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BURNOUT DISTRIBUTION IN SM-1 (APPR-1) CONTROL ROD ELEMENTS FIXED ELEMENT No. 57 AND ABSORBER SECTIONS AT 10.5 MWYRS

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APAE MEMO NO. 199

BURNOUT DISTRIBUTION IN SM-1 (APPR-1) CONTROL ROD ELEMENTS, FIXED ELEMENT # 57 AND ABSORBER SECTIONS AT 10.5 MWYRS

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Army Package Power Reactor

Contract #AT (30-3)-326

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632 001

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FIGURES

ABSTRACT

An analytical prediction of the burnout distributions in particular SM-1 fuel elements and absorber sections after 10.5 MWYR of core energy release is given. The distributions are based on the results of a one-shot, non-uniform burnout calculation, and are presented for both fuel and boron -10 depletion. Particular emphasis is placed on those elements removed from the SM-1 core in March, 1959, since their subsequent burnup analysis by ORNL should provide a valuable check on the analytical models employed.

I. INDTRODUCTION

During the period of March 12-26, 1959, an on-site examination of SM-1 fuel elements and absorber sections was performed at Ft. Belvoir, Virginia. On the basis of the in-core examination (1) it was decided to replace the boron absorbers in rods 1, 2, 3, 4 and C, with europium absorbers, to replace the control rod fuel element in one of the eccentric rods (rod 4) with a new control rod fuel element, and to replace the fixed element in position #57 with a partially burned fixed fuel element from another core position (#56). See Figure 1. (2). Hot cell examination of the removed elements will be performed at ORNL; this examination will include burnup analysis of fuel and boron. This memo is an attempt to predict the burnup distributions from the analytical models currently employed in the reactor analysis section at ALCO Products, Inc.

The value of core energy release at the time of removal, as measured by the ^{O}F-day meter, was 10.5 MWYR. Since much of the analytical work is dependant on the results of a previously reported one-shot, non-uniform burnout calculation (3), it was decided to use the nearest calculated curves of 9.84 MWYR, and apply a factor of $\frac{10.5}{9.84} = 1.07$ to all subsequently calculated burnup distributions.

Comparison of the predicted and measured burnouts should provide a valuable check on the analytical model. Calculations for the centerline element, Rod C, have been included here as an indication of the maximum expected burnout distribution.

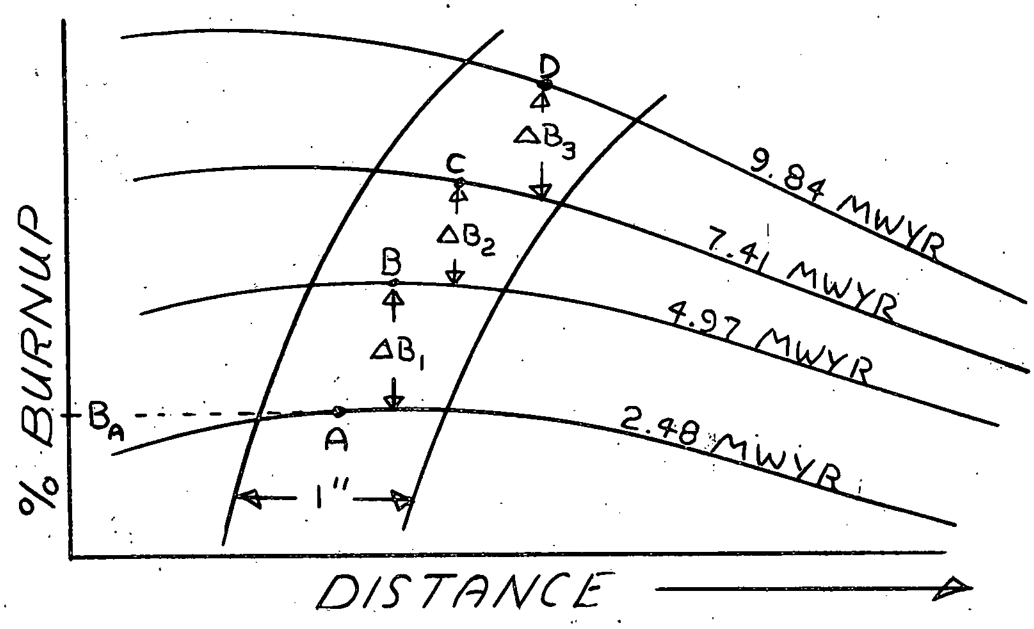
II. CONTROL ROD FUEL ELEMENTS

A. Method of Calculation - Fuel Burnout

The calculation of burnup distribution is based upon the results of the one-shot, non-uniform core burnup described in APAE Memo 126 (3). Figures 2, 3, and 5 are taken directly from APAE Memo 126 and are strictly valid only for stationary elements. In order to account for the control rod fuel elements which move up into the core as the absorber is withdrawn, the following approach was used.

Figure 4, which shows the average fuel burnup in the core as a function of axial position at various values of core energy release, was replotted from Figure 2. The bank motion over 9.84 MWYRS (Figure 3) was then superimposed on Figure 4 and one-inch increments along the control rod fuel element and absorber sections were plotted.

The average burnup over 9.84 MWYRS in a one-inch increment of control rod fuel element was calculated from Figure 4 as follows:



Section of Figure 4

C32 005

In 2.48MWYR the average burnup in this particular inch of fuel element is B_A. From 2.48 MWYR to 4.97 MWYR the increase in burnup is ΔB_i, where point "B" is the average burnup per increment over 4.94 MWYR. The total average fuel burnup in the one inch segment over 9.84 MWYR is then

$$\%B_{9.84} \doteq B_A + \Delta B_1 + \Delta B_2 + \Delta B_3$$

The average fuel burnup distribution over the inserted portion of the control rod fuel element was calculated in this manner.

Figure 2 was obtained from Windowshade calculations which consider an axial flux distribution at an average radial position. The burnup distribution in a particular control rod fuel element was assumed to vary as the ratio of the radial thermal flux at the element to the average radial thermal flux, i.e.

$$\% B(r) = \frac{\overline{\Phi}(r)}{\overline{\overline{\Phi}}_{th}} \times \overline{B}$$

Figure 5 $\quad \overline{\Phi}^{th}_{\mathrm{CL}} / \overline{\overline{\Phi}}_{th} = 1.478$ and $\overline{\Phi}^{th}_{ecc.} / \overline{\overline{\Phi}}_{th} = 1.183.$

It should be noted that the term fuel burnup is defined here as the fraction of original U-235 atoms removed by all processes. The fraction of original U-235 atoms lost through fission is then $\%$ Burnup/ $(1 + \alpha_{eff})$ where

$1 + \alpha_{eff}$ = effective capture to fission ratio

$= \beta (1 + \alpha_{th}) + (1 - \beta)(1 + \alpha_f)$

$1 + \alpha_{th}$ = thermal capture to fission ratio = 1.18

$1 + \alpha_f$ = fast capture to fission ratio = 1.42

β = percent thermal fissions as a function of average fuel burnout (3)

$= 0.7514 + (0.2060)(\% B)$

C32 006

$1 + \alpha_{eff} = 1.23$ for ℄ element and eccentric element and element #57

B. Water Hole Peaking

Task VII measurements (4) indicate that the flux peak in the water hole is not completely suppressed by the Haynes combs (See Figure 6). This flux peak is thought to be a local effect over the top 1/2" of the fuel, with the maximum occurring between the two middle plates and falling off towards the outer plates. To account for this effect, a local water hole peaking factor should be applied to the average fuel burnup at the bank position (top of fuel section) which varies from 1.2 for plate #8. to 1.0 for plate #1

C. Local Plate Factors

Measurements, and P_3 calculations indicate the presence of thermal flux peaks at the corners of the elements. In an attempt to relate the average fuel burnup distribution in an element to a local fuel burnup on a particular plate, a local plate factor, F_j^n, has been defined.

$$F_j^n = \bar{\beta} \left[\frac{\Phi^{th}(r)}{\bar{\Phi}^{th}} \right] + (1 - \bar{\beta})$$

where

$\bar{\beta}$ = average percent thermal fissions in element

$\Phi^{th}(r)$ = relative thermal flux at position " r " from the ℄ of element.

$\bar{\Phi}^{th}$ = relative average thermal flux

Both $\Phi^{th}(r)$ and $\bar{\Phi}^{th}$ were obtained from the cylindrical P-3 calculation as plotted in Figure 7.

The local fuel burnup on a particular plate is then

$$B_j^n = \bar{B} \times F_j^n$$

C32 007

where: n = plate No. (1, 5 or 8)

j = plate section (A, or B)

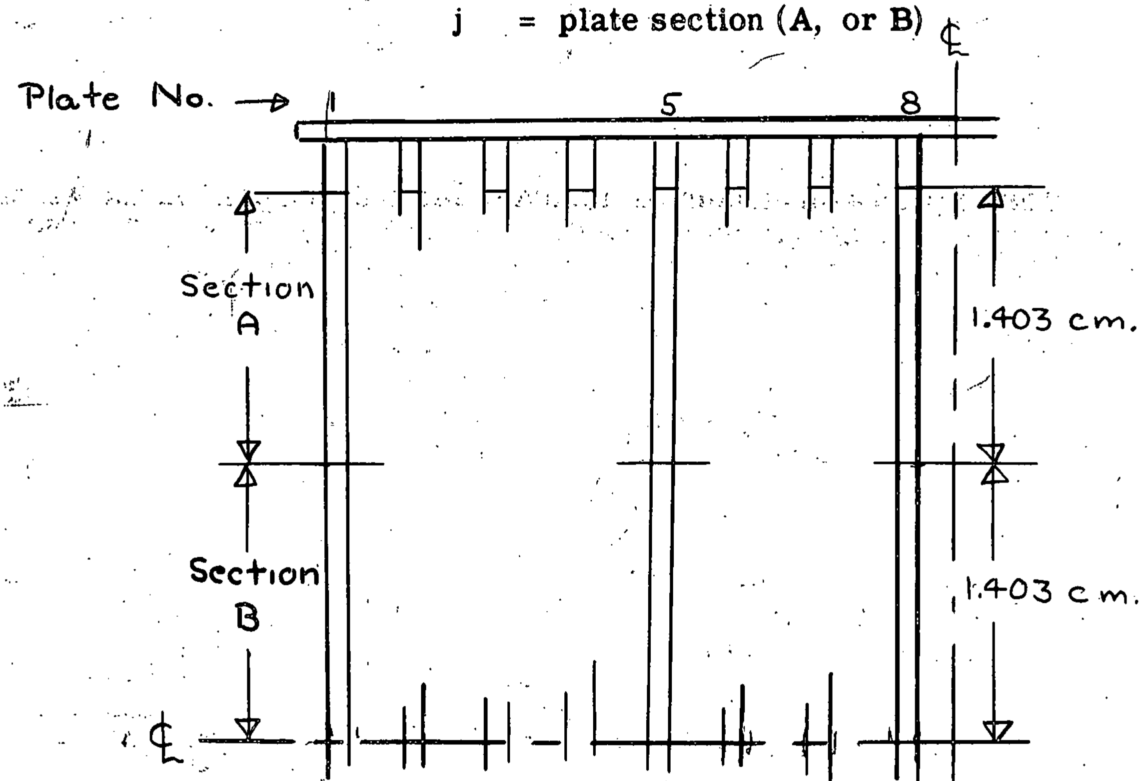


Figure 8 Quadrant of CRFE

TABLE 1 Local Plate Factors for Control Rod Fuel Elements

	℄ Element	Eccentric Element
F_A^1	1.154	1.151
F_B^1	1.111	1.109
F_A^5	1.004	1.004
F_B^5	.945	.946
F_A^8	.965	.965
F_B^8	.918	.920

The local burnup in a control rod fuel element is then found by the following recipe:

1. Determine the average fuel burnup in the element from Figure 9 or 10 as a function of axial position.

2. Apply a water hole peaking factor (1.2 for plate #8, 1.1 for plate #5, 1.0 for plate #1) if the axial position is within 0.5" from the top of the fuel section.

3. Apply the local plate factor for the particular plate and section from Table 1.

III. Burnup of Burnable Poision

The burnup of the B-10 in the control rod fuel elements and in element #57 may be related to the fuel burnup (where again the fuel burnup is based on U-235 depletion due to all processes) by the following equation.

$$(1-B^{B-10}) = (1-B^{fuel})^{g'} \qquad (3)$$

where

$$g' = \frac{\left(\frac{\sigma_{th}^{a-B}}{\sigma_{th}^{a}}\right)(\beta)(1+\alpha_{th}) + \left(\frac{\sigma_{f}^{a-B}}{\sigma_{f}^{a}}\right)(1-\beta)(1+\alpha_{f})}{(\beta)(1+\alpha_{th}) + (1-\beta)(1+\alpha_{f})}$$

Values of "g'" corresponding to the average fuel burnup in the element have been used in this calculation

$$\overline{g'}_{℄} = 5.8199$$

$$\overline{g'}_{ecc.} = 5.8036$$

$$\overline{g'}_{\#57} = 5.7948$$

The burnup distributions of the burnable poison are also included in Figures 9 and 10.

III. BURNUP DISTRIBUTION IN ABSORBER SECTION

A. Method of Calculation

The term "burnup" in the absorber sections has been defined as the number of absorptions per original number of B-10 atoms, i.e. $B = {}^{A}/_{No.}$

The number of thermal absorptions in the total rodded region of the core in a time "t" can be found from the Windowshade model.

$$\text{Absorptions} = \overline{\overline{\Phi}}^{th} \times \frac{\overline{\Phi}_R}{\overline{\Phi}_C} \times \Sigma_p \times V \times t \times \epsilon \tag{6}$$

where $\overline{\overline{\Phi}}^{th}$ = average absolute thermal flux in the core = 1.36×10^{13} neutrons/cm^2 sec. (7)

$\overline{\Phi}_C$ = relative average Windowshade flux in core = 4.83 (3)

$\overline{\Phi}_R$ = relative average Windowshade flux over initial rodded region of core = 2.15 (3)

Σ_P = Equivalent uniform absorption cross section which represents the effect of the 5 rod bank = 0.07755 cm^{-1} (3)

V = Volume of core associated with Σ_p, and is volume of portion of core into which absorber is inserted

t = time in seconds.

ϵ = fraction of Σ_p associated with the insertion of absorbers = .8863 (8)

The number of absorptions in a particular one-inch section of inserted absorber bank (5 rods) over 10.5 MWYRS may be calculated from Figure 4, thus:

$$\text{Absorption/inch} = \overline{\overline{\Phi}}^{th} \times \frac{\overline{\Phi}_R}{\overline{\Phi}_C} \times \Sigma_p \times V^* \times \epsilon \times 1.07 \times \left[\frac{B_1^i}{\overline{B}_1}\Delta t_1 + \frac{B_1^i + \Delta B_2^i}{\overline{B}_2}\Delta t_2 + \frac{B_2^i + \Delta B_3^i}{\overline{B}_3}\Delta t_3 + \frac{B_3^i + \Delta B_4^i}{\overline{B}_4}\Delta t_4 \right] \equiv A^i_{total}$$

C32 010

where V^* = Volume of a 1 inch slab of core

Bn^i = % fuel burnup in inch "i" of absorber section after "n" MWYRS

$\overline{Bn}$ = average fuel burnup in rodded region of the core over "n" MWYRS

ΔBn^i = increase in % fuel burnup in inch "i" of absorber from "n-1" to "n" MWYRS

Δt_n = time increment between "n-1" and "n" MWYRS, seconds

n	
1	= 2.48 MWYRS
2	= 4.97 "
3	= 7.41 "
4	= 9.84 "

From Figure 5

$$\Phi_{\text{¢}} / \overline{\Phi} = 1.478 \text{ and}$$
$$\Phi_{ecc} / \overline{\Phi} = 1.183$$

The number of absorptions in a particular rod is then

$$A^i_{\text{¢}} = \frac{(1.478)\, A^i_{total}}{1.478 + (4)(1.183)} = 0.238\, A^i_{total}$$

$$A^i_{ecc} = \frac{(1.183)\, A^i_{total}}{1.478 + (4)(1.183)} = 0.191\, A^i_{total}$$

Assuming an initial loading of 14.1 gm. B-10 per plate, the original number of B-10 atoms per inch of absorber element (4 plates) is, 0.163228×10^{24}. Figures 9 and 10 show the absorber burnup (absorptions per atom) for the "in-core" sections of the rods.

B. Comparison with K_{ex} Method

An alternative method of calculating the total number of absorptions in the rods over 10.5 MWYRS is the excess reactivity method (6). In this model a fraction "λ" of the excess neutrons (K_{eff} - 1) are assumed to be captured in the rods. From reactivity measurements on Rod A (4) $\lambda = 0.543$.

Number of absorption in rods $= P \delta \nu t \lambda \overline{K_{ex}}$

where: P = Reactor Power $= 10^7$ watts

δ = Fission/watt sec. $= 3.24 \times 10^{10}$

ν = neutrons/fission = 2.46

t = time increment, seconds

$\overline{K_{ex}}$ = average excess reactivity in time "t"

For purposes of comparison, the total number of absorptions in the rods in 10.5 MWYR was calculated using the Σ p method and the excess reactivity method. The average K_{ex} over 10.5 MWYRS was 0.065 and the average bank position was 9.86 inches.

		Absorptions in Rods
K_{ex} method	=	0.931×10^{24}
Σ_p method	=	0.985×10^{24}

Chemical analysis of the boron absorber removed from the SM-1 should provide a check of these figures.

IV. BURNUP DISTRIBUTION IN FIXED ELEMENT NUMBER 57

Figure 11 shows the axial fuel burnout distribution at 10.5 MWYR at an average radial position in the core. To relate this average burnup distribution to a local burnup in element #57 a local factor "L_j^n" was defined such that

$$B(n, j, z) = B(z) \times L_j^n$$

where: $B(n, j, z)$ = local fuel burnup in element #57 at position (n, j, z)

$B(z)$ = Average axial fuel burnout distribution from Figure 11.

The local factor, "L_j^n" may be written as

$$L_j^n = A \times F(x) \times F(y) \times P(j)$$

n = number of fuel plate in element #57 (See Figure 12)

j = Section of fuel plate in element #57 (See Figure 12)

A = Normalization constant = 1.49 - Valprod relative power at centerline.

$F(x)$, $F(y)$ = Geometrical power factors which result from an attempt to separate a radial power distribution into x and y components. (9)

where $x = f(n)$, $y = f(j)$

$P(j)$ = side plate peaking factor obtained from P_3 calculations.

TABLE II - Side Plate Peaking Factors

j	P (j)
A	1.05

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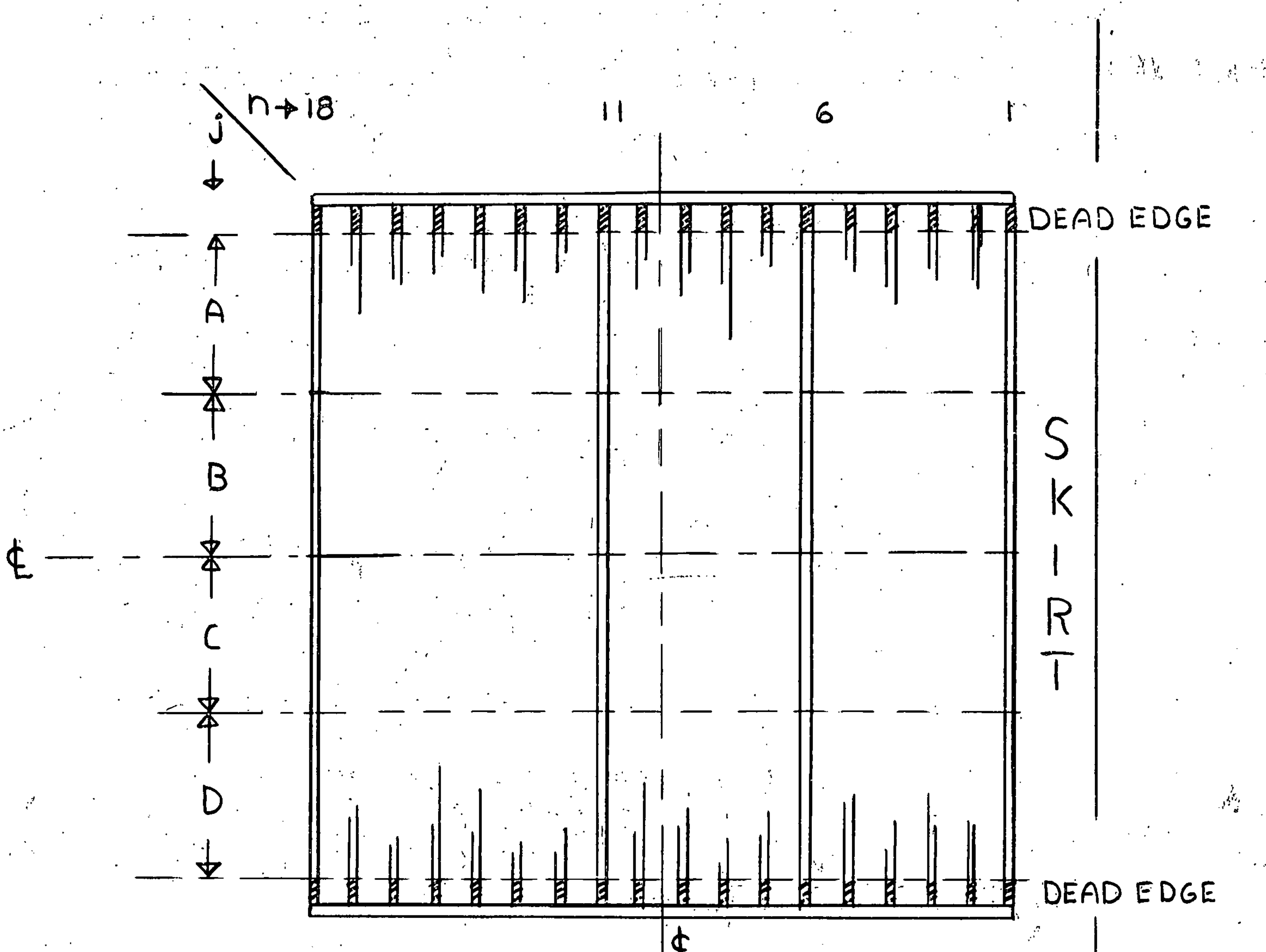
TABLE II (Cont.)

j	P (j)
B	0.95
C	0.95
D	1.05



Fuel Element #57

Fig. 12

TABLE III Local Burnup Factors for Element #57

L_j^n		
L_A^{18}	=	.973
L_C^{18}	=	.841
L_D^{18}	=	.897
L_A^{11}	=	.827
L_C^{11}	=	.715
L_A^{6}	=	.830
L_C^{6}	=	.717
L_A^{1}	=	1.621
L_C^{1}	=	1.400
L_D^{1}	=	1.494

To determine the local burnup in Element #57:

1. Determine average axial burnup, $B(z)$, in element from Figure 11.

2. Apply local burnup factor "L_j^n" from Table III (note there is no water hole peaking factor for fixed element #57).

It should again be emphasized that the fuel burnups are calculated as % U-235 depleted by all processes. To determine % U-235 depleted by fission the burnout should be divided by the effective capture to fission ratio, $1 + \alpha_{eff}$.

$$1 + \alpha_{eff} = 1.23 \text{ for element \#57}$$

V. RESULTS

The burnout distributions are shown in Figures 9, 10 and 11. Figures 9, 10 and 11 include the factor of 1.07 to account for the difference between available calculated curves (9.84 MWYR) and the energy release at the time of removal from the core. (10.5 MWYR) In the control rod fuel elements, the maximum fuel burnup occurs in Section A of plate #1 approximately 5 inches below the top of the fuel. In element #57 the maximum fuel burnup occurs in Section A of the outer plate (plate #1) approximately 5 inches from the bottom of the core. Applying local peaking factor F_A^1 to the control rod fuel elements and L'_A to element #57, the average and maximum fuel burnups are

Element	Average Fuel Burnup in the Element	Maximum Local Fuel Burnup
(Rod C)	0.25	0.68
Ecc. Rod (Rod 4)	0.21	0.54
#57	0.19	0.63

Figures 9 and 10 also indicate that the burnable poison is almost completely burned out over a large portion of the inserted control rod fuel elements - based upon the average fuel burnouts in these elements. In element #57, the burnout distribution of the B-10 was calculated using Figure 11 and an average local factor $\overline{L}_j^n$ for the entire element ($\overline{L}_{57} = 0.809$). The average boron burnup distribution is shown in Figure 12. The maximum average burnup of the B-10 in this element is $\simeq$ 0.90.

The maximum burnup in the control rod absorber sections occurs at the lower tip of the absorber, and is $\simeq$ 0.27 for the ℄ rod, and $\simeq$ 0.22 for an eccentric rod.

APPENDIX I

Fuel Accountability Calculations

Rod 4

Initial loading per plate	26.11 grams U-235 (10)
Average Fuel Burnup	0.211
Grams U-235 Burned per plate	5.51
Grams U-235 Remaining per plate	20.60
Grams U-235 Remaining per element	329.60

Element #57

Initial loading per plate	28.62 grams U-235 (10)
Average Fuel Burnup (Fig. 10 x $\overline{L}_{57}$) per plate	0.235 x .809 = 0.190
Grams U-235 Burned per plate	5.44
Grams U-235 Remaining per plate	23.18
Grams U-235 Remaining in element	417.24

APPENDIX II

Comparison With Estimated Fuel Burnups at End of Core Life

	Core Energy 10.5 MWYR	Release 15 MWYR
% Burnup[1], U-235		
Average	23%	33%
Gross Maximum[2].	54%	73%
Local Maximum[3].	68%	93%
Atom % Burnup of Meat[4]. (Due to U-235 Depletion)		
Average	1.3%	1.9%
Gross Maximum	3.0%	4.1%
Local Maximum	3.8%	5.2%

1) Burnup based upon depletion of U-235 By all Processes.

2) Gross maximum burnup determined by synthesis of one dimensional radial and axial calculation. This is applicable over areas of approximately 2 in.^{2} or larger.

3) Local maximum burnup determined by methods described in this report. These are point burnups applicable over areas of roughly 0.5 inches^2.

4) Based upon 5.61 At. % U-235 in meat.

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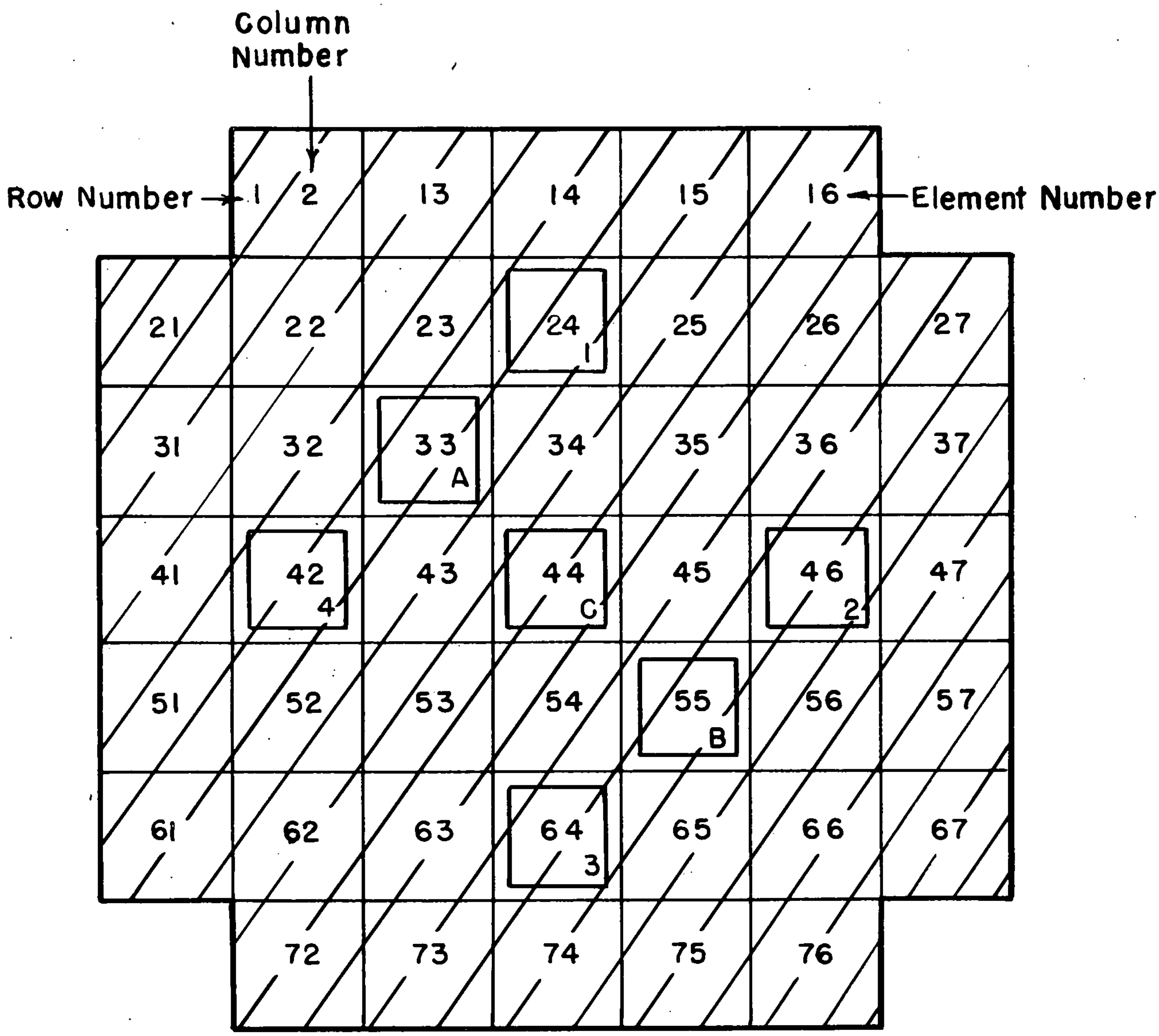
FIG. I

CORE ARRAY



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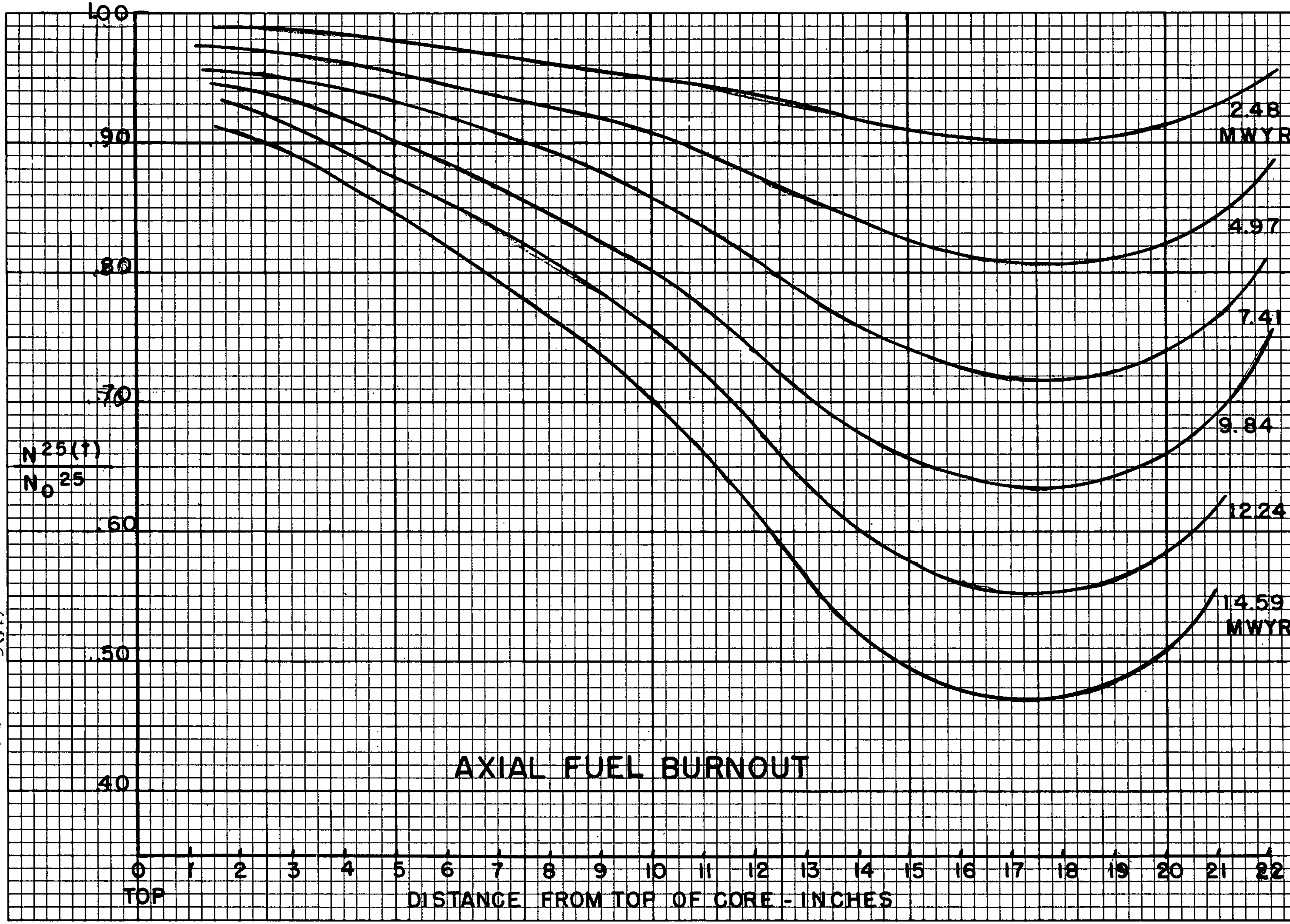
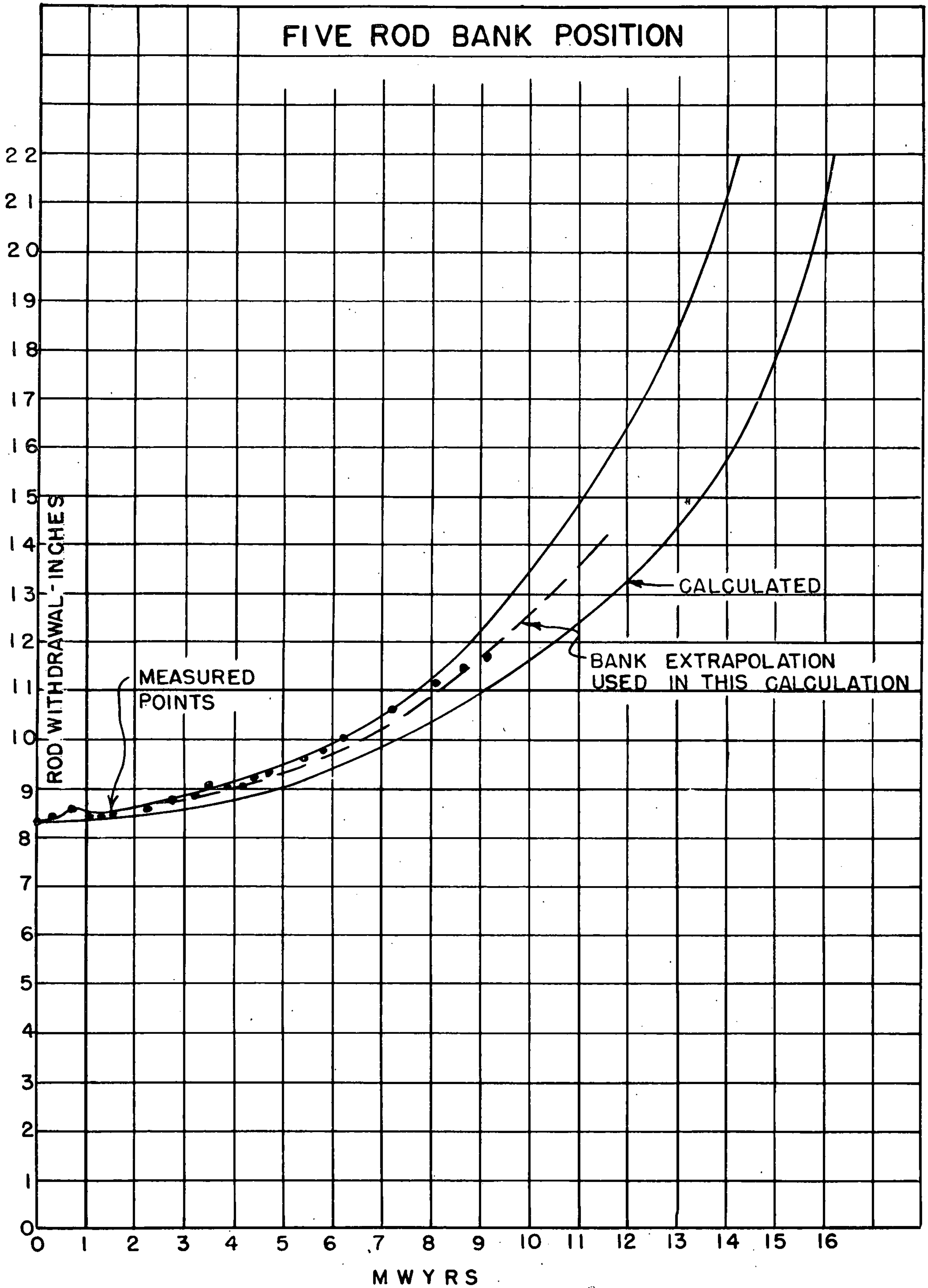

AXIAL FUEL BURNOUT
$\frac{N^{25}(t)}{N_0^{25}}$
1.00
.90
.80
.70
.60
.50
.40
0
TOP
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
DISTANCE FROM TOP OF CORE - INCHES
2.48
MWYR
4.97
7.41
9.84
12.24
14.59
MWYR

FIG.2

FIG. 3


FIVE ROD BANK POSITION
ROD WITHDRAWAL - INCHES
MEASURED POINTS
CALCULATED
BANK EXTRAPOLATION USED IN THIS CALCULATION
MWYRS
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
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19
20
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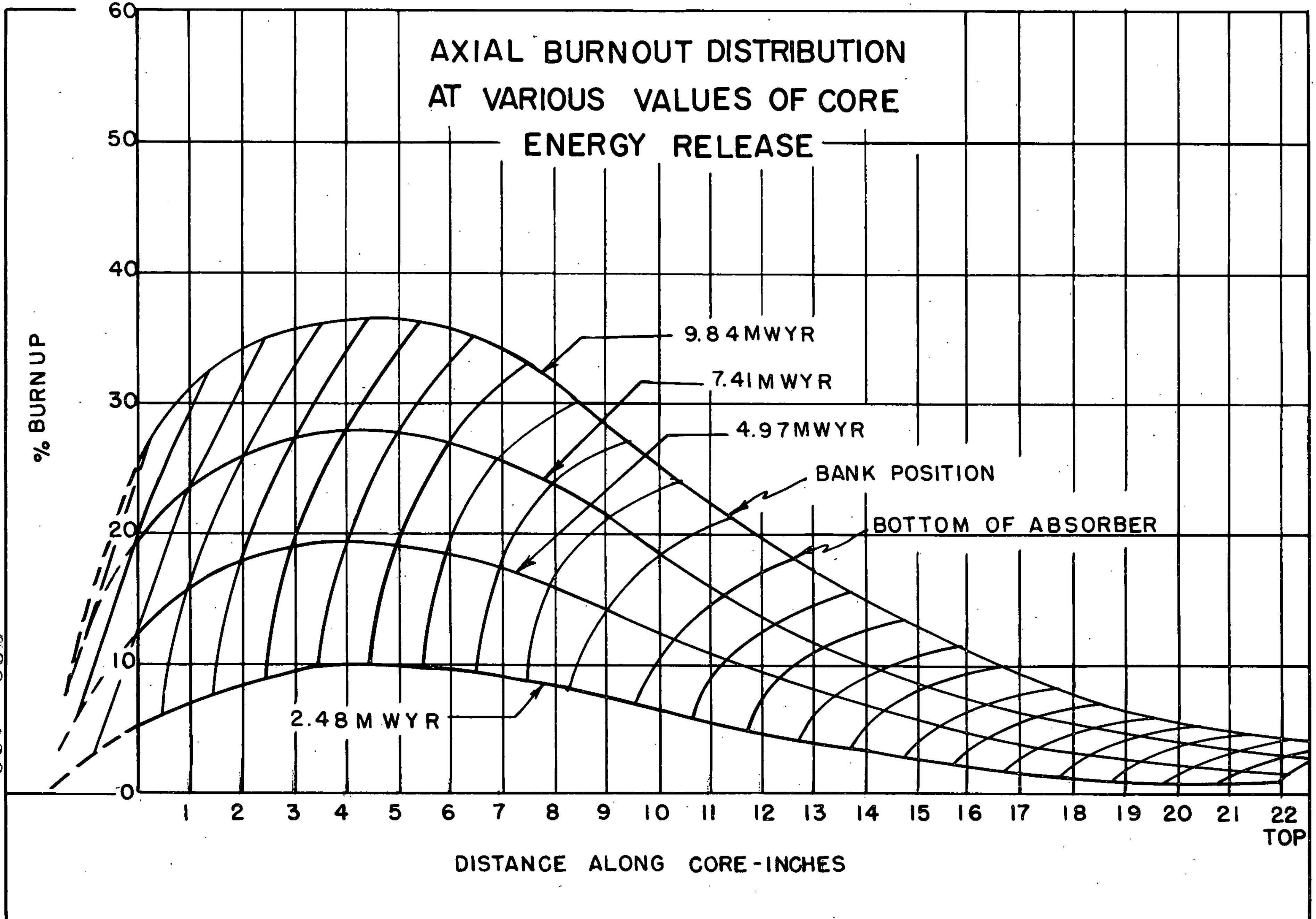
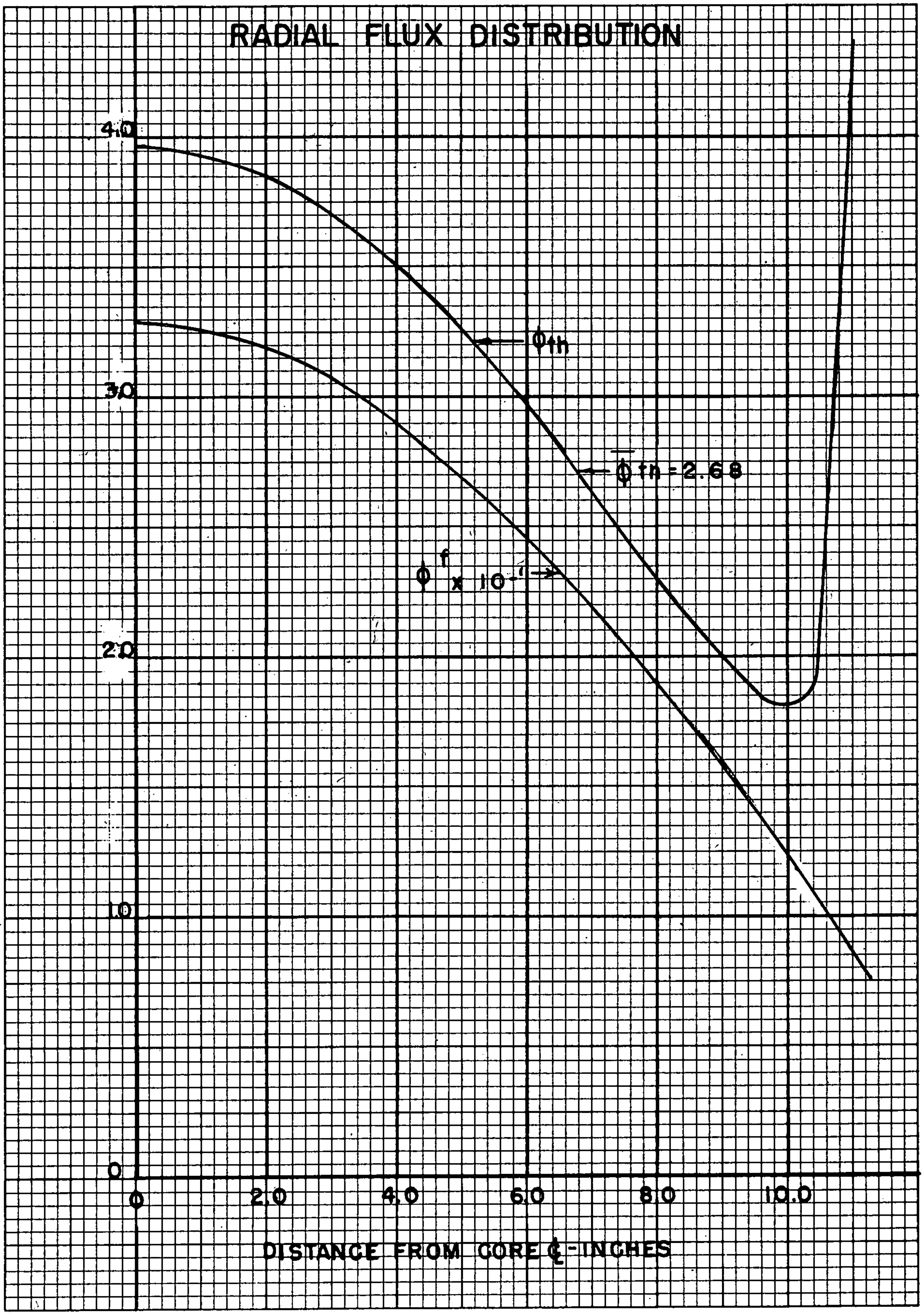

AXIAL BURNOUT DISTRIBUTION
AT VARIOUS VALUES OF CORE
ENERGY RELEASE
% BURNUP
60
50
40
30
20
10
0
9.84 MWYR
7.41 MWYR
4.97 MWYR
BANK POSITION
BOTTOM OF ABSORBER
2.48 MWYR
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22
TOP
DISTANCE ALONG CORE-INCHES

FIG. 4

FIG. 5


RADIAL FLUX DISTRIBUTION
4.0
3.0
2.0
1.0
0
0
2.0
4.0
6.0
8.0
10.0
ϕ_{th}
$\bar{\phi}_{th} = 2.68$
$\phi_f \times 10^{-1}$
DISTANCE FROM CORE ℄ - INCHES