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THEORY OF RESONANCE ABSORPTION OF NEUTRONS IN A LUMP

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ABSTRACT

We have derived from the Boltzmann equation a new integral equation governing the slowing down of neutrons in a lump, assuming a spatially uniform neutron flux inside it. In the first approximation we have solved the equation using the asymptotic energy dependence of the neutron flux and proposed a new resonance integral formula. In the limit of both the narrow resonance and the wide resonance, the formula tends to be equal to that derived in the conventional narrow and wide resonance approximations. In the second approximation we calculated effects of the deviation of the Placzek function from its asymptotic form by a method similar to that used by Weinberg and Wigner for homogeneous systems. We have shown that the resonance integrals of the usual narrow and wide resonance approximations become equal for a certain value of Γ_n/Γ, irrespective of the resonance width. Some numerical results are given.

INTRODUCTION

The resonance absorption of neutrons in a lump has been calculated by two approximate methods: the narrow resonance approximation and the wide resonance approximation, referred to hereinafter as the N.R.A. and the W.R.A., respectively.[1] In the N.R.A., we assume that the values of ξ (the average lethargy change per collision) for both the moderator and the absorber are very much larger than the lethargy extent of the resonance line under consideration. Thus, resonance neutrons which collide elastically with moderating and absorbing nuclei are slowed down well below the resonance region. On the other hand, in the W.R.A. we assume that the width of the resonance in lethargy greatly exceeds ξ of the absorbing nucleus, and the slowing down of neutrons by collisions with absorbing nuclei can be neglected. Recently, Dresner[2] proposed a new resonance integral formula in the W.R.A., taking into account multiple scattering of neutrons in the lump.

However, the width of the resonance line is finite and the above-mentioned approximations correspond to the two extreme cases. We must consider the slowing down of neutrons by absorbing nuclei correctly to improve the approximate formulas. Spinney[3] proposed the modified narrow resonance formula considering this effect and obtained improved results in a homogeneous system. Corngold[4] estimated errors due to the assumption of an infinite mass absorber for the resonance absorption of a slab lattice. Recently, Spinrad, Chernick, and Corngold[5] derived an approximate formula for a lump immersed in an infinite moderating system, assuming a flat flux inside the lump.

As Weinberg and Wigner[6] indicated for the resonance absorption in a homogeneous system, neutron absorptions are equivalent to negative sources, and the strong absorption induces the flux oscillation represented by the Placzek function.[7] They obtained the second-order corrections to the resonance escape probability by introducing the correction term $\xi_1(u)$ (u is lethargy) to ξ.

In this paper we shall derive a new integral equation for the neutron flux inside a lump immersed in a strongly moderating infinite system, considering correctly the neutron slowing down by absorbing nuclei and assuming the spatially flat flux. This equation corresponds to an extension to a heterogeneous system of an equation derived by Weinberg and Wigner[6] and by Corngold[8] for a homogeneous system. In the first approximation, we shall derive a new resonance integral formula assuming the asymptotic energy dependence of the neutron flux in which the neutron slowing down by collisions with absorbing nuclei and multiple scatterings inside the lump are taken into account. In the limit of both the narrow resonance and the wide resonance, the formula tends to that derived from the N.R.A. and the W.R.A., respectively. Since we do not make any assumption concerning the width of the resonance line considered, it gives an improved formula compared to the usual ones. In the second approximation, we shall consider the deviation of the Placzek function from its asymptotic form by making use of a treatment similar to that of Weinberg and Wigner.[6] In the heterogeneous system, the flux oscillation is induced not only by neutron absorptions but also by variations in the first-collision density of neutrons impinging from the outside of the lump and by variations of the neutron leakage from the lump.

In the last section, we shall discuss the validity of the flat flux assumption and show that resonance integrals of the N.R.A. and W.R.A. become equal for a certain value of Γ_n/Γ, irrespective of the resonance width. For this value of Γ_n/Γ the correcting term derived from our formula to the N.R.A. or W.R.A. vanishes. For a simple case of a square resonance cross section, comparisons of our results to that of the N.R.A. and W.R.A. are made.

I. MATHEMATICAL FORMULATION

We shall consider the resonance absorption of neutrons in a single line inside a metallic lump immersed in an infinite moderating system. We assume that the value of ξ for the moderator is very much larger than the lethargy extent of the resonance line under consideration and that resonance neutrons are slowed down well below the resonance region by only one collision with a moderating nucleus. As the resonance energy is very much lower than that of fission neutrons, the neutron flux in the moderator at the resonance energy can be considered to be spatially flat and to have the asymptotic energy variation. We normalize the flux in the moderator to unit magnitude.

The neutron flux inside the lump, $\zeta(\vec{r},u)$, satisfies the following Boltzmann equation:

$$\zeta(\vec{r},u) = P(\vec{r},u) + \int_0^u du' K_m(u - u')\Sigma_s(u') \int_V d\vec{r}\,' \frac{\exp\left[-\Sigma_t(u)\left|r - r'\right|\right]}{4\pi\left|\vec{r} - \vec{r}\,'\right|^2} \zeta(\vec{r}\,',u'), \quad (1)$$

where the integration with respect to $\vec{r}\,'$ is to be extended over the lump whose volume and surface are denoted by V and S, respectively. For the origin of lethargy, $u = 0$, we choose an energy which is very much lower than the fission energy and very much higher than the resonance energy but otherwise arbitrary. $P(\vec{r},u)$ is the first-collision density of incident neutrons from the outside of the lump and is given by[9]

$$P(\vec{r},u) = \int_S dS'(\vec{n},\vec{\lambda}) \frac{\exp\left[-\Sigma_t(u)\left|\vec{r}-\vec{r}'\right|\right]}{4\pi\left|\vec{r}-\vec{r}'\right|^2}, \tag{2}$$

where $\vec{\lambda} = (\vec{r}-\vec{r}')/\left|\vec{r}-\vec{r}'\right|$ and the point $\vec{r}'$ is on the surface of the lump. In Eqs. (1) and (2) we have used the following notations:[10]

$$K_m(u) = \frac{\exp(-u)}{(1-\alpha_m)} \quad \text{for } 0 < u < q_m,$$

$$= 0 \qquad \text{otherwise;}$$

$q_m = \log\left[(m+1)/(m-1)\right]^2$, $1 - \alpha_m = 4m/(m+1)^2$;

m = mass of the absorbing nucleus;

$\Sigma_t(u)$ = macroscopic total cross section in the lump;

$\Sigma_a(u)$ = absorption cross section;

$\Sigma_s(u)$ = scattering cross section;

$\Sigma_{p.s.}$ = potential scattering cross section.

In the following, we assume the neutron flux inside the lump is spatially flat and denote it by $\mathcal{S}(u)$. We shall discuss the validity of this assumption in the last section. Integrating over $\vec{r}$, Eq. (2) becomes

$$\mathcal{S}(u) = P(u) + \frac{1}{\Sigma_t(u)}\left[1 - P(u)\right]\int_0^u du' K_m(u-u')\Sigma_s(u')\mathcal{S}(u'), \tag{3}$$

where $P(u) = (1/V) \int d\vec{r} P(\vec{r},u)$ and we have used the following relation:

$$\frac{1}{V} \int_V d\vec{r} \int_V d\vec{r}\,' \frac{\exp\left[-\Sigma_t(u)\left|\vec{r} - \vec{r}\,'\right|\right]}{4\pi\left|\vec{r} - \vec{r}\,'\right|^2} = \frac{1}{\Sigma_t(u)} \left[1 - P(u)\right] . \tag{4}$$

Equation (3) is identical with the fundamental equation of Spinrad, Chernick, and Corngold.[5] Putting

$$\zeta(u) = \frac{1 + \bar{\ell}\Sigma_{p.s.}}{1 + \bar{\ell}\,\Sigma_t(u)} U(u) \tag{5}$$

and expanding $U(u')$ on the right-hand side of Eq. (3) in Taylor series, they transformed the integral equation (3) into a differential equation with respect to $U(u)$ by retaining at most linear terms in u'. $\bar{\ell}$ is the mean chord length of the lump and is defined by $\bar{\ell} = (4V/S)$. However, these methods seem to be inadequate for the problem in which cross sections vary rapidly with energy.[6]

In the following we shall transform Eq. (3) into a new and completely equivalent equation by using a method similar to that used by Weinberg and Wigner[6] and by Corngold[8] in homogeneous systems. We rewrite Eq. (3) as follows:

$$X(u) = \frac{\Sigma_t(u)}{\left[1 - P(u)\right]} P(u) + \int_0^u du' K_m(u - u') \left[1 - P(u')\right] \frac{\Sigma_s(u')}{\Sigma_t(u')} X(u') , \tag{6}$$

where

$$X(u) = \frac{\Sigma_t(u)}{\left[1 - P(u)\right]} \zeta(u) . \tag{7}$$

We denote the Laplace transformation of f(u) by $L\left\{f(u)\right\}$ and apply the Laplace transformation to Eq. (6) to obtain

$$L\left\{X(u)\right\} = L\left\{\frac{\Sigma_t(u)}{\left[1 - P(u)\right]} P(u)\right\} + L\left\{K_m(u)\right\} L\left\{\left[1 - P(u)\right] \frac{\Sigma_s(u)}{\Sigma_t(u)} X(u)\right\} . \tag{8}$$

Here we have used the convolution theorem:

$$L\left\{\int_0^u du' f(u - u') g(u')\right\} = L\left\{f(u)\right\} L\left\{g(u)\right\} . \tag{9}$$

To obtain the final equation we rewrite Eq. (8) as follows:

$$\left[1 - L\left\{K_m(u)\right\}\right] L\left\{X(u)\right\} = L\left\{\frac{\Sigma_t(u)}{\left[1 - P(u)\right]} P(u)\right\} \tag{10}$$

$$- L\left\{K_m(u)\right\} \cdot L\left\{\left[1 - \left[1 - P(u)\right] \frac{\Sigma_s(u)}{\Sigma_t(u)}\right] X(u)\right\} .$$

Dividing both sides of Eq. (10) by $1 - L\left\{K_m(u)\right\}$ we have

$$L\left\{X(u)\right\} = L\left\{\frac{\Sigma_t(u)}{\left[1 - P(u)\right]} P(u)\right\} + \frac{L\left\{K_m(u)\right\}}{\left[1 - L\left\{K_m(u)\right\}\right]} L\left\{\frac{\Sigma_t(u)}{\left[1 - P(u)\right]} P(u)\right\} \tag{11}$$

$$- \frac{L\left\{K_m(u)\right\}}{\left[1 - L\left\{K_m(u)\right\}\right]} L\left\{\left[P(u) + \left[1 - P(u)\right]\frac{\Sigma_a(u)}{\Sigma_t(u)}\right] X(u)\right\}.$$

Here we have made use of the following relations:

$$\left[1 - P(u)\right] \frac{\Sigma_s(u)}{\Sigma_t(u)} - 1 = -\left\{P(u) + \left[1 - P(u)\right] \frac{\Sigma_a(u)}{\Sigma_t(u)}\right\}, \tag{12}$$

$$\frac{1}{\left[1 - L\left\{K_m(u)\right\}\right]} L\left\{F(u)\right\} = L\left\{F(u)\right\} + \frac{L\left\{K_m(u)\right\}}{\left[1 - L\left\{K_m(u)\right\}\right]} L\left\{F(u)\right\}.$$

$L\left\{K_m(u)\right\}/\left[1 - L\left\{K_m(u)\right\}\right]$ is the Laplace transform of the Placzek function $\phi_m(u)$,[8,10] i.e., the neutron flux in an infinite no absorbing system with a neutron source at u = 0. Thus we obtain by inverting Eq. (11):

$$X(u) = \frac{\Sigma_t(u)}{\left[1 - P(u)\right]} P(u) + \int_0^u du'\phi_m(u - u') \frac{\Sigma_t(u')}{\left[1 - P(u')\right]} P(u') \tag{13}$$

$$- \int_0^u du'\phi_m(u - u') \left[P(u') + \left[1 - P(u')\right] \frac{\Sigma_a(u')}{\Sigma_t(u')}\right] X(u'),$$

which is the new equation we have been seeking. Equation (13) is just equivalent to Eq. (3). However, as we shall show later, unlike Eq. (3), good approximate solutions to the new equation may be constructed. Equation (13) is the extension to a heterogeneous medium of the fundamental equation for a homogeneous system derived by Weinberg and Wigner,[6] and by Corngold.[8] In fact, we obtain their equation by setting P(u), the neutron escape probability from the lump, equal to zero, except in the inhomogeneous term in Eq. (13). It is interesting to note that the quantity $\Sigma_a(u)/\Sigma_t(u)$ for the homogeneous system corresponds to $P(u) + \left[1 - P(u)\right] \Sigma_a(u)/\Sigma_t(u)$ of the heterogeneous system. The physical meaning of this fact is that the neutron leakage from the lump given P(u) is equivalent to the actual neutron absorption rate $\left[1 - P(u)\right] \Sigma_a(u)/\Sigma_t(u)$.

We now decompose the Placzek function into two parts,[6]

$$\phi_m(u) = \frac{1}{\xi} \left[1 - P_m(u)\right] , \tag{14}$$

where P_m gives the deviation of ϕ_m from its asymptotic form. By inserting Eq. (14) into Eq. (13), we have

$$X(u) = \frac{\Sigma_t(u)}{\left[1 - P(u)\right]} P(u) + \frac{1}{\xi} \int_0^u du' \frac{\Sigma_t(u')}{\left[1 - P(u)\right]} P(u') - \frac{1}{\xi} \int_0^u du' P_m(u - u') \qquad (15)$$

$$\times \frac{\Sigma_t(u')}{\left[1 - P(u')\right]} P(u') - \frac{1}{\xi} \int_0^u du' \left[P(u') + \left[1 - P(u')\right] \frac{\Sigma_a(u')}{\Sigma_t(u')}\right] X(u')$$

$$+ \frac{1}{\xi} \int_0^u du' P_m(u - u') \left[P(u') + \left[1 - P(u')\right] \frac{\Sigma_a(u')}{\Sigma_t(u')}\right] X(u').$$

Terms containing P_m as a factor in Eq. (15) give rise to the flux oscillation. The flux oscillation is induced not only by neutron absorptions but also by incident and escaping neutrons.

II. RESONANCE INTEGRAL USING APPROXIMATE SOLUTION

First-Order Approximation

We are not able to obtain exact solutions of Eq. (13), just as we are unable to solve Eq. (3) exactly, however, Eq. (13) or (15) is superior to Eq. (3) for making clear the physical meaning and to get better approximate solutions. Equation (13) can be solved by the iterative method to give after appropriate rearrangements,

$$X(u) \equiv \frac{\Sigma_t(u)}{\left[1 - P(u)\right]} \zeta(u) = \frac{\Sigma_t(u)}{\left[1 - P(u)\right]} P(u) \tag{16}$$

$$- \sum_{\ell=1}^{\infty} \left(\frac{-1}{\xi}\right)^{\ell} \int_0^u du_1 \left[1 - P_m(u - u_1)\right] Q(u_1) \int_0^{u_1} du_2 \cdots$$

$$\times \int_0^{u_{\ell-2}} du_{\ell-1} \left[1 - P_m(u_{\ell-2} - u_{\ell-1})\right] Q(u_{\ell-1}) \int_0^{u_{\ell-1}} du_\ell \left[1 - P_m(u_{\ell-1} - u_\ell)\right]$$

$$\times P(u_\ell) \Sigma_s(u_\ell),$$

where we have used the following abbreviation:

$$Q(u) = P(u) + \left[1 - P(u)\right] \frac{\Sigma_a(u)}{\Sigma_t(u)} . \tag{17}$$

The convergence of the series solution will be rapid when the width of the resonance line is narrow.

In particular, when we set all the $P_m(u)$ in Eq. (16) equal to zero, i.e., when we replace the Placzek function $\phi_m(u)$ by its asymptotic form $1/\xi$, the first-order approximate solution of $X(u)$, $X^{(1)}(u)$, becomes

$$X^{(1)}(u) = \frac{\Sigma_t(u)}{\left[1 - P(u)\right]} P(u) + \frac{1}{\xi} \int_0^u du' P(u') \Sigma_s(u') \tag{18}$$

$$\times \exp\left[- \frac{1}{\xi} \int_{u'}^{u} du'' Q(u'')\right],$$

where we have used the following relation:

$$\int_0^u du_1 Q(u_1) \int_0^{u_1} du_2 \cdots \int_0^{u_{n-1}} du_n Q(u_n) \int_0^{u_n} du_{n+1} P(u_{n+1}) \Sigma_s(u_{n+1}) \tag{19}$$

$$= \frac{1}{n!} \int_0^u du' P(u') \Sigma_s(u') \left[\int_{u'}^u du_1 Q(u_1) \right]^n \quad .$$

Of course, we can easily obtain $X^{(1)}(u)$ directly from Eq. (15) by setting $P_m(u) = 0$.

To show that our method is consistent, we shall obtain the neutron flux above the resonance energy from Eq. (18) for the case of no absorption, i.e., $\Sigma_a = 0$, $\Sigma_t = \Sigma_{p.s.}$. Denoting P(u) for this case by P_o and noting $Q(u) = P_o$, we have

$$\frac{\Sigma_{p.s.}}{\left[1 - P_o\right]} \rho^{(1)}(u) = \frac{\Sigma_{p.s.}}{\left[1 - P_o\right]} P_o + \Sigma_{p.s.} \left[1 - \exp\left(- \frac{P_o}{\xi} u\right)\right] . \tag{20}$$

As we have chosen the origin of the lethargy at the energy very much higher than the resonance energy, the term containing the exponential factor can be neglected and $\rho^{(1)}(u) = 1$. In the case of no absorption and constant scattering cross section, the neutron flux inside and outside the lump become equal, as they should be. This result is important, and will not hold when we choose an incorrect solution.

Next, we shall show that our solution given by Eq. (18) coincides with the usual one in the wide resonance limit. When ξ is very small, the integrand in the second term of Eq. (18) remains finite only for u' very near to u and we may replace $Q(u'')$ and $P(u')\Sigma_s(u')$ in the integrand by $Q(u)$ and $P(u)\Sigma_s(u)$ to obtain

$$\frac{\Sigma_t(u)}{\left[1 - P(u)\right]} \zeta^{(1)}(u) = \frac{\Sigma_t(u)}{\left[1 - P(u)\right]} P(u) \tag{21}$$

$$+ P(u)\Sigma_s(u) \frac{1}{Q(u)} \left[1 - \exp\left\{-\frac{1}{\xi} Q(u)u\right\}\right] .$$

As we have mentioned above, the exponential term on the right hand side of Eq. (21) can be neglected for a suitable value of u and employing the definition of $Q(u)$ by Eq. (17), $\zeta^{(1)}(u)$ can be written

$$\zeta^{(1)}(u) = \frac{P(u)}{P(u) + \left[1 - P(u)\right] \dfrac{\Sigma_a(u)}{\Sigma_t(u)}} . \tag{22}$$

On the other hand, when we neglect the slowing down of neutrons by the elastic collision with fuel nuclei, Eq. (1) becomes[2]

$$\zeta(\vec{r},u) = P(\vec{r},u) + \Sigma_s(u) \int_V d\vec{r}' \frac{\exp\left[-\Sigma_t(u)|r - r'|\right]}{4\pi|\vec{r} - \vec{r}'|^2} \zeta(\vec{r}',u) . \tag{23}$$

Making the flat flux approximation and using the relation given by Eq. (4), we easily obtain Eq. (22). Dresner[2] derived the same result in the W.R.A. by making use of a variational principle. The result may also be obtained by the successive generations method.

To derive the resonance integral formula in the narrow resonance limit, it is convenient to rewrite Eq. (18) in another form. Suppose the absorption begins from the lethargy $u = u_1$. We now decompose the integral on the right hand side of Eq. (18) into two regions, i.e., (o,u_1) and (u_1,u), and after integrations we have

$$X^{(1)}(u) = \frac{\Sigma_t(u)}{\left[1 - P(u)\right]} P(u) + \Sigma_{p.s.} \exp\left[-\frac{1}{\xi}\int_{u_1}^{u} du'Q(u')\right] \tag{24}$$

$$+ \frac{1}{\xi}\int_{u_1}^{u} du'P(u')\Sigma_s(u') \exp\left[-\frac{1}{\xi}\int_{u'}^{u} du''Q(u'')\right],$$

where we have neglected the term of order of $\exp\left[-(P_o/\xi)u_1\right]$.

When the absorption occurs in the region (u_1,u_2), the resonance integral, R.I., is defined by the following expression:

$$\text{R.I.} = \frac{1}{N}\int_{u_1}^{u_2} du\Sigma_a(u)\mathfrak{X}(u), \tag{25}$$

where N is the number of absorbing nuclei per cubic centimeter of the lump. From Eqs. (24) and (25) we have the resonance integral in the first-order approximation:

$$\text{R.I.}^{(1)} = \frac{1}{N} \int_{u_1}^{u_2} du \Sigma_a(u) P(u) \tag{26}$$

$$+ \frac{1}{N} \Sigma_{p.s.} \int_{u_1}^{u_2} du \left[1 - P(u)\right] \frac{\Sigma_a(u)}{\Sigma_t(u)} \exp\left[-\frac{1}{\xi} \int_{u_1}^{u} du' Q(u')\right]$$

$$+ \frac{1}{N} \frac{1}{\xi} \int_{u_1}^{u_2} du \left[1 - P(u)\right] \frac{\Sigma_a(u)}{\Sigma_t(u)} \int_{u_1}^{u} du' \Sigma_s(u') P(u') \exp\left[-\frac{1}{\xi} \int_{u'}^{u} du'' Q(u'')\right],$$

where $Q(u) = P(u) + \left[1 - P(u)\right] \frac{\Sigma_a(u)}{\Sigma_t(u)}$ as defined by Eq. (17). When we make ξ tend to infinity in Eq. (26) assuming the N.R.A., the R.I. becomes

$$\text{R.I.} = \frac{1}{N} \int_{u_1}^{u_2} du \Sigma_a(u) + \frac{1}{N} \Sigma_{p.s.} \int du \frac{\Sigma_a(u)}{\Sigma_t(u)} \left[1 - P(u)\right], \tag{27}$$

which just coincides with the usual narrow resonance formula.[1]

It is interesting to consider the physical meaning of each term on the right hand side of Eq. (26). The first term is clearly the absorption rate of resonance neutrons incident from the outside of the lump by the first collision with absorbing nuclei. The second term is the absorption rate of off-resonance neutrons which are slowed by elastic collision with absorbing nuclei to become resonance neutrons and which are absorbed before escaping to the outside of the lump. The resonance neutron flux is then reduced by the factor,

$$\exp\left[-\frac{1}{\xi}\int_{u_1}^{u} du'\left\{P(u') + \left[1 - P(u')\right]\frac{\Sigma_a(u')}{\Sigma_t(u')}\right\}\right]$$

because of the absorption and the escape. This factor corresponds to the well-known factor,

$$\exp\left[-\frac{1}{\xi}\int du'\,\frac{\Sigma_a(u')}{\Sigma_t(u')}\right]$$

for a homogeneous system. The third term represents the absorption rate of the incident resonance neutrons which are absorbed after colliding elastically with absorbing nuclei. The above mentioned three processes exhaust all of the possible ones.

We shall rewrite Eq. (26) in another form to see the relation between the N.R.A. and W.R.A.:

$$\mathrm{R.I.}^{(1)} = \frac{1}{N}\int_{u_1}^{u_2} du\,\Sigma_a(u)P(u) + \frac{1}{N}\Sigma_{p.s.}\int_{u_1}^{u_2} du\left[1 - P(u)\right]\frac{\Sigma_a(u)}{\Sigma_t(u)} \tag{28}$$

$$-\frac{1}{N}\Sigma_{p.s.}\int_{u_1}^{u_2} du\left[1 - P(u)\right]\frac{\Sigma_a(u)}{\Sigma_t(u)}\left\{1 - \exp\left[-\frac{1}{\xi}\int_{u_1}^{u} du'Q(u')\right]\right\}$$

$$+\frac{1}{N}\frac{1}{\xi}\int_{u_1}^{u_2} du\left[1 - P(u)\right]\frac{\Sigma_a(u)}{\Sigma_t(u)}\int_{u_1}^{u_2} du'\Sigma_s(u')P(u')\exp\left[-\frac{1}{\xi}\int_{u'}^{u} du''Q(u'')\right],$$

or, denoting the resonance integral of the N.R.A. and W.R.A. by R.I.(N.R.) and R.I.(W.R.), respectively, we have

$$\mathrm{R.I.} = \mathrm{R.I.(N.R.)}$$
$$- \frac{1}{N}\frac{1}{\xi}\int_{u_1}^{u_2} du \left[1 - P(u)\right] \frac{\Sigma_a(u)}{\Sigma_t(u)} \int_{u_1}^{u} du' \left\{\Sigma_{p.s.} Q(u') - \Sigma_s(u')P(u')\right\} \tag{29}$$
$$\times \exp\left[-\frac{1}{\xi}\int_{u'}^{u} du'' Q(u'')\right] ,$$

where we have used the following relation:

$$1 - \exp\left[-\frac{1}{\xi}\int_{u_1}^{u} du' Q(u')\right] = \frac{1}{\xi}\int_{u_1}^{u} du' Q(u') \exp\left[-\frac{1}{\xi}\int_{u'}^{u} du'' Q(u'')\right] . \tag{30}$$

The second term of Eq. (29) gives a correction term to the R.I.(N.R.). On the other hand, we can easily show that

$$\mathrm{R.I.(N.R.)} - \mathrm{R.I.(W.R.)} = \frac{1}{N}\int_{u_1}^{u_2} du \frac{\Sigma_a(u)}{\Sigma_t(u)} \frac{1}{Q(u)} \left[1 - P(u)\right] \tag{31}$$
$$\times \left\{\Sigma_{p.s.} Q(u) - \Sigma_s(u) P(u)\right\} .$$

From Eqs. (29) and (31) we can conclude that the R.I. (N.R.) and R.I. (W.R.) become equal each other and our corrections to the R.I. (N.R.) and R.I. (W.R.) vanish when the following equation holds:

$$\Sigma_{p.s.}Q(u) - \Sigma_s(u)P(u) = 0. \tag{32}$$

Moreover, the R.I. (N.R.) overestimates (underestimates) and the R.I. (W.R.) underestimates (overestimates) when $\Sigma_{p.s.}Q(u) - \Sigma_s(u)P(u) > (<) 0$. When we neglect the interference effect between the resonance and potential scattering, Eq. (32) becomes as follows:

$$1 - \frac{\Gamma_n}{\Gamma}\left\{1 + \frac{1}{\Sigma_{p.s.}}\,\Sigma_t(u)\,\frac{P(u)}{\left[1 - P(u)\right]}\right\} = 0\,. \tag{33}$$

Here, Γ_γ, Γ_n, and Γ are the radiative, neutron, and total widths of the resonance line.

The physical meaning of Eq. (32) or (33) can be understood as follows: The contribution to the R.I. from neutrons incident on the lump from the moderator at resonance energies is greater in the W.R.A. than in the N.R.A. because of the multiple scattering present in the former. In the N.R.A. there is, however, an additional contribution from neutrons which enter the resonance band due to last collisions in the lump. When Eq. (32) or (33) holds, these contributions just cancel and the N.R.A. and W.R.A. become equal. Spinney[3] discussed the validity of the N.R.A. and W.R.A. in homogeneous mixtures of U and H and derived a relation corresponding to Eq. (32).

Second-Order Approximation

In this section we shall consider the effect of the deviation of the Placzek function $\phi_m(u)$ from its asymptotic form as the second-order approximation. From Eq. (18) we have for $u > u'$

$$X(u') = X(u)\,\exp\left[\frac{1}{\xi}\int_{u'}^{u} du''Q(u'')\right] \tag{34}$$

$$- \frac{\Sigma_t(u)}{\left[1 - P(u)\right]}\,P(u)\exp\left[\frac{1}{\xi}\int_{u'}^{u} du''Q(u'')\right] + \frac{\Sigma_t(u')}{\left[1 - P(u')\right]}\,P(u')$$

$$- \frac{1}{\xi}\int_{u'}^{u} du''P(u'')\Sigma_s(u'')\exp\left[-\frac{1}{\xi}\int_{u''}^{u'} du'''Q(u''')\right].$$

This expression would be accurate if the asymptotic Placzek function were used. Substituting of $X(u')$ given by Eq. (34) into the fifth term on the right hand side of Eq. (15), we have, after some rearrangements, the following integral equation:

$$\left[\xi - \xi_1(u)\right]\,X(u) = \left[\xi - \xi_1(u)\right]\,\frac{\Sigma_t(u)}{\left[1 - P(u)\right]}\,P(u) \tag{35}$$

$$+ \int_0^u du'\,\frac{\Sigma_t(u')}{\left[1 - P(u')\right]}\,P(u') - \int_0^u du'P_m(u - u')\Sigma_s(u')P(u')$$

$$- \frac{1}{\xi}\int_0^u du'\Sigma_s(u')P(u')\int_0^{u'} du''P_m(u - u'')Q(u'')\exp\left[\frac{1}{\xi}\int_{u''}^{u'} du'''Q(u''')\right]$$

$$- \int_0^u du' Q(u') X(u'),$$

where we have used the following definition:

$$\xi_1(u) = \int_0^u du' P_m(u - u') Q(u') \exp\left[\frac{1}{\xi} \int_{u'}^u du'' Q(u'')\right] . \tag{36}$$

As $P(u) = 0$ and $Q(u) = \dfrac{\Sigma_a(u)}{\Sigma_t(u)}$ for the infinite system, $\xi_1(u)$ just coincides with the quantity defined by Weinberg and Wigner[6] for the homogeneous system. Equation (36) gives the extension to the lump system of $\xi_1(u)$.

The integral equation (35) can be solved by differentiation and the second-order solution $X^{(2)}(u)$ becomes

$$X^{(2)}(u) = \frac{\Sigma_t(u)}{\left[1 - P(u)\right]} P(u) \tag{37}$$

$$+ \frac{1}{[\xi - \xi_1(u)]} \int_0^u du' P(u') \Sigma_s(u') \exp\left[- \int_{u'}^u du'' \frac{Q(u'')}{[\xi - \xi_1(u'')]}\right]$$

$$- \frac{1}{[\xi - \xi_1(u)]} \left\{\omega(u) - \int_0^u du' \omega(u') \frac{Q(u')}{[\xi - \xi_1(u')]}\right.$$

$$\left. \times \exp\left[- \int_{u'}^u du'' \frac{Q(u'')}{[\xi - \xi_1(u'')]}\right]\right\},$$

where

$$\omega(u) = \int_{0}^{u} du' P_m(u - u') \left[\Sigma_s(u')P(u') + \frac{1}{\xi} Q(u') \int_{u'}^{u} du'' \Sigma_s(u'')P(u'') \right. \tag{38}$$

$$\left. \times \exp\left\{ \frac{1}{\xi} \int_{u'}^{u''} du''' Q(u''') \right\} \right] .$$

The second term of $X^{(2)}(u)$ is obtained from the second term of $X^{(1)}(u)$ given by Eq. (18) by substitution of $\xi - \xi_1(u)$ for ξ and is very similar to the case of the homogeneous system. The third term originates from the third term on the right hand side of Eq. (15). This term gives rise to the flux oscillation due to variations in the first collision density of incident neutrons. In a way similar to that for the first-order solution, we shall show in the appendix A that $X^{(2)}(u)$ gives the correct asymptotic neutron flux in the absence of absorption. The resonance integral of the second order approximation becomes

$$R.I.^{(2)} = \frac{1}{N} \int_{u_1}^{u_2} du \Sigma_a(u)P(u) \tag{39}$$

$$+ \frac{1}{N} \int_{u_1}^{u_2} du \frac{[1 - P(u)]}{[\xi - \xi_1(u)]} \frac{\Sigma_a(u)}{\Sigma_t(u)} \int_{0}^{u} du' P(u') \Sigma_s(u') \exp\left[- \int_{u'}^{u} du'' \frac{Q(u'')}{[\xi - \xi_1(u'')]} \right]$$

$$- \frac{1}{N} \int_{u_1}^{u} du \, \frac{[1 - P(u)]}{[\xi - \xi_1(u)]} \, \frac{\Sigma_a(u)}{\Sigma_t(u)} \left[\omega(u) - \int_{0}^{u} du' \omega(u') \, \frac{Q(u')}{[\xi - \xi_1(u')]} \right.$$

$$\left. \times \ \exp\left\{ - \int_{u'}^{u} du'' \, \frac{Q(u'')}{[\xi - \xi_1(u'')]} \right\} \right] .$$

III. RATIONAL APPROXIMATION AND DISCUSSION

We have derived the new formula of the resonance integral for the lump system by making use of the flat flux assumption inside the lump. At first we shall discuss the validity of our fundamental assumption.

As already pointed out by Spinrad, Chernick, and Corngold,[5] the flat flux assumption is clearly valid when $\bar{\ell}\,\Sigma_t(u) \ll 1$, i.e., for a small lump or at the wing of the resonance region. On the other hand, in the case $\Sigma_a \gg \Sigma_s$ neutrons are absorbed by their first collisions with absorbing nuclei and resonance neutrons slowed down by elastic collisions with absorbing nuclei are absorbed in their original spatial form. Therefore our assumption is also valid in this case. When $\bar{\ell}\,\Sigma_t(u) \gg 1$ and $\Sigma_a \approx \Sigma_s$, the neutron flux at the center of the lump may be reduced. However, Spinrad, Chernick, and Corngold showed that the attenuation length of the surface absorption averaged over the resonance region is of the order of a potential scattering mean free path for the extreme case of a semi-infinite absorber. Therefore, we conclude that the flat flux assumption is fairly good for not very large $\bar{\ell}$.

Dresner[2] has proposed a new resonance integral formula in the W.R.A. which takes into account the spatial variation of the flux inside the lump. He has shown that for lumps for which $\Sigma_t \bar{\ell} >> 1$ at the line center and $\Sigma_{p.s.} \bar{\ell} \lesssim 1$, his formula underestimates the resonance integral, but not by more than 10.6%. Finally, he has compared his formula with Eq. (22) for the Breit-Wigner line shape in small lumps in which potential scattering may be neglected.* Equation (22) is always larger, except for $\Gamma_\gamma/\Gamma = 1$, when the two are equal. The relative difference is 3.3% at $\Gamma_\gamma/\Gamma = 0.75$, 8.6% at $\Gamma_\gamma/\Gamma = 0.50$, and 19.7% at $\Gamma_\gamma/\Gamma = 0.25$. Thus the flat flux approximation appears valid even for situations in which there is considerable multiple scattering of neutrons in the lump. However, it is true that the flat flux approximation overestimates the resonance integral.

We shall conclude this section comparing the resonance integral calculated by our first order formula given by Eq. (26) or (29) with that of the W.R.A. and N.R.A. using the rational approximation for the escape probability P(u).[1,2,5] In the rational approximation P(u) is

$$P(u) = \frac{1}{1 + \bar{\ell}\,\Sigma_t(u)} . \tag{40}$$

Upon substitution of this expression into Eq. (26), we obtain

*L. Dresner, private communication.

$$\text{R.I.}^{(1)} = \frac{1}{N} \int_{u_1}^{u_2} du \, \frac{\Sigma_a(u)}{\left[1 + \bar{\ell}\, \Sigma_t(u)\right]} \tag{41}$$

$$+ \frac{1}{N} \Sigma_{p.s.} \int_{u_1}^{u_2} du \, \frac{\bar{\ell}\, \Sigma_a(u)}{\left[1 + \bar{\ell}\, \Sigma_t(u)\right]} \exp\left[- \frac{1}{\xi} \int_{u_1}^{u} du' \left\{ \frac{1 + \bar{\ell}\, \Sigma_a(u')}{1 + \bar{\ell}\, \Sigma_t(u')} \right\} \right]$$

$$+ \frac{1}{N} \frac{1}{\xi} \int_{u_1}^{u_2} du \, \frac{\bar{\ell}\, \Sigma_a(u)}{\left[1 + \bar{\ell}\, \Sigma_t(u)\right]} \int_{u_1}^{u} du' \, \frac{\Sigma_s(u')}{\left[1 + \bar{\ell}\, \Sigma_t(u')\right]}$$

$$\times \exp\left[- \frac{1}{\xi} \int_{u'}^{u} du'' \left\{ \frac{1 + \bar{\ell}\, \Sigma_a(u'')}{1 + \bar{\ell}\, \Sigma_t(u'')} \right\} \right] .$$

For the square cross section, the resonance integral can be easily calculated and it gives almost all of the important features of our formula. We assume the following cross sections:

$$\Sigma_a = N\sigma_o \frac{\Gamma_\gamma}{\Gamma} \qquad \text{for } E_o - \frac{\Delta}{2} < E < E_o + \frac{\Delta}{2} ,$$

$$= 0 \qquad \text{otherwise;}$$

$$\Sigma_{r.s.} = N\sigma_o \frac{\Gamma_n}{\Gamma} \qquad \text{for } E_o - \frac{\Delta}{2} < E < E_o + \frac{\Delta}{2} ,$$

$$= 0 \qquad \text{otherwise,}$$

$$\Sigma_{p.s.} = N\sigma_{p.s.} ,$$

where $\Sigma_{r.s.}$ is the resonance scattering cross section and σ_o is the height of the total resonance cross section. Then the resonance integral becomes

$$\text{R.I.} = \sigma_{p.s.}(b - 1) \frac{\frac{\Gamma_\gamma}{\Gamma}}{\left[\frac{\Gamma_\gamma}{\Gamma} + \frac{\sigma_{p.s.}}{\sigma_o}(b - 1)\right]} \log\left[\frac{1 + \frac{1}{2}\frac{\Delta}{E_o}}{1 - \frac{1}{2}\frac{\Delta}{E_o}}\right] \tag{42}$$

$$+ \left(1 - b\frac{\Gamma_n}{\Gamma}\right) \frac{\xi\sigma_{p.s.}\frac{\Gamma_\gamma}{\Gamma}}{\left[\frac{\Gamma_\gamma}{\Gamma} + \frac{\sigma_{p.s.}}{\sigma_o}(b - 1)\right]^2} \left[1 - \left(\frac{1 - \frac{1}{2}\frac{\Delta}{E_o}}{1 + \frac{1}{2}\frac{\Delta}{E_o}}\right)^{(\lambda/\xi)}\right],$$

where

$$b = 1 + \frac{1}{\bar{l}\Sigma_{p.s.}}, \tag{43}$$

$$\lambda = \left[\frac{\frac{\Gamma_\gamma}{\Gamma} + \frac{\sigma_{p.s.}}{\sigma_o}(b - 1)}{1 + \frac{\sigma_{p.s.}}{\sigma_o}b}\right]. \tag{44}$$

We can rewrite Eq. (42) in another form in the narrow resonance limit, i.e., $\frac{\Delta}{E_o} \ll \xi$,

$$\text{R.I.} = \sigma_{p.s.} b \frac{\frac{\Gamma_\gamma}{\Gamma}}{\left[1 + \frac{\sigma_{p.s.}}{\sigma_o}b\right]} \log\left[\frac{1 + \frac{1}{2}\frac{\Delta}{E_o}}{1 - \frac{1}{2}\frac{\Delta}{E_o}}\right] \tag{45}$$

$$- \left(1 - b\frac{\Gamma_n}{\Gamma}\right) \frac{\sigma_{p.s.}\frac{\Gamma_\gamma}{\Gamma}}{\left[1 + \frac{\sigma_{p.s.}}{\sigma_o}b\right]^2} \frac{1}{\lambda} \left[\log\left\{\frac{1 + \frac{1}{2}\frac{\Delta}{E_o}}{1 - \frac{1}{2}\frac{\Delta}{E_o}}\right\} - \frac{\xi}{\lambda}\left\{1 - \left(\frac{1 - \frac{1}{2}\frac{\Delta}{E_o}}{1 + \frac{1}{2}\frac{\Delta}{E_o}}\right)^{(\lambda/\xi)}\right\}\right],$$

or

$$\text{R.I.} = \sigma_{p.s.}\, b \frac{\frac{\Gamma_\gamma}{\Gamma}}{\left[1 + \frac{\sigma_{p.s.}}{\sigma_o} b\right]} \frac{\Delta}{E_o}$$

$$- \left(1 - b\frac{\Gamma_n}{\Gamma}\right) \frac{\sigma_{p.s.} \frac{\Gamma_\gamma}{\Gamma}}{\left[1 + \frac{\sigma_{p.s.}}{\sigma_o} b\right]^2} \frac{\Delta}{E_o} \left\{ \frac{1}{\xi} \left(\frac{\Delta}{2E_o}\right) - \frac{2}{3} \lambda \frac{1}{\xi^2} \left(\frac{\Delta}{2E_o}\right)^2 + \cdots \right\} .$$

As easily verified, the first term on the right hand side of Eq. (42) is the resonance integral of the W.R.A. and the second term represents the correction term to it. Similarly, the first term on the right hand side of Eq. (45) is nothing but the resonance integral of the N.R.A. and the correction term is of the order of $\frac{1}{\xi} \frac{\Delta}{E_o}$. Dresner's formula for the wide resonance gives

$$\text{R.I.(D)} = \sigma_{p.s.}(b - 1) \frac{\frac{\Gamma_\gamma}{\Gamma}}{\left[\left(1 - \frac{\sqrt{3}}{4}\right) \frac{\Gamma_\gamma}{\Gamma} + \frac{\sqrt{3}}{4} \sqrt{\frac{\Gamma_\gamma}{\Gamma} + \frac{\sigma_{p.s.}}{\sigma_o}(b - 1)}\right]} \tag{46}$$

$$\times \log \left(\frac{1 + \frac{1}{2}\frac{\Delta}{E_o}}{1 - \frac{1}{2}\frac{\Delta}{E_o}} \right) .$$

It is very important to notice the fact that the resonance integral we proposed can be written in either of the following two forms:

$$\text{R.I.} = \text{R.I.(N.R.)} \left[1 - \left(1 - \frac{\Gamma_n}{\Gamma} b\right) \cdot F\left(\frac{\Gamma_\gamma}{\Gamma}, \sigma_o, \sigma_{p.s.}, \xi, \ldots\right) \right] , \tag{47}$$

or

$$R.I. = R.I.(W.R.)\left[1 + \left(1 - \frac{\Gamma_n}{\Gamma} b\right) \cdot G\left(\frac{\Gamma_\gamma}{\Gamma}, \sigma_o, \sigma_{p.s.}, \xi, \ldots\right)\right], \tag{48}$$

where F and G are the positive functions of $\bar{\ell}$, ξ and the resonance parameters. Therefore we can conclude that the N.R. formula overestimates (underestimates) and the W.R. formula underestimates (overestimates) when $1 - \frac{\Gamma_n}{\Gamma} b > (<) 0$ irrespective of the resonance width. Moreover, both of R.I.(N.R.) and R.I.(W.R.) coincide with our R.I. irrespective of the width when

$$1 - \frac{\Gamma_n}{\Gamma} b = 0. \tag{49}$$

Equation (49) is the specialization of Eq. (32) or (33) to the rational approximation of P(u). As we shall show in the following, the relations given by Eqs. (47), (48), and (49) are valid not only for the square cross section but also for more general resonance cross sections, for example, for the Breit-Wigner one level formula taking into account of the Doppler broadening. We shall assume the following cross sections.

$$\Sigma_a(u) = N\sigma_o \frac{\Gamma_\gamma}{\Gamma} \psi(u), \tag{50}$$

$$\Sigma_{r.s.}(u) = N\sigma_o \frac{\Gamma_n}{\Gamma} \psi(u), \tag{51}$$

where $\psi(u)$ represents the energy dependence of the resonance cross sections. For this cross section we obtain

$$\text{R.I.(N.R.)} = \sigma_{\text{p.s.}}\, b\, \frac{\Gamma_\gamma}{\Gamma} \int_{u_1}^{u_2} du\, \frac{\psi(u)}{\left[\frac{\sigma_{\text{p.s.}}}{\sigma_0}\, b + \psi(u)\right]}, \tag{52}$$

$$\text{R.I.(W.R.)} = \sigma_{\text{p.s.}} (b - 1) \int_{u_1}^{u_2} du\, \frac{\psi(u)}{\left[\frac{\sigma_{\text{p.s.}}}{\sigma_0} (b - 1) \frac{\Gamma}{\Gamma_\gamma} + \psi(u)\right]}. \tag{53}$$

R.I.(N.R.) and R.I.(W.R.) become equal each other when Eq. (49) holds. Moreover, we can show from Eq. (29) using the rational approximation that our resonance integral formula becomes

$$\text{R.I.} = \text{R.I.(N.R.)} - \left(1 - b\, \frac{\Gamma_n}{\Gamma}\right)$$

$$\times\ \sigma_{\text{p.s.}} \frac{\Gamma_\gamma}{\Gamma} \frac{1}{\xi} \int_{u_1}^{u_2} du\, \frac{\psi(u)}{\left[\frac{\sigma_{\text{p.s.}}}{\sigma_0}\, b + \psi(u)\right]} \int_{u_1}^{u} du'\, \frac{\psi(u')}{\left[\frac{\sigma_{\text{p.s.}}}{\sigma_0}\, b + \psi(u')\right]} \tag{54}$$

$$\times\ \exp\left[-\frac{1}{\xi} \int_{u'}^{u} du'' \left\{ \frac{\frac{\sigma_{\text{p.s.}}}{\sigma_0} (b - 1) + \frac{\Gamma_\gamma}{\Gamma} \psi(u'')}{\frac{\sigma_{\text{p.s.}}}{\sigma_0}\, b + \psi(u'')} \right\}\right].$$

The R.I. given by Eq. (54) has just a similar form as that of Eq. (47). We can also rewrite Eq. (54) in a form similar to Eq. (48).

When we take into account of the interference effect between the resonance and potential scatterings[2], Eq. (49) is replaced by

$$1 - \frac{\Gamma_n}{\Gamma}\, b \left[1 - \frac{1}{b^2} (b - 1) g_J\right] = 0, \tag{55}$$

where g_J is the statistical weight of the compound state. It is very interesting to notice the fact that Eq. (49) or (55) does not depend on the width of the resonance line.

The resonance integrals of a U^{238} metallic lump ($\bar{\ell}$ = 2 cm) for the square cross section, as a function of the resonance width Δ and Γ_γ/Γ are given in Figs. 1 and 2. Parameters we have used are that of 6.6 ev resonance line of U^{238} (Ref. 11) and the height σ_o is determined so as to give the same area as the actual resonance line.

IV. CONCLUSIONS

We have proposed an improved resonance integral formula which contains the N.R. and W.R. formulas as the extreme cases. However, there remain many things still to be done, e.g., the estimation of the second order correction, and the improvement of the flat flux approximation.

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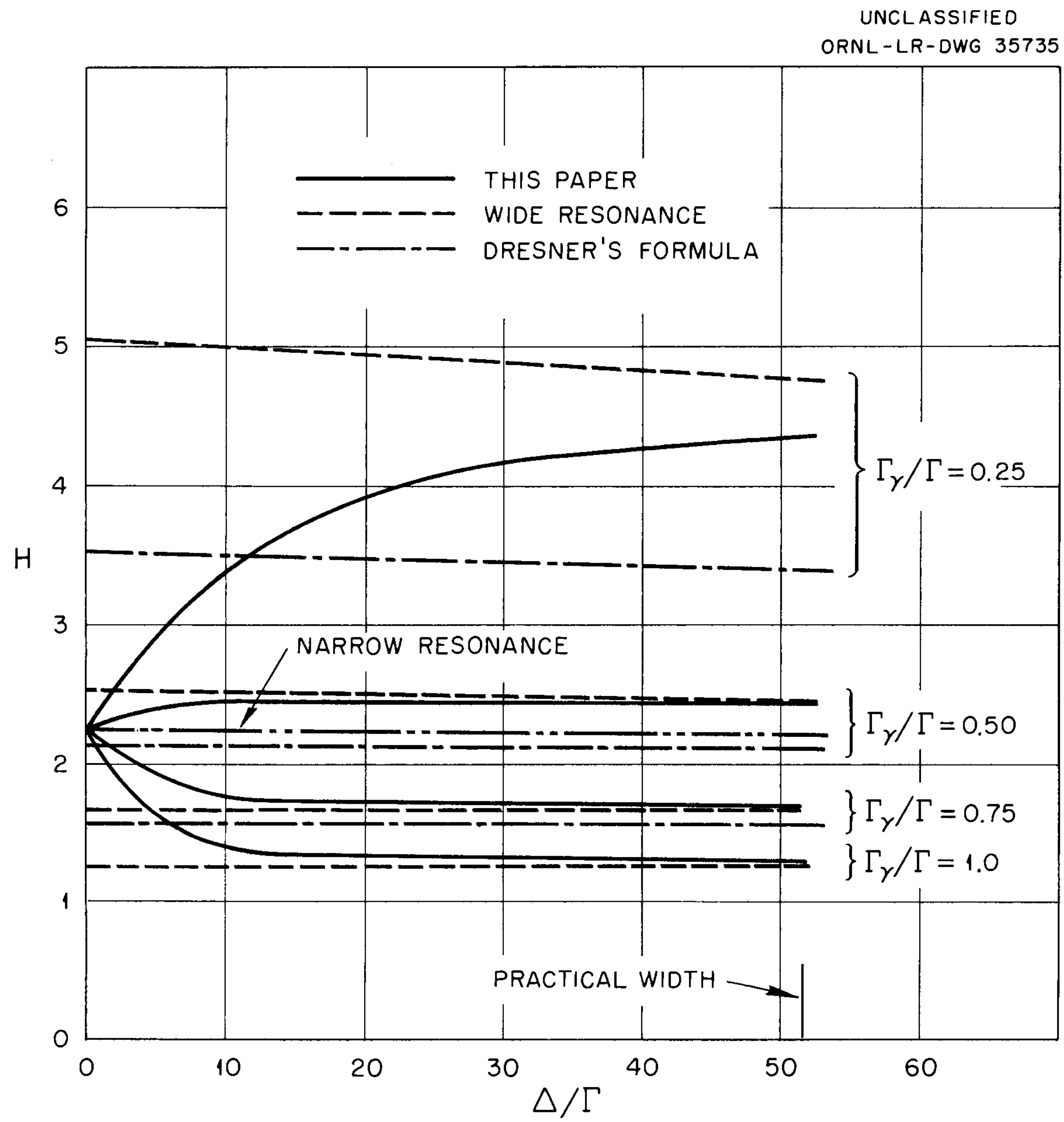

Fig. 1. The Resonance Integral of a U^{238}-Metallic Lump for Square Cross Section vs. the Width Δ. $\Gamma = 26.5$ mv, $E_0 = 6.6$ ev. The height is determined so as to give the same area as 6.6-ev resonance line and $\bar{\ell} =$ 2 cm.

$$R.I. = \sigma_{p.s.} \frac{\Gamma_\gamma}{\Gamma} \log \left[\frac{1 + \Delta/2E_0}{1 - \Delta/2E_0} \right] \cdot H$$

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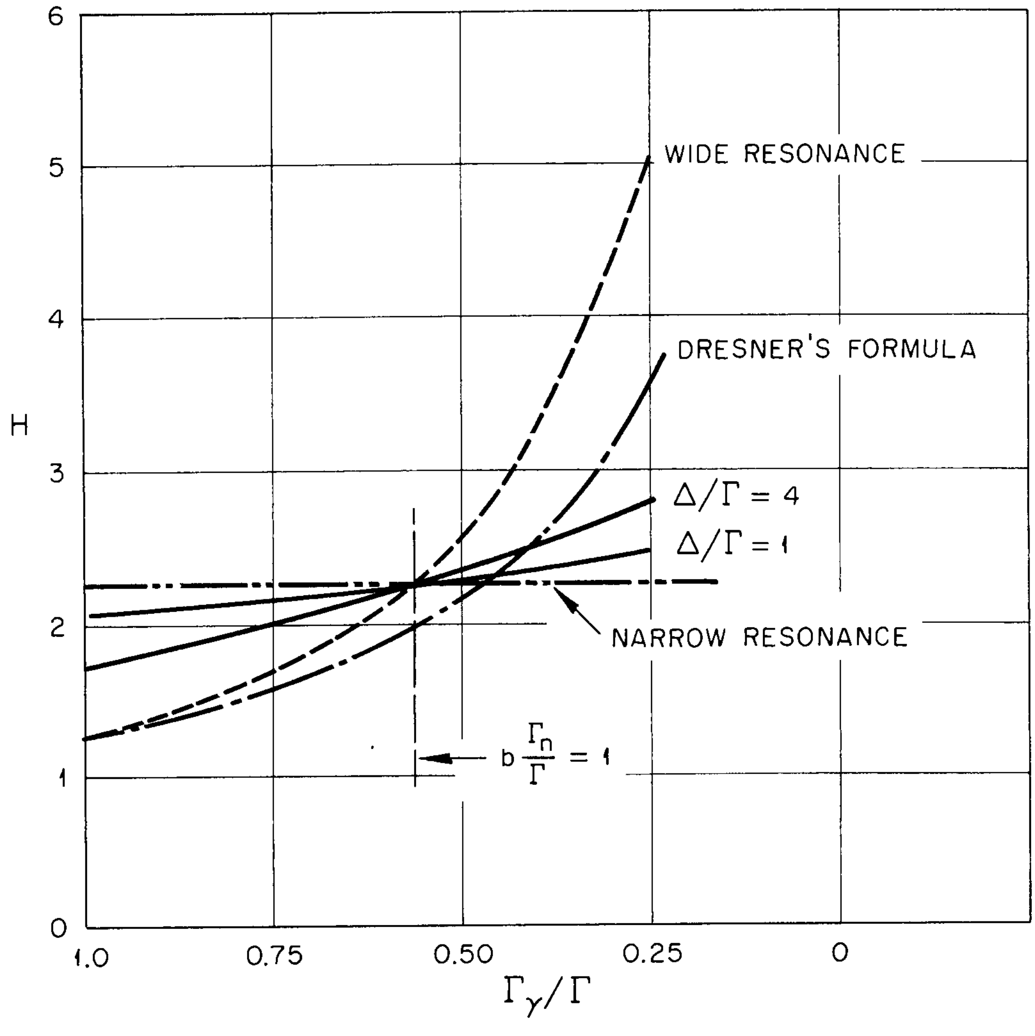

Fig. 2. The Resonance Integral vs. Γ_γ/Γ for $0 \leq \frac{\Delta}{\Gamma} \leq 4$. Parameters are the same as in Fig. 1. $b = 1 + \frac{1}{\ell\,\Sigma_{p.s.}}$

APPENDIX A

We shall prove $X^{(2)}(u)$ given by Eq. (37) represents the correct asymptotic form in the absence of absorption, i.e., $\Sigma_a = 0$, $\Sigma_t(u) = \Sigma_{p.s.}$. In this case P(u) becomes constant and we denote it by P_o. From Eqs. (36) and (38), $\xi_1(u)$ and $\omega(u)$ become

$$\xi_1(u) = P_o \int_o^u du' P_m(u - u') \exp\left[\frac{P_o}{\xi}(u - u')\right], \qquad (A.1)$$

$$\omega(u) = \Sigma_{p.s.}\xi_1(u). \qquad (A.2)$$

Substituting Eqs. (A.1) and (A.2) into Eq. (37) we have

$$X^{(2)}(u) = \frac{P_o}{[1 - P_o]}\Sigma_{p.s.} + \frac{P_o\Sigma_{p.s.}}{[\xi - \xi_1(u)]}\int_o^u du' \exp\left[- P_o\int_{u'}^u du'' \frac{1}{\{\xi - \xi_1(u'')\}}\right] \qquad (A.3)$$

$$- \frac{\Sigma_{p.s.}}{[\xi - \xi_1(u)]}\xi_1(u) + \frac{P_o\Sigma_{p.s.}}{[\xi - \xi_1(u)]}\int_o^u du' \frac{\xi_1(u')}{[\xi - \xi_1(u')]}$$

$$\times \exp\left[- P_o\int_{u'}^u du'' \frac{1}{\{\xi - \xi_1(u'')\}}\right].$$

We rewrite the last term on the right hand side of Eq. (A.3) as follows:

$$- \frac{P_o \Sigma_{p.s.}}{[\xi - \xi_1(u)]} \int_0^u du' \left[1 - \frac{\xi}{\{\xi - \xi_1(u')\}} \right] \exp \left[- P_o \int_{u'}^u du'' \frac{1}{\{\xi - \xi_1(u'')\}} \right] ,$$

by integration we obtain

$$- \frac{P_o \Sigma_{p.s.}}{[\xi - \xi_1(u)]} \int_0^u du' \exp \left[- P_o \int_{u'}^u du'' \frac{1}{\{\xi - \xi_1(u'')\}} \right] + \frac{\Sigma_{p.s.}}{[\xi - \xi_1(u)]} \xi \tag{A.4}$$

$$- \frac{\Sigma_{p.s.}}{[\xi - \xi_1(u)]} \xi \exp \left[- P_o \int_0^u du' \frac{1}{\{\xi - \xi_1(u')\}} \right].$$

From Eqs. (A.3) and (A.4) we have

$$X^{(2)}(u) = \frac{\Sigma_{p.s.}}{[1 - P_o]} - \frac{\Sigma_{p.s.}}{[\xi - \xi_1(u)]} \xi \exp \left[- P_o \int_0^u du' \frac{1}{\{\xi - \xi_1(u')\}} \right] . \tag{A.5}$$

As the second term of the right hand side of Eq. (A.5) is of the order of $\exp \left[- \frac{P_o}{\xi} u \right]$, it can be neglected and $\zeta^{(2)}(u) = 1$ as we expected.

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