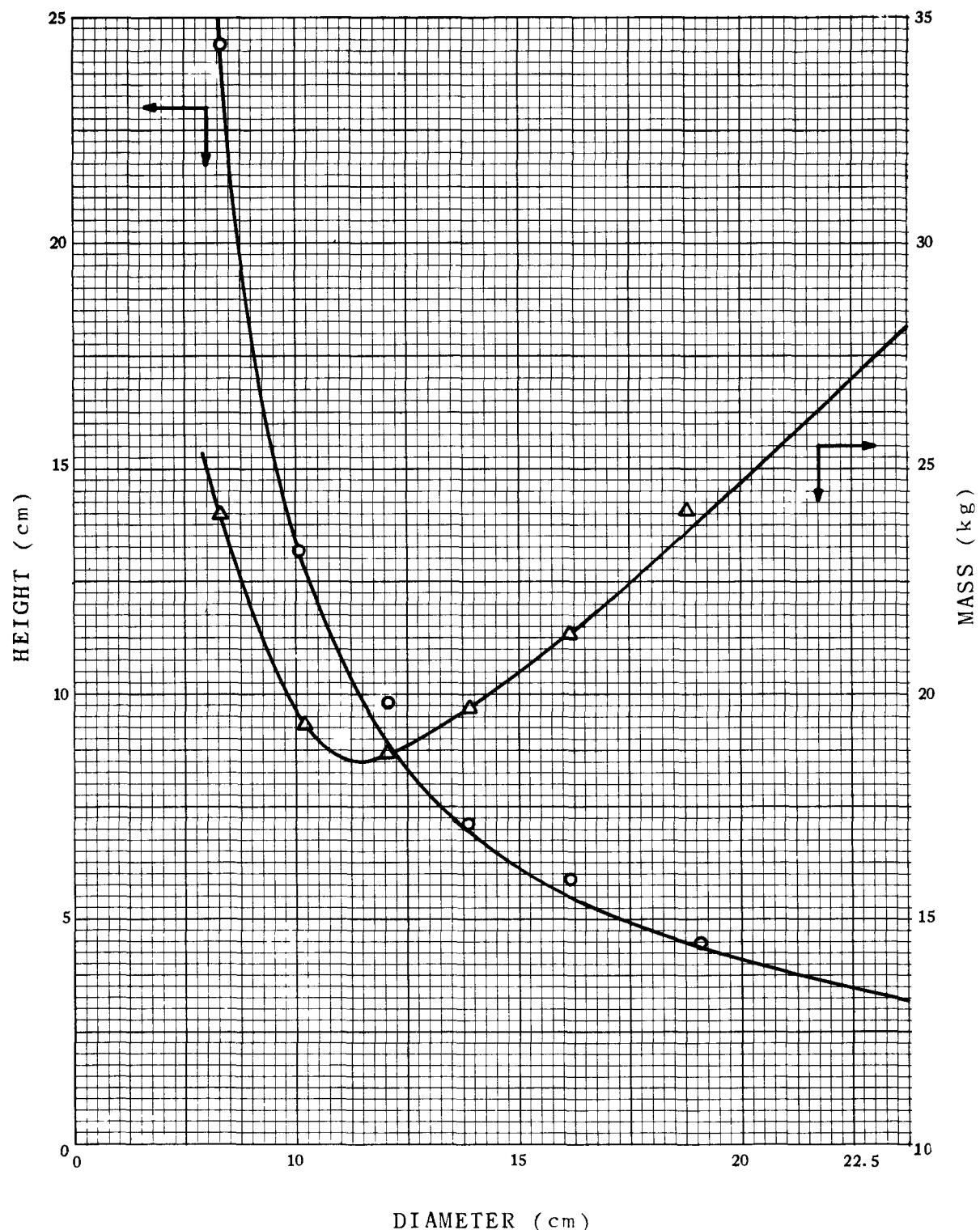


FIGURE 3. CRITICAL MASS AND HEIGHT OF GRAPHITE REFLECTED URANIUM METAL vs CYLINDER DIAMETER

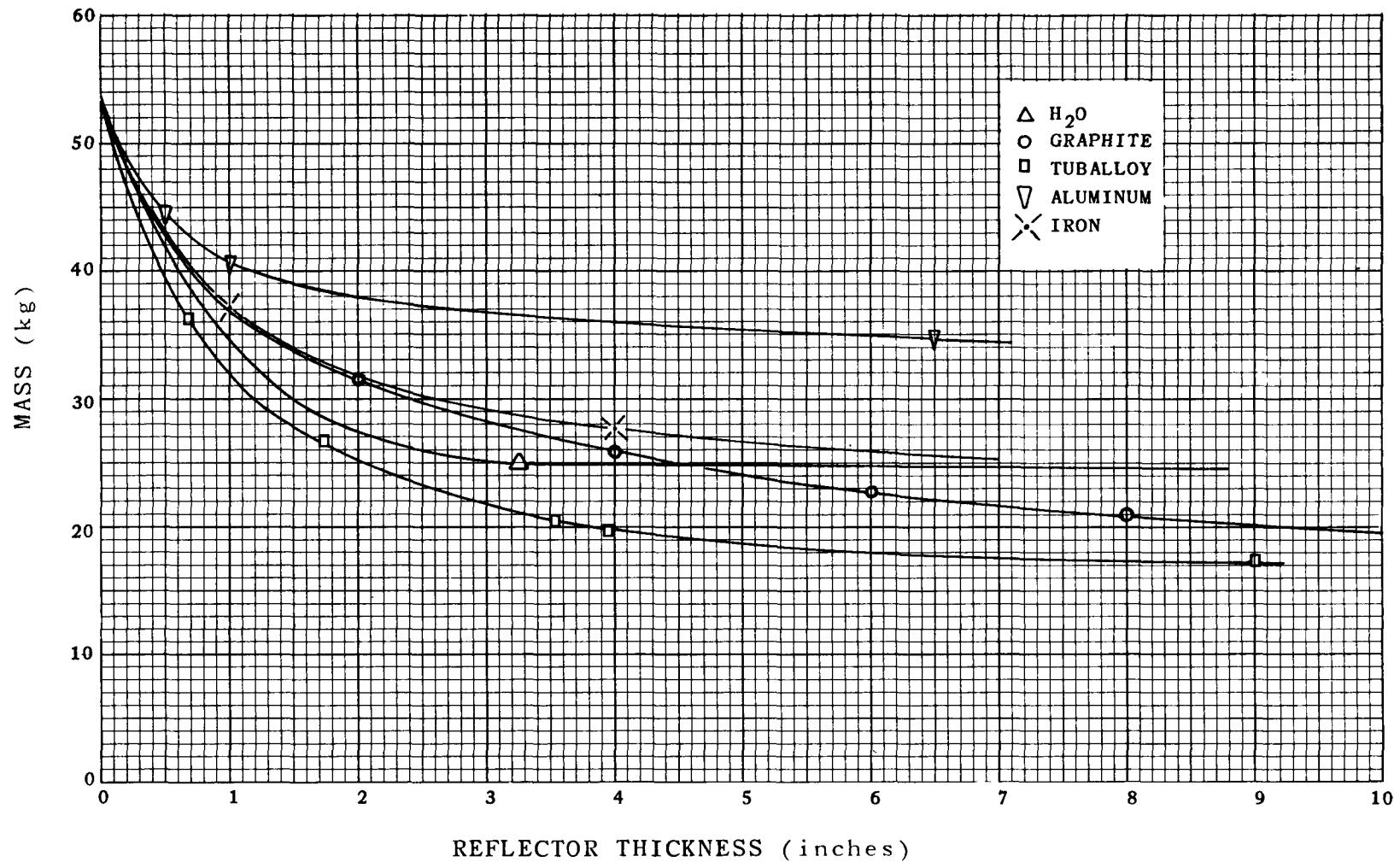


FIGURE 4. CRITICAL MASS OF SPHERICAL URANIUM METAL CORE vs REFLECTOR THICKNESS

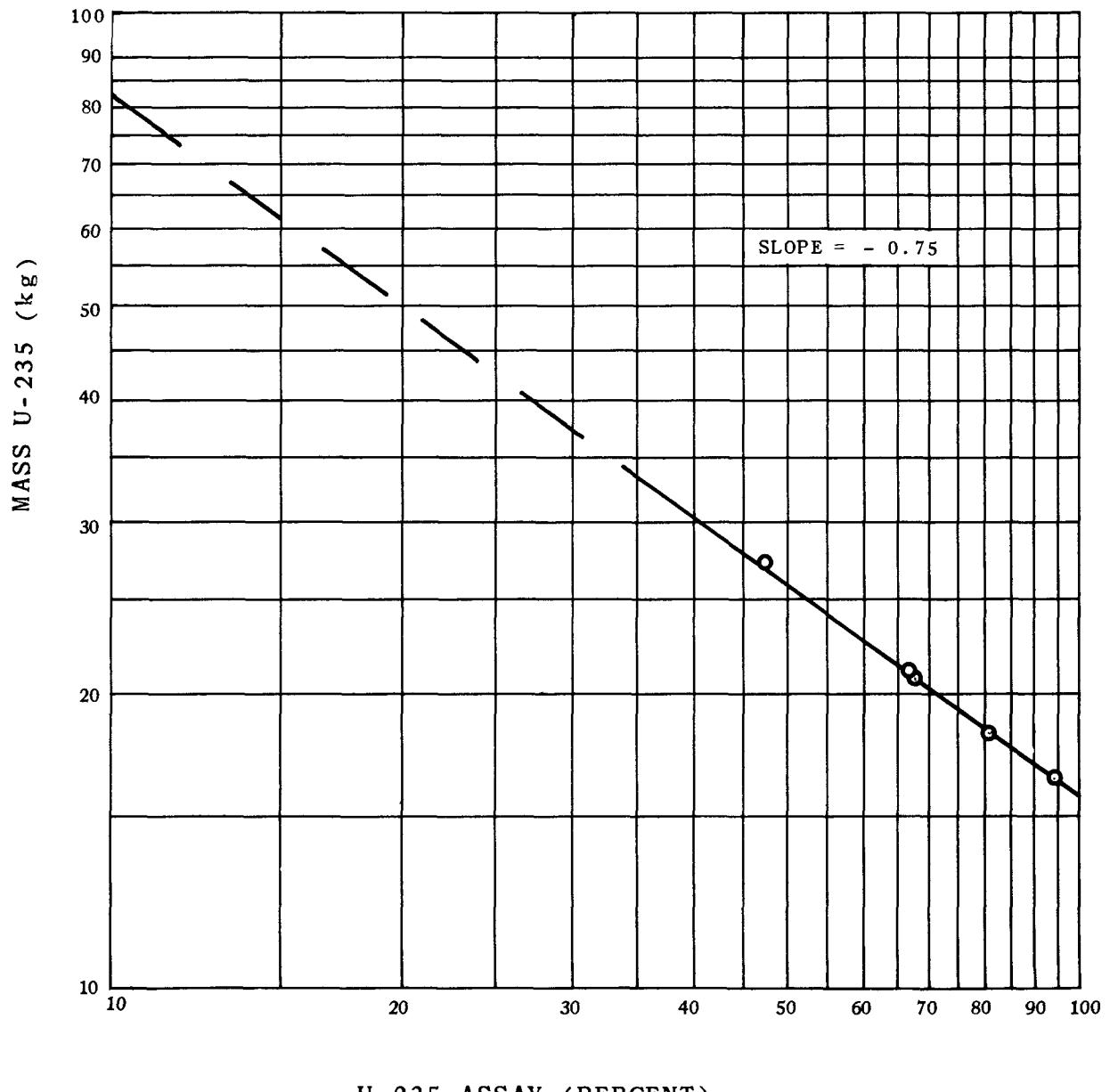


FIGURE 5. CRITICAL MASS OF TUBALLOY-REFLECTED U-235 vs U-235 ASSAY (SPHERICAL CORE)

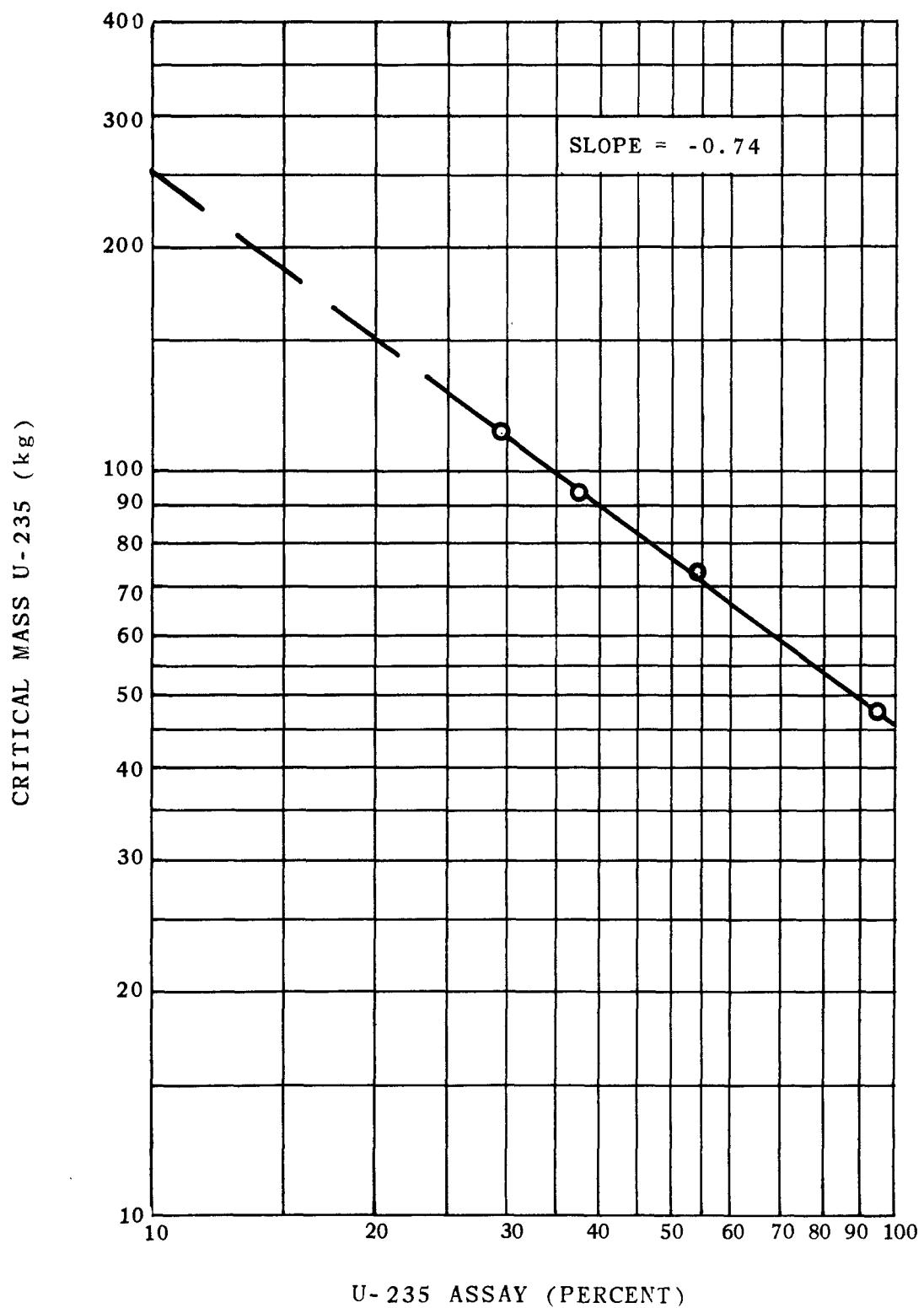


FIGURE 6. CRITICAL MASS OF UNREFLECTED URANIUM METAL AS A FUNCTION OF U-235 ASSAY (SPHERICAL CORE)

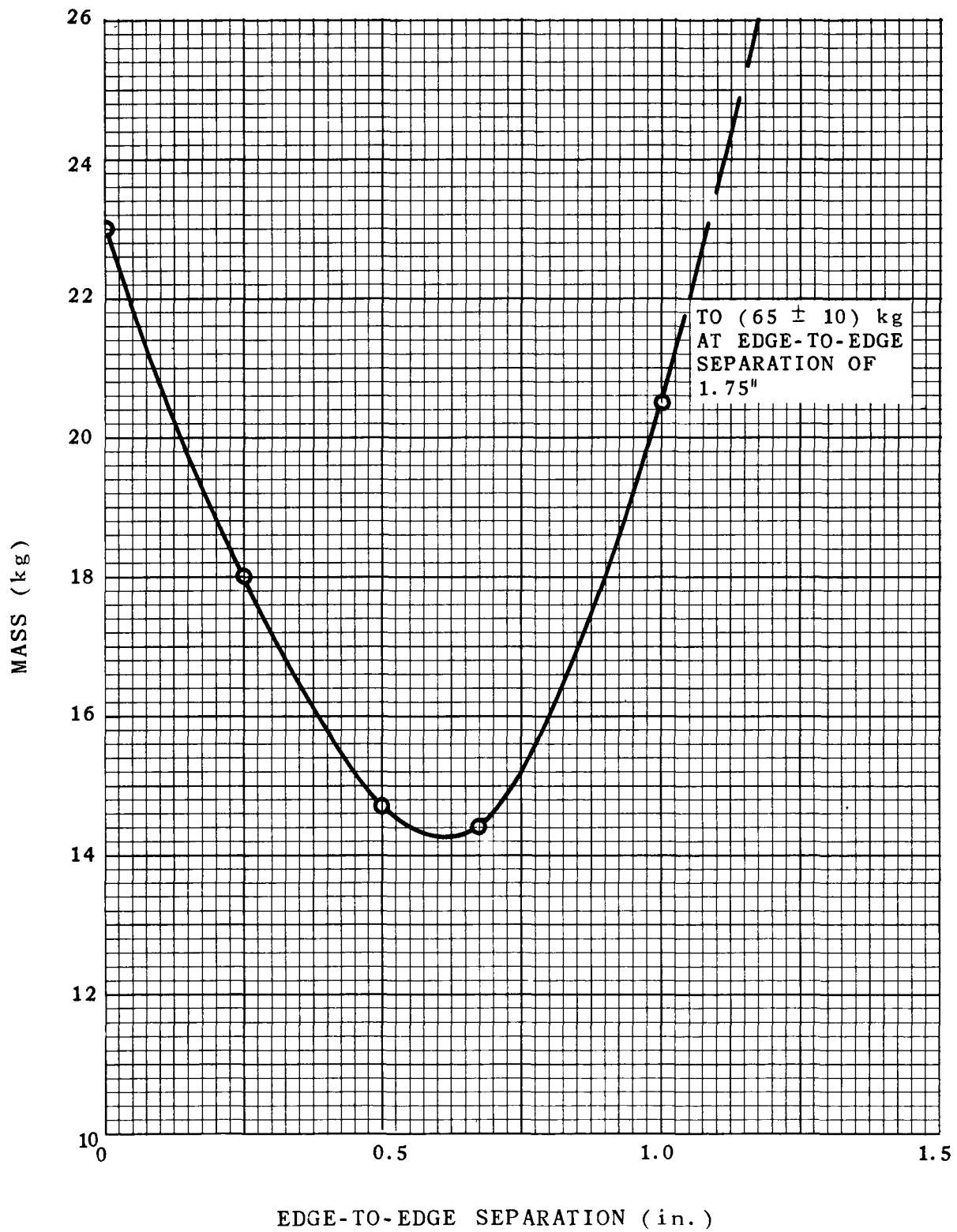


FIGURE 7. CRITICAL MASS OF  $1/2$  CUBES OF URANIUM METAL vs EDGE-TO-EDGE SPACING IN A WATER FLOODED SIMPLE LATTICE

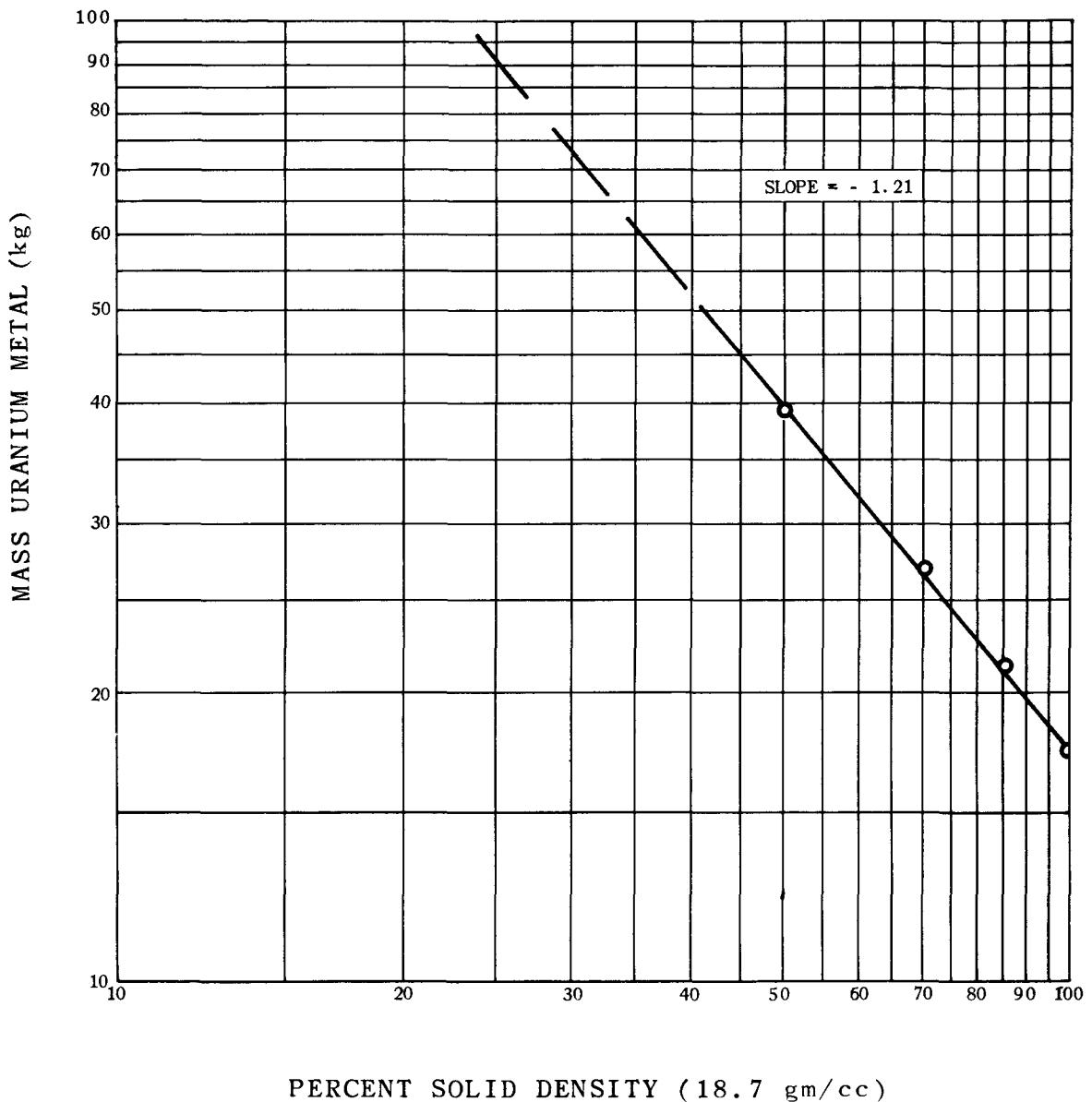


FIGURE 8. CRITICAL MASS OF TUBALLOY-REFLECTED URANIUM METAL AS A FUNCTION OF CORE DENSITY (SPHERICAL CORE)