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ANALYSIS OF  
HIGH CONVERSION CRITICAL EXPERIMENTS

by

E. M. Pennington

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E. M. Pennington

Reactor Physics Division

March 1967

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# ANALYSIS OF HIGH CONVERSION CRITICAL EXPERIMENTS

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E. M. Pennington

## ABSTRACT

Critical experiments performed in the ZPR-7 facility using Hi-C or BORAX-V fuel and light-water moderator are analyzed. The Hi-C fuel consisted of 3.04 w/o enriched  $\text{UO}_2$  pellets of 0.935 cm diameter, stacked to a height of 122 cm in either aluminum or stainless steel tubing. Experiments with BORAX-V fuel used  $\text{UO}_2$  pellets, which were 4.95 w/o enriched, 0.871 cm in diameter, and stacked 61 cm high in stainless steel tubing. All lattices analyzed consisted of fuel rods uniformly spaced in either square or triangular arrays. The H-to- $\text{U}^{238}$  atom ratios in the cores varied from roughly 4.6 to 0.5 so that investigations extended far into the undermoderated region. Calculations of critical bucklings, reflector savings, and various microparameters are compared with experimental values. Agreement with experiment is in general reasonably good, except for the  $\text{U}^{238}$ -to- $\text{U}^{235}$  fission ratio.

## I. INTRODUCTION

The High Conversion Critical Experiments Program (Hi-C) was initiated to extend the range of investigations of light-water-moderated, slightly enriched, uranium oxide cores to lower hydrogen-to- $\text{U}^{238}$  atom ratios than had been studied previously. Uniform lattices having H-to- $\text{U}^{238}$  atom ratios from about 4.6 to 0.5 were studied. These H-to- $\text{U}^{238}$  atom ratios correspond to water-to-fuel volume ratios ranging from 1.5 to 0.16. The various H-to- $\text{U}^{238}$  atom ratios were obtained by using square and triangular grids of different pitches, except for the tightest loading, which was achieved without gridwork.

As the name of the program suggests, tightly packed cores have high conversion ratios, which retard reactivity losses with fuel depletion and therefore lead to long core lifetimes. Also, heavily loaded cores have large surface areas available for heat transfer, thereby allowing high power densities. The close spacing of the rods is ultimately limited by lack of

space for coolant flow and structural support. Tightly packed zones can be used as driver zones surrounding cores of more conventional design, leading to a net increase in the conversion ratio of the system. Thus the study of highly undermoderated cores is of interest for design of light-water-moderated power reactors.

Two lattices were studied using the 4.95 w/o enriched  $\text{UO}_2$  BORAX-V fuel. The other lattices consisted of the 3.04 w/o enriched  $\text{UO}_2$  Hi-C fuel. Initially, approximately 3200 aluminum-clad and 2100 stainless steel-clad Hi-C fuel pins were available. Later, all but 400 stainless steel-clad fuel pins were dejacketed, the pellets being transferred to aluminum cladding. This enabled the achievement of criticality for tighter lattices with aluminum-clad fuel. Four uniform Hi-C lattices of aluminum-clad fuel and one of stainless steel-clad fuel attained criticality. In addition, one other Hi-C uniform lattice with aluminum-clad fuel and five others with stainless steel-clad fuel were investigated. In these cases, criticality was attained by surrounding the central core region by one or two driver regions. Often the region next to the central core consisted of aluminum-clad fuel with the same pitch as the stainless steel-clad fuel in the central region, while the second driver zone, when present, contained fuel more loosely spaced. Microparameter measurements (initial conversion ratio, thermal disadvantage factor, etc.) were made in the central zone. All cores were constructed in a tank roughly 2 m in diameter, which provided an effectively infinite radial light-water reflector. The water level in the tank could be adjusted to any desired level to achieve criticality.

Reference 1 describes in detail the ZPR-7 critical facility, the Hi-C and BORAX-V fuel pins and cladding, the experimental techniques involved, and the experimental results obtained. Some early Hi-C experimental results were presented at technical meetings.<sup>2,3</sup> Later experimental work was described in Reactor Physics Division Annual Reports.<sup>4,5</sup> Experimental studies of thermal disadvantage factors<sup>6</sup> and cadmium ratios<sup>7</sup> were also reported previously. References 8 and 9 have outlined some of the theoretical work. A short comparison of experimental and calculated results has also been presented.<sup>10</sup>

The next section of this report describes briefly the fuel, cladding, and lattice dimensions involved. Following sections describe the calculational methods in detail and present the numerical results. Most of the calculations were performed in the framework of four-group diffusion theory, constants for three fast groups being obtained from the GAM-I code,<sup>11</sup> and thermal group constants being derived from the THERMOS code.<sup>12</sup> Experimental and theoretical values are compared whenever possible.

## II. FUEL, CLADDING, AND LATTICE DESCRIPTIONS

Table I presents data on the fuel and cladding dimensions and atomic number densities of these components, which were used in all calculations. The values in Table I are in substantial agreement with those quoted in Table III of Ref. 1. The number densities of hydrogen and oxygen in the light-water moderator were taken to be  $0.06694 \times 10^{24}$  and  $0.03347 \times 10^{24}$  atoms/cm<sup>3</sup>, respectively.

TABLE I. Hi-C and BORAX-V Fuel and Cladding Data

|                                         | BORAX-V                     |                            | Hi-C                         |
|-----------------------------------------|-----------------------------|----------------------------|------------------------------|
| <u>Fuel</u>                             |                             |                            |                              |
| Enrichment, w/o                         | 4.95                        |                            | 3.0422                       |
| Radius, cm                              | 0.43561                     |                            | 0.46736                      |
| Length, cm                              | 60.96                       |                            | 121.92                       |
| Oxide density, g/cm <sup>3</sup>        | 10.2                        |                            | 10.2                         |
| <sup>235</sup> N, atoms/cm <sup>3</sup> | 0.001141 x 10 <sup>24</sup> |                            | 0.0007013 x 10 <sup>24</sup> |
| <sup>238</sup> N, atoms/cm <sup>3</sup> | 0.02163 x 10 <sup>24</sup>  |                            | 0.02207 x 10 <sup>24</sup>   |
| <sup>16</sup> O, atoms/cm <sup>3</sup>  | 0.04554 x 10 <sup>24</sup>  |                            | 0.04554 x 10 <sup>24</sup>   |
| <u>Cladding</u>                         |                             |                            |                              |
| Type                                    | 304 SS                      | 6061-T6 Al                 | 304 SS                       |
| Inner radius, cm                        | 0.43942                     | 0.48445                    | 0.47964                      |
| Outer radius, cm                        | 0.47752                     | 0.52900                    | 0.52850                      |
| Density, g/cm <sup>3</sup>              | 7.806                       | 2.70                       | 7.806                        |
| N <sup>Fe</sup> , atoms/cm <sup>3</sup> | 0.06063 x 10 <sup>24</sup>  | -                          | 0.06063 x 10 <sup>24</sup>   |
| N <sup>Cr</sup> , atoms/cm <sup>3</sup> | 0.01718 x 10 <sup>24</sup>  | -                          | 0.01718 x 10 <sup>24</sup>   |
| N <sup>Ni</sup> , atoms/cm <sup>3</sup> | 0.00722 x 10 <sup>24</sup>  | -                          | 0.00722 x 10 <sup>24</sup>   |
| N <sup>Al</sup> , atoms/cm <sup>3</sup> | -                           | 0.06025 x 10 <sup>24</sup> | -                            |

Table II lists the volume fractions, unit-cell outer radii, H-to-U<sup>238</sup> atom ratios, and H<sub>2</sub>O-to-fuel volume ratios for all the Hi-C and BORAX-V uniform lattices. The core codes given in the first column of Table II will be used in referring to the lattices throughout the rest of this report.



TABLE II. HI-C and BORAX-V Lattice Descriptions

| Core Code | Lattice Pitch, cm | Fuel    | Cladding | Unit-cell Radius, cm | Volume Fractions |         |          |         | H <sub>2</sub> O/UO <sub>2</sub> Volume Ratio | H/U <sup>238</sup> Atom Ratio |
|-----------|-------------------|---------|----------|----------------------|------------------|---------|----------|---------|-----------------------------------------------|-------------------------------|
|           |                   |         |          |                      | Fuel             | Void    | Cladding | Water   |                                               |                               |
| B1.27□S   | 1.27 square       | BORAX-V | SS       | 0.71650              | 0.36963          | 0.00649 | 0.06805  | 0.55583 | 1.504                                         | 4.654                         |
| B1.27△S   | 1.27 triangular   | BORAX-V | SS       | 0.66680              | 0.42679          | 0.00750 | 0.07857  | 0.48714 | 1.141                                         | 3.533                         |
| H1.349□A  | 1.349 square      | Hi-C    | Al       | 0.76109              | 0.37708          | 0.02808 | 0.07794  | 0.51690 | 1.371                                         | 4.158                         |
| H1.349□S  | 1.349 square      | Hi-C    | SS       | 0.76109              | 0.37708          | 0.02007 | 0.08504  | 0.51781 | 1.373                                         | 4.165                         |
| H1.24□A   | 1.24 square       | Hi-C    | Al       | 0.69960              | 0.44628          | 0.03323 | 0.09225  | 0.42824 | 0.9596                                        | 2.911                         |
| H1.24□S   | 1.24 square       | Hi-C    | SS       | 0.69960              | 0.44628          | 0.02376 | 0.10064  | 0.42932 | 0.9620                                        | 2.918                         |
| H1.27△A   | 1.27 triangular   | Hi-C    | Al       | 0.66680              | 0.49126          | 0.03658 | 0.10155  | 0.37061 | 0.7544                                        | 2.289                         |
| H1.27△S   | 1.27 triangular   | Hi-C    | SS       | 0.66680              | 0.49126          | 0.02615 | 0.11079  | 0.37180 | 0.7568                                        | 2.296                         |
| H1.166△A  | 1.166 triangular  | Hi-C    | Al       | 0.61220              | 0.58279          | 0.04340 | 0.12047  | 0.25334 | 0.4347                                        | 1.319                         |
| H1.166△S  | 1.166 triangular  | Hi-C    | SS       | 0.61220              | 0.58279          | 0.03103 | 0.13143  | 0.25475 | 0.4371                                        | 1.326                         |
| H1.127△A  | 1.127 triangular  | Hi-C    | Al       | 0.59172              | 0.62383          | 0.04646 | 0.12895  | 0.20076 | 0.3218                                        | 0.9760                        |
| H1.127△S  | 1.127 triangular  | Hi-C    | SS       | 0.59172              | 0.62383          | 0.03321 | 0.14069  | 0.20227 | 0.3242                                        | 0.9833                        |
| H1.069△S  | 1.069 triangular  | Hi-C    | SS       | 0.56127              | 0.69336          | 0.03691 | 0.15637  | 0.11336 | 0.1635                                        | 0.4959                        |

### III. CALCULATION OF THERMAL PARAMETERS

Thermal lattice parameters were computed with the aid of the THERMOS code<sup>12</sup> on the IBM-704. This code calculates the scalar thermal-neutron spectrum as a function of position in a lattice cell by solving numerically the integral transport equation with isotropic scattering. Thirty thermal groups and 20 space points are allowed in the code. Free-gas kernels are used, except for hydrogen, for which the Nelkin, Brown-St. John, and free-gas kernels are all available. The slowing-down source for the problems reported here was taken as resulting from hydrogen only, and was spatially flat in the H<sub>2</sub>O region. The transport kernels in THERMOS are derived on the basis of mirror-image reflection at the cell boundary. Newmarch<sup>13</sup> has shown that this boundary condition leads to a gross overestimate of the disadvantage factor in a cylindrical cell having a moderator region that is thin in terms of mean free paths. Thus the method of Honeck<sup>14</sup> was used in which an extra region consisting of a heavy scatterer was placed outside the actual cell boundary. This method produces an essentially isotropic distribution of the neutrons returning to the actual cell. The extra region was three mean free paths thick in all problems. A mass of  $10^5$  amu was assigned to the heavy scatterer so that neutrons would be returned to the cell in the same velocity group in which they left. Since void regions are not allowed in this version of THERMOS, the inner radius of the cladding was set equal to the outer radius of the fuel, and the atomic number densities of the clad constituents were adjusted to yield the correct total number of atoms.

Calculations using the Nelkin kernel for hydrogen were carried out for all the lattices. Also, problems using both Brown-St. John and free-gas kernels for hydrogen were run for some lattices. The velocity limits for the 30 thermal groups were the same as those quoted in the THERMOS manual.<sup>12</sup> The cross-section library used was based on BNL-325 and the supplement.<sup>15</sup> Since the upper velocity limit of Group 27 corresponds to an energy of 0.415 eV, which is almost exactly the same as the lower energy limit of 0.414 eV in GAM-I, edits were obtained for both 27 and 30 groups. (The group number in THERMOS increases with increasing velocity.) The upper energy limit of Group 30 is 0.785 eV. It is assumed that, above this limit, the flux per unit energy interval,  $\phi(E)$ , has a  $1/E$  dependence.

In addition to the 30-group problems, THERMOS one-group calculations were done for all lattices using cross sections from the 27-group edits of the Nelkin-kernel multigroup problems. These one-group cross sections were also used in the three-region B692/RP collision probability code.<sup>16,17</sup> This code assumes that neutrons incident on the boundary of the unit cell from the inside are returned isotropically. Average thermal fluxes are computed in B692/RP in the framework of the flat-flux approximation.

The thermal-diffusion coefficient is not calculated in the version of THERMOS used here. Thus a scheme was devised for its calculation. First the value of  $(1 - \mu) \sigma_S^H$  for hydrogen in  $H_2O$  according to the Nelkin kernel was calculated from

$$\frac{(1 - \mu^H) \sigma_S^H}{(1 - \mu^H) \sigma_S^H} = \frac{\sum_{i=1}^{27} (\sigma_{S0i}^H - \sigma_{S1i}^H) V_i N(V_i) \Delta V_i}{\sum_{i=1}^{27} V_i N(V_i) \Delta V_i}, \quad (1)$$

where  $V_i$  is the velocity of group  $i$ , which has a velocity interval of  $\Delta V_i$ ,  $N(V_i)$  is the neutron number density of group  $i$ , and  $\sigma_{S0i}^H$  and  $\sigma_{S1i}^H$  are the  $P_0$  and  $P_1$  components of the hydrogen scattering cross section. Although the  $P_1$  component is not used in THERMOS itself, it is calculated by the GAKER code, which is one of the THERMOS family of codes, and appears in the THERMOS library. The neutron number densities used in Eq. 1 were those for the outermost mesh interval in  $H_2O$  in the cell. A spatial average of  $(1 - \mu^H) \sigma_S^H$  would differ little from the outer mesh-interval value. Values of  $(1 - \mu) \sigma_S$  for all other materials were calculated simply as

$$(1 - \mu) \sigma_S = \left(1 - \frac{2}{3A}\right) \sigma_S,$$

where  $A$  is the mass number and  $\sigma_S$  is the constant, "high-energy," scattering cross section used in the free-gas kernel. The macroscopic values of  $(1 - \mu) \Sigma_S$  were calculated for each cell region by summing the products of the microscopic  $(1 - \mu) \sigma_S$  and the corresponding material number densities. Next the macroscopic values of  $(1 - \mu) \Sigma_S$  for each cell region were flux-weighted using the 27-group edits of the Nelkin-kernel problems to yield the value of  $(1 - \mu) \Sigma_S$  for the cell. Then  $\Sigma_{tr}$  for the cell was calculated as  $\Sigma_{tr} = \Sigma_a + (1 - \mu) \Sigma_S$ , where  $\Sigma_a$  is the absorption cross section of the 27-group edits. The diffusion constant was finally calculated from  $D = 1/(3\Sigma_{tr})$ , and the thermal-diffusion area was computed as  $L^2 = D/\Sigma_a$ .

A THERMOS Nelkin-kernel problem with one mesh interval was run for  $H_2O$  in order to yield values of  $\Sigma_a$ ,  $D$ , and  $L^2$  to be used in the radial reflector savings calculations described in Section VI.

Table III presents flux and neutron-number-density ratios for all the lattices based on the 27-group edits of the Nelkin-kernel problems. Average velocities are in dimensionless units based on 2200 m/sec. Flux ratios from the Nelkin and B692/RP one-group problems are given.

Subscripts 1, 2, and 3 in Table III denote fuel, cladding, and water regions, respectively. Reference 17 contains some of the flux ratios.

TABLE III. Thermal-flux and Neutron-number-density Ratios and Average Velocities

|                             |                    | Lattice |         |          |          |         |         |         |         |          |          |          |          |          |
|-----------------------------|--------------------|---------|---------|----------|----------|---------|---------|---------|---------|----------|----------|----------|----------|----------|
| Quantity <sup>a</sup>       | Description        | B1.27□S | B1.27△S | H1.349□A | H1.349□S | H1.24□A | H1.24□S | H1.27△A | H1.27△S | H1.166△A | H1.166△S | H1.127△A | H1.127△S | H1.069△S |
| $\bar{\phi}_2/\bar{\phi}_1$ | Nelkin             | 1.1150  | 1.1124  | 1.0910   | 1.0837   | 1.0894  | 1.0819  | 1.0880  | 1.0807  | 1.0853   | 1.0781   | 1.0831   | 1.0772   | 1.0730   |
| $\bar{\phi}_3/\bar{\phi}_1$ | 27-group           | 1.2535  | 1.2299  | 1.1737   | 1.1905   | 1.1572  | 1.1709  | 1.1486  | 1.1615  | 1.1286   | 1.1375   | 1.1271   | 1.1367   | 1.1167   |
| $\bar{n}_2/\bar{n}_1$       | Nelkin             | 1.1590  | 1.1565  | 1.1242   | 1.1143   | 1.1230  | 1.1129  | 1.1218  | 1.1116  | 1.1179   | 1.1077   | 1.1147   | 1.1057   | 1.0986   |
| $\bar{n}_3/\bar{n}_1$       | 27-group           | 1.3554  | 1.3235  | 1.2411   | 1.2648   | 1.2184  | 1.2381  | 1.2063  | 1.2246  | 1.1774   | 1.1903   | 1.1729   | 1.1858   | 1.1570   |
| $\bar{v}_1$                 | Nelkin<br>27-group | 1.5900  | 1.6693  | 1.4922   | 1.5135   | 1.5788  | 1.6067  | 1.6507  | 1.6747  | 1.8120   | 1.8519   | 1.9151   | 1.9374   | 2.1446   |
| $\bar{v}_2$                 |                    | 1.5295  | 1.6057  | 1.4481   | 1.4719   | 1.5314  | 1.5619  | 1.6009  | 1.6281  | 1.7593   | 1.8024   | 1.8607   | 1.8874   | 2.0946   |
| $\bar{v}_3$                 |                    | 1.4703  | 1.5514  | 1.4112   | 1.4246   | 1.4995  | 1.5195  | 1.5719  | 1.5886  | 1.7371   | 1.7697   | 1.8403   | 1.8572   | 2.0699   |
| $\bar{\phi}_2/\bar{\phi}_1$ | Nelkin             | 1.1125  | 1.1105  | 1.0903   | 1.0829   | 1.0890  | 1.0815  | 1.0879  | 1.0807  | 1.0865   | 1.0789   | 1.0848   | 1.0785   | 1.0767   |
| $\bar{\phi}_3/\bar{\phi}_1$ | 1-group            | 1.2462  | 1.2249  | 1.1696   | 1.1861   | 1.1551  | 1.1687  | 1.1477  | 1.1607  | 1.1299   | 1.1391   | 1.1299   | 1.1395   | 1.1233   |
| $\bar{\phi}_2/\bar{\phi}_1$ | B692/RP            | 1.1173  | 1.1142  | 1.0924   | 1.0859   | 1.0902  | 1.0836  | 1.0887  | 1.0824  | 1.0870   | 1.0801   | 1.0858   | 1.0798   | 1.0786   |
| $\bar{\phi}_3/\bar{\phi}_1$ | 1-group            | 1.2394  | 1.2203  | 1.1641   | 1.1802   | 1.1497  | 1.1634  | 1.1413  | 1.1544  | 1.1299   | 1.1397   | 1.1260   | 1.1363   | 1.1334   |

<sup>a</sup>Subscripts 1, 2, and 3 denote fuel, cladding, and water regions, respectively.

<sup>b</sup>Average velocities are in dimensionless units based on 2200 m/sec.

The thermal-flux ratios in Table III calculated by Nelkin 27-group, Nelkin one-group, or B692/RP one-group methods are all quite close to each other. Agreement between the Nelkin and B692/RP one-group calculations suggests that the use of the extra scattering region mocks up the isotropic return boundary condition well, and that the flat flux approximation is adequate for such closely packed lattices. Inspection of the velocities indicates the progressive hardening of the neutron spectrum as the H-to- $U^{238}$  atom ratio is decreased.

Table IV lists thermal parameters for all the lattices, based on the Nelkin-kernel problems with 27-group edits. Notation used in the table is standard. The values of  $D$ ,  $\bar{\Sigma}_a$ , and  $\nu\bar{\Sigma}_f$  are those used in the buckling and reflector savings calculations of Sections V and VI.

TABLE IV. Thermal Parameters Based on Nelkin-kernel THERMOS Problems

|                                        | Lattice |         |          |          |         |         |         |         |          |          |          |          |          |
|----------------------------------------|---------|---------|----------|----------|---------|---------|---------|---------|----------|----------|----------|----------|----------|
| Parameter                              | B1.27□S | B1.27△S | H1.349□A | H1.349□S | H1.24□A | H1.24□S | H1.27△A | H1.27△S | H1.166△A | H1.166△S | H1.127△A | H1.127△S | H1.069△S |
| $k^{25}$                               | 0.82539 | 0.83604 | 0.81884  | 0.74013  | 0.83605 | 0.75436 | 0.84465 | 0.76139 | 0.85818  | 0.77247  | 0.86278  | 0.77641  | 0.78231  |
| $k^{fuel}$                             | 0.89089 | 0.90259 | 0.92613  | 0.83724  | 0.94602 | 0.85373 | 0.95606 | 0.86196 | 0.97198  | 0.87506  | 0.97749  | 0.87966  | 0.88664  |
| $k^{clad}$                             | 0.05849 | 0.05930 | 0.00605  | 0.10009  | 0.00620 | 0.10229 | 0.00627 | 0.10341 | 0.00638  | 0.10511  | 0.00642  | 0.10559  | 0.10603  |
| $k^{H_2O}$                             | 0.05062 | 0.03811 | 0.06782  | 0.06267  | 0.04778 | 0.04398 | 0.03767 | 0.03463 | 0.02164  | 0.01983  | 0.01609  | 0.01475  | 0.00733  |
| $\eta^{25}$                            | 2.0620  | 2.0593  | 2.0647   | 2.0642   | 2.0619  | 2.0611  | 2.0596  | 2.0588  | 2.0542   | 2.0528   | 2.0506   | 2.0499   | 2.0430   |
| $\eta^{fuel}$                          | 1.9103  | 1.9075  | 1.8256   | 1.8248   | 1.8223  | 1.8212  | 1.8196  | 1.8186  | 1.8137   | 1.8121   | 1.8100   | 1.8093   | 1.8026   |
| $\bar{\Sigma}_a$ , cm <sup>-1</sup>    | 0.18099 | 0.20076 | 0.12813  | 0.13857  | 0.14254 | 0.15429 | 0.14970 | 0.16288 | 0.16152  | 0.17513  | 0.16328  | 0.17917  | 0.18040  |
| $\bar{\Sigma}_f$ , cm <sup>-1</sup>    | 0.30803 | 0.34565 | 0.21663  | 0.21170  | 0.24572 | 0.23990 | 0.26042 | 0.25532 | 0.28473  | 0.27771  | 0.28888  | 0.28517  | 0.28834  |
| $\bar{\Sigma}_S$ , cm <sup>-1</sup>    | 1.7966  | 1.5948  | 1.6424   | 1.7126   | 1.3753  | 1.4570  | 1.2061  | 1.2991  | 0.88265  | 0.99114  | 0.74730  | 0.86749  | 0.66384  |
| $\bar{\Sigma}_{tr}$ , cm <sup>-1</sup> | 1.5654  | 1.4259  | 1.3965   | 1.4727   | 1.1998  | 1.2887  | 1.0757  | 1.1776  | 0.84625  | 0.96414  | 0.74992  | 0.88216  | 0.74699  |
| D, cm                                  | 0.2129  | 0.2338  | 0.2387   | 0.2263   | 0.2778  | 0.2587  | 0.3099  | 0.2831  | 0.3939   | 0.3457   | 0.4445   | 0.3779   | 0.4462   |
| L <sup>2</sup> , cm <sup>2</sup>       | 1.176   | 1.165   | 1.863    | 1.633    | 1.949   | 1.677   | 2.070   | 1.738   | 2.439    | 1.974    | 2.722    | 2.109    | 2.473    |

Table V compares thermal quantities for three of the lattices, based on the use of Nelkin, free-gas, and Brown-St. John kernels for hydrogen in the THERMOS problems. The three scattering kernels yield close results for most quantities. The Nelkin kernel produces the hardest spectrum of the three kernels; the free-gas kernel gives a scattering cross section that is considerably smaller than that for the other two.

TABLE V. Comparisons of Thermal Quantities Based on Various Scattering Kernels for Hydrogen

| Scattering Kernel <sup>a</sup>     | Lattice |          |                |         |          |                |          |          |                |
|------------------------------------|---------|----------|----------------|---------|----------|----------------|----------|----------|----------------|
|                                    | B1.27□S |          |                | H1.24□A |          |                | H1.127△S |          |                |
|                                    | Nelkin  | Free Gas | Brown-St. John | Nelkin  | Free Gas | Brown-St. John | Nelkin   | Free Gas | Brown-St. John |
| $\bar{\phi}_2/\bar{\phi}_1$        | 1.1150  | 1.1247   | 1.1200         | 1.0894  | 1.0966   | 1.0934         | 1.0772   | 1.0816   | 1.0797         |
| $\bar{\phi}_3/\bar{\phi}_1$        | 1.2535  | 1.2506   | 1.2667         | 1.1572  | 1.1584   | 1.1656         | 1.1367   | 1.1421   | 1.1410         |
| $\bar{n}_2/\bar{n}_1$              | 1.1590  | 1.1752   | 1.1673         | 1.1230  | 1.1358   | 1.1300         | 1.1057   | 1.1161   | 1.1120         |
| $\bar{n}_3/\bar{n}_1$              | 1.3554  | 1.3523   | 1.3759         | 1.2184  | 1.2224   | 1.2324         | 1.1858   | 1.2002   | 1.1971         |
| $\bar{V}_1$                        | 1.5900  | 1.5413   | 1.5342         | 1.5788  | 1.5455   | 1.5170         | 1.9374   | 1.8841   | 1.8855         |
| $\bar{V}_2$                        | 1.5295  | 1.4751   | 1.4718         | 1.5314  | 1.4921   | 1.4680         | 1.8874   | 1.8260   | 1.8308         |
| $\bar{V}_3$                        | 1.4703  | 1.4255   | 1.4124         | 1.4995  | 1.4646   | 1.4350         | 1.8572   | 1.7930   | 1.7972         |
| $\rho^{25}$                        | 0.82539 | 0.82538  | 0.82506        | 0.83605 | 0.83642  | 0.83624        | 0.77641  | 0.77606  | 0.77640        |
| $\rho_{\text{fuel}}$               | 0.89089 | 0.89062  | 0.89024        | 0.94602 | 0.94599  | 0.94570        | 0.87966  | 0.87894  | 0.87931        |
| $\rho_{\text{clad}}$               | 0.05849 | 0.05908  | 0.05863        | 0.00620 | 0.00625  | 0.00620        | 0.10559  | 0.10619  | 0.10585        |
| $\rho_{\text{H}_2\text{O}}$        | 0.05062 | 0.05030  | 0.05113        | 0.04778 | 0.04776  | 0.04810        | 0.01475  | 0.01487  | 0.01484        |
| $\eta^{25}$                        | 2.0620  | 2.0631   | 2.0633         | 2.0619  | 2.0625   | 2.0633         | 2.0499   | 2.0512   | 2.0511         |
| $\eta_{\text{fuel}}$               | 1.9103  | 1.9119   | 1.9122         | 1.8223  | 1.8236   | 1.8246         | 1.8093   | 1.8112   | 1.8110         |
| $\bar{\Sigma}_a, \text{cm}^{-1}$   | 0.18099 | 0.18758  | 0.18723        | 0.14254 | 0.14594  | 0.14847        | 0.17917  | 0.18458  | 0.18446        |
| $\bar{\Sigma}_{f, \text{cm}^{-1}}$ | 0.30803 | 0.31941  | 0.31873        | 0.24572 | 0.25176  | 0.25618        | 0.28517  | 0.29383  | 0.29375        |
| $\bar{\Sigma}_S, \text{cm}^{-1}$   | 1.7966  | 1.3181   | 1.8737         | 1.3753  | 1.0177   | 1.4348         | 0.86749  | 0.73341  | 0.88599        |

<sup>a</sup>Subscripts 1, 2, and 3 denote fuel, cladding, and water regions, respectively.

Table VI lists thermal-flux and neutron-number-density ratios obtained from both 27-group (0-0.415 eV) and 30-group (0-0.785 eV) edits of the Nelkin-kernel problems. Extending the energy range by 0.37 eV does

TABLE VI. Thermal-flux and Neutron-number-density Ratios for 27-group and 30-group Edits

| Lattice Groups <sup>a</sup> | B1.27□S  |        | B1.27△S  |        | H1.349□A |        | H1.349□S |        | H1.24□A  |        |
|-----------------------------|----------|--------|----------|--------|----------|--------|----------|--------|----------|--------|
|                             | 27       | 30     | 27       | 30     | 27       | 30     | 27       | 30     | 27       | 30     |
| $\bar{\phi}_2/\bar{\phi}_1$ | 1.1150   | 1.1022 | 1.1124   | 1.0980 | 1.0910   | 1.0832 | 1.0837   | 1.0763 | 1.0894   | 1.0802 |
| $\bar{\phi}_3/\bar{\phi}_1$ | 1.2535   | 1.2256 | 1.2299   | 1.2008 | 1.1737   | 1.1589 | 1.1905   | 1.1738 | 1.1572   | 1.1413 |
| $\bar{n}_2/\bar{n}_1$       | 1.1590   | 1.1520 | 1.1565   | 1.1478 | 1.1242   | 1.1202 | 1.1143   | 1.1105 | 1.1230   | 1.1178 |
| $\bar{n}_3/\bar{n}_1$       | 1.3554   | 1.3395 | 1.3235   | 1.3055 | 1.2411   | 1.2333 | 1.2648   | 1.2558 | 1.2184   | 1.2093 |
| Lattice Groups <sup>a</sup> | H1.24□S  |        | H1.27△A  |        | H1.27△S  |        | H1.166△A |        | H1.166△S |        |
|                             | 27       | 30     | 27       | 30     | 27       | 30     | 27       | 30     | 27       | 30     |
| $\bar{\phi}_2/\bar{\phi}_1$ | 1.0819   | 1.0731 | 1.0880   | 1.0775 | 1.0807   | 1.0710 | 1.0853   | 1.0727 | 1.0781   | 1.0662 |
| $\bar{\phi}_3/\bar{\phi}_1$ | 1.1709   | 1.1529 | 1.1486   | 1.1315 | 1.1615   | 1.1428 | 1.1286   | 1.1097 | 1.1375   | 1.1170 |
| $\bar{n}_2/\bar{n}_1$       | 1.1129   | 1.1078 | 1.1218   | 1.1155 | 1.1116   | 1.1057 | 1.1179   | 1.1090 | 1.1077   | 1.0991 |
| $\bar{n}_3/\bar{n}_1$       | 1.2381   | 1.2275 | 1.2063   | 1.1957 | 1.2246   | 1.2127 | 1.1774   | 1.1641 | 1.1903   | 1.1754 |
| Lattice Groups <sup>a</sup> | H1.127△A |        | H1.127△S |        | H1.069△S |        |          |        |          |        |
|                             | 27       | 30     | 27       | 30     | 27       | 30     |          |        |          |        |
| $\bar{\phi}_2/\bar{\phi}_1$ | 1.0831   | 1.0696 | 1.0772   | 1.0637 | 1.0730   | 1.0581 |          |        |          |        |
| $\bar{\phi}_3/\bar{\phi}_1$ | 1.1271   | 1.1072 | 1.1367   | 1.1143 | 1.1167   | 1.0923 |          |        |          |        |
| $\bar{n}_2/\bar{n}_1$       | 1.1147   | 1.1044 | 1.1057   | 1.0953 | 1.0986   | 1.0852 |          |        |          |        |
| $\bar{n}_3/\bar{n}_1$       | 1.1729   | 1.1577 | 1.1858   | 1.1684 | 1.1570   | 1.1353 |          |        |          |        |

<sup>a</sup>Subscripts 1, 2, and 3 denote fuel, cladding, and water regions, respectively.



not lower the flux and number-density ratios very much. This suggests that comparisons of experimental and theoretical disadvantage factors (essentially  $\bar{n}_3/\bar{n}_1$ ) should not be very sensitive to thicknesses of cadmium covers as far as the effect of changing cutoff energy is concerned.

As mentioned above, the slowing-down source term in the THERMOS problems was taken as being due to hydrogen only, and was spatially flat in the  $H_2O$  region. For the H1.069 $\Delta$ S lattice, a B692/RP problem was run with the source density in each region taken proportional to  $\xi\Sigma_S$  of that region. These source densities were 0.01604, 0.01791, and 1.000 in fuel, cladding, and  $H_2O$  respectively. Table VII compares fluxes from this problem with those from the problem with source in  $H_2O$  only. Table VII also gives B692/RP fluxes for the H1.069 $\Delta$ S lattice, obtained by using cross sections for the H1.127 $\Delta$ S lattice.

TABLE VII. B692/RP Problems for the H1.069 $\Delta$ S Lattice

| Flux<br>Ratio <sup>a</sup>                                | A      | B      | C      |
|-----------------------------------------------------------|--------|--------|--------|
| $\bar{\phi}_2/\bar{\phi}_1$                               | 1.0786 | 1.0713 | 1.0874 |
| $\bar{\phi}_3/\bar{\phi}_1$                               | 1.1334 | 1.1194 | 1.1480 |
| A: Standard problem                                       |        |        |        |
| B: Source densities proportional to $\xi\Sigma_S$         |        |        |        |
| C: Cross sections for the H1.127 $\Delta$ S lattice used. |        |        |        |

<sup>a</sup>Subscripts 1, 2, and 3 denote fuel, cladding, and water regions, respectively.

As expected, the use of regional source densities proportional to  $\xi\Sigma_S$  flattens the flux somewhat in comparison with the case in which the source is in  $H_2O$  only. However, the effect for the H1.069 $\Delta$ S lattice is not large and would be smaller for all other lattices. The use of H1.127 $\Delta$ S cross sections in place of the H1.069 $\Delta$ S cross sections increases flux peaking slightly because of the larger absorption cross section for the softer spectrum of the looser lattice. In fact, the flux ratios for the H1.069 $\Delta$ S lattice using the H1.127 $\Delta$ S cross sections are somewhat higher than the corresponding flux ratios for the H1.127 $\Delta$ S lattice, rather than lower, as was the case for the standard problem.

Experimental disadvantage factors were measured in the lattices with stainless steel cladding. The experimental values are ratios of the average subcadmium  $U^{235}$  fission-product activity per unit weight of a

highly enriched foil in the  $H_2O$  moderator to that of a highly enriched foil in the fuel. Reference 1 gives details of the experimental techniques and the experimental results.

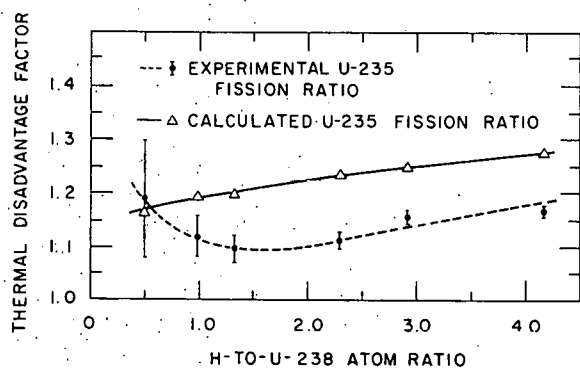
Table VIII presents  $U^{235}$  fission ratios for the stainless steel-clad lattices obtained from 27-group edits of the Nelkin-kernel THERMOS problems. These fission ratios are not greatly different from the corresponding neutron-number-density ratios given in Table III. This is because the fission activation is proportional to  $\int \Sigma_f(V) N(V) V dV$ , where  $\Sigma_f(V)$  is the fission cross section, and  $N(V)$  is the neutron number density at velocity  $V$ . Since  $U^{235}$  is nearly a  $1/V$  absorber in the thermal-energy range, the integral is almost proportional to  $\int N(V) dV$ .

TABLE VIII.  $U^{235}$  Fission Ratios for Lattices with Stainless Steel Cladding

| Fission Ratios <sup>a</sup> | Lattice |         |          |         |         |          |          |          |
|-----------------------------|---------|---------|----------|---------|---------|----------|----------|----------|
|                             | B1.27□S | B1.27△S | H1.349□S | H1.24□S | H1.27△S | H1.166△S | H1.127△S | H1.069△S |
| $F_2/F_1$                   | 1.1655  | 1.1629  | 1.1188   | 1.1174  | 1.1162  | 1.1119   | 1.1097   | 1.1021   |
| $F_3/F_1$                   | 1.3705  | 1.3372  | 1.2755   | 1.2479  | 1.2337  | 1.1979   | 1.1927   | 1.1624   |

<sup>a</sup>Subscripts 1, 2, and 3 denote fuel, cladding, and water regions, respectively.

Figure 1 shows both the experimental disadvantage factors and the calculated values of Table VIII for the Hi-C cores, plotted against the H-to- $U^{238}$  atom ratio. The THERMOS values are larger than experimental values, except for the tightest lattice. In some cases, the calculated value of  $\xi - 1$  (where  $\xi$  is the disadvantage factor) is about twice as large as the measured  $\xi - 1$ . Honeck<sup>18</sup> has previously observed this trend for water-uranium oxide lattices with water-to-fuel volume ratios greater than unity.



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Fig. 1. Experimental and Theoretical Thermal Disadvantage Factors in Hi-C Cores

yield a disadvantage factor that is monotonically decreasing with decreasing lattice pitch. Weiss and Stamm'ler<sup>19</sup> have shown that the disadvantage factor, calculated in a one-group framework, does not decrease monotonically with decreasing moderator thickness, but possesses a minimum and will increase as the moderator thickness decreases beyond a certain value. These considerations of Weiss and Stamm'ler assume the use of cross sections that are independent of lattice pitch. It was pointed out above, in reference to Table VII, that the B692/RP flux ratios for the H1.069△S lattice, calculated

The experimental results indicate a minimum in the disadvantage factor, whereas calculations

using the H1.127 $\Delta$ S cross sections; are higher than the flux ratios for the H1.127 $\Delta$ S lattice. Thus the progressive hardening of the cross sections with decreasing lattice pitch in the THERMOS problems overbalances the tendency, based on spatial-transport-theory effects, for the disadvantage factor to have a minimum.

The use of a unit cell with the isotropic-return boundary condition may not be very good for the extremely closely packed lattices. Hardy *et al.*<sup>20</sup> predicted a minimum for the disadvantage factor in metal lattices using a 36-group Monte Carlo program that treated the cell geometry explicitly. Also, Fukai<sup>21</sup> pointed out that the cylindrical-cell, isotropic-return boundary condition breaks down somewhat for very tightly packed lattices, in that the disadvantage factor does not increase rapidly enough with decreasing pitch.

Another source of calculational error is the assumption of isotropic scattering. Honeck,<sup>18</sup> however, demonstrated that anisotropic scattering effects are rather small.

A significant effect has been discovered<sup>22</sup> in connection with the interpretation of thermal-disadvantage-factor measurements. Three-dimensional Monte Carlo calculations, explicitly including foils, indicate that the flux in a foil is often considerably different from the flux in the medium in which it is placed, even for optically thin foils. The flux peaking or dipping depends strongly on the medium surrounding the foil.

Calculations were carried out<sup>22</sup> using a 32-group, thermal Monte Carlo code, DRAM, both for some lattices at Bettis Atomic Power Laboratory and for the Hi-C lattices. Disadvantage factors were computed both for the foil regions and for unperturbed regions in the rod and moderator. For all lattices, the disadvantage factors for the unperturbed regions were higher than those for the foil regions. Disadvantage factors for foil regions were in better agreement with the experimental values than were those for the unperturbed regions. Thus, the results of these calculations explain the fact that calculated disadvantage factors tend to be higher than measured ones.

Disadvantage factors for a small cell represent a severe test of both calculational and experimental techniques. In the case of the calculations, the isotropic-return boundary condition breaks down somewhat for the very tightly packed lattices. The large error bars in Fig. 1 and the discussion in Ref. 1 demonstrate experimental difficulties. Also, the fact that fluxes in foils do not necessarily represent fluxes in the surrounding unperturbed media very well, introduces complications into a comparison of theory and experiment. Thus the discrepancy between experiment and theory shown in Fig. 1 is not surprising.

#### IV. CALCULATION OF NONTHERMAL PARAMETERS

Parameters for three fast groups with lower boundary energies at 1.35 MeV, 1.23 keV, and 0.414 eV were obtained using the GAM-I code.<sup>11</sup> These group limits are such that most  $U^{238}$  fission occurs in the first group, and most resonance absorption occurs in the third group. The second group is mainly a slowing-down group, although considerable  $U^{238}$  capture does occur in this group.

The GAM-I code calculates fast-neutron spectra and associated multigroup constants using either the  $P_1$  or  $B_1$  approximation. The GAM-I library tape<sup>23</sup> contains data for 68 fine groups, having lethargy widths of  $\Delta u = 0.25$ , ranging from 10 MeV to 0.414 eV. In the work reported here, the cross sections on the original GAM-I tape were used for hydrogen, oxygen, aluminum, iron, nickel, and chromium. For  $U^{235}$  and  $U^{238}$ , the cross sections on the original tape led to calculated bucklings that were much higher than experimental. The cross sections of  $U^{235}$  and  $U^{238}$  were therefore replaced by versions obtained from Hanford, based on the data in Ref. 24. The new  $U^{238}$  cross sections differ from the original in the values of  $\sigma_c$ ,  $\sigma_f$ ,  $\nu$ , and resonance parameters, with transfer cross sections remaining unchanged. For  $U^{235}$ , all cross sections are changed from the original, and resonance parameters are included.

In the GAM-I code, it is assumed that the materials involved form a homogeneous mixture, except for the materials with resonance parameters. For such materials ( $U^{238}$  and  $U^{235}$ , in our case), the methods of Adler, Hinman, and Nordheim<sup>25</sup> are used. Resonance integrals for individual resonances are calculated using either the narrow resonance (NR) or narrow resonance-infinite mass absorber (NRIMA) approximation, depending on whether the average energy loss in a collision with the absorber atom is larger or smaller than the practical width.<sup>25</sup> The escape probability for the fuel rod is computed using the formula for an isolated rod, the mean chord length,  $\bar{l}$ , being replaced by  $\bar{l}/(1 - C)$ , where  $C$  is the Dancoff factor. Dancoff factors for the GAM-I input were determined from the table<sup>26</sup> calculated by Carlvik. The cladding, void, and  $H_2O$  cross sections were homogenized to yield the moderator cross section used in looking up the Dancoff factors in the table. Since the table involves two parallel rods only, an approximate method was used for determining the effect of shielding by rods intervening between pairs of rods. The contribution for two rods partly shielded by intervening rods was determined by multiplying the  $C$  value for the two unshielded rods by  $d_{\perp}/d$ . Here  $d$  is the rod diameter, while  $d_{\perp}$  is defined as follows: Consider a plane cutting the rods perpendicular to the rod axes. Draw a line in the plane joining the axes of the rods for which the Dancoff factor is being calculated. Then draw two more lines in the plane parallel to the first line and tangent to the intervening rods on the sides nearest the first line. The perpendicular distance between these two lines is  $d_{\perp}$ . Table IX lists the scattering cross

sections used in the Dancoff-factor determinations. Table X presents the Dancoff factors, calculated as just described, along with values computed by revised versions of the B692/RP collision-probability codes.<sup>16,17</sup> The two-region version evaluates  $C$  from Eq. 17 of Ref. 17; the three-region version uses the three-region extension of Eq. 17, the clad and void regions being homogenized into one region.

TABLE IX. Scattering Cross Sections Used in Dancoff-factor Calculations

| Material | $\sigma_p$ , barns | Material         | $\sigma_p$ , barns |
|----------|--------------------|------------------|--------------------|
| Hydrogen | 20.0               | Nickel           | 17.0               |
| Oxygen   | 3.66               | Chromium         | 4.2                |
| Aluminum | 1.42               | U <sup>235</sup> | 10.7               |
| Iron     | 11.0               | U <sup>238</sup> | 10.7               |

TABLE X. Dancoff Factors

| Core Code | Ref. 26 | B692/RP<br>2-region | B692/RP<br>3-region |
|-----------|---------|---------------------|---------------------|
| B1.27□S   | 0.250   | 0.22934             | 0.23418             |
| B1.27△S   | 0.307   | 0.29903             | 0.30413             |
| H1.349□A  | 0.272   | 0.25211             | 0.26688             |
| H1.349□S  | 0.250   | 0.22812             | 0.23518             |
| H1.24□A   | 0.365   | 0.35370             | 0.36910             |
| H1.24□S   | 0.331   | 0.31674             | 0.32405             |
| H1.27△A   | 0.425   | 0.42608             | 0.44094             |
| H1.27△S   | 0.383   | 0.37908             | 0.38610             |
| H1.166△A  | 0.576   | 0.58655             | 0.59809             |
| H1.166△S  | 0.513   | 0.51512             | 0.52049             |
| H1.127△A  | 0.652   | 0.66358             | 0.67280             |
| H1.127△S  | 0.577   | 0.57939             | 0.58365             |
| H1.069△S  | 0.690   | 0.69195             | 0.69407             |

Three-region values of  $C$  from the B692/RP code are a little larger than two-region ones, the largest difference being less than 6%. Differences between three- and two-region values are greater for aluminum-clad fuel than for stainless steel-clad fuel, since the cross section of aluminum differs from that of water by more than the stainless steel cross section does. Comparison of the values based on Carlvik's table and the B692/RP values shows that the Carlvik values tend to be a little lower than those from B692/RP for tight lattices, and a little higher for looser lattices. At any rate, the use of a Dancoff factor as in the GAM-I code implies that the cladding is treated in the narrow-resonance approximation. This may not be very well justified for broad low-lying resonances, especially in the case of stainless steel cladding.



Table XI gives resonance integrals from the GAM-I output for  $U^{238}$  absorption and for  $U^{235}$  absorption and fission. These values include only contributions from resolved and unresolved resonances and not the contribution from the smoothly varying part of the cross sections. The decrease in the resonance integrals with decreasing lattice pitch is apparent in Table XI, as is the fact that resonance integrals for stainless steel-clad rods are higher than for aluminum-clad rods, especially for the tighter lattices. All the  $U^{235}$  resonance integrals are greater than 80% of the corresponding infinite-dilution values, so that  $U^{235}$  is not strongly self-shielded even in the tightest lattices.

TABLE XI. Resonance Integrals from GAM-I Output

| Core Code | Resonance Integrals, barns |                         |                      |
|-----------|----------------------------|-------------------------|----------------------|
|           | $U^{238}$<br>Absorption    | $U^{235}$<br>Absorption | $U^{235}$<br>Fission |
| B1.27□S   | 19.104                     | 271.48                  | 168.96               |
| B1.27△S   | 18.253                     | 269.27                  | 167.68               |
| H1.349□A  | 17.841                     | 282.65                  | 175.51               |
| H1.349□S  | 18.158                     | 283.29                  | 175.88               |
| H1.24□A   | 16.686                     | 279.65                  | 173.80               |
| H1.24□S   | 17.187                     | 280.81                  | 174.46               |
| H1.27△A   | 15.793                     | 277.42                  | 172.52               |
| H1.27△S   | 16.418                     | 279.01                  | 173.43               |
| H1.166△A  | 14.660                     | 270.85                  | 168.79               |
| H1.166△S  | 15.645                     | 274.14                  | 170.65               |
| H1.127△A  | 13.297                     | 265.99                  | 166.04               |
| H1.127△S  | 14.644                     | 270.79                  | 168.75               |
| H1.069△S  | 12.730                     | 263.10                  | 164.40               |
| Infinite  |                            |                         |                      |
| Dilution  | 273.80                     | 319.70                  | 197.22               |
| Smooth    | 5.167                      | 92.85                   | 78.33                |

GAM-I problems were run for all lattices in the  $P_1$  approximation and for most lattices in the  $B_1$  approximation. The input bucklings used were the experimental values of Table XII of Ref. 1 for lattices that went critical without an external driver zone. Estimated values were used for lattices requiring a driver zone to attain criticality. No  $B_1$  problems were run for the two most tightly packed stainless steel-clad Hi-C lattices because the buckling,  $B^2$ , is negative for these lattices, and the  $B_1$  approximation requires  $\sqrt{B^2}$  in the input. GAM-I problems were also run for water, using  $B^2 = 0.0001 \text{ cm}^{-2}$  in order to obtain parameters for the radial-reflector-savings calculations.

Table XII lists constants from the GAM-I output for the three fast groups in the  $P_1$  approximation. The diffusion equation for a group may be written as

$$D_i \nabla^2 \phi_i - \Sigma_{Ri} \phi_i + \sum_{j < i} \Sigma_{ji} \phi_j + \Sigma_{ii}^{n,2n} \phi_i + \chi_i \sum_j \nu_j \Sigma_{fj} \phi_j = 0. \quad (2)$$

Here  $\Sigma_{Ri}$  equals  $\Sigma_{ci} + \Sigma_{fi} + \Sigma_{out\ i}$ , where  $\Sigma_{ci}$  and  $\Sigma_{fi}$  are the capture and fission cross sections in group  $i$ ; and  $\Sigma_{out\ i}$ , which equals

$$\sum_{j > i} \left( \Sigma_{ij}^{el} + \Sigma_{ij}^{inel} + \Sigma_{ij}^{n,2n} \right),$$

is the total removal cross section from group  $i$  by elastic, inelastic, and  $(n,2n)$  processes. The transfer cross section  $\Sigma_{ij}$  is given by the sum

$$\Sigma_{ij}^{el} + \Sigma_{ij}^{inel} + 2\Sigma_{i,j}^{n,2n},$$

and is the total cross section for transfer of neutrons to group  $j$  from processes in group  $i$ . Other notation used in Eq. 2 is self-explanatory. The group-energy limits used here are such that  $\chi_1 = 0.57309$ ,  $\chi_2 = 0.42691$ , and  $\chi_3 = 0$ . The  $(n,2n)$  reaction occurs only in group 1, so that  $\Sigma_{ii}^{n,2n}$  equals zero for  $i \neq 1$ .

Table XII shows that scattering other than to the next lower group is quite small, and that the  $(n,2n)$  reaction contributes little. Capture and fission processes in group 3 and fast fission in group 1 are all quite important.

Table XIII compares  $P_1$  and  $B_1$  constants for two lattices. The constants from the two approximations are almost the same, except for the diffusion constants. This is because the diffusion constants involve current weighting with the fine-group currents in GAM-I, whereas the other quantities are computed using flux weighting. The differences between  $P_1$  and  $B_1$  constants are greater for the B1.27□S lattice than for the H1.127△A lattice since the input  $B^2$  values were 0.010734 and 0.0010  $\text{cm}^{-2}$ , respectively. In the limit of zero buckling, the  $P_1$  and  $B_1$  equations become identical.

TABLE XII. Constants from GAM-I for Three Fast Groups in the  $P_1$  Approximation

| Parameter                            | Lattice                  |                          |                          |                          |                          |                          |                          |                          |                          |                          |                          |                          |                          |                          |
|--------------------------------------|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|
|                                      | B1.27□S                  | B1.27ΔS                  | H1.349□A                 | H1.349□S                 | H1.24□A                  | H1.24□S                  | H1.27ΔA                  | H1.27ΔS                  | H1.166ΔA                 | H1.166ΔS                 | H1.127ΔA                 | H1.127ΔS                 | H1.069ΔS                 | Water                    |
| D1, cm                               | 2.0344                   | 2.0137                   | 2.1606                   | 2.0555                   | 2.1574                   | 2.0329                   | 2.1560                   | 2.0195                   | 2.1545                   | 1.9918                   | 2.1544                   | 1.9816                   | 1.9650                   | 2.2472                   |
| D2, cm                               | 0.99514                  | 0.98236                  | 1.0400                   | 1.0011                   | 1.0327                   | 0.98756                  | 1.0284                   | 0.97895                  | 1.0213                   | 0.96468                  | 1.0182                   | 0.95582                  | 0.94040                  | 1.0949                   |
| D3, cm                               | 0.54907                  | 0.54839                  | 0.62441                  | 0.55454                  | 0.64206                  | 0.55603                  | 0.65600                  | 0.55814                  | 0.69110                  | 0.56509                  | 0.70964                  | 0.56859                  | 0.57373                  | 0.59511                  |
| $\Sigma R1, \text{cm}^{-1}$          | 0.095708                 | 0.093294                 | 0.089762                 | 0.092912                 | 0.085808                 | 0.089600                 | 0.083185                 | 0.087374                 | 0.077776                 | 0.082802                 | 0.075318                 | 0.080653                 | 0.076920                 | 0.10706                  |
| $\Sigma R2, \text{cm}^{-1}$          | 0.061156                 | 0.055170                 | 0.057226                 | 0.058104                 | 0.049412                 | 0.050223                 | 0.044338                 | 0.045158                 | 0.033911                 | 0.034374                 | 0.029183                 | 0.029775                 | 0.021455                 | 0.10422                  |
| $\Sigma R3, \text{cm}^{-1}$          | 0.11529                  | 0.10632                  | 0.10465                  | 0.10642                  | 0.092159                 | 0.094145                 | 0.083691                 | 0.085887                 | 0.066173                 | 0.068647                 | 0.057406                 | 0.060371                 | 0.045569                 | 0.17149                  |
| $\nu_1 \Sigma f_1, \text{cm}^{-1}$   | 0.012514                 | 0.014418                 | 0.012409                 | 0.012404                 | 0.014650                 | 0.014629                 | 0.016097                 | 0.016064                 | 0.019018                 | 0.018939                 | 0.020320                 | 0.020225                 | 0.022341                 | 0                        |
| $\nu_2 \Sigma f_2, \text{cm}^{-1}$   | 0.0022992                | 0.0026493                | 0.0015432                | 0.0015415                | 0.0018155                | 0.0018126                | 0.0019891                | 0.0019854                | 0.0023356                | 0.0023278                | 0.0024884                | 0.0024792                | 0.0027329                | 0                        |
| $\nu_3 \Sigma f_3, \text{cm}^{-1}$   | 0.026415                 | 0.029738                 | 0.017296                 | 0.017343                 | 0.019910                 | 0.019975                 | 0.021438                 | 0.021523                 | 0.023805                 | 0.023908                 | 0.024420                 | 0.024574                 | 0.024322                 | 0                        |
| $\Sigma^{n,2n}_{11}, \text{cm}^{-1}$ | $0.62516 \times 10^{-4}$ | $0.70911 \times 10^{-4}$ | $0.64271 \times 10^{-4}$ | $0.65222 \times 10^{-4}$ | $0.74340 \times 10^{-4}$ | $0.74821 \times 10^{-4}$ | $0.80425 \times 10^{-4}$ | $0.80622 \times 10^{-4}$ | $0.91118 \times 10^{-4}$ | $0.89893 \times 10^{-4}$ | $0.95037 \times 10^{-4}$ | $0.93674 \times 10^{-4}$ | $0.97840 \times 10^{-4}$ | 0                        |
| $\Sigma_{12}, \text{cm}^{-1}$        | 0.090169                 | 0.087023                 | 0.084260                 | 0.087421                 | 0.079445                 | 0.083256                 | 0.076264                 | 0.080477                 | 0.069725                 | 0.074970                 | 0.066759                 | 0.072139                 | 0.067572                 | 0.10617                  |
| $\Sigma_{13}, \text{cm}^{-1}$        | $0.53238 \times 10^{-4}$ | $0.46942 \times 10^{-4}$ | $0.49377 \times 10^{-4}$ | $0.49635 \times 10^{-4}$ | $0.41200 \times 10^{-4}$ | $0.41542 \times 10^{-4}$ | $0.35832 \times 10^{-4}$ | $0.36210 \times 10^{-4}$ | $0.24812 \times 10^{-4}$ | $0.25251 \times 10^{-4}$ | $0.19816 \times 10^{-4}$ | $0.20237 \times 10^{-4}$ | $0.11632 \times 10^{-4}$ | $0.90140 \times 10^{-4}$ |
| $\Sigma_{14}, \text{cm}^{-1}$        | $0.17830 \times 10^{-7}$ | $0.15707 \times 10^{-7}$ | $0.16532 \times 10^{-7}$ | $0.16618 \times 10^{-7}$ | $0.13775 \times 10^{-7}$ | $0.13889 \times 10^{-7}$ | $0.11966 \times 10^{-7}$ | $0.12092 \times 10^{-7}$ | $0.82507 \times 10^{-8}$ | $0.33965 \times 10^{-8}$ | $0.65664 \times 10^{-8}$ | $0.67062 \times 10^{-8}$ | $0.33048 \times 10^{-8}$ | $0.30277 \times 10^{-7}$ |
| $\Sigma_{23}, \text{cm}^{-1}$        | 0.057060                 | 0.050435                 | 0.053469                 | 0.054250                 | 0.044956                 | 0.045652                 | 0.039419                 | 0.040110                 | 0.028018                 | 0.028346                 | 0.022824                 | 0.023257                 | 0.014056                 | 0.10418                  |
| $\Sigma_{24}, \text{cm}^{-1}$        | $0.18712 \times 10^{-4}$ | $0.16472 \times 10^{-4}$ | $0.17538 \times 10^{-4}$ | $0.17757 \times 10^{-4}$ | $0.14665 \times 10^{-4}$ | $0.14849 \times 10^{-4}$ | $0.12796 \times 10^{-4}$ | $0.12973 \times 10^{-4}$ | $0.89442 \times 10^{-5}$ | $0.39950 \times 10^{-5}$ | $0.71880 \times 10^{-5}$ | $0.72671 \times 10^{-5}$ | $0.41537 \times 10^{-5}$ | $0.34615 \times 10^{-4}$ |
| $\Sigma_{34}, \text{cm}^{-1}$        | 0.080037                 | 0.067591                 | 0.076403                 | 0.076802                 | 0.060995                 | 0.061223                 | 0.051229                 | 0.051386                 | 0.031821                 | 0.031564                 | 0.023747                 | 0.023463                 | 0.010572                 | 0.17029                  |

TABLE XIII. A Comparison of  $P_1$  and  $B_1$  Constants from GAM-I

| Parameter                            | B1.27□S                  |                          | H1.127ΔA                 |                          |
|--------------------------------------|--------------------------|--------------------------|--------------------------|--------------------------|
|                                      | $B_1$                    | $P_1$                    | $B_1$                    | $P_1$                    |
| D1, cm                               | 1.8904                   | 2.0344                   | 2.1367                   | 2.1544                   |
| D2, cm                               | 0.97309                  | 0.99514                  | 1.0159                   | 1.0182                   |
| D3, cm                               | 0.54655                  | 0.54907                  | 0.70892                  | 0.70964                  |
| $\Sigma R1, \text{cm}^{-1}$          | 0.095523                 | 0.095708                 | 0.075316                 | 0.075318                 |
| $\Sigma R2, \text{cm}^{-1}$          | 0.061179                 | 0.061156                 | 0.029183                 | 0.029183                 |
| $\Sigma R3, \text{cm}^{-1}$          | 0.11530                  | 0.11529                  | 0.057406                 | 0.057406                 |
| $\nu_1 \Sigma f_1, \text{cm}^{-1}$   | 0.012532                 | 0.012514                 | 0.020321                 | 0.020320                 |
| $\nu_2 \Sigma f_2, \text{cm}^{-1}$   | 0.0022993                | 0.0022992                | 0.0024884                | 0.0024884                |
| $\nu_3 \Sigma f_3, \text{cm}^{-1}$   | 0.026415                 | 0.026415                 | 0.024420                 | 0.024420                 |
| $\Sigma^{n,2n}_{11}, \text{cm}^{-1}$ | $0.63838 \times 10^{-4}$ | $0.62516 \times 10^{-4}$ | $0.95059 \times 10^{-4}$ | $0.95037 \times 10^{-4}$ |
| $\Sigma_{12}, \text{cm}^{-1}$        | 0.089974                 | 0.090169                 | 0.066757                 | 0.066759                 |
| $\Sigma_{13}, \text{cm}^{-1}$        | $0.53104 \times 10^{-4}$ | $0.53238 \times 10^{-4}$ | $0.19815 \times 10^{-4}$ | $0.19316 \times 10^{-4}$ |
| $\Sigma_{14}, \text{cm}^{-1}$        | $0.17785 \times 10^{-7}$ | $0.17830 \times 10^{-7}$ | $0.65661 \times 10^{-8}$ | $0.65664 \times 10^{-8}$ |
| $\Sigma_{23}, \text{cm}^{-1}$        | 0.057082                 | 0.057060                 | 0.022825                 | 0.022824                 |
| $\Sigma_{24}, \text{cm}^{-1}$        | $0.18719 \times 10^{-4}$ | $0.18712 \times 10^{-4}$ | $0.71882 \times 10^{-5}$ | $0.71380 \times 10^{-5}$ |
| $\Sigma_{34}, \text{cm}^{-1}$        | 0.080045                 | 0.080037                 | 0.023747                 | 0.023747                 |

## V. CRITICALITY CALCULATIONS

A FORTRAN code, BUCKLE, was written for the CDC-160A computer to calculate critical bucklings and fluxes by fundamental-mode methods. This code uses the set of equations, Eq. 2, with  $\nabla^2 \phi_i$  replaced by  $-B^2 \phi_i$ . The quantity

$$\sum_j \nu_j \Sigma_{fj} \phi_j$$

is set equal to unity, and values of  $\phi_1, \phi_2, \dots$ , are calculated in turn using an input value of  $B^2$ . Then the process is repeated for other values of  $B^2$  until

$$\sum_j \nu_j \Sigma_{fj} \phi_j$$

calculated using the computed fluxes, is equal to unity within an input-convergence criterion.

Calculations using BUCKLE were done for all lattices using the  $P_1$  constants of Table XII for the three fast groups, and the constants of Table IV for the thermal group. Experimental values, where available, were used for the input buckling. Problems were also run using the  $B_1$  constants for the three fast groups. Table XIV gives the results of these calculations. Here  $B_m^2$  is the material buckling from BUCKLE, and  $B_g^2$  is the experimental geometric buckling from Table XII of Ref. 1. The quantity  $k_{eff}$  is defined by

$$k_{eff} = \sum_j \nu_j \Sigma_{fj} \phi_j$$

Figure 2 shows the calculated bucklings, based on the  $P_1$  approximation in GAM-I, along with the experimental bucklings.

The calculated bucklings are higher than the experimental ones, except for the H1.349A lattice in the  $P_1$  approximation. The average values of  $k_{eff}$  for the lattices that went critical without a driver zone are 1.0044 and 1.0107 for the  $P_1$  and  $B_1$  cases, respectively.

TABLE XIV. Bucklings and Fluxes from Fundamental-mode Calculations

| Core Code | GAM-I<br>Option | $B_m^2$ , cm <sup>-2</sup> | $B_g^2$ , cm <sup>-2</sup> | $k_{eff}(B_g^2)$ | $\phi_1$ | $\phi_2$ | $\phi_3$ | $\phi_4$ |
|-----------|-----------------|----------------------------|----------------------------|------------------|----------|----------|----------|----------|
| B1.27□S   | P <sub>1</sub>  | 0.010873                   | 0.010734                   | 1.0038           | 4.8664   | 12.0277  | 5.6619   | 2.4734   |
|           | B <sub>1</sub>  | 0.011270                   |                            | 1.0143           | 4.9081   | 12.0382  | 5.6597   | 2.4716   |
| B1.27△S   | P <sub>1</sub>  | 0.009301                   | 0.009147                   | 1.0045           | 5.1191   | 13.5659  | 6.1429   | 2.0471   |
|           | B <sub>1</sub>  | 0.009589                   |                            | 1.0127           | 5.1514   | 13.5730  | 6.1401   | 2.0457   |
| H1.349□A  | P <sub>1</sub>  | 0.008994                   | 0.009182                   | 0.9940           | 5.2514   | 13.0579  | 6.3343   | 3.7166   |
|           | B <sub>1</sub>  | 0.009303                   |                            | 1.0038           | 5.2951   | 13.0692  | 6.3322   | 3.7137   |
| H1.349□S  | P <sub>1</sub>  | 0.006635                   | -                          | -                | 5.3819   | 13.8602  | 6.8319   | 3.7477   |
|           | B <sub>1</sub>  | 0.006792                   |                            | -                | 5.4019   | 13.8654  | 6.8308   | 3.7464   |
| H1.24□A   | P <sub>1</sub>  | 0.007111                   | 0.007076                   | 1.0013           | 5.6699   | 15.4584  | 7.1872   | 3.0351   |
|           | B <sub>1</sub>  | 0.007302                   |                            | 1.0079           | 5.7007   | 15.4655  | 7.1844   | 3.0331   |
| H1.24□S   | P <sub>1</sub>  | 0.004968                   | 0.004747                   | 1.0076           | 5.7526   | 16.4314  | 7.7432   | 3.0487   |
|           | B <sub>1</sub>  | 0.005052                   |                            | 1.0104           | 5.7645   | 16.4343  | 7.7420   | 3.0479   |
| H1.27△A   | P <sub>1</sub>  | 0.005777                   | 0.005538                   | 1.0094           | 5.9972   | 17.5876  | 7.9275   | 2.6823   |
|           | B <sub>1</sub>  | 0.005899                   |                            | 1.0140           | 6.0191   | 17.5922  | 7.9248   | 2.6809   |
| H1.27△S   | P <sub>1</sub>  | 0.003764                   | -                          | -                | 6.0392   | 18.6911  | 8.5230   | 2.6729   |
|           | B <sub>1</sub>  | 0.003807                   |                            | -                | 6.0457   | 18.6923  | 8.5221   | 2.6725   |
| H1.166△A  | P <sub>1</sub>  | 0.002649                   | 0.002436                   | 1.0105           | 6.8723   | 24.7451  | 10.1977  | 1.9975   |
|           | B <sub>1</sub>  | 0.002674                   |                            | 1.0117           | 6.8780   | 24.7454  | 10.1962  | 1.9972   |
| H1.166△S  | P <sub>1</sub>  | 0.000752                   | -                          | -                | 6.8054   | 26.6636  | 10.9447  | 1.9710   |
|           | B <sub>1</sub>  | 0.000756                   |                            | -                | 6.8063   | 26.6630  | 10.9446  | 1.9710   |
| H1.127△A  | P <sub>1</sub>  | 0.001380                   | -                          | -                | 7.3288   | 29.9514  | 11.7110  | 1.6982   |
|           | B <sub>1</sub>  | 0.001386                   |                            | -                | 7.3301   | 29.9508  | 11.7106  | 1.6981   |
| H1.127△S  | P <sub>1</sub>  | -0.000480                  | -                          | -                | 7.1989   | 32.2765  | 12.4929  | 1.6390   |
| H1.069△S  | P <sub>1</sub>  | -0.002773                  | -                          | -                | 8.0295   | 51.4392  | 16.4428  | 0.97145  |

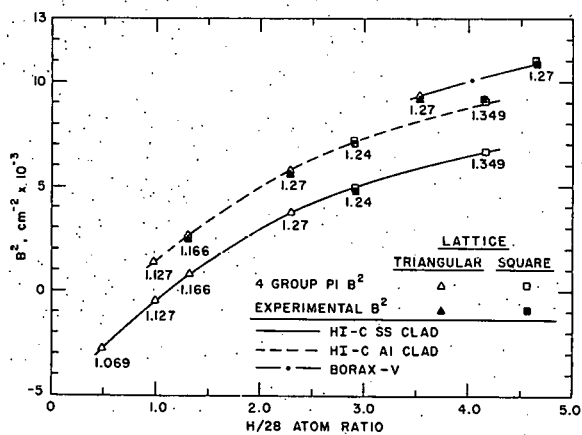
Fig. 2  
Theoretical and Experimental Bucklings

Table XV gives the parameters involved in the two-group criticality equation used in the analysis of Brookhaven light-water-moderated lattices.<sup>27</sup> This equation is

$$\frac{(\eta f)_1(1 - p_1)}{1 + \tau B^2} + \frac{(\eta f)_2 p_1}{(1 + \tau B^2)(1 + L^2 B^2)} = 1, \quad (3)$$

where the subscripts 1 and 2 designate fast and thermal groups, respectively, and the definitions used are



$$(\eta f)_1 = \frac{\nu_1 \Sigma_{f1}}{\Sigma'_{a1}}, \quad (\eta f)_2 = \frac{\nu_2 \Sigma_{f2}}{\Sigma_{a2}}, \quad p_1 = \frac{\Sigma_{12}}{\Sigma'_{a1} + \Sigma_{12}},$$

$$\Sigma'_{a1} = \Sigma_{a1} - \Sigma_{11}^{n,2n}, \quad \tau = \frac{D_1}{\Sigma'_{a1} + \Sigma_{12}}, \quad \text{and} \quad L^2 = \frac{D_2}{\Sigma_{a2}}.$$

These definitions differ slightly from those used in the Brookhaven work in that the  $(n, 2n)$  reaction was not considered there. The fast-group constants involved in Table XV are based on the one-group edits of the GAM-I problems; the thermal constants are based on Table IV. Values of  $B^2$  in Table XV are slightly different from the  $B_m^2$  in Table XIV, since two-group and four-group bucklings would agree only if the exact theoretical critical buckling was used in the GAM-I input.

TABLE XV. Two-group Parameters

| Parameter             | B1.27□S  | B1.27△S  | H1.349□A | H1.349□S | H1.24□A  | H1.24□S  | H1.27△A  | H1.27△S  |
|-----------------------|----------|----------|----------|----------|----------|----------|----------|----------|
| $p_1$                 | 0.62246  | 0.55482  | 0.65399  | 0.64865  | 0.57211  | 0.56485  | 0.51405  | 0.50566  |
| $(\eta f)_1$          | 0.86548  | 0.87682  | 0.76138  | 0.72618  | 0.77482  | 0.73432  | 0.78359  | 0.73886  |
| $(\eta f)_2$          | 1.7019   | 1.7217   | 1.6907   | 1.5278   | 1.7239   | 1.5549   | 1.7396   | 1.5675   |
| $\tau, \text{cm}^2$   | 34.266   | 36.026   | 39.066   | 35.460   | 42.785   | 38.311   | 45.706   | 40.469   |
| $L^2, \text{cm}^2$    | 1.1763   | 1.1646   | 1.8630   | 1.6331   | 1.9489   | 1.6767   | 2.0701   | 1.7381   |
| $B^2, \text{cm}^{-2}$ | 0.010879 | 0.009309 | 0.008983 | 0.006641 | 0.007113 | 0.004974 | 0.005787 | 0.003775 |
| $\lambda_1^a$         | 0.2380   | 0.2923   | 0.1950   | 0.2065   | 0.2542   | 0.2684   | 0.3012   | 0.3169   |
| $\lambda_2^a$         | 0.7620   | 0.7077   | 0.8050   | 0.7935   | 0.7458   | 0.7316   | 0.6988   | 0.6831   |

| Parameter             | H1.166△A | H1.166△S | H1.127△A | H1.127△S  | H1.069△S  |
|-----------------------|----------|----------|----------|-----------|-----------|
| $p_1$                 | 0.37130  | 0.35721  | 0.30155  | 0.28620   | 0.14425   |
| $(\eta f)_1$          | 0.78282  | 0.73204  | 0.78684  | 0.72781   | 0.70016   |
| $(\eta f)_2$          | 1.7628   | 1.5857   | 1.7692   | 1.5916    | 1.5983    |
| $\tau, \text{cm}^2$   | 53.729   | 47.396   | 58.787   | 50.932    | 61.316    |
| $L^2, \text{cm}^2$    | 2.4387   | 1.9740   | 2.7223   | 2.1092    | 2.4734    |
| $B^2, \text{cm}^{-2}$ | 0.002652 | 0.000763 | 0.001379 | -0.000481 | -0.002751 |
| $\lambda_1^a$         | 0.4308   | 0.4541   | 0.5084   | 0.5326    | 0.7207    |
| $\lambda_2^a$         | 0.5692   | 0.5459   | 0.4916   | 0.4674    | 0.2793    |

<sup>a</sup> $\lambda_1$  and  $\lambda_2$  are the first and second terms of the left member of Eq. 3, representing fast and thermal fissions, respectively.

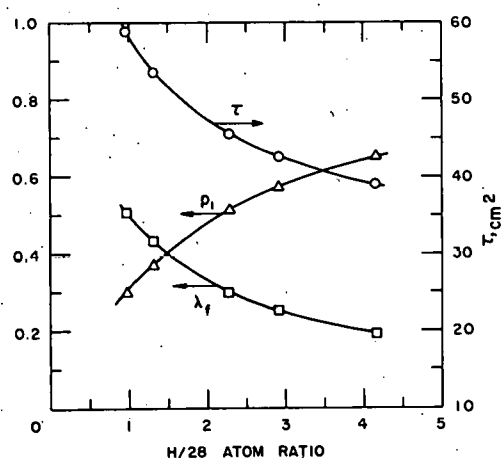


Table XV shows clearly the increasing importance of nonthermal events as the water-to-fuel ratio is decreased, the parameters  $p_1$  and  $\lambda_1$  being especially significant. Figure 3 shows the variation of  $p_1$ ,  $\lambda_1$ , and  $\tau$  with H-to- $\text{U}^{238}$  ratio for the aluminum-clad Hi-C lattices.

Fig. 3. Two-group Parameters for Hi-C Aluminum-clad Lattices

## VI. RADIAL REFLECTOR SAVINGS

Radial reflector savings were calculated for the seven lattices that went critical without an external driver zone. The multigroup, one-dimensional, diffusion-theory code, RP-122 (REX), was used. Both four-group and two-group computations were carried out using both  $P_1$  and  $B_1$  nonthermal constants from GAM-I and thermal constants from THERMOS. To determine criticality, the REX code iterates on the axial buckling,  $B_Z^2$ . The critical  $B_Z^2$  from the REX output was subtracted from the BUCKLE value of  $B_m^2$  to obtain the radial buckling,  $B_R^2$ . Hence the radial reflector savings,  $\lambda_R$ , were found from

$$\lambda_R = \frac{2.4048}{B_R} - R.$$

The core radii used in the REX problems were essentially those of the experiments. A convergence criterion was used such that  $k_{eff}$  for the converged  $B_Z^2$  differs from unity by less than 0.0001. Table XVI compares the calculated values of  $\lambda_R$  with experimental results from Table XII of Ref. 1.

TABLE XVI. Radial Reflector Savings

| Core Code | Reflector Savings, cm |       |           |       |                  |
|-----------|-----------------------|-------|-----------|-------|------------------|
|           | Four-group            |       | Two-group |       | Experimental     |
|           | $P_1$                 | $B_1$ | $P_1$     | $B_1$ |                  |
| B1.27□S   | 7.21                  | 6.95  | 7.30      | 7.04  | $7.35 \pm 0.27$  |
| B1.27△S   | 7.57                  | 7.35  | 7.63      | 7.40  | $7.55 \pm 0.10$  |
| H1.24□S   | 7.76                  | 7.64  | 7.69      | 7.58  | $7.69 \pm 0.08$  |
| H1.349□A  | 7.73                  | 7.48  | 7.81      | 7.56  | $7.38 \pm 0.12$  |
| H1.24□A   | 8.45                  | 8.25  | 8.46      | 8.25  | $8.13 \pm 0.16$  |
| H1.27△A   | 9.02                  | 8.85  | 8.94      | 8.76  | $8.77 \pm 0.17$  |
| H1.166△A  | 10.86                 | 10.72 | 10.32     | 10.23 | $10.73 \pm 0.14$ |

Values of  $\lambda_R$  using  $P_1$  constants are always higher than the corresponding  $B_1$  values. In some cases, a four-group calculation yields a larger  $\lambda_R$  than a two-group calculation; in other cases, the reverse situation is true. The four-group values of  $\lambda_R$  involving  $P_1$  constants agree with experimental values within the quoted experimental error in four cases out of seven.

No axial reflector savings were calculated because of the complicated structure of the bottom reflector, which consists of Lucite and water. Also,  $\lambda_R$  is of more interest than  $\lambda_Z$  because the radial buckling contributes considerably more to the experimental  $B_g^2$  than does the axial buckling.

## VII. MICROPARAMETER CALCULATIONS

A. Initial and Modified Conversion Ratios and  $\alpha^{25}$ 

The initial conversion ratio (ICR) is defined as the ratio of the captures in  $U^{238}$  to absorptions in  $U^{235}$  in a fresh core and so represents the ratio of  $Pu^{239}$  production to  $U^{235}$  destruction. The modified conversion ratio (MCR) is the ratio of  $U^{238}$  captures to  $U^{235}$  fissions and is more closely related to what was actually measured in the experiments.<sup>1</sup> The parameter  $\alpha^{25}$  is the ratio of captures to fissions in  $U^{235}$ . Thus the relation  $ICR = MCR/(1 + \alpha^{25})$  is true.

Table XVII presents values of ICR, MCR, and  $\alpha^{25}$  calculated using the  $P_1$  cross sections from GAM-I and the thermal cross sections from the THERMOS Nelkin-kernel problems. Use of the  $B_1$  parameters from GAM-I would make little difference in the results. In Table XVII,  $C_f^{28}$  and  $C_t^{28}$  are the fast and thermal captures in  $U^{238}$ , respectively. The term "fast" refers to processes in the three GAM-I groups; "thermal" refers to those in the THERMOS groups. Likewise,  $A_f^{25}$ ,  $A_t^{25}$ ,  $C_f^{25}$ ,  $C_t^{25}$ ,  $F_f^{25}$ ,  $F_t^{25}$ ,  $\alpha_f^{25}$ , and  $\alpha_t^{25}$  refer to absorptions, captures, fissions, and  $\alpha$  in fast and thermal-energy ranges for  $U^{235}$ .

TABLE XVII. Initial and Modified Conversion Ratios and  $\alpha^{25}$ 

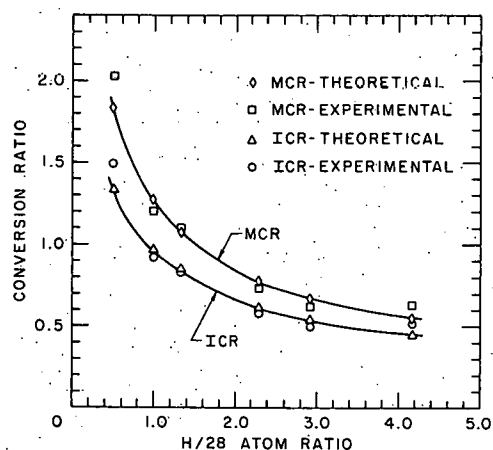
| Parameter       | Lattice  |          |          |          |          |          |          |          |          |          |          |          |          |
|-----------------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|
|                 | B1.27□S  | B1.27△S  | H1.349□A | H1.349□S | H1.24□A  | H1.24□S  | H1.27△A  | H1.27△S  | H1.166△A | H1.166△S | H1.127△A | H1.127△S | H1.069△S |
| $C_f^{28}$      | 0.13402  | 0.16156  | 0.14687  | 0.15991  | 0.18728  | 0.20521  | 0.21829  | 0.24008  | 0.31390  | 0.35113  | 0.36685  | 0.41474  | 0.60034  |
| $C_t^{28}$      | 0.02932  | 0.02735  | 0.05109  | 0.05043  | 0.04758  | 0.04674  | 0.04474  | 0.04378  | 0.03672  | 0.03541  | 0.03181  | 0.03032  | 0.01828  |
| $A_f^{25}$      | 0.10846  | 0.13367  | 0.07883  | 0.08502  | 0.10388  | 0.11201  | 0.12427  | 0.13383  | 0.18176  | 0.19592  | 0.21762  | 0.23394  | 0.32499  |
| $A_t^{25}$      | 0.36949  | 0.34360  | 0.38994  | 0.38436  | 0.36169  | 0.35484  | 0.33916  | 0.33148  | 0.27688  | 0.26664  | 0.23923  | 0.22800  | 0.13710  |
| $C_f^{25}$      | 0.034240 | 0.042183 | 0.025105 | 0.027141 | 0.033056 | 0.035746 | 0.039524 | 0.042725 | 0.057680 | 0.062519 | 0.068898 | 0.074568 | 0.10288  |
| $C_t^{25}$      | 0.055956 | 0.052415 | 0.058619 | 0.057860 | 0.054790 | 0.053868 | 0.051698 | 0.050637 | 0.042819 | 0.041391 | 0.037351 | 0.035664 | 0.021835 |
| $F_f^{25}$      | 0.074215 | 0.091493 | 0.053724 | 0.057880 | 0.070826 | 0.076253 | 0.084744 | 0.091106 | 0.12409  | 0.13340  | 0.14872  | 0.15938  | 0.22211  |
| $F_t^{25}$      | 0.31353  | 0.29118  | 0.33132  | 0.32650  | 0.30691  | 0.30097  | 0.28746  | 0.28084  | 0.23406  | 0.22525  | 0.20188  | 0.19234  | 0.11527  |
| $\alpha_f^{25}$ | 0.4614   | 0.4611   | 0.4673   | 0.4689   | 0.4667   | 0.4689   | 0.4664   | 0.4690   | 0.4648   | 0.4687   | 0.4633   | 0.4679   | 0.4632   |
| $\alpha_t^{25}$ | 0.1785   | 0.1800   | 0.1769   | 0.1772   | 0.1785   | 0.1790   | 0.1798   | 0.1803   | 0.1829   | 0.1838   | 0.1850   | 0.1854   | 0.1894   |
| ICR             | 0.3418   | 0.3958   | 0.4223   | 0.4481   | 0.5045   | 0.5397   | 0.5676   | 0.6100   | 0.7645   | 0.8357   | 0.8726   | 0.9635   | 1.3387   |
| MCR             | 0.4213   | 0.4937   | 0.5141   | 0.5472   | 0.6218   | 0.6679   | 0.7067   | 0.7632   | 0.9790   | 1.0778   | 1.1371   | 1.2654   | 1.8336   |
| $\alpha^{25}$   | 0.2326   | 0.2472   | 0.2174   | 0.2211   | 0.2326   | 0.2376   | 0.2451   | 0.2510   | 0.2806   | 0.2897   | 0.3031   | 0.3134   | 0.3697   |

Table XVIII compares experimental and calculated conversion ratios for all lattices for which experiments were performed. The experimental values are from Table XIX of Ref. 1. Values of  $\alpha^{25}$  used in deriving the experimental ICR from the experimental MCR are slightly different from the values in Table XVII. The conversion ratios in Table XVIII for the Hi-C cores are also plotted against the H-to- $U^{238}$  atom ratio in Fig. 4.

Agreement between theory and experiment in the case of the conversion ratios is reasonably good. The initial conversion ratio is greater than unity only for the tightest lattice, which has a negative buckling.

TABLE XVIII. Calculated and Experimental Conversion Ratios

| Lattice  | Calculated |        | Experimental      |                   |
|----------|------------|--------|-------------------|-------------------|
|          | MCR        | ICR    | MCR               | ICR               |
| B1.27□S  | 0.4213     | 0.3418 | $0.367 \pm 0.018$ | $0.297 \pm 0.015$ |
| B1.27△S  | 0.4937     | 0.3958 | $0.413 \pm 0.021$ | $0.330 \pm 0.016$ |
| H1.349□S | 0.5472     | 0.4481 | $0.635 \pm 0.007$ | $0.518 \pm 0.006$ |
| H1.24□S  | 0.6679     | 0.5397 | $0.622 \pm 0.006$ | $0.500 \pm 0.005$ |
| H1.27△S  | 0.7632     | 0.6100 | $0.730 \pm 0.021$ | $0.581 \pm 0.017$ |
| H1.166△S | 1.0778     | 0.8357 | $1.082 \pm 0.006$ | $0.835 \pm 0.005$ |
| H1.127△S | 1.2654     | 0.9635 | $1.203 \pm 0.025$ | $0.913 \pm 0.019$ |
| H1.069△S | 1.8336     | 1.3387 | $2.023 \pm 0.047$ | $1.485 \pm 0.034$ |



112-6731

Fig. 4. Conversion Ratios for Hi-C Stainless Steel-clad Cores

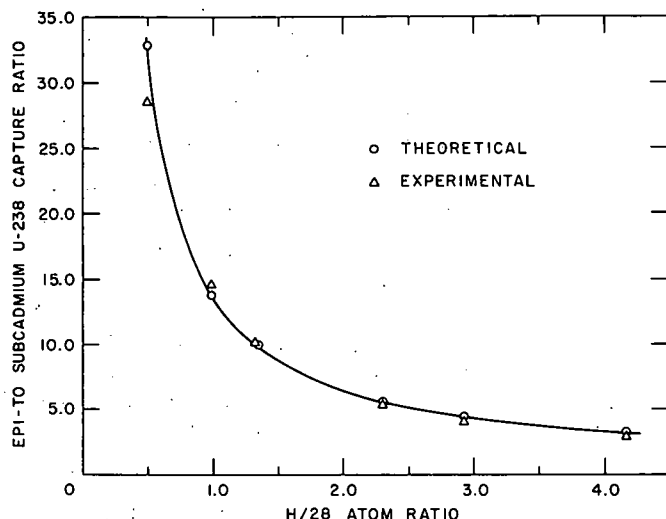
B.  $U^{238}$  Capture Cadmium Ratios

Experimental  $U^{238}$  capture cadmium ratios were measured for the lattices with stainless steel cladding. Values were also calculated for all the lattices on the assumption that captures in the three GAM-I groups represent epicadmium capture, and captures in the THERMOS group represent subcadmium capture. Thus no consideration was given as to whether the experimental cadmium cutoff energies were in agreement with the 0.414-eV lower limit of the GAM-I calculations. Table XIX presents both calculated and experimental values.

TABLE XIX. Calculated and Experimental  $U^{238}$  Capture Cadmium Ratios

| Lattice  | Calculated |             | Experimental      |                 |
|----------|------------|-------------|-------------------|-----------------|
|          | $CR^{28}$  | $\rho^{28}$ | $CR^{28}$         | $\rho^{28}$     |
| B1.27□S  | 1.2188     | 4.571       | $1.182 \pm 0.041$ | $5.49 \pm 1.24$ |
| B1.27△S  | 1.1693     | 5.907       | $1.188 \pm 0.083$ | $5.32 \pm 2.35$ |
| H1.349□A | 1.3479     | 2.875       | -                 | -               |
| H1.349□S | 1.3154     | 3.171       | $1.328 \pm 0.004$ | $3.05 \pm 0.04$ |
| H1.24□A  | 1.2541     | 3.936       | -                 | -               |
| H1.24□S  | 1.2278     | 4.390       | $1.243 \pm 0.002$ | $4.12 \pm 0.03$ |
| H1.27△A  | 1.2050     | 4.879       | -                 | -               |
| H1.27△S  | 1.1824     | 5.484       | $1.185 \pm 0.003$ | $5.40 \pm 0.09$ |
| H1.166△A | 1.1170     | 8.548       | -                 | -               |
| H1.166△S | 1.1008     | 9.916       | $1.100 \pm 0.001$ | $10.0 \pm 0.1$  |
| H1.127△A | 1.0867     | 11.53       | -                 | -               |
| H1.127△S | 1.0731     | 13.68       | $1.069 \pm 0.002$ | $14.5 \pm 0.4$  |
| H1.069△S | 1.0304     | 32.84       | $1.035 \pm 0.001$ | $28.6 \pm 0.8$  |

The experimental values are from Table XX of Ref. 1. In Table XIX,  $CR^{28}$  is the  $U^{238}$  capture cadmium ratio, while  $\rho^{28}$  is the epicadmium-to-subcadmium capture ratio. Thus,  $\rho^{28} = 1/(CR^{28} - 1)$ . Figure 5 is a plot of experimental and theoretical values of  $\rho^{28}$  versus the H-to- $U^{238}$  atom ratio for the Hi-C stainless steel-clad cores. Satisfactory agreement exists between theory and experiment.



112-6732

Fig. 5.  $U^{238}$  Epicadmium to Subcadmium Capture Ratios for Hi-C Stainless Steel-clad Cores

of  $U^{235}$  fission cadmium ratio versus H-to- $U^{238}$  atom ratio falls on essentially a straight line, as suggested in Ref. 7, although the slopes of the theoretical and experimental lines are somewhat different.

### C. $U^{235}$ Fission Cadmium Ratios

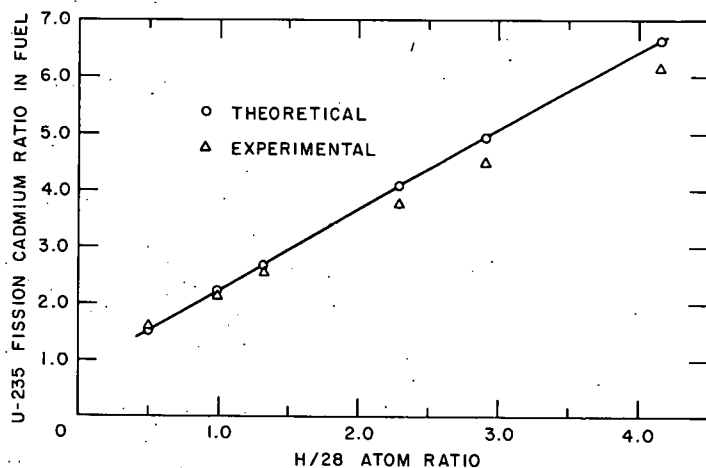
Fission cadmium ratios,  $CR_f^{25}$ , were measured for the stainless steel-clad lattices. As for the  $U^{238}$  capture cadmium ratios,  $U^{235}$  fission cadmium ratios were calculated on the assumption that the GAM-I groups and the THERMOS group correspond to epicadmium and subcadmium energy ranges, respectively. Table XX presents experimental values from Table XXIII of Ref. 1 along with calculated values. Figure 6 shows the data in graphical form for the Hi-C stainless steel-clad cores. The data in the plot

TABLE XX. Calculated and Experimental  $U^{235}$  Fission Cadmium Ratios

| Lattice  | Calculated $CR_f^{25}$ | Experimental $CR_f^{25}$ | Lattice  | Calculated $CR_f^{25}$ | Experimental $CR_f^{25}$ |
|----------|------------------------|--------------------------|----------|------------------------|--------------------------|
| B1.27□S  | 5.225                  | $4.94 \pm 0.20$          | H1.27△A  | 4.392                  | -                        |
| B1.27△S  | 4.183                  | $3.90 \pm 0.16$          | H1.27△S  | 4.083                  | $3.75 \pm 0.08$          |
| H1.349□A | 7.167                  | -                        | H1.166△A | 2.886                  | -                        |
| H1.349□S | 6.641                  | $6.13 \pm 0.20$          | H1.166△S | 2.689                  | $2.55 \pm 0.06$          |
| H1.24□A  | 5.333                  | -                        | H1.127△A | 2.357                  | -                        |
| H1.24□S  | 4.947                  | $4.49 \pm 0.11$          | H1.127△S | 2.207                  | $2.15 \pm 0.05$          |
|          |                        |                          | H1.069△S | 1.519                  | $1.59 \pm 0.03$          |

### D. Gold and Indium Cadmium Ratios

Both gold and indium cadmium ratios were measured in almost all the lattices. Calculated values were obtained using the B512/RP foil-activation program.<sup>28</sup>



112-8006

Fig. 6.  $U^{235}$  Fission Cadmium Ratios for Hi-C Stainless Steel-clad Cores

covers. The flux has the energy dependence of a Maxwellian plus a  $1/E$  tail. More specifically, the energy dependence is given by

$$\phi(X) = \frac{2r}{\sqrt{\pi}} X e^{-X} + \frac{X_0^{1/2}}{X} \Delta(X), \quad (4)$$

where  $r$  is a constant,  $X = E/E_M$ ,  $X_0 = E_{2200}/E_M$ , and  $\Delta(X)$  is zero for  $X < \mu$  and unity for  $X \geq \mu$ . Here,  $E$  is the neutron energy,  $E_M$  is the energy at the peak of the Maxwellian, and  $E_{2200}$  is the energy corresponding to a neutron velocity of 2200 m/sec. In most of the calculations, the parameter  $\mu$  was taken to be 5. Scattering by the foil and covers is neglected, so that their cross sections represent absorption only. The energy dependence of the cross sections is a sum of single-level Breit-Wigner resonances without Doppler broadening and a  $1/V$  component. For cadmium and indium, the cross sections were considered to be given completely by the resonances at 0.178 and 1.457 eV, respectively. For gold, a  $1/V$  component yielding a cross section of 5.05 barns at 0.0253 eV was added to the cross section produced by the 4.906 eV resonance. The resonance parameters used are from Ref. 15.

Values of  $E_M$  were determined, using output from the THERMOS calculations, from the maxima in the plots of the neutron number density per unit velocity,  $N(V)$ , versus the velocity. Plots using the  $N(V)$  at the fuel center and at the outer mesh interval in  $H_2O$  were assumed to be adequate to represent the fuel and water regions, respectively. The values of  $r$  were found by equating the ratio  $\phi_4/\phi_3$  to the ratio of the integral of the flux of Eq. 4 over the energy range of the THERMOS group to the corresponding integral over the energy range of the third GAM-I group. Here  $\phi_4$  and  $\phi_3$  are the fluxes determined by the fundamental-mode calculations,  $\phi_4$  being adjusted to yield the average flux in fuel or  $H_2O$ , rather than in the unit cell.

The B512/RP program calculates both the neutron absorption in a bare foil of thickness  $t$  and the absorption in the same foil between two identical covers (usually cadmium) of thickness  $l_0$  in slab geometry. The cadmium cut-off energy,  $E_{cut}$ , is also computed. Flux depression in the foil and covers, resulting from neutron absorption, is considered. The flux is assumed to impinge isotropically on the outer surface of the bare foil or on the outer surface of the

The experimental-foil thicknesses were 1.1, 1.2, and 20 mils for gold, indium, and cadmium, respectively. Tables XXI and XXII present results of the calculations for gold and indium, respectively, along with experimental values from Table XXV of Ref. 1.

TABLE XXI. Gold Cadmium Ratios

| Lattice  | Region            | $E_M$ , eV | $r$    | $E_{cut}$ , eV | CR         |               |
|----------|-------------------|------------|--------|----------------|------------|---------------|
|          |                   |            |        |                | Calculated | Experimental  |
| B1.27□S  | Fuel <sup>a</sup> | 0.0415     | 1.625  | 0.5309         | 1.378      | 1.413 ± 0.025 |
| B1.27□S  | H <sub>2</sub> O  | 0.0323     | 2.250  | 0.5869         | 1.525      | 1.520 ± 0.045 |
| B1.27△S  | Fuel <sup>a</sup> | 0.0434     | 1.170  | 0.5323         | 1.286      | 1.288 ± 0.030 |
| B1.27△S  | H <sub>2</sub> O  | 0.0352     | 1.553  | 0.5873         | 1.382      | 1.354 ± 0.040 |
| H1.349□A | Fuel              | 0.0370     | 2.537  | 0.5863         | 1.572      | 1.62 ± 0.02   |
| H1.349□S | Fuel              | 0.0377     | 2.305  | 0.5864         | 1.525      | 1.58 ± 0.02   |
| H1.24□A  | Fuel              | 0.0402     | 1.693  | 0.5866         | 1.401      | 1.455 ± 0.008 |
| H1.24□A  | Fuel <sup>b</sup> | 0.0402     | 1.334  | 0.5870         | 1.373      | 1.455 ± 0.008 |
| H1.24□A  | Fuel <sup>c</sup> | 0.0402     | 2.181  | 0.6085         | 1.454      | 1.455 ± 0.008 |
| H1.24□A  | Fuel <sup>d</sup> | 0.0402     | 1.693  | 0.7298         | 1.139      | -             |
| H1.24□A  | H <sub>2</sub> O  | 0.0346     | 2.085  | 0.5869         | 1.488      | 1.503 ± 0.015 |
| H1.24□S  | Fuel              | 0.0408     | 1.534  | 0.5867         | 1.369      | 1.435 ± 0.008 |
| H1.24□S  | H <sub>2</sub> O  | 0.0346     | 1.917  | 0.5870         | 1.455      | 1.442 ± 0.008 |
| H1.27△A  | Fuel              | 0.0421     | 1.277  | 0.5868         | 1.316      | 1.361 ± 0.015 |
| H1.27△S  | Fuel              | 0.0428     | 1.146  | 0.5869         | 1.289      | 1.346 ± 0.015 |
| H1.166△A | Fuel              | 0.0489     | 0.618  | 0.5869         | 1.175      | 1.221 ± 0.008 |
| H1.166△S | Fuel              | 0.0496     | 0.546  | 0.5870         | 1.160      | 1.201 ± 0.005 |
| H1.127△A | Fuel              | 0.0517     | 0.399  | 0.5872         | 1.128      | 1.177 ± 0.010 |
| H1.127△S | Fuel              | 0.0525     | 0.340  | 0.5873         | 1.115      | 1.152 ± 0.015 |
| H1.069△S | Fuel              | 0.0600     | 0.0793 | 0.5888         | 1.055      | 1.099 ± 0.008 |

<sup>a</sup>15-mil, rather than 20 mil, cadmium covers.

<sup>b</sup> $\mu = 3$ , rather than 5.

<sup>c</sup> $\mu = 10$ , rather than 5.

<sup>d</sup>Foil thickness,  $t = 0$ .

TABLE XXII. Indium Cadmium Ratios

| Lattice  | Region            | $E_M$ , eV | $r$    | $E_{cut}$ , eV | CR         |               |
|----------|-------------------|------------|--------|----------------|------------|---------------|
|          |                   |            |        |                | Calculated | Experimental  |
| B1.27□S  | Fuel <sup>a</sup> | 0.0415     | 1.625  | 0.6757         | 1.416      | 1.470 ± 0.020 |
| B1.27□S  | H <sub>2</sub> O  | 0.0323     | 2.250  | 0.7623         | 1.570      | 1.558 ± 0.040 |
| B1.27△S  | Fuel <sup>a</sup> | 0.0434     | 1.170  | 0.6764         | 1.326      | 1.382 ± 0.030 |
| B1.27△S  | H <sub>2</sub> O  | 0.0352     | 1.553  | 0.7625         | 1.433      | 1.412 ± 0.040 |
| H1.349□A | Fuel              | 0.0370     | 2.537  | 0.7619         | 1.619      | 1.61 ± 0.03   |
| H1.349□S | Fuel              | 0.0377     | 2.305  | 0.7620         | 1.574      | 1.58 ± 0.03   |
| H1.24□A  | Fuel              | 0.0402     | 1.693  | 0.7620         | 1.453      | 1.482 ± 0.015 |
| H1.24□A  | Fuel <sup>b</sup> | 0.0402     | 1.334  | 0.7623         | 1.430      | 1.482 ± 0.015 |
| H1.24□A  | Fuel <sup>c</sup> | 0.0402     | 2.181  | 0.7764         | 1.493      | 1.482 ± 0.015 |
| H1.24□A  | Fuel <sup>d</sup> | 0.0402     | 1.693  | 1.203          | 1.222      | -             |
| H1.24□A  | H <sub>2</sub> O  | 0.0346     | 2.085  | 0.7623         | 1.536      | 1.518 ± 0.020 |
| H1.24□S  | Fuel              | 0.0408     | 1.534  | 0.7621         | 1.422      | 1.469 ± 0.008 |
| H1.24□S  | H <sub>2</sub> O  | 0.0346     | 1.917  | 0.7624         | 1.503      | 1.483 ± 0.020 |
| H1.27△A  | Fuel              | 0.0421     | 1.277  | 0.7621         | 1.369      | 1.398 ± 0.015 |
| H1.27△S  | Fuel              | 0.0428     | 1.146  | 0.7621         | 1.343      | 1.373 ± 0.007 |
| H1.166△A | Fuel              | 0.0489     | 0.618  | 0.7620         | 1.231      | 1.260 ± 0.008 |
| H1.166△S | Fuel              | 0.0496     | 0.546  | 0.7621         | 1.216      | 1.234 ± 0.005 |
| H1.127△A | Fuel              | 0.0517     | 0.399  | 0.7621         | 1.184      | 1.204 ± 0.020 |
| H1.127△S | Fuel              | 0.0525     | 0.340  | 0.7622         | 1.171      | -             |
| H1.069△S | Fuel              | 0.0600     | 0.0793 | 0.7632         | 1.108      | 1.135 ± 0.008 |

<sup>a</sup>15-mil, rather than 20-mil, cadmium covers.

<sup>b</sup> $\mu = 3$ , rather than 5.

<sup>c</sup> $\mu = 10$ , rather than 5.

<sup>d</sup>Foil thickness,  $t = 0$ .

The calculated gold cadmium ratios are usually somewhat lower than the experimental ones, the discrepancy becoming greater as the lattice pitch is decreased. Calculated values of CR-1 are about 10% too low for the looser Hi-C lattices and 45% too low for the tightest lattice. Agreement between theory and experiment is a little better for indium, although the trend of calculated values being too low for tightly packed lattices continues. The value of CR-1 for indium in the tightest lattice is only about 20% too low. The effect of flux depression in the gold or indium foils is quite important, even for the thin foils used here, since the cross sections of the dominant resonances are very large. This effect is seen by comparing the calculated cadmium ratios of the H1.24□A lattice for the actual value of  $t = 20$  mils and for  $t = 0$ .

The calculations described here have a number of deficiencies. The treatment of the energy dependence of the flux as a Maxwellian plus a  $1/E$  tail is not very accurate. There is some departure from Maxwellian behavior, especially for the tighter lattices. The high absorption causes the tail to depart from the  $1/E$  form. Also, the cutoff of  $\mu = 5$  is not strongly justified. An idea of the effect of changing  $\mu$  is obtained by comparing the cadmium ratios for  $\mu = 3, 5$ , and  $10$  for the H1.24□A lattice in Tables XXI and XXII. The consideration of the flux at the center of the fuel and at the outer mesh interval in  $H_2O$  as being representative of fuel and  $H_2O$  average values is also approximate.

Other calculational deficiencies include the use of only one resonance in representing the cross sections, the neglect of Doppler broadening, and the neglect of scattering by the foils. In view of the approximate nature of the calculations, the discrepancies between theory and experiment are not surprising.

#### E. $U^{238}$ -to- $U^{235}$ Fission Ratios

The ratio of the number of fissions in  $U^{238}$  to those in  $U^{235}$  can be calculated from the GAM-I and THERMOS cross sections, along with the fluxes from BUCKLE, as

$$\delta^{28} = \frac{N^{28} \sum_{i=1}^4 \sigma_{fi}^{28} \phi_i}{N^{25} \sum_{i=1}^4 \sigma_{fi}^{25} \phi_i} \quad (5)$$

Most of the contribution to the numerator in Eq. 5 comes from Group 1, none coming from below Group 2.



An approximate formula for  $\delta^{28}$  may be derived as follows. The diffusion equation for Group 1 is

$$D_1 \nabla^2 \phi_1 - \Sigma_{R1} \phi_1 + \Sigma_{11}^{n,2n} \phi_1 + \chi_1 \nu_1 \Sigma_{f1} \phi_1 + \chi_1 \sum_{i=2}^4 \nu_i \Sigma_{fi} \phi_i = 0, \quad (6)$$

where the notation is as in Eq. 2. On using  $\nabla^2 \phi_1 = -B^2 \phi_1$ , we can rewrite Eq. 6 as

$$\phi_1 = \frac{\chi_1 \sum_{i=2}^4 \nu_i \Sigma_{fi} \phi_i}{\Sigma_{R1} - \Sigma_{11}^{n,2n} - \chi_1 \nu_1 \Sigma_{f1} + D_1 B^2} \quad (7)$$

Now make the following approximations:

1. All  $U^{238}$  fissions occur in Group 1.
2. All  $U^{235}$  fissions occur below Group 1, except that  $U^{235}$  Group 1 fissions are included in the  $\chi_1 \nu_1 \Sigma_{f1}$  terms in the denominator of Eq. 7.
3. For  $U^{235}$  fission,  $\nu_2 = \nu_3 = \nu_4 = \nu_t = 2.43$ .

These approximations lead to the equation

$$\delta^{28} = \frac{\nu_t \chi_1 \Sigma_{f1}^{28}}{\Sigma_{R1} - \Sigma_{11}^{n,2n} - \chi_1 \nu_1 \Sigma_{f1} + D_1 B^2} \quad (8)$$

Equation 8 has the advantage of involving only Group 1 cross sections. The first of the above three approximations is the poorest, causing the values of  $\delta^{28}$  given by Eq. 8 to be a few percent too low.

Values of  $\delta^{28}$  were calculated for all the lattices using Eq. 5. Also, approximate values were computed from Eq. 8 using three sets of cross sections. These were the Group 1 cross sections from the GAM-I output, and cross sections obtained by averaging the first eight fine groups, having lethargy widths of 0.25, of the GAM-I (Ref. 23) and MUFT-4 (Ref. 29) libraries over a  $U^{235}$  fission spectrum. In the case of the GAM-I library, the versions of  $U^{235}$  and  $U^{238}$  from Hanford were used, as mentioned in Section IV above.

The calculations outlined above were done in the homogeneous approximation; that is, no allowance was made for the spatial variation of the fast flux in the unit cell. This is reasonably well justified in the energy

region of importance for fast fission since the cross sections are relatively small, and the mean free paths are therefore long. To study heterogeneous effects, average fluxes were calculated for the three regions (fuel, cladding, and water) of the unit cell with the B692/RP collision-probability code.<sup>16,17</sup> Both GAM-I and MUFT-4 fission-spectrum-averaged cross sections were used. The transport approximation was used for elastic scattering. Average regional fluxes from B692/RP were combined with the cross sections for the individual cell regions to yield homogenized constants for Eq. 8.

Table XXIII presents values of  $\delta^{28}$ , calculated by all the methods described above, along with the experimental values measured for lattices with stainless steel cladding. The experimental values are from Table XXI of Ref. 1. Figure 7 shows the experimental and calculated four-group values of  $\delta^{28}$  for the Hi-C stainless steel-clad cores. The GAM-I and MUFT-4 cross sections, averaged over a fission spectrum from 1.35 to 10 MeV, are presented in Tables XXIV and XXV, respectively. Table XXVI gives the average fluxes in Group 1, calculated using the GAM-I fission-spectrum cross sections.

TABLE XXIII. U<sup>238</sup>-to-U<sup>235</sup> Fission Ratios

| Method                                                         | Lattice        |                |          |                 |         |                 |         |                 |          |                 |                 |                 |
|----------------------------------------------------------------|----------------|----------------|----------|-----------------|---------|-----------------|---------|-----------------|----------|-----------------|-----------------|-----------------|
|                                                                | HI.27DS        | HI.27AS        | HI.349DA | HI.349DS        | HI.24DA | HI.24DS         | HI.27DA | HI.27AS         | HI.166DA | HI.166AS        | HI.127DA        | HI.127AS        |
| Four-group GAM-I (P <sub>1</sub> , homogeneous)                | 0.05424        | 0.06674        | 0.06136  | 0.06302         | 0.07991 | 0.08121         | 0.09440 | 0.09513         | 0.1334   | 0.1320          | 0.1555          | 0.1524          |
| Four-group GAM-I (P <sub>1</sub> , heterogeneous) <sup>a</sup> | 0.05686        | 0.06908        | 0.06419  | 0.06606         | 0.08230 | 0.08379         | 0.09649 | 0.09742         | 0.1349   | 0.1337          | 0.1566          | 0.1537          |
| One-group GAM-I (P <sub>1</sub> , homogeneous)                 | 0.05132        | 0.06309        | 0.05794  | 0.05945         | 0.07535 | 0.07644         | 0.08892 | 0.08943         | 0.1253   | 0.1234          | 0.1459          | 0.1422          |
| One-group GAM-I (fission spectrum, homogeneous)                | 0.05373        | 0.06630        | 0.06056  | 0.06213         | 0.07909 | 0.08040         | 0.09363 | 0.09447         | 0.1327   | 0.1317          | 0.1551          | 0.1525          |
| One-group GAM-I (fission spectrum, heterogeneous)              | 0.05633        | 0.06862        | 0.06336  | 0.06513         | 0.08146 | 0.08295         | 0.09570 | 0.09674         | 0.1342   | 0.1334          | 0.1562          | 0.1538          |
| One-group MUFT-4 (fission spectrum, homogeneous)               | 0.05564        | 0.06914        | 0.06304  | 0.06489         | 0.08323 | 0.08486         | 0.09942 | 0.1006          | 0.1448   | 0.1437          | 0.1717          | 0.1685          |
| One-group MUFT-4 (fission spectrum, heterogeneous)             | 0.05837        | 0.07159        | 0.06603  | 0.06814         | 0.08579 | 0.08767         | 0.1017  | 0.1031          | 0.1465   | 0.1456          | 0.1731          | 0.1701          |
| Experimental                                                   | 0.0650 ± 0.006 | 0.1190 ± 0.010 | -        | 0.0742 ± 0.0011 | -       | 0.1011 ± 0.0004 | -       | 0.1160 ± 0.0017 | -        | 0.1539 ± 0.0039 | 0.1889 ± 0.0131 | 0.1877 ± 0.0020 |
|                                                                |                |                |          |                 |         |                 |         |                 |          |                 |                 | 0.2622 ± 0.0036 |

<sup>a</sup>Approximate four-group heterogeneous values were calculated by multiplying the four-group homogeneous values by the ratio of the  $\delta^{28}$  value for the one-group, GAM-I, fission-spectrum heterogeneous case to that for the corresponding homogeneous case.

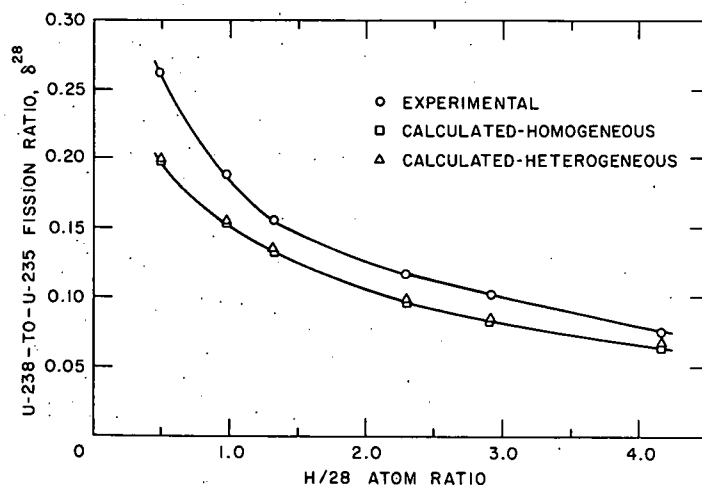


Fig. 7  
U<sup>238</sup>-to-U<sup>235</sup> Fission Ratios  
for Hi-C Stainless Steel-clad Cores

TABLE XXIV. GAM-I Cross Sections Averaged over a Fission Spectrum from 1.35 to 10 MeV

| Type of Cross Section   | Cross Sections, barns |         |          |         |         |          |                  |                  |
|-------------------------|-----------------------|---------|----------|---------|---------|----------|------------------|------------------|
|                         | Hydrogen              | Oxygen  | Aluminum | Iron    | Nickel  | Chromium | U <sup>238</sup> | U <sup>235</sup> |
| $\sigma_c$              | 0                     | 0.0208  | 0.00610  | 0.00166 | 0.00197 | 0.0020   | 0.0355           | 0.0506           |
| $\sigma_f$              | 0                     | 0       | 0        | 0       | 0       | 0        | 0.5149           | 1.276            |
| $\nu\sigma_f$           | 0                     | 0       | 0        | 0       | 0       | 0        | 1.395            | 3.598            |
| $\sigma_{out}^{inel}$   | 0                     | 0.00188 | 0.238    | 0.650   | 0.522   | 0.336    | 2.307            | 1.557            |
| $\sigma_{out}^{el}$     | 1.515                 | 0.221   | 0.118    | 0.043   | 0.062   | 0.062    | 0.019            | 0.021            |
| $\sigma_{out}^{n,2n}$   | 0                     | 0       | 0        | 0       | 0       | 0        | 0.0167           | 0.0075           |
| $\sigma_{II}^{inel}$    | 0                     | 0.00169 | 0.188    | 0.348   | 0.392   | 0.311    | 0.261            | 0.209            |
| $\sigma_{II}^{el}(P_0)$ | 0.9642                | 1.7297  | 2.2903   | 2.1321  | 2.3411  | 2.6850   | 4.684            | 4.391            |
| $\sigma_{II}^{el}(P_1)$ | 2.4809                | 1.1287  | 2.7950   | 2.9436  | 2.6780  | 3.3635   | 8.937            | 4.031            |
| $\sigma_{II}^{el}(tr)$  | 0.1372                | 1.337   | 1.359    | 1.151   | 1.448   | 1.564    | 1.705            | 3.047            |
| $\sigma_{II}^{n,2n}$    | 0                     | 0       | 0        | 0       | 0       | 0        | 0.0082           | 0.0042           |

TABLE XXV. MUFT-4 Cross Sections Averaged over a Fission Spectrum from 1.35 to 10 MeV

| Type of Cross Section   | Cross Sections, barns |        |          |         |        |          |                  |                  |
|-------------------------|-----------------------|--------|----------|---------|--------|----------|------------------|------------------|
|                         | Hydrogen              | Oxygen | Aluminum | Iron    | Nickel | Chromium | U <sup>238</sup> | U <sup>235</sup> |
| $\sigma_c$              | 0.0000328             | 0.0529 | 0.0131   | 0.00327 | 0      | 0        | 0.0344           | 0.1020           |
| $\sigma_f$              | 0                     | 0      | 0        | 0       | 0      | 0        | 0.524            | 1.147            |
| $\nu\sigma_f$           | 0                     | 0      | 0        | 0       | 0      | 0        | 1.414            | 3.258            |
| $\sigma_{out}^{inel}$   | 0                     | 0.0028 | 0.234    | 0.450   | 1.098  | 0.880    | 1.906            | 1.609            |
| $\sigma_{out}^{el}$     | 1.500                 | 0.206  | 0.091    | 0.040   | 0.043  | 0.068    | 0.013            | 0.013            |
| $\sigma_{out}^{n,2n}$   | 0                     | 0      | 0        | 0       | 0      | 0        | 0                | 0                |
| $\sigma_{II}^{inel}$    | 0                     | 0.0030 | 0.215    | 0.541   | 0.0405 | 0.0328   | 0.306            | 0.243            |
| $\sigma_{II}^{el}(P_0)$ | 1.002                 | 1.604  | 2.179    | 2.166   | 2.141  | 2.880    | 4.203            | 4.288            |
| $\sigma_{II}^{el}(P_1)$ | 2.571                 | 1.291  | 3.036    | 3.150   | 3.057  | 4.044    | 7.731            | 6.495            |
| $\sigma_{II}^{el}(tr)$  | 0.145                 | 1.174  | 1.167    | 1.116   | 1.122  | 1.532    | 1.626            | 2.123            |
| $\sigma_{II}^{n,2n}$    | 0                     | 0      | 0        | 0       | 0      | 0        | 0                | 0                |

TABLE XXVI. Average Group 1 Fluxes Calculated Using  
GAM-I Fission-spectrum Cross Sections

| Lattice  | $\bar{\phi}_2/\bar{\phi}_1(a)$ | $\bar{\phi}_3/\bar{\phi}_1(a)$ | Lattice  | $\bar{\phi}_2/\bar{\phi}_1(a)$ | $\bar{\phi}_3/\bar{\phi}_1(a)$ |
|----------|--------------------------------|--------------------------------|----------|--------------------------------|--------------------------------|
| B1.27□S  | 0.95910                        | 0.93577                        | H1.27ΔA  | 0.97486                        | 0.96243                        |
| B1.27ΔS  | 0.96655                        | 0.94961                        | H1.27ΔS  | 0.97185                        | 0.96009                        |
| H1.349□A | 0.95991                        | 0.93682                        | H1.166ΔA | 0.98355                        | 0.97615                        |
| H1.349□S | 0.95700                        | 0.93450                        | H1.166ΔS | 0.98047                        | 0.97372                        |
| H1.24□A  | 0.96968                        | 0.95383                        | H1.127ΔA | 0.98688                        | 0.98118                        |
| H1.24□S  | 0.96671                        | 0.95150                        | H1.127ΔS | 0.98376                        | 0.97869                        |
|          |                                |                                | H1.069ΔS | 0.98883                        | 0.98609                        |

(a) Subscripts 1, 2, and 3 denote fuel, cladding, and water regions, respectively.

Table XXIII and Fig. 7 show that heterogeneous effects on  $\delta^{28}$  are small and become smaller as the lattice pitch is decreased, as would be expected from the results in Table XXVI. The heterogeneous  $\delta^{28}$  is less than 5% larger than the corresponding homogeneous value, even for the loosest lattice. One-group values of  $\delta^{28}$  are smaller than four-group values, as anticipated above, because of the neglect of  $U^{238}$  fission below 1.35 MeV. Averaging GAM-I cross sections over a fission spectrum results in  $\delta^{28}$  values somewhat above those obtained using cross sections averaged over the spectrum generated by GAM-I. This is because the value of  $\Sigma_{R1}$  for the GAM-I spectrum is larger than the corresponding fission-spectrum value. MUFT-4 values of  $\delta^{28}$  are higher than the corresponding GAM-I values. The smaller MUFT-4 inelastic-scattering cross section for  $U^{238}$ , seen by comparing Tables XXIV and XXV, is mainly responsible for this.

Since all experimental values are considerably higher than the corresponding theoretical values, it is apparent that agreement between theory and experiment on  $\delta^{28}$  is not good. The reason for the discrepancy is not known. It may be caused by inadequacies in the cross sections or by some systematic experimental error. In investigating a very tight lattice of 1.3% enrichment, the workers at Westinghouse<sup>30</sup> also found that the greatest discrepancy between theory and experiment was in  $\delta^{28}$ .

Computations were made to determine the reactivity effects of using heterogeneous rather than homogeneous cross sections in Group 1, and also of using MUFT-4 rather than GAM-I cross sections in Group 1. Fast constants obtained using both GAM-I and MUFT-4 fission-spectrum cross sections in both homogeneous and heterogeneous calculations were substituted for the original GAM-I  $P_1$  constants in the first group of the four-group BUCKLE problems. Results for the three lattices studied are presented in Table XXVII in terms of reactivity differences.

TABLE XXVII. Reactivity Changes on Substitution of Various Group 1 Cross Sections into Four-group BUCKLE Problems

| Substitution                                 | Reactivity Changes, % |          |          |
|----------------------------------------------|-----------------------|----------|----------|
|                                              | B1.27□S               | H1.127ΔA | H1.069ΔS |
| GAM-I homogeneous →<br>GAM-I heterogeneous   | +0.134                | +0.059   | +0.045   |
| MUFT-4 homogeneous →<br>MUFT-4 heterogeneous | +0.104                | +0.063   | +0.052   |
| GAM-I homogeneous →<br>MUFT-4 homogeneous    | -1.579                | -0.749   | +0.702   |

Table XXVII shows that use of heterogeneous rather than homogeneous cross sections increases reactivity by little more than 0.1%, even for one of the looser lattices. Thus, the use of homogeneous constants in the criticality calculations of Section V is justified. Use of MUFT-4 cross sections in place of GAM-I cross sections decreases reactivity for most lattices, but increases it for the H1.069ΔS lattice. Section VIII presents a more systematic study of the effects on reactivity of changing important cross sections.

## VIII. EFFECTS OF CROSS-SECTION CHANGES ON REACTIVITY

It was mentioned in Section IV that use of the original GAM-I library cross sections for  $U^{235}$  and  $U^{238}$  led to calculated bucklings that were much higher than experimental ones, and that versions of  $U^{235}$  and  $U^{238}$  obtained from Hanford were therefore substituted for the original ones. To investigate reactivity effects of cross-section changes, calculations were done, using the BUCKLE code described in Section V, in which certain cross sections were changed by 10%, the others being kept constant. The standard cross sections are those in Tables IV and XII. Table XXVIII presents results in terms of  $\Delta k_{eff}/k_{eff}$ , where  $k_{eff}$  is unity for the critical buckling determined by the standard cross sections. No cross-section changes were considered in the thermal group, since thermal cross sections are rather well known. The effects of the indicated cross-section changes on the diffusion coefficients were not treated.

TABLE XXVIII. Effects of Cross-section Changes  
on Reactivity ( $\Delta k_{eff}/k_{eff}$ , %)

| Lattice  | Cross Section with +10% Change |               |                                  |                    |                    |                    |                    |
|----------|--------------------------------|---------------|----------------------------------|--------------------|--------------------|--------------------|--------------------|
|          | $\Sigma_{c1}$                  | $\Sigma_{f1}$ | $\Sigma_{out},$<br>$\Sigma_{12}$ | $\sigma_{c2}^{28}$ | $\sigma_{c3}^{28}$ | $\sigma_{c3}^{25}$ | $\sigma_{f3}^{25}$ |
| B1.27□S  | -0.047                         | +0.393        | +0.597                           | -0.367             | -1.311             | -0.415             | +0.667             |
| B1.27ΔS  | -0.051                         | +0.473        | +0.444                           | -0.468             | -1.502             | -0.495             | +0.832             |
| H1.349□A | -0.049                         | +0.420        | +0.537                           | -0.410             | -1.399             | -0.301             | +0.499             |
| H1.349□S | -0.052                         | +0.424        | +0.303                           | -0.424             | -1.422             | -0.301             | +0.586             |
| H1.24□A  | -0.057                         | +0.529        | +0.331                           | -0.561             | -1.676             | -0.382             | +0.669             |
| H1.24□S  | -0.062                         | +0.524        | +0.086                           | -0.583             | -1.718             | -0.386             | +0.780             |
| H1.27ΔA  | -0.067                         | +0.604        | +0.162                           | -0.693             | -1.852             | -0.449             | +0.807             |
| H1.27ΔS  | -0.065                         | +0.600        | +0.080                           | -0.714             | -1.901             | -0.446             | +0.944             |
| H1.166ΔA | -0.083                         | +0.795        | -0.305                           | -1.098             | -2.282             | -0.599             | +1.225             |
| H1.166ΔS | -0.082                         | +0.773        | -0.559                           | -1.149             | -2.385             | -0.594             | +1.423             |
| H1.127ΔA | -0.090                         | +0.894        | -0.549                           | -1.385             | -2.401             | -0.682             | +1.478             |
| H1.127ΔS | -0.091                         | +0.862        | -0.802                           | -1.462             | -2.544             | -0.674             | +1.708             |
| H1.069ΔS | -0.111                         | +1.026        | -1.333                           | -2.487             | -2.790             | -0.808             | +2.339             |

Table XXVIII shows that the calculated reactivities are quite sensitive to nonthermal cross-section changes in general, the sensitivity increasing as the pitch is decreased and the lattices become more undermoderated. As might be expected, the sensitivity to the  $U^{238}$  capture cross section is quite pronounced. The major reason for the large reactivity differences obtained by substitution of the Hanford cross sections is that the original GAM-I cross sections yield values of  $\sigma_{c2}^{28}$  only about half as large as those based on the Hanford cross sections. Table XXVIII shows that such a difference represents a reactivity change of the order of 5% for the H1.166ΔA lattice. Although the reactivity is even more sensitive to  $\sigma_{c3}^{28}$  than to  $\sigma_{c2}^{28}$ , the differences between Hanford and original GAM-I values of  $\sigma_{c3}^{28}$  are small and therefore do not contribute much to the large reactivity differences.

## IX. DISCUSSION AND CONCLUSIONS

Agreement between theory and experiment for the Hi-C and BORAX-V lattices is in general reasonably good, except for  $\delta^{28}$ . The Hi-C lattices provide a severe test of calculational methods. This is because the close spacing of the fuel rods makes theoretical treatment difficult in general and accurate treatment of nonthermal events quite important. Several improvements could be made in the theoretical methods, although those used should be reasonably good. Some possible improvements are discussed below.

In the thermal-energy region, the use of the isotropic return boundary condition, as mocked up by an extra scattering region at the outer boundary of a circular unit cell, may not be very good for the most tightly packed lattices. A Monte Carlo calculation using the actual square or hexagonal cell would be better than this one-dimensional THERMOS calculation. Other defects in the thermal calculations are the use of isotropic scattering, the treatment of the flux above the thermal cutoff as  $1/E$ , and the treatment of the slowing-down source as being due to hydrogen only. Also, the prescription used for the calculation of the thermal diffusion coefficient is rather rough. The revised version of THERMOS,<sup>31</sup> in which the thermal-diffusion coefficient is calculated more accurately, was not available when the calculations reported here were made.

In the resonance region, a treatment of resonance absorption involving neither the extreme NR nor NR1A approximations but using numerical integration of the transport equation would be an improvement. Another possibility would be the use of a Monte Carlo code. Such treatments could also allow for interference effects between resonances lying close together. Another approximation involved is the use, in the resonance calculations, of the escape probability for an isolated rod with the mean chord length,  $\bar{l}$ , replaced by  $\bar{l}/(1 - C)$ , where  $C$  is the Dancoff factor. The worst feature of this approximation is probably that the clad is treated in the narrow-resonance approximation in the calculation of the Dancoff factor. This may not be well justified for broad, low-lying resonances, especially for stainless steel-clad rods. Another resonance-region inaccuracy is caused by the fact that all absorption in a given resonance is assigned to the fine GAM-I group in which the resonance energy lies. The treatment of resonance absorption in the unresolved region in GAM-I is also rather approximate. In the case of fissionable isotopes, the Breit-Wigner single-level formula is not adequate, although parameters for multilevel formulas are available only over small-energy ranges. This makes accurate treatment of resonance absorption in  $U^{235}$  difficult.

The reason for the large discrepancy between theory and experiment in the case of  $\delta^{28}$  is not understood. The existence of inaccuracies in the high-energy cross-section data would seem to be the most plausible explanation, although a systematic experimental error is also possible. The reactivity of the Hi-C lattices is quite sensitive to changes in cross sections, as

was shown in Section VIII. The rather good agreement between theory and experiment on reactivity may be a result of cancellations of errors arising both from inaccurate cross sections and from the use of approximate theoretical techniques. Such errors might tend to cancel well in the calculation of reactivity, but not as well in the calculation of another quantity, such as  $\delta^{28}$ .

The Hi-C program extended investigations of  $\text{UO}_2\text{-H}_2\text{O}$  lattices into a range of lower hydrogen-to- $\text{U}^{238}$  atom ratios than had been studied previously. Such investigations are of interest because reactors with high conversion ratios and consequently high burnup are expected to become increasingly important in the utilization of nuclear energy. Experimental studies with fuel rods of enrichments other than the nominal 3% used here would also be of interest.



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