SCTM 215-59(51)

TABULATION OF THE HYPERGEOMETRIC PROBABILITY DISTRIBUTION FOR LOT SIZES LESS THAN OR EQUAL TO 50

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ABSTRACT

This memorandum describes a tabulation (too large to be included in the memorandum) of the hypergeometric probability distribution for lot sizes up to and including 50. Uses of the tabulation in sampling inspection and reliability are described. The tabulation provides a simple way of testing the equality of two proportions when the total number of observations on the proportions does not exceed 50.

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DISCLAIMER

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Introduction

A tabulation of the hypergeometric probability distribution has been made at Sandia Corporation. This tabulation has many applications, some of which are described below. Since the nomenclature of sampling inspection was used to set up the tabulation described in this memorandum, this nomenclature will be used here. The following symbols are defined:

N = Number of items in a lot,

N* = Number of items in a sample taken from the lot,

K* = Number of defective items in the lot, and

X = Number of defective items observed in the sample.

Then

$$\Pr\{\text{Exactly } X \text{ defectives in the sample}\} = PX^*$$

$$= \frac{K^*!\,N^*!}{(K^* - X)!\,(N^* - X)!\,X!}\;\frac{(N - K^*)!\,(N - N^*)!}{N!\,(N - K^* - N^* + X)!},$$

where X is an integer such that $\max[0,\ N^* + K^* - N] \leq X \leq \min[N^*,\ K^*]$,

and

$$\Pr\{X \text{ defectives or less in the sample}\} = PX$$

$$= \sum_{i=\max[0,\,N^*+K^*-N]}^{X} \frac{K^*!\,N^*!}{(K^* - i)!\,(N^* - i)!\,i!}\;\frac{(N - K^*)!\,(N - N^*)!}{N!\,(N - K^* - N^* + i)!}.$$

Since N* and K* may be interchanged in either of the probabilities PX and PX* without changing the value of the probabilities, it is necessary to tabulate only for $K^* \leq N^*$. If $N^* < K^*$, it is necessary to enter the table with N* and K* interchanged, and the probabilities may be read directly from the table.

The table that has been made tabulates exhaustively all hypergeometric probabilities for $N \leq 50$, with the restriction that $K^* \leq N^*$. In view of the preceding paragraph, this, of course, is no restriction at all, and it is possible to say that all hypergeometric probabilities for $N \leq 50$ have been tabulated.

Because of the large size of the table only a few copies were made. Copies were available in Organizations 1592-1 (B. Ostle), 5511-3 (J. C. Connell), and 5125 (D. B. Owen) when this memorandum was written. The appendix, page 23, is a copy of the first 10 pages of the hypergeometric table. The entire table contains 3231 pages.

Three additional symmetries in the hypergeometric distribution were not used when the table was prepared. These are:

If

$$P^*(N, N^*, K^*, X) = PX^* = \frac{K^*!\,N^*!}{(K^* - X)!\,(N^* - X)!\,X!} \; \frac{(N - K^*)!\,(N - N^*)!}{N!\,(N - K^* - N^* + X)!},$$

then

$$\begin{aligned} P^*(N, N^*, K^*, X) &= P^*(N, N^*, N - K^*, N^* - X) \\ &= P^*(N, N - N^*, K^*, K^* - X) \\ &= P^*(N, N - N^*, N - K^*, N - N^* - K^* + X). \end{aligned}$$

Similarly, if

$$P\{N, N^*, K^*, X\} = PX$$

$$= \sum_{i=\max[0,\, N^*+K^*-N]}^{X} \frac{K^*!\,N^*!}{(K^* - i)!\,(N^* - i)!\,i!} \; \frac{(N - K^*)!\,(N - N^*)!}{N!\,(N - K^* - N^* + i)!},$$

then

$$\begin{aligned} P\{N, N^*, K^*, X\} &= P\{N, N - N^*, N - K^*, N - N^* - K^* + X\} \\ &= 1 - P\{N, N^*, N - K^*, N^* - X - 1\} \\ &= 1 - P\{N, N - N^*, K^*, K^* - X - 1\}. \end{aligned}$$

The symmetry mentioned above involving the interchangeability of N^* and K^* may be written

$$P^*\{N, N^*, K^*, X\} = P^*\{N, K^*, N^*, X\},$$

and

$$P\{N, N^*, K^*, X\} = P\{N, K^*, N^*, X\}.$$

The preceding symmetries mean that most (but not all) of the entries in the table appear four times. For example, the following entries are noted:

N	N*	K*	X	PX	PX*
50	20	10	0	0.00292486	0.00292486
50	40	20	20	1.00000039	0.00292486
50	30	10	10	1.00000057	0.00292486
50	40	30	20	0.00292486	0.00292486

For these entries, PX* is the same. Because of the method of computation, however, these values would not have to be exactly the same, and in some instances equivalent entries may differ as much as five in the seventh decimal place.

The error in any entry in the table for PX and PX* is not more than two in the sixth decimal place. This is more than accurate enough for most applications.

As a further example of the duplication of entries, consider the following four sets of equivalent entries:

Set I

N	N*	K*	X	PX	PX*
16	10	4	0	0.00824176	0.00824176
16	10	4	1	0.11813180	0.10989004
16	10	4	2	0.48901094	0.37087914
16	10	4	3	0.88461533	0.39560439
16	10	4	4	0.99999996	0.11538463

Set II

N	N*	K*	X	PX	PX*
16	6	4	0	0.11538459	0.11538459
16	6	4	1	0.51098891	0.39560431
16	6	4	2	0.88186796	0.37087908
16	6	4	3	0.99175803	0.10989004
16	6	4	4	0.99999978	0.00824175

Set III

N	N*	K*	X	PX	PX*
16	12	6	2	0.00824176	0.00824176
16	12	6	3	0.11813180	0.10989004
16	12	6	4	0.48901094	0.37087914
16	12	6	5	0.88461533	0.39560439
16	12	6	6	0.99999996	0.11538463

Set IV

N	N*	K*	X	PX	PX*
16	12	10	6	0.11538463	0.11538463
16	12	10	7	0.51098902	0.39560439
16	12	10	8	0.88186815	0.37087914
16	12	10	9	0.99175820	0.10989004
16	12	10	10	0.99999996	0.00824176

Clearly, when one of the sets of values is given, it would be easy to obtain the other three. Also, since the entries differ in the last places, it is clear that independent and not entirely equivalent calculations were made.

Applications to Sampling Inspection Plans

For a lot of size N = 12, a sample N* = 5 is taken and the lot is accepted if X = 0 defectives are found in the sample. The probability of acceptance for various proportions defective in the lot is needed. From the table one can read:

N	N*	K*	X	PX
12	5	0	0	1.0000
12	5	1	0	0.5833
12	5	2	0	0.3182
12	5	3	0	0.1591
12	5	4	0	0.0707
12	5	5	0	0.0265
12	5	6	0	0.0076
12	5	7	0	0.0013

If K* gets above 7 = N - N* + X, the probability of acceptance is zero, since then it is impossible to get five nondefective items in the sample. Also, if K* = 0, the probability of acceptance is one, since then the lot is surely accepted. Hence the operating characteristic for this plan is obtained by plotting PX against K*/N. The result is shown in Figure 1.

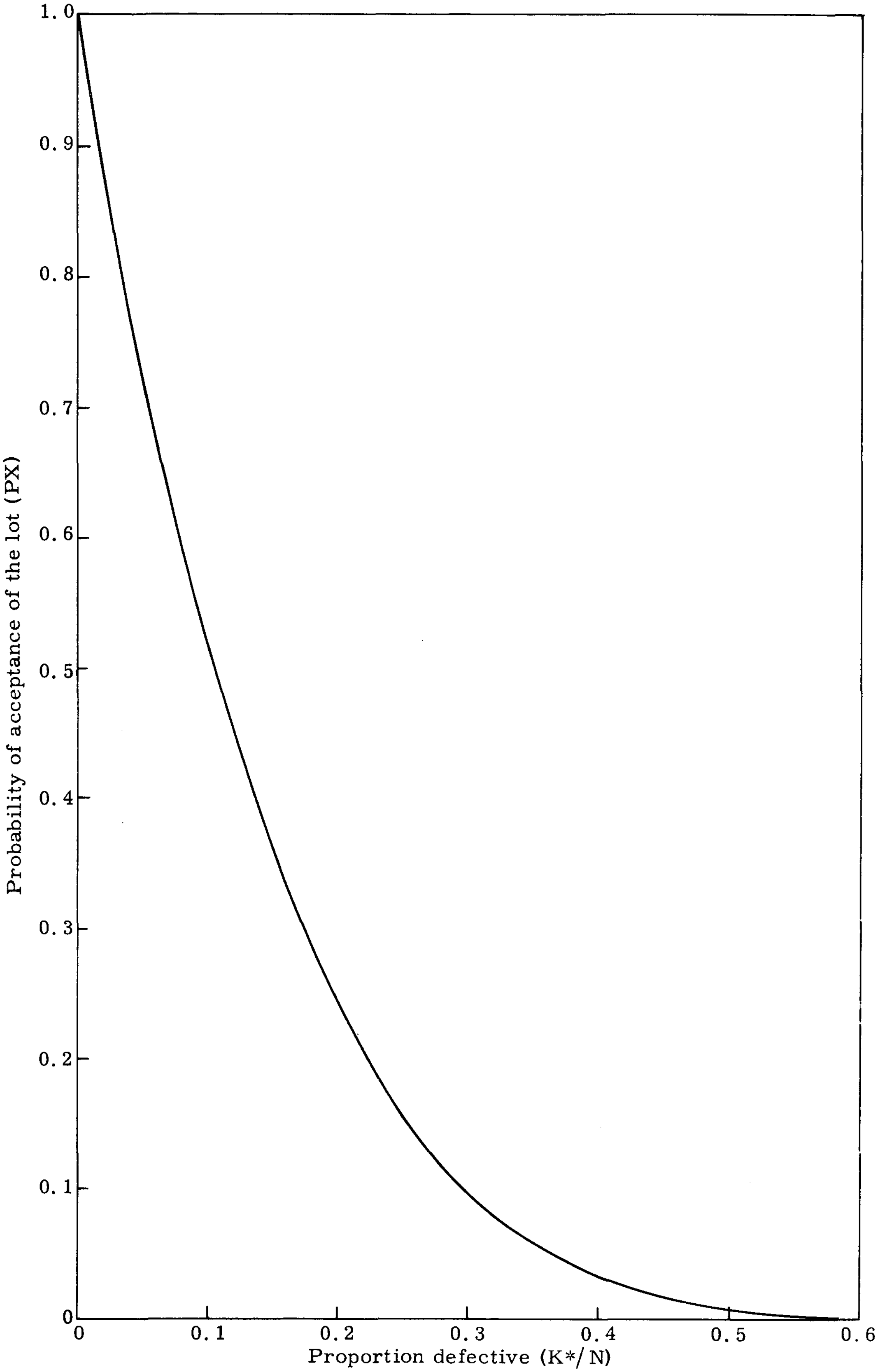

Figure 1. Operating-Characteristic Curve for N = 12, N* = 5, and X = 0

As a second example, consider a lot of size N = 25, where a sample of size N* = 10 is taken and the lot is accepted if X = 0 or 1 defectives are found in the sample. The following entries are found in the table.

N	N*	K*	X	PX
25	10	0	1	1.0000
25	10	1	1	1.0000
25	10	2	1	0.8500
25	10	3	1	0.6543
25	10	4	1	0.4676
25	10	5	1	0.3134
25	10	6	1	0.1978
25	10	7	1	0.1175
25	10	8	1	0.0654
25	10	9	1	0.0339
25	10	10	1	0.0162
25	10	11	1	0.0070
25	10	12	1	0.0027
25	10	13	1	0.0009
25	10	14	1	0.0002
25	10	15	1	0.0000
25	10	16	1	0.0000

If K* > N - N* + X, then PX is zero; and if K* ≤ X, then PX is one. Hence the values above K* = 16 have been omitted, and the values for K* = 0 and 1 have been added for convenience in plotting. Figure 2 shows the graph of the operating-characteristic curve of this example.

Applications to Tests of the Equality of Two Proportions (Two-by-Two Tables)

A two-by-two contingency table may be represented as follows:

	Characteristic I		
Characteristic II	Has	Does not have	Totals
Has	X	K* - X	K*
Does not have	N* - X	N - N* - K* + X	N - K*
Totals	N*	N - N*	N

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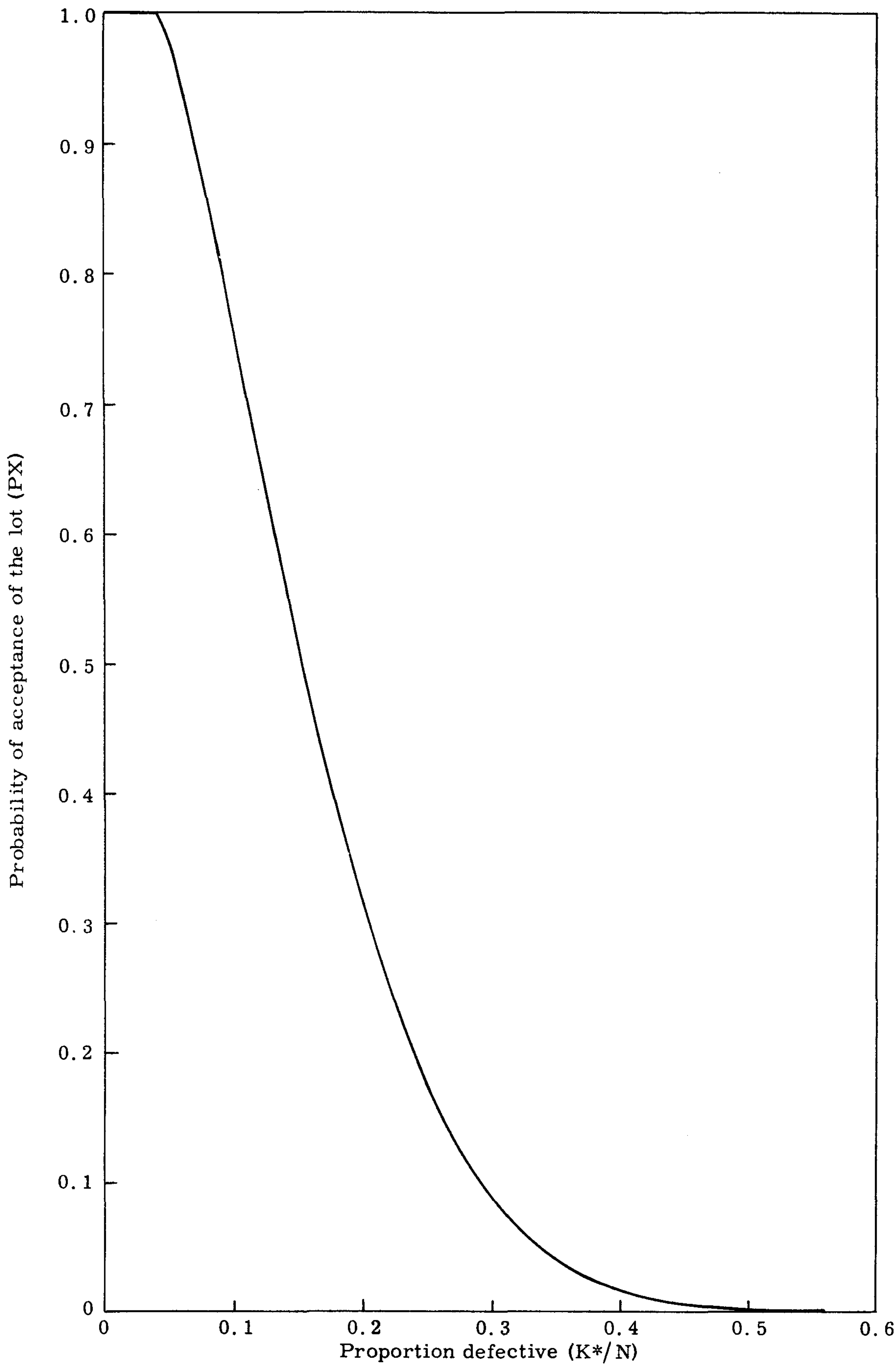

Figure 2. Operating-Characteristic Curve for N = 25, N* = 10, and X = 0 or 1

An example will make clear the usefulness of the hypergeometric table in testing a two-by-two table. The problem given on page 19 of Reference 1 will be solved again here. The problem is presented as follows:

Performance of Encapsulated SA-1's

Treatment	Failure	Success	Totals
Encapsulation Method I	9	6	15
Encapsulation Method II	3	11	14
Totals	12	17	29

Hence, take N* = 12, K* = 15, X = 9, and N = 29. But to read this from the hypergeometric table, it is necessary to interchange K* and N*. Page 382 of the table contains the following entries:

N	N*	K*	X	PX	PX*
29	15	12	0	0.00000175	0.00000175
29	15	12	1	0.00010696	0.00010521
29	15	12	2	0.00213226	0.00202530
29	15	12	3	0.01968488	0.01755261
29	15	12	4	0.09867170	0.07898681
29	15	12	5	0.29726710	0.19859540
29	15	12	6	0.58688549	0.28961839
29	15	12	7	0.83512970	0.24824421
29	15	12	8	0.95925176	0.12412206
29	15	12	9	0.99435702	0.03510526
29	15	12	10	0.99962280	0.00526578
29	15	12	11	0.99999103	0.00036823
29	15	12	12	0.99999979	0.00000876

According to Reference 1 (page 19):

> "The question to be answered is: Does encapsulation Method II have a better effect on performance than encapsulation Method I? This is a one-sided test since the outcome is interesting only if Method II is better than Method I. ... the relative proportion of successes using Method II is 11/14 = 0.79 and for Method I is 6/15 = 0.40. Hence, in this sample, Method II shows a better performance than Method I. The question now is: Is this due to chance or is Method II really better than Method I? Next, the statistical test will be performed. If Method II were worse than Method I in the sample, then no further statistical test would be performed because the hypothesis that Method II is no better than Method I is automatically accepted.
>
> "(Note: The procedure outlined above enables one to make a one-tailed test. For a two-tailed test no preliminary look at the proportions is necessary.)"

The probability of observing exactly nine failures is then PX* = 0.03510526. But it is necessary to find the probability of nine or more failures (a deviation as extreme as, or more extreme than, that observed), and this can be obtained from the table by taking

$$1 - \Pr\{X \leq 8\} = 1 - 0.95925176 = 0.04074824.$$

Since this probability is less than 0.05, there is a significant difference between Encapsulation Method I and Encapsulation Method II at the 95-percent level of significance.

If in the problem solved above there were no prior understanding that the outcome be interesting only if Method II were better than Method I, than a two-sided test should be run, and looking at the table quoted above one should say there is a significant difference at the 95-percent level of significance only if $X \leq 3$ or if $X \geq 10$, since

$$\Pr\{X \leq 3\} + \Pr\{X \geq 10\} = 0.01968488 + 1 - 0.99435702$$
$$= 0.02532786 < 0.05.$$

The tabulation on page 12 shows that X cannot be raised to 4, since then PX = 0.09867170. But X could be lowered to 9 at the upper end if X were lowered to 2 at the lower end. That is, another two-sided rule for calling significance at the 95-percent significance level is if $X \leq 2$ or if $X \geq 9$, since

$$\Pr\{X \leq 2\} + \Pr\{X \geq 9\} = 0.00213226 + 1 - 0.95925176$$
$$= 0.04288050 < 0.05.$$

Both rules, $X \leq 3$ or $X \geq 10$, and $X \leq 2$ or $X \geq 9$, are equally good. The second might be preferred over the first since the actual test probability is closer to 0.05. The first might be preferred to the second since both tails of the distribution are below 0.025. The difficulty, of course, arises because of the discreteness of X. For further discussion of this point see Reference 1.

Applications to the Distribution of the Number of Exceedances

Consider a random sample of size n taken from a continuous distribution. Let another random sample, of size m independent of the first sample, be drawn from the same population. The probability that X observations among the observations of the second sample will exceed the r^{th} largest observation in the first sample is given by

$$Pr\{X \text{ among m future trials will exceed the } r^{th} \text{ largest observation in a sample of } n\}$$

$$= \binom{m+n-r-X}{n-r}\binom{X+r-1}{r-1} \Big/ \binom{m+n}{n}$$

$$= \frac{n}{m+n} P^*(m+n-1,\ m,\ X+r-1,\ X)$$

$$= \frac{r}{X+r} P^*(m+n,\ m,\ X+r,\ X),$$

where the P*( ) is the quantity defined in Section 1 and is equal to PX* in the hypergeometric table.

The probability that the largest among n past observations will be exceeded X times or less in m future trials is given by

$$Pr\{X \text{ or less among m future trials will exceed the largest among n observations}\}$$

$$= \sum_{y=0}^{X} \binom{m+n-y-1}{n-1} \Big/ \binom{m+n}{n} = 1 - \binom{m+n-1-X}{n} \Big/ \binom{m+n}{n}.$$

The summation of the binomial coefficients was accomplished by means of Equation 12. 6 (page 61) of Reference 2. Hence, the

$$Pr\{X \text{ or less among m future trials will exceed the largest among n observations}\}$$

$$= 1 - P^*\{m+n,\ n,\ X+1,\ 0\}, \quad \text{for } 0 \leq X \leq m,$$

and

$$Pr\{X \text{ or more among m future trials will exceed the largest among n observations}\}$$

$$= P^*\{m+n,\ n,\ X,\ 0\}, \quad \text{for } 0 \leq X \leq m.$$

Also,

$$Pr\{X \text{ or less among m future trials will exceed the smallest observation in a sample of } n\}$$

$$= \sum_{y=0}^{X} \binom{y+n-1}{n-1} \Big/ \binom{m+n}{n} = \binom{n+X}{n} \Big/ \binom{m+n}{n}.$$

The summation of the binomial coefficients was accomplished by means of Equation 12. 8 (page 62) of Reference 2.

Then,

$$\Pr\{X \text{ or less among m future trials will exceed the smallest observation in a sample of n}\} = P^*\{m + n,\ n,\ m - X,\ 0\}, \quad \text{for } 0 \le X \le m.$$

Consider the following example of the use of the hypergeometric table. What is the probability that the largest flood in the past 20 years will be exceeded at least once during the next 25 years?

The answer is (assuming no major change in weather patterns)

$$\Pr\{\text{one or more among 25 future years will have a flood which exceeds the largest flood in the past 20 years}\} = P^*\{45,\ 20,\ 1,\ 0\} = 0.55556.$$

It is also possible to list the probability of X or more exceedances for all values of X, $0 \le X \le 25$. These are

$P^*\{45, 20, 0, 0\}$	=	1.00000
$P^*\{45, 20, 1, 0\}$	=	0.55556
$P^*\{45, 20, 2, 0\}$	=	0.30303
$P^*\{45, 20, 3, 0\}$	=	0.16209
$P^*\{45, 20, 4, 0\}$	=	0.08490
$P^*\{45, 20, 5, 0\}$	=	0.04349
$P^*\{45, 20, 6, 0\}$	=	0.02174
$P^*\{45, 20, 7, 0\}$	=	0.01059
$P^*\{45, 20, 8, 0\}$	=	0.00502
$P^*\{45, 20, 9, 0\}$	=	0.00231
$P^*\{45, 20, 10, 0\}$	=	0.00102
$P^*\{45, 20, 11, 0\}$	=	0.00044
$P^*\{45, 20, 12, 0\}$	=	0.00018
$P^*\{45, 20, 13, 0\}$	=	0.00007
$P^*\{45, 20, 14, 0\}$	=	0.00003
$P^*\{45, 20, 15, 0\}$	=	0.00001

and the rest of the probabilities are zero to five decimal places. Using this table makes it possible to pick a value of X such that the probability of X or more exceedances is less than some set probability.

As a second example, consider the following. In a sample of 15 of the SA-2's, all met a certain criterion on voltage; i.e., all were below a stated voltage. What is the probability that in an additional sample of 20 all will also be below the stated voltage?

The method given here gives the following bound on the required probability:

$$\Pr\{\text{zero among 20 future trials will exceed the largest among 15 observations}\}$$

$$= 1 - P^*\{35,\ 15,\ 1,\ 0\} = 1 - 0.57142849 = 0.42857151.$$

The probability requested is less than this value; i.e.,

$$\Pr\{\text{all observations in the second sample will be below the stated voltage}\} \leq 0.42857151.$$

For a probability like this to be small, the first sample must be large compared to the second sample.

As a third example, what is the probability that one additional observation from a population will be between the extremes of a sample of size n? The answer is obtained by specializing the following:

$$\Pr\{X \text{ or more among } m \text{ future trials will exceed the largest among } n \text{ observations}\}$$

$$= P^*\{m + n,\ n,\ X,\ 0\} \quad \text{for } 0 \leq X \leq m.$$

For the problem at hand, $X = 1$ and $m = 1$ and, because of symmetry,

$$\Pr\{\text{one future trial will be below the smallest among } n \text{ observations}\}$$

$$= P^*\{n + 1,\ n,\ 1,\ 0\} \text{ also.}$$

Hence,

$$\Pr\{\text{one additional observation will be between the extremes of a sample of size } n\}$$

$$= 1 - 2\,P^*\{n + 1,\ n,\ 1,\ 0\} = 1 - \frac{2}{n+1} = \frac{n-1}{n+1}.$$

Another question can now be answered from this last equation. That is, how large a sample is needed to be 95-percent sure that the next observation is between the extremes of the sample? The answer is obtained from

$$\frac{n - 1}{n + 1} = 0.95,$$

or

$$n = 39.$$

It is interesting to compare this result with distribution-free tolerance limits. Using Table IV (page 11) of Reference 6 makes it possible to say with 95-percent confidence that at least 88.37 percent of the future observations will lie between the extremes of the sample.

Applications to a Sequential Procedure

Given a lot of N items containing k defectives, a frequent question is: how many items must be sampled from the lot to produce n nondefectives?

The solution to this problem may be obtained as follows:

$$\Pr\{X + n \text{ trials or less will be required to produce n nondefectives}\}$$

$$= \frac{(N - k)!\,(N - n)!}{(N - k - n)!\,N!}\left[1 + n\,\frac{k}{N - n} + \frac{n(n + 1)}{2}\,\frac{k(k - 1)}{(N - n)(N - n - 1)} + \dots\right.$$

$$\left. + \frac{n(n + 1)\dots(n + X - 1)}{X!}\,\frac{k(k - 1)\dots(k - X + 1)}{(N - n)(N - n - 1)\dots(N - n - X + 1)}\right],$$

where $0 \leq X \leq k$ and $N \geq k + n$.

Now by an argument exactly equivalent to the argument which connects negative binomial sums to binomial sums given in Appendix A of Reference 7, it can be shown that this probability reduces to

$$1 - P\{N,\ X + n,\ N - k,\ n - 1\} = P\{N,\ n + X,\ k,\ X\}.$$

For example, suppose a lot of 50 items contains 10 defectives and it is necessary to obtain 20 nondefective items from the lot. The sampling will stop when the 20 nondefectives are obtained. What is the probability that the 20 nondefective items can be obtained with a sample of 25 or less?

The answer is $P\{50,\ 25,\ 10,\ 5\} = 0.63739897$.

The entire probability distribution for the possible sample sizes may be obtained from the hypergeometric table also. The distribution is

Sample size	Look up
20	$P\{50,\ 20,\ 10,\ 0\} = 0.00292486$
21	$P\{50,\ 21,\ 10,\ 1\} = 0.02242396$
22	$P\{50,\ 22,\ 10,\ 2\} = 0.08596408$

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Sample size	Look up
23	$P\{50, 23, 10, 3\} = 0.21909597$
24	$P\{50, 24, 10, 4\} = 0.41756099$
25	$P\{50, 25, 10, 5\} = 0.63739897$
26	$P\{50, 26, 10, 6\} = 0.82059771$
27	$P\{50, 27, 10, 7\} = 0.93400604$
28	$P\{50, 28, 10, 8\} = 0.98393041$
29	$P\{50, 29, 10, 9\} = 0.99805093$
30	$P\{50, 30, 10, 10\} = 1.00000000$

Hence, for example, about 66 times out of 1000 it will be necessary to take a sample of 27 to obtain 20 nondefectives from the lot.

As a second example, suppose a lot of N = 35 items is at hand, and it is necessary to obtain 20 nondefectives from this lot and then sampling will cease. When sampling is stopped, it is necessary to make a statement about the number of defectives in the original lot; e.g., the number of defectives in the lot is no more than k with 90-percent assurance.

This problem may be solved by solving $P\{35, X + 20, k, X\} \leq 0.10$ for k. The results are:

Sample size taken	X	k	Actual probability
20	0	3	0.070
21	1	5	0.071
22	2	7	0.050
23	3	8	0.070
24	4	9	0.084
25	5	10	0.089
26	6	11	0.084
27	7	12	0.070
28	8	13	0.050
29	9	14	0.028
30	10	14	0.070
31	11	15	0.026
32	12	15	0.070

Now, for example, it can be said that if sampling stopped with a sample of 25 items to produce 20 nondefective items, one is at least 90-percent sure that there were no more than 10 defective items in the lot before sampling; or equivalently, one is 90-percent sure that there are no more than five defective items in the remaining ten items.

It might be instructive to compare these results with the case of an infinite lot. Suppose a sample of 25 is taken from a continuous production process to produce 20 nondefective items. With 90-percent assurance, what is the upper bound on the proportion defective coming from this process?

The answer is obtained by using Section 9 of Reference 7 (page 12) and is given by

$$1 - \frac{n + 1}{n + 1 + XF_{\gamma, 2X, 2n+2}},$$

where F_γ is an upper γ-percentage point of the F distribution based on 2X degrees of freedom for the numerator, and 2n + 2 degrees of freedom for the denominator. Here n = 20 and X = 5, and hence $F_{0.90} = 1.75$, and the upper bound on the proportion defective produced by the process is 1 - 0.706 = 0.294, with 90-percent assurance. For the finite lot with N = 35, as pointed out above, the proportion defective in the original lot is less than 10/35 = 0.286 with 90-percent assurance, or the proportion defective in the remaining part of the lot is less than 5/10 = 0.500 with 90-percent assurance.

Number of Entries in the Table

In this section a formula for the number of entries in the table as presently constructed is derived, then a second formula is given for determining the number of entries if all the symmetry mentioned in Section 1 is used.

If all of the sets of entries began with X equal to zero, i.e., the counts started with zero even when they should have started with N* + K* - N, then the number of entries would be

$$S_1 = \sum_{N=2}^{p} \sum_{N^*=1}^{N-1} \sum_{K^*=1}^{N-N^*} \sum_{X=0}^{K^*} 1 = \frac{(p-1)p(p+1)(p+6)}{24},$$

where p is the last value of N that is to be included in the table and all possible entries in the hypergeometric table are included except those for which K* > N*.

But all of the sets of entries did not start at zero, and the number of entries to be subtracted is

$$S_2 = \sum_N \sum_{N^*} \sum_{K^*=N-N^*+1}^{N^*} \sum_{X=0}^{N^*+K^*-N-1} 1.$$

If N is even, N* goes from $\frac{N}{2} + 1$ to N - 1; and if N is odd, N* goes from $\frac{N+1}{2}$ to N - 1.

For just the even values of N, part of S_2 reduces to (with N = 2R)

$$S_3 = \sum_{R=2}^{M} \sum_{N^*=R+1}^{2R-1} \left(2N^{*2} + N^* - 2\,N^*N - R + 2R^2\right) = \frac{M(M-1)(M+1)^2}{6} ..$$

For just the odd values of N, part of S_2 reduces to (with N = 2R + 1)

$$S_4 = \sum_{R=1}^{Q} \sum_{N^*=R+1}^{2R} \left(2N^{*2} + N^* - 4N^*R - 2N^* + 2R^2 + R\right) = \frac{Q^2(Q+1)(Q+2)}{6} .$$

Now if p is even, then the number of entries is obtained by taking S_1 and subtracting S_3' and S_4',

where

$$S_3' = \frac{p(p-2)(p+2)^2}{96} ,$$

and

$$S_4' = \frac{(p-2)^2 p(p+2)}{96} .$$

Hence,

$$S = \frac{p(p^3 + 12p^2 + 2p - 12)}{48}$$

for even values of p.

Similarly, for odd values of p,

$$S_3'' = \frac{(p-3)(p-1)(p+1)^2}{96} ,$$

and

$$S_4'' = \frac{(p-1)^2(p+1)(p+3)}{96} .$$

Hence,

$$S = \frac{p^4 + 12p^3 + 2p^2 - 12p - 3}{48}$$

for odd values of p.

For example, if p = 50, S = 161,550 entries, and since there are 50 entries per page, there are 3231 pages in the table up through N = 50.

If p = 37, S = 51,756, and since there are 50 entries per page, there are 1035 pages plus six entries in the table up through N = 37.

In summary, the number of entries in the table may be obtained from

$$S = \frac{p^4 + 12p^3 + 2p^2 - 12p - \begin{cases} 0 \\ 3 \end{cases}}{48},$$

where the 0 is used if p is even, the 3 is used if p is odd, and p is the last value of N to be included in the table.

If the additional symmetries of the hypergeometric function mentioned in Section 1 are used, it is possible to restrict N* as follows:

$$K^* \leq N^* \leq N/2 .$$

Note that now all the X's start at zero.

Then the number of items in the table becomes

$$S^* = \sum_{N=2}^{p} \sum_{N^*=1}^{[N/2]} \sum_{K^*=1}^{N^*} \sum_{X=0}^{K^*} 1,$$

where the bracket indicates the greatest integer less than or equal to N/2.

Then,

$$S^* = \sum_{N=2}^{p} \sum_{N^*=1}^{[N/2]} \sum_{K^*=1}^{N^*} (K^* + 1) = \sum_{N=2}^{p} \sum_{N^*=1}^{[N/2]} \frac{N^*(N^* + 3)}{2}.$$

If N = 2R,

$$S_1^* = \sum_{N=2}^{p} \sum_{N^*=1}^{R} \frac{1}{2}\left(N^{*2} + 3N^*\right) = \sum_{N=2}^{p} \frac{R(R + 1)(R + 5)}{6}.$$

If N = 2Q + 1,

$$S_2^* = \sum_{N=2}^{p} \sum_{N^*=1}^{Q} \frac{1}{2}\left(N^{*2} + 3N^*\right) = \sum_{N=2}^{p} \frac{Q(Q + 1)(Q + 5)}{6}.$$

Now if p = 2s,

$$S^* = \sum_{R=1}^{s} \frac{R(R + 1)(R + 5)}{6} + \sum_{Q=1}^{s-1} \frac{Q(Q + 1)(Q + 5)}{6}$$

$$= \frac{1}{24}\Big[s(s + 1)(s + 2)(s + 7) + (s - 1)s(s + 1)(s + 6)\Big]$$

$$= \frac{1}{12}\Big[s(s + 1)(s^2 + 7s + 4)\Big];$$

hence, if p is even,

$$S^* = \frac{1}{192}\Big[p(p + 2)(p^2 + 14p + 16)\Big].$$

If p = 2s + 1,

$$S^* = \sum_{R=1}^{s} \frac{R(R + 1)(R + 5)}{6} + \sum_{Q=1}^{s} \frac{Q(Q + 1)(Q + 5)}{6} = \frac{1}{12}\Big[s(s + 1)(s + 2)(s + 7)\Big];$$

and hence if p is odd,

$$S^* = \frac{1}{192}\Big[(p - 1)(p + 1)(p + 3)(p + 13)\Big].$$

In summary, if all the symmetry of the hypergeometric function is used,

$$S^* = \frac{1}{192}\Big[p(p + 2)(p^2 + 14p + 16)\Big] \text{ for even values of p,}$$

and

$$S^* = \frac{1}{192}\Big[(p - 1)(p + 1)(p + 3)(p + 13)\Big] \text{ for odd values of p.}$$

If p = 50, S* = 43, 550 entries, or 871 pages at 50 entries per page; and for p = 100, S* = 606, 475 entries, or 12, 129. 5 pages at 50 entries per page.

APPENDIX

Sample Pages from the Hypergeometric Table

N	N*	K*	X	PX	PX*
2	1	1	0	0.50000000	0.50000000
2	1	1	1	1.00000000	0.50000000
3	1	1	0	0.66666666	0.66666666
3	1	1	1	0.99999999	0.33333332
3	2	1	0	0.33333332	0.33333332
3	2	1	1	0.99999999	0.66666666
3	2	2	1	0.66666666	0.66666666
3	2	2	2	0.99999999	0.33333332
4	1	1	0	0.74999999	0.74999999
4	1	1	1	0.99999999	0.25000000
4	2	1	0	0.50000000	0.50000000
4	2	1	1	1.00000000	0.50000000
4	2	2	0	0.16666666	0.16666666
4	2	2	1	0.83333332	0.66666666
4	2	2	2	0.99999998	0.16666666
4	3	1	0	0.25000000	0.25000000
4	3	1	1	0.99999999	0.74999999
4	3	2	1	0.50000000	0.50000000
4	3	2	2	1.00000000	0.50000000
4	3	3	2	0.75000002	0.75000002
4	3	3	3	1.00000001	0.25000000
5	1	1	0	0.79999999	0.79999999
5	1	1	1	0.99999999	0.20000000
5	2	1	0	0.59999999	0.59999999
5	2	1	1	0.99999996	0.39999997
5	2	2	0	0.29999999	0.29999999
5	2	2	1	0.89999995	0.59999996
5	2	2	2	0.99999993	0.09999998
5	3	1	0	0.39999995	0.39999995
5	3	1	1	0.99999995	0.59999999
5	3	2	0	0.09999996	0.09999996
5	3	2	1	0.69999993	0.59999996
5	3	2	2	0.99999992	0.29999999
5	3	3	1	0.29999999	0.29999999
5	3	3	2	0.89999995	0.59999996
5	3	3	3	0.99999992	0.09999996
5	4	1	0	0.19999997	0.19999997
5	4	1	1	0.99999992	0.79999995
5	4	2	1	0.39999992	0.39999992
5	4	2	2	0.99999992	0.59999999
5	4	3	2	0.59999999	0.59999999
5	4	3	3	0.99999992	0.39999992
5	4	4	3	0.79999991	0.79999991
5	4	4	4	0.99999987	0.19999997

Sample Pages from the Hypergeometric Table (continued)

N	N*	K*	X	PX	PX*
6	1	1	0	0.83333333	0.83333333
6	1	1	1	0.99999999	0.16666666
6	2	1	0	0.66666666	0.66666666
6	2	1	1	0.99999999	0.33333332
6	2	2	0	0.39999997	0.39999997
6	2	2	1	0.93333327	0.53333330
6	2	2	2	0.99999993	0.06666666
6	3	1	0	0.50000000	0.50000000
6	3	1	1	1.00000000	0.50000000
6	3	2	0	0.20000000	0.20000000
6	3	2	1	0.80000000	0.59999999
6	3	2	2	1.00000000	0.20000000
6	3	3	0	0.05000000	0.05000000
6	3	3	1	0.50000000	0.45000000
6	3	3	2	0.95000000	0.45000000
6	3	3	3	1.00000000	0.05000000
6	4	1	0	0.33333332	0.33333332
6	4	1	1	0.99999999	0.66666666
6	4	2	0	0.06666664	0.06666664
6	4	2	1	0.59999994	0.53333330
6	4	2	2	0.99999990	0.39999995
6	4	3	1	0.19999997	0.19999997
6	4	3	2	0.79999996	0.59999999
6	4	3	3	0.99999993	0.19999997
6	4	4	2	0.39999995	0.39999995
6	4	4	3	0.93333314	0.53333319
6	4	4	4	0.99999978	0.06666664
6	5	1	0	0.16666666	0.16666666
6	5	1	1	1.00000004	0.83333340
6	5	2	1	0.33333327	0.33333327
6	5	2	2	0.99999991	0.66666664
6	5	3	2	0.50000004	0.50000004
6	5	3	3	1.00000009	0.50000004
6	5	4	3	0.66666664	0.66666664
6	5	4	4	0.99999991	0.33333327
6	5	5	4	0.83333340	0.83333340
6	5	5	5	1.00000004	0.16666666
7	1	1	0	0.85714285	0.85714285
7	1	1	1	0.99999996	0.14285711
7	2	1	0	0.71428570	0.71428570
7	2	1	1	0.99999998	0.28571428
7	2	2	0	0.47619046	0.47619046
7	2	2	1	0.95238090	0.47619043
7	2	2	2	0.99999993	0.04761904
7	3	1	0	0.57142854	0.57142854
7	3	1	1	0.99999997	0.42857143
7	3	2	0	0.28571428	0.28571428
7	3	2	1	0.85714281	0.57142854
7	3	2	2	0.99999993	0.14285711

Sample Pages from the Hypergeometric Table (continued)

N	N*	K*	X	PX	PX*
7	3	3	0	0.11428571	0.11428571
7	3	3	1	0.62857142	0.51428571
7	3	3	2	0.97142854	0.34285712
7	3	3	3	0.99999997	0.02857143
7	4	1	0	0.42857143	0.42857143
7	4	1	1	0.99999997	0.57142854
7	4	2	0	0.14285713	0.14285713
7	4	2	1	0.71428566	0.57142854
7	4	2	2	0.99999998	0.28571431
7	4	3	0	0.02857143	0.02857143
7	4	3	1	0.37142855	0.34285712
7	4	3	2	0.88571428	0.51428573
7	4	3	3	0.99999999	0.11428571
7	4	4	1	0.11428571	0.11428571
7	4	4	2	0.62857144	0.51428573
7	4	4	3	0.97142856	0.34285712
7	4	4	4	0.99999999	0.02857143
7	5	1	0	0.28571428	0.28571428
7	5	1	1	1.00000004	0.71428578
7	5	2	0	0.04761904	0.04761904
7	5	2	1	0.52380948	0.47619043
7	5	2	2	0.99999996	0.47619049
7	5	3	1	0.14285713	0.14285713
7	5	3	2	0.71428566	0.57142854
7	5	3	3	0.99999998	0.28571431
7	5	4	2	0.28571431	0.28571431
7	5	4	3	0.85714285	0.57142854
7	5	4	4	0.99999998	0.14285713
7	5	5	3	0.47619043	0.47619043
7	5	5	4	0.95238087	0.47619043
7	5	5	5	0.99999991	0.04761904
7	6	1	0	0.14285713	0.14285713
7	6	1	1	1.00000001	0.85714289
7	6	2	1	0.28571428	0.28571428
7	6	2	2	1.00000004	0.71428578
7	6	3	2	0.42857146	0.42857146
7	6	3	3	1.00000000	0.57142854
7	6	4	3	0.57142849	0.57142849
7	6	4	4	0.99999996	0.42857146
7	6	5	4	0.71428578	0.71428578
7	6	5	5	1.00000004	0.28571428
7	6	6	5	0.85714289	0.85714289
7	6	6	6	1.00000001	0.14285713
8	1	1	0	0.87500001	0.87500001
8	1	1	1	0.99999999	0.12499998
8	2	1	0	0.74999999	0.74999999
8	2	1	1	0.99999999	0.25000000
8	2	2	0	0.53571426	0.53571426

Sample Pages from the Hypergeometric Table (continued)

N	N*	K*	X	PX	PX*
8	2	2	1	0.96428566	0.42857140
8	2	2	2	0.99999993	0.03571428
8	3	1	0	0.62499999	0.62499999
8	3	1	1	0.99999996	0.37499998
8	3	2	0	0.35714284	0.35714284
8	3	2	1	0.89285710	0.53571426
8	3	2	2	0.99999993	0.10714284
8	3	3	0	0.17857145	0.17857145
8	3	3	1	0.71428574	0.53571429
8	3	3	2	0.98214288	0.26785714
8	3	3	3	1.00000000	0.01785713
8	4	1	0	0.49999997	0.49999997
8	4	1	1	0.99999994	0.49999997
8	4	2	0	0.21428568	0.21428568
8	4	2	1	0.78571417	0.57142849
8	4	2	2	0.99999985	0.21428568
8	4	3	0	0.07142855	0.07142855
8	4	3	1	0.49999999	0.42857143
8	4	3	2	0.92857142	0.42857143
8	4	3	3	0.99999997	0.07142855
8	4	4	0	0.01428570	0.01428570
8	4	4	1	0.24285707	0.22857137
8	4	4	2	0.75714278	0.51428571
8	4	4	3	0.98571415	0.22857137
8	4	4	4	0.99999985	0.01428570
8	5	1	0	0.37499999	0.37499999
8	5	1	1	1.00000000	0.62500001
8	5	2	0	0.10714284	0.10714284
8	5	2	1	0.64285713	0.53571429
8	5	2	2	0.99999996	0.35714284
8	5	3	0	0.01785713	0.01785713
8	5	3	1	0.28571427	0.26785714
8	5	3	2	0.82142860	0.53571434
8	5	3	3	1.00000004	0.17857145
8	5	4	1	0.07142854	0.07142854
8	5	4	2	0.49999997	0.42857143
8	5	4	3	0.92857140	0.42857143
8	5	4	4	0.99999994	0.07142854
8	5	5	2	0.17857143	0.17857143
8	5	5	3	0.71428569	0.53571426
8	5	5	4	0.98214282	0.26785713
8	5	5	5	0.99999995	0.01785713
8	6	1	0	0.24999997	0.24999997
8	6	1	1	0.99999994	0.74999997
8	6	2	0	0.03571428	0.03571428
8	6	2	1	0.46428562	0.42857134
8	6	2	2	0.99999991	0.53571429
8	6	3	1	0.10714284	0.10714284
8	6	3	2	0.64285713	0.53571429

Sample Pages from the Hypergeometric Table (continued)

N	N*	K*	X	PX	PX*
8	6	3	3	0.99999996	0.35714284
8	6	4	2	0.21428566	0.21428566
8	6	4	3	0.78571402	0.57142836
8	6	4	4	0.99999967	0.21428566
8	6	5	3	0.35714277	0.35714277
8	6	5	4	0.89285703	0.53571426
8	6	5	5	0.99999987	0.10714284
8	6	6	4	0.53571426	0.53571426
8	6	6	5	0.96428557	0.42857131
8	6	6	6	0.99999985	0.03571428
8	7	1	0	0.12499995	0.12499995
8	7	1	1	0.99999988	0.87499993
8	7	2	1	0.24999995	0.24999995
8	7	2	2	0.99999987	0.74999992
8	7	3	2	0.37499998	0.37499998
8	7	3	3	0.99999994	0.62499996
8	7	4	3	0.49999987	0.49999987
8	7	4	4	0.99999975	0.49999987
8	7	5	4	0.62499996	0.62499996
8	7	5	5	0.99999994	0.37499998
8	7	6	5	0.74999992	0.74999992
8	7	6	6	0.99999987	0.24999995
8	7	7	6	0.87499988	0.87499988
8	7	7	7	0.99999983	0.12499995
9	1	1	0	0.88888889	0.88888889
9	1	1	1	0.99999999	0.11111110
9	2	1	0	0.77777776	0.77777776
9	2	1	1	0.99999997	0.22222221
9	2	2	0	0.58333331	0.58333331
9	2	2	1	0.97222216	0.38888886
9	2	2	2	0.99999993	0.02777776
9	3	1	0	0.66666666	0.66666666
9	3	1	1	0.99999999	0.33333332
9	3	2	0	0.41666666	0.41666666
9	3	2	1	0.91666663	0.49999997
9	3	2	2	0.99999995	0.08333332
9	3	3	0	0.23809522	0.23809522
9	3	3	1	0.77380951	0.53571429
9	3	3	2	0.98809519	0.21428568
9	3	3	3	0.99999995	0.01190475
9	4	1	0	0.55555554	0.55555554
9	4	1	1	0.99999999	0.44444444
9	4	2	0	0.27777774	0.27777774
9	4	2	1	0.83333325	0.55555551
9	4	2	2	0.99999988	0.16666663
9	4	3	0	0.11904762	0.11904762
9	4	3	1	0.59523805	0.47619043
9	4	3	2	0.95238093	0.35714287
9	4	3	3	0.99999996	0.04761904

Sample Pages from the Hypergeometric Table (continued)

N	N*	K*	X	PX	PX*
9	4	4	0	0.03968253	0.03968253
9	4	4	1	0.35714284	0.31746031
9	4	4	2	0.83333333	0.47619049
9	4	4	3	0.99206348	0.15873015
9	4	4	4	0.99999998	0.00793650
9	5	1	0	0.44444441	0.44444441
9	5	1	1	0.99999996	0.55555554
9	5	2	0	0.16666663	0.16666663
9	5	2	1	0.72222210	0.55555546
9	5	2	2	0.99999984	0.27777774
9	5	3	0	0.04761903	0.04761903
9	5	3	1	0.40476190	0.35714287
9	5	3	2	0.88095234	0.47619043
9	5	3	3	0.99999996	0.11904762
9	5	4	0	0.00793650	0.00793650
9	5	4	1	0.16666662	0.15873012
9	5	4	2	0.64285711	0.47619049
9	5	4	3	0.96031742	0.31746031
9	5	4	4	0.99999995	0.03968253
9	5	5	1	0.03968253	0.03968253
9	5	5	2	0.35714275	0.31746022
9	5	5	3	0.83333316	0.47619040
9	5	5	4	0.99206328	0.15873012
9	5	5	5	0.99999978	0.00793650
9	6	1	0	0.33333332	0.33333332
9	6	1	1	0.99999996	0.66666664
9	6	2	0	0.08333332	0.08333332
9	6	2	1	0.58333327	0.49999995
9	6	2	2	0.99999993	0.41666666
9	6	3	0	0.01190475	0.01190475
9	6	3	1	0.22619043	0.21428568
9	6	3	2	0.76190477	0.53571434
9	6	3	3	0.99999999	0.23809522
9	6	4	1	0.04761903	0.04761903
9	6	4	2	0.40476187	0.35714284
9	6	4	3	0.88095227	0.47619040
9	6	4	4	0.99999989	0.11904762
9	6	5	2	0.11904762	0.11904762
9	6	5	3	0.59523802	0.47619040
9	6	5	4	0.95238086	0.35714284
9	6	5	5	0.99999989	0.04761903
9	6	6	3	0.23809522	0.23809522
9	6	6	4	0.77380956	0.53571434
9	6	6	5	0.98809524	0.21428568
9	6	6	6	0.99999999	0.01190475
9	7	1	0	0.22222221	0.22222221
9	7	1	1	0.99999990	0.77777769
9	7	2	0	0.02777776	0.02777776

N	N*	K*	X	PX	PX*
9	7	2	1	0.41666657	0.38888881
9	7	2	2	0.99999981	0.58333325
9	7	3	1	0.08333332	0.08333332
9	7	3	2	0.58333327	0.49999995
9	7	3	3	0.99999993	0.41666666
9	7	4	2	0.16666663	0.16666663
9	7	4	3	0.72222210	0.55555546
9	7	4	4	0.99999984	0.27777774
9	7	5	3	0.27777774	0.27777774
9	7	5	4	0.83333320	0.55555546
9	7	5	5	0.99999984	0.16666663
9	7	6	4	0.41666666	0.41666666
9	7	6	5	0.91666660	0.49999995
9	7	6	6	0.99999993	0.08333332
9	7	7	5	0.58333325	0.58333325
9	7	7	6	0.97222205	0.38888881
9	7	7	7	0.99999981	0.02777776
9	8	1	0	0.11111110	0.11111110
9	8	1	1	1.00000010	0.88888901
9	8	2	1	0.22222221	0.22222221
9	8	2	2	0.99999997	0.77777776
9	8	3	2	0.33333336	0.33333336
9	8	3	3	1.00000003	0.66666667
9	8	4	3	0.44444441	0.44444441
9	8	4	4	1.00000000	0.55555560
9	8	5	4	0.55555560	0.55555560
9	8	5	5	1.00000000	0.44444441
9	8	6	5	0.66666667	0.66666667
9	8	6	6	1.00000003	0.33333336
9	8	7	6	0.77777787	0.77777787
9	8	7	7	1.00000007	0.22222221
9	8	8	7	0.88888926	0.88888926
9	8	8	8	1.00000039	0.11111113
10	1	1	0	0.90000001	0.90000001
10	1	1	1	0.99999999	0.09999998
10	2	1	0	0.80000000	0.80000000
10	2	1	1	1.00000000	0.20000000
10	2	2	0	0.62222222	0.62222222
10	2	2	1	0.97777776	0.35555553
10	2	2	2	0.99999998	0.02222222
10	3	1	0	0.70000001	0.70000001
10	3	1	1	1.00000000	0.29999999
10	3	2	0	0.46666663	0.46666663
10	3	2	1	0.93333326	0.46666663
10	3	2	2	0.99999993	0.06666666
10	3	3	0	0.29166663	0.29166663
10	3	3	1	0.81666663	0.52500000
10	3	3	2	0.99166659	0.17499996
10	3	3	3	0.99999991	0.00833333

Sample Pages from the Hypergeometric Table (continued)

N	N*	K*	X	PX	PX*
10	4	1	0	0.59999999	0.59999999
10	4	1	1	0.99999996	0.39999997
10	4	2	0	0.33333332	0.33333332
10	4	2	1	0.86666662	0.53333330
10	4	2	2	0.99999994	0.13333332
10	4	3	0	0.16666663	0.16666663
10	4	3	1	0.66666663	0.50000000
10	4	3	2	0.96666662	0.29999999
10	4	3	3	0.99999995	0.03333332
10	4	4	0	0.07142855	0.07142855
10	4	4	1	0.45238087	0.38095232
10	4	4	2	0.88095234	0.42857146
10	4	4	3	0.99523802	0.11428569
10	4	4	4	0.99999992	0.00476190
10	5	1	0	0.50000004	0.50000004
10	5	1	1	1.00000009	0.50000004
10	5	2	0	0.22222221	0.22222221
10	5	2	1	0.77777775	0.55555554
10	5	2	2	0.99999996	0.22222221
10	5	3	0	0.08333332	0.08333332
10	5	3	1	0.50000001	0.41666669
10	5	3	2	0.91666671	0.41666669
10	5	3	3	1.00000003	0.08333332
10	5	4	0	0.02380951	0.02380951
10	5	4	1	0.26190476	0.23809525
10	5	4	2	0.73809525	0.47619049
10	5	4	3	0.97619050	0.23809525
10	5	4	4	1.00000001	0.02380951
10	5	5	0	0.00396825	0.00396825
10	5	5	1	0.10317459	0.09920634
10	5	5	2	0.49999995	0.39682535
10	5	5	3	0.89682530	0.39682535
10	5	5	4	0.99603164	0.09920634
10	5	5	5	0.99999989	0.00396825
10	6	1	0	0.39999995	0.39999995
10	6	1	1	0.99999995	0.59999999
10	6	2	0	0.13333332	0.13333332
10	6	2	1	0.66666657	0.53333326
10	6	2	2	0.99999990	0.33333332
10	6	3	0	0.03333332	0.03333332
10	6	3	1	0.33333326	0.29999994
10	6	3	2	0.83333326	0.50000000
10	6	3	3	0.99999990	0.16666663
10	6	4	0	0.00476190	0.00476190
10	6	4	1	0.11904758	0.11428569
10	6	4	2	0.54761898	0.42857140
10	6	4	3	0.92857122	0.38095225
10	6	4	4	0.99999978	0.07142855
10	6	5	1	0.02380951	0.02380951

Sample Pages from the Hypergeometric Table (continued)

N	N*	K*	X	PX	PX*
10	6	5	2	0.26190474	0.23809522
10	6	5	3	0.73809515	0.47619040
10	6	5	4	0.97619037	0.23809522
10	6	5	5	0.99999989	0.02380951
10	6	6	2	0.07142855	0.07142855
10	6	6	3	0.45238080	0.38095225
10	6	6	4	0.88095219	0.42857140
10	6	6	5	0.99523788	0.11428569
10	6	6	6	0.99999978	0.00476190
10	7	1	0	0.29999999	0.29999999
10	7	1	1	0.99999995	0.69999996
10	7	2	0	0.06666666	0.06666666
10	7	2	1	0.53333329	0.46666663
10	7	2	2	0.99999993	0.46666663
10	7	3	0	0.00833333	0.00833333
10	7	3	1	0.18333329	0.17499996
10	7	3	2	0.70833329	0.52500000
10	7	3	3	0.99999992	0.29166663
10	7	4	1	0.03333332	0.03333332
10	7	4	2	0.33333326	0.29999994
10	7	4	3	0.83333321	0.49999995
10	7	4	4	0.99999984	0.16666663
10	7	5	2	0.08333332	0.08333332
10	7	5	3	0.49999998	0.41666666
10	7	5	4	0.91666663	0.41666666
10	7	5	5	0.99999996	0.08333332
10	7	6	3	0.16666663	0.16666663
10	7	6	4	0.66666658	0.49999995
10	7	6	5	0.96666653	0.29999994
10	7	6	6	0.99999984	0.03333332
10	7	7	4	0.29166663	0.29166663
10	7	7	5	0.81666663	0.52500000
10	7	7	6	0.99166659	0.17499996
10	7	7	7	0.99999991	0.00833333
10	8	1	0	0.19999997	0.19999997
10	8	1	1	1.00000003	0.80000007
10	8	2	0	0.02222222	0.02222222
10	8	2	1	0.37777776	0.35555553
10	8	2	2	1.00000001	0.62222227
10	8	3	1	0.66666666	0.06666666
10	8	3	2	0.53333329	0.46666663
10	8	3	3	0.99999993	0.46666663
10	8	4	2	0.13333332	0.13333332
10	8	4	3	0.66666657	0.53333326
10	8	4	4	0.99999993	0.33333336
10	8	5	3	0.22222221	0.22222221
10	8	5	4	0.77777781	0.55555560
10	8	5	5	1.00000001	0.22222221
10	8	6	4	0.33333336	0.33333336

Sample Pages from the Hypergeometric Table (continued)

N	N*	K*	X	PX	PX*
10	8	6	5	0.86666661	0.53333326
10	8	6	6	0.99999996	0.13333334
10	8	7	5	0.46666673	0.46666673
10	8	7	6	0.93333346	0.46666673
10	8	7	7	1.00000013	0.06666668
10	8	8	6	0.62222243	0.62222243
10	8	8	7	0.97777797	0.35555553
10	8	8	8	1.00000018	0.02222222
10	9	1	0	0.09999998	0.09999998
10	9	1	1	0.99999995	0.89999997
10	9	2	1	0.19999997	0.19999997
10	9	2	2	1.00000003	0.80000007
10	9	3	2	0.29999999	0.29999999
10	9	3	3	0.99999995	0.69999996
10	9	4	3	0.39999992	0.39999992
10	9	4	4	0.99999992	0.59999999
10	9	5	4	0.50000004	0.50000004
10	9	5	5	1.00000009	0.50000004
10	9	6	5	0.59999999	0.59999999
10	9	6	6	0.99999992	0.39999992
10	9	7	6	0.69999996	0.69999996
10	9	7	7	0.99999999	0.30000003
10	9	8	7	0.80000018	0.80000018
10	9	8	8	1.00000010	0.19999994
10	9	9	8	0.89999984	0.89999984
10	9	9	9	0.99999982	0.09999998
11	1	1	0	0.90909091	0.90909091
11	1	1	1	0.99999998	0.09090907
11	2	1	0	0.81818181	0.81818181
11	2	1	1	0.99999998	0.18181816
11	2	2	0	0.65454542	0.65454542
11	2	2	1	0.98181814	0.32727272
11	2	2	2	0.99999995	0.01818181
11	3	1	0	0.72727271	0.72727271
11	3	1	1	0.99999997	0.27272726
11	3	2	0	0.50909089	0.50909089
11	3	2	1	0.94545452	0.43636362
11	3	2	2	0.99999996	0.05454545
11	3	3	0	0.33939391	0.33939391
11	3	3	1	0.84848481	0.50909090
11	3	3	2	0.99393935	0.14545455
11	3	3	3	0.99999996	0.00606060
11	4	1	0	0.63636363	0.63636363
11	4	1	1	0.99999999	0.36363637
11	4	2	0	0.38181816	0.38181816
11	4	2	1	0.89090901	0.50909085
11	4	2	2	0.99999991	0.10909090
11	4	3	0	0.21212120	0.21212120
11	4	3	1	0.72121204	0.50909085

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