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TRANSIENT HYDRAULIC ANALYSIS FOR REACTOR CORES(1)

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DISCLAIMER

The objective of this paper is to present mathematical solution approaches for lumped-parameter equations used in nuclear rod-bundle thermal-hydraulic analysis. In the lumped-parameter approach the cross section is divided into subchannels which are further subdivided axially to form a three-dimensional array of control volumes for analysis.(1,2,3,4)

The lumped parameter mathematical models are written in various degrees of sophistication for both single-and two-phase flows for water, liquid-metal and gas-cooled reactors. For purposes of discussion, this paper is limited to a homogeneous thermal-equilibrium flow without sonic velocity propagation. This allows fluid mixture density to be expressed in terms of fluid enthalpy.

The essential features of the flow equations required for this paper may be put into the vector form

$$A \frac{\partial \rho}{\partial t} + \frac{\partial m}{\partial x} + D^T w = 0 \tag{1}$$

$$\frac{\partial m}{\partial t} + A \frac{\partial p}{\partial x} = f_1 (\rho, m, w) \tag{2}$$

$$\frac{\partial w}{\partial t} - Dp = f_2 (\rho, m, w) \tag{3}$$

which are the equations of continuity, axial-momentum and lateral-momentum, respectively.(4) The functions on the right side contain the momentum flux and external forces on the control volume. The lateral momentum equations are each vector directed but are not cross coupled by the lateral velocities.

The matrix D in the above equations is an operator to order the differential equations in the lateral direction.(3) D takes the difference between quantities of adjacent subchannels and is analogous to a lateral differential operator. The matrix D also implicitly contains the lateral boundary conditions.

The pressure-velocity and vorticity-velocity methods(5) are two general approaches for solving the above equations. For the pressure-velocity method,(5,6) Equations (2) and (3) are combined to eliminate the time derivatives. The result is

$$\frac{\partial^2 p}{\partial x^2} + DA^{-1}D^T p = f_3\ (\rho,\ m,\ w). \qquad (4)$$

This is analogous to a Poisson equation for the pressure field and may be solved by using two-point boundary conditions for pressure. Once the pressure solution is obtained, Equations (2) and (3) may be used to advance the flow solution. Special provision must be included in this approach to satisfy the continuity equation since it is not insured in the calculation of flows. This can be done by requiring very tight convergence on the pressure solution and/or correcting the advanced flow solution by using a "potential-like" function(7) derived from the continuity error.

In the second method,(5) the momentum equations are combined to eliminate pressure. The result is

$$\frac{\partial}{\partial t}\left(\frac{\partial w}{\partial x} - DA^{-1}m\right) = f_3\ (\rho,\ m,\ w). \qquad (5)$$

The quantity on the left in parenthesis may be defined as Ω which is recognized as being analogous to vorticity. From Equations (5) Ω may be advanced in time by using an explicit finite difference equation. With Ω known, it is possible to determine the lateral velocity by differentiating Ω and substituting $\partial m/\partial x$ from Equation (1) to give

$$\frac{\partial^2 w}{\partial x^2} + DA^{-1}D^T w = \frac{\partial \Omega}{\partial x} - D\,\frac{\partial \rho}{\partial t} \tag{6}$$

This is again analogous to a Poisson equation which may be solved by using two-point boundary conditions on w. Once w has been obtained, m is calculated by integrating Equation (1) using m_o as a boundary condition.

The boundary condition m_o may be specified directly or it may be calculated from specified pressure boundary conditions by using the momentum integral technique.(8) An integration of Equation (2) over the length of the core gives an equation for the channel average flow rate

$$A^{-1}\,\frac{d\hat{m}}{dt} = \frac{\Delta p}{L} + \frac{1}{L}\int_0^L A^{-1} f_1\ (\rho,\ m,\ w)\ dx \tag{7}$$

The right side can be evaluated since Δp is a boundary condition and the data for the integration of f_1 is known. Integrating the continuity equation once gives

$$m = m_o - \int_0^x (D^T w + A\,\frac{\partial \rho}{\partial t})\ dx' \tag{8}$$

and again from 0 to L gives

$$\hat{m} = m_o - \frac{1}{L}\int_0^L \int_o^x (D^T \dot{w} + A \frac{\partial \rho}{\partial t})\, dx'\, dx \tag{9}$$

which may be solved for m_o, the required inlet flow boundary condition.

The previous methods have been incorporated into the COBRA computer program. These methods are providing important new computational capabilities required for reactor safety analysis.

NOMENCLATURE

A - Subchannel area

D - Matrix operator

m - Subchannel axial flow rate

p - Pressure

w - Intersubchannel crossflow rate per unit length

ρ - Density

t - Time

x - Axial distance

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