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Cross-Section Sensitivity of the D-T Fusion Probability and the D-T and T-T Reaction Rates

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CROSS-SECTION SENSITIVITY OF THE D-T FUSION PROBABILITY AND THE D-T AND T-T REACTION RATES*

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Abstract

The cross-section sensitivity of the fusion probability has been calculated for various conditions of incident deuteron energy and plasma electron temperature. The fusion probability is most sensitive to the D-T cross section at the higher energies ($\gtrsim$ 50 keV), and, based on the reported errors in the cross section, the errors in the calculated fusion probabilities should be $\lesssim$ 10%.

The cross-section sensitivities of the D-T reaction rate in a D-T plasma and the T-T reaction rate in a tritium plasma have also been calculated for various assumed values of the plasma ion temperature.

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I. INTRODUCTION

The probability for producing thermonuclear power from the thermalization of fast deuterons in a tritium plasma has been studied by several researchers.[1,2] Reliable estimates of the fusion probability are needed by the designers of the Tokamak Fusion Test Reactor (TFTR)[3] and of other two-component fusion test reactors. It has recently been suggested that the available cross-section data may be in significant error and that, therefore, the calculated fusion probability may be quite uncertain.[4] To provide some insight into the influence of the D-T cross section in specific energy ranges on the fusion probability, the sensitivity of this probability to cross-section uncertainties has been calculated for various conditions of incident deuteron energies and plasma temperatures.

The cross-section sensitivities of the D-T reaction rate in a D-T plasma and the T-T reaction rate in a tritium plasma have also been calculated for various assumed values of the plasma ion temperatures. The T-T reaction rate, in particular, is of interest because of the suggestion that the tritium temperature in the TFTR, which will be built at Princeton, might be inferred from the measurements on neutrons produced in this reaction.

The method of calculating the cross-section sensitivities of the fusion probability and of the D-T and T-T reaction rates is discussed in Sec. II and the results are discussed in Sec. III.

II. CALCULATIONAL METHOD

The change in the fusion probability, f, due to a fractional change, $\delta\sigma/\sigma$, in the D-T cross section in the energy range of $E_{i+1} - E_i$ may be calculated from the equation

$$\delta f = \delta\sigma/\sigma \int_{E_{i+1}}^{E_i} I(E')dE' \quad , \tag{1}$$

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where I(E'), the fusion probability per unit energy, is directly proportional to the D-T cross section and inversely proportional to the energy loss per unit time of a deuteron in a tritium plasma. Following Pistunovich,[2] when the lifetime of the deuteron in the plasma is greater than the slowing-down time,

$$I(E) = (2/m_d)^{\frac{1}{2}} n_i \sigma(E_d) E_d^{\frac{1}{2}} (dE_d/dt)^{-1} , \qquad (2)$$

where

m_d is the deuteron mass,

n_i is the density of the tritium plasma,

$\sigma(E_d)$ is the cross section for the D-T reaction,

E_d is the energy of the deuteron, and

(dE_d/dt) is the rate of energy loss of the deuteron in the plasma.

A deuteron moving with velocity v_d in a tritium plasma slows down in the plasma by Coulomb scattering. The mean rate of change of the kinetic energy of the deuteron in a tritium plasma of density n_i is given by[5]

$$\frac{1}{n_i}\frac{dE_d}{dt} = \left(\frac{4\pi e^4}{v_d}\right) \Lambda Z_d^2 Z_i^2 \left[\left\{\frac{\phi(b_e v)}{m_e} - \frac{2b_e v(m_d + m_e)}{\sqrt{\pi}\, m_d m_e} \exp(-b_e^2 v^2)\right\} + m_i^{-1}\right] , \qquad (3)$$

where

Z_d and m_d are the atomic number and mass of the deuteron,

Z_i and m_i are the atomic number and mass of the tritium,

e and m_e are the electron charge and mass,

Λ is the Coulomb logarithm, and

$\phi(b_e v)$ is the error integral

$$\phi(b_e v) = 2\pi^{-\frac{1}{2}} \int_0^{b_e v} \exp(-\alpha^2)d\alpha$$

where

$$b_e v = \left(\frac{m_e}{2T}\right)^{1/2} v \quad ,$$

v is the velocity, and

T is the temperature of the plasma electrons.

In deriving this equation, it is assumed that the deuterium and tritium ions are singly charged, the tritium plasma is at rest relative to the deuterons, and that the plasma electron temperature may be described by a Maxwellian distribution.

The change in the reaction rate per unit volume due to a change in the reaction cross section in a given energy range may also be calculated using Eq. (1) when I(E') is the reaction rate per unit volume per unit energy. For the calculations reported here,

$$I(E) = \kappa\, n_1\, n_2 \left(\frac{m^*}{kT}\right)^{3/2} \left(\frac{2}{\pi}\right)^{1/2} \left(\frac{2}{m_i}\right) \exp\left(-\frac{m^*}{m_i}\frac{E}{kT}\right) E\,\sigma(E) \quad , \qquad (4)$$

where

m^* is the reduced mass of the interacting plasma ions,

$\sigma(E)$ is the reaction cross section, and

κ is a constant that is equal to unity for T-T reactions and equal to 1/2 for D-T reactions.

This equation was derived with classical techniques for the encounter of a plasma of density n_1 with a plasma of density n_2, each having a Maxwellian velocity distribution and moving with respect to each other with relative velocity v_r, and has been written for ions of mass m_i and energy E interacting with a plasma of ion temperature kT.

The D-T cross section, used in estimating the sensitivity of the fusion probability and the sensitivity of the D-T reaction rate, and the T-T cross section, used to estimate the sensitivity of the T-T reaction rate, are shown in Fig. 1. These cross sections were obtained using the analytic fits to experimental data given by Duane.[6] A detailed discussion of these cross sections and comparisons with other analytic and experimental data are given by Stewart and Hale.[4] For most fusion calculations, the D-T cross section used is either that given by Duane[6] (Fig. 1) or that given by Artsimovich.[7]

The T-T cross section is not well known, particularly at low energies ($<$ 0.1 MeV). Stewart and Hale[4] point out the need for more precise measurements down to 20 keV. Comparison of the analytic fits of the T-T cross section by Duane[6] and by Strel'nikov *et al.*[8] and experimental data indicates fairly reasonable agreement between ~ 0.20 MeV and ~ 1 MeV for the two parameterizations but rather poor agreement at lower energies where the experimental data are sparse.

III. DISCUSSION OF RESULTS

The sensitivity per unit lethargy of the fusion probability, $[(\delta f/f)/(\delta\sigma/\sigma)/\delta u]$, is shown in Fig. 2 as a function of energy when 200- and 500-keV deuterons are thermalized in tritium plasmas having electron temperatures of 5 and 10 keV. The results given in the figure are independent of ion density since the density enters into the calculation only through the Coulomb logarithm. When the histogram values shown are multiplied by the lethargy interval, the percentage change in the fusion probability for a 1% change in the D-T cross section in a particular lethargy interval is obtained. The value in a given lethargy interval is directly proportional to $\delta\sigma/\sigma$, so the results for any assumed change in the cross section can be obtained by multiplication.

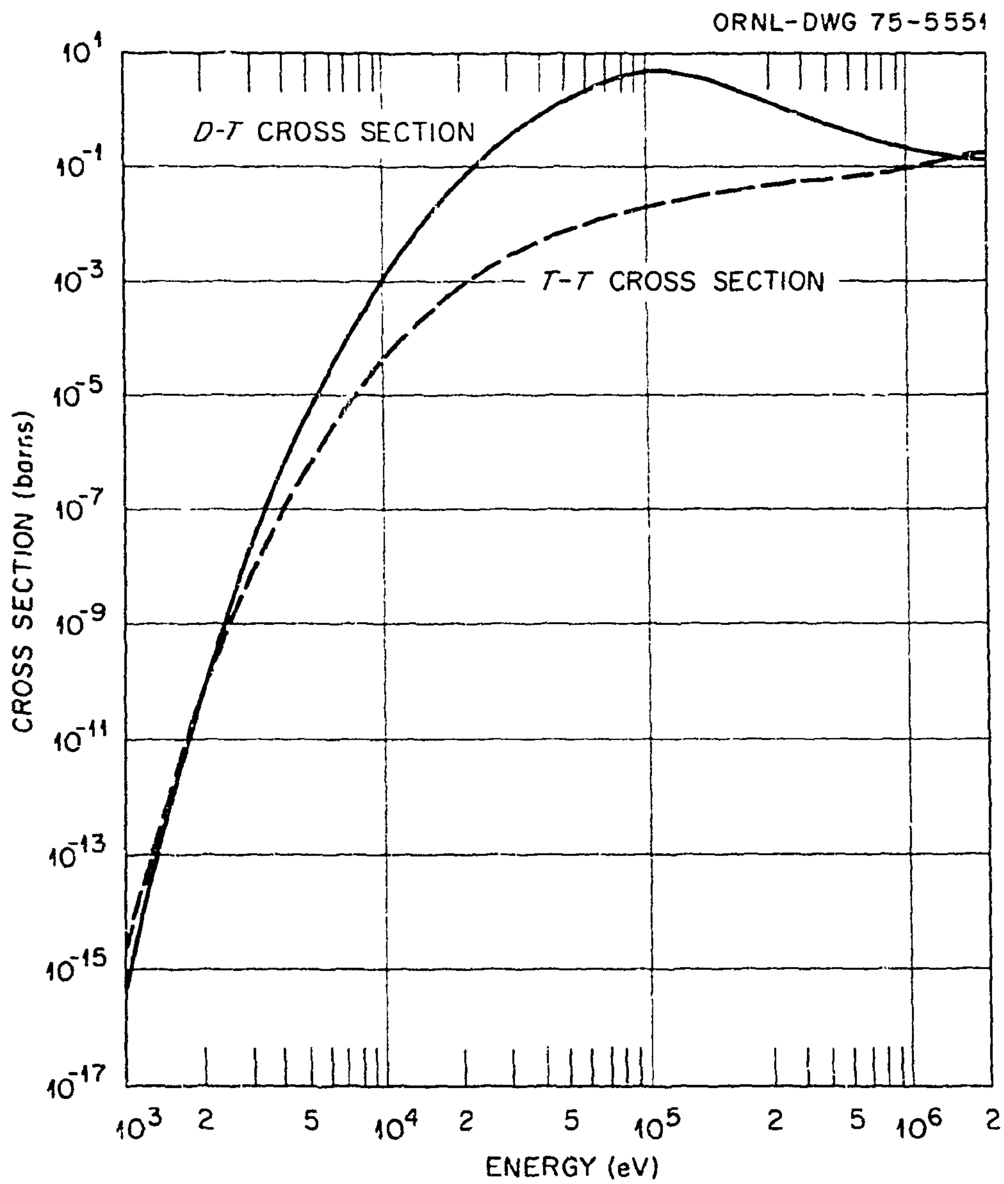

Fig. 1. D-T and T-T cross sections as a function of energy.

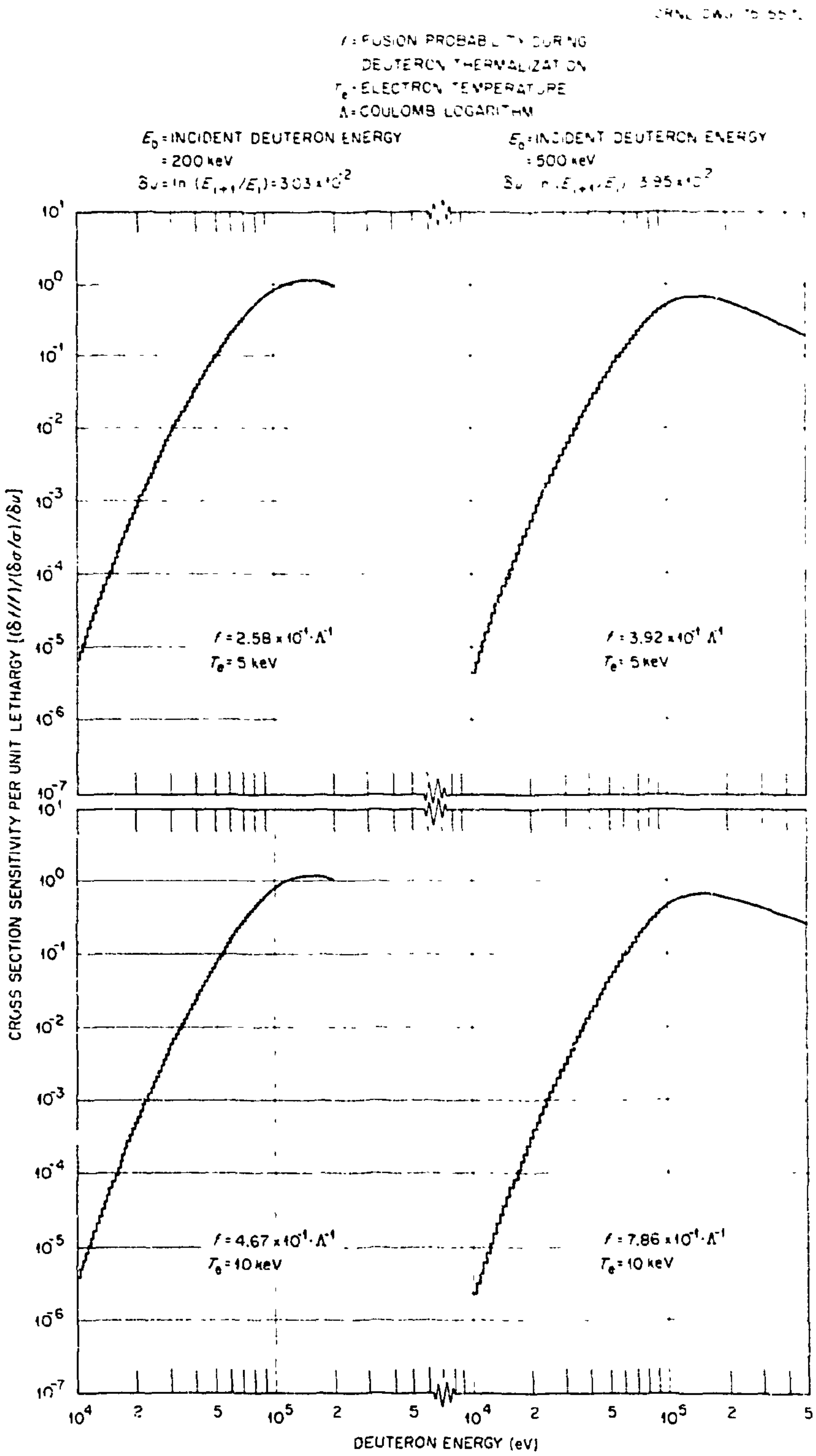

Fig. 2. Sensitivity per unit lethargy of the D-T fusion probability vs deuteron energy.

For the deuteron energies and plasma electron temperatures considered, the fusion probability is most sensitive ($\gtrsim 10^{-1}$) to the D-T cross section at deuteron energies of $\gtrsim$ 50 keV. Based on the reported errors in the cross section in this energy range,[4] the errors in the fusion probability should be $\lesssim$ 10%. For deuteron energies below ~ 50 keV, the sensitivity of the fusion probability to the D-T cross section becomes increasingly small, so even if the uncertainties in the data are large, the errors in the calculated fusion probability are still small.

The sensitivities per unit lethargy of the D-T and the T-T reaction rates per unit volume, $[(\delta R/R)/(\delta\sigma/\sigma)/\delta u]$, are shown in Figs. 3 and 4, respectively, as a function of incident ion energy. In both cases, estimates of the reaction rates were made for plasma ion temperatures of 0.3, 1.0, and 5.0 keV. The sensitivity profiles given in these figures are independent of the plasma ion density since the ion density enters the calculation through the product $n_D \cdot n_T$ for the D-T reaction and n_T^2 for the T-T reaction, where n_D and n_T are the deuteron and triton densities, respectively.

At a plasma ion temperature of 5 keV, the largest fraction of the D-T reaction rate occurs at deuteron energies of ~ 20-150 keV. At energies above ~ 100 keV, the uncertainties in the D-T cross section[4] will introduce errors into the calculated reaction rate of ~ 10%. At energies below ~ 100 keV, the fit to the cross section used in calculating the reaction rate is based on a single experiment, so the sensitivity of the reaction rate to the D-T cross section may be quite uncertain. Since all of the contribution to the reaction rates for the plasmas having ion temperatures of 0.3 and 1.0 keV, also shown in Fig. 3, occur at deuteron energies of < 100 keV, these profiles are correspondingly subject to the same uncertainties.

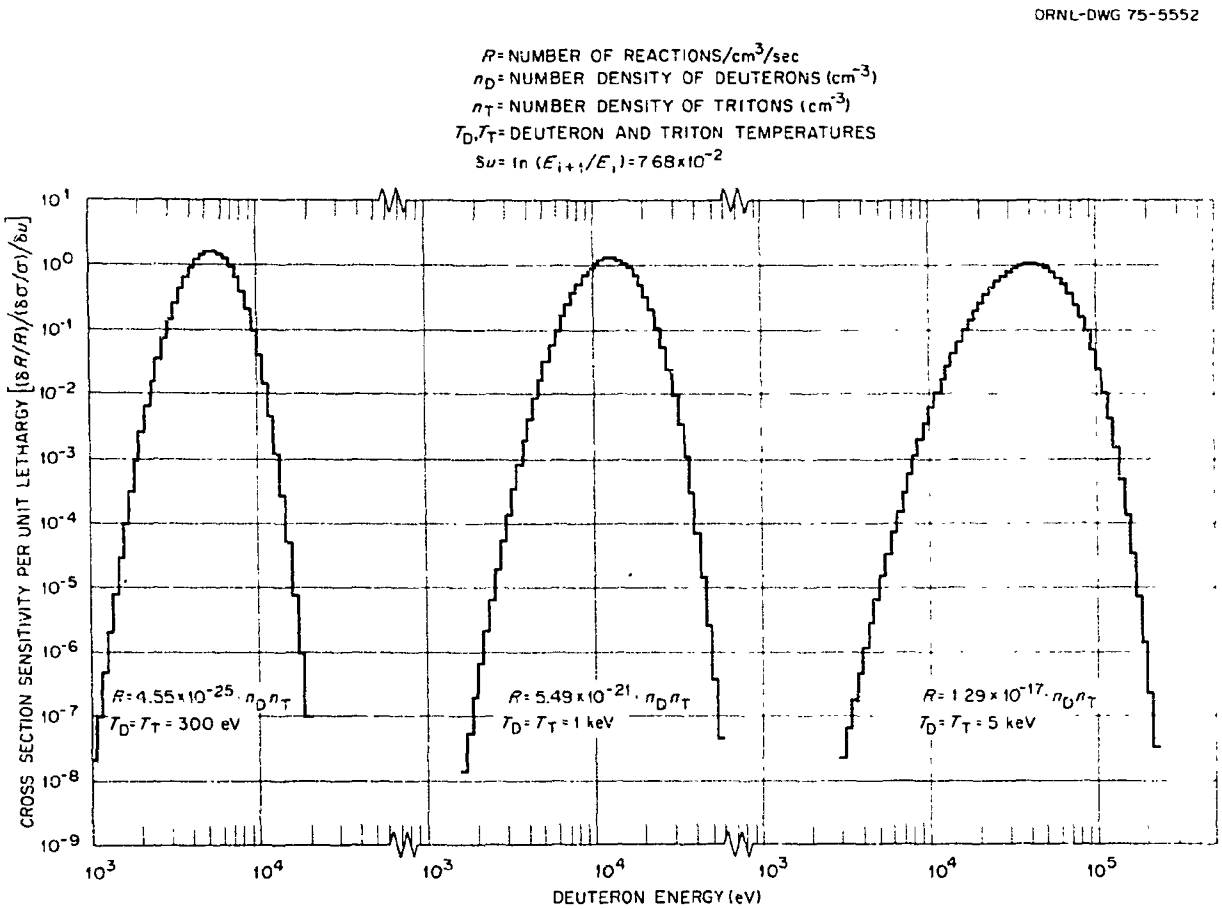

Fig. 3. Sensitivity per unit lethargy of the D-T reaction rate vs deuteron energy.

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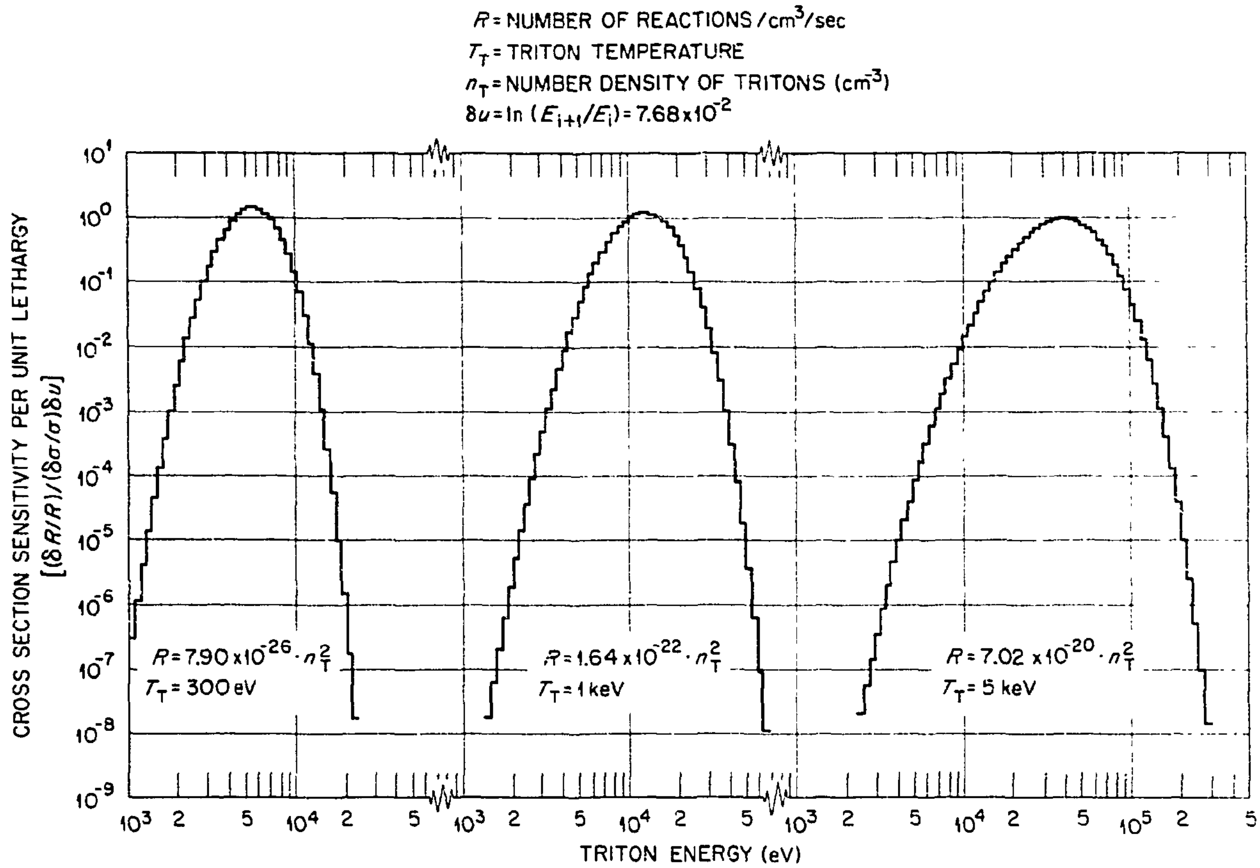

Fig. 4. Sensitivity per unit lethargy of the T-T reaction rate vs triton energy.

For the three plasma ion temperatures considered in estimating the T-T reaction rate, shown in Fig. 4, all of the contribution to this reaction rate stems from an energy region where there are no experimental data on the T-T cross section. The results given in the figure are, therefore, quite uncertain, especially in the peak region of the profiles where errors in the cross section could introduce large uncertainties into the estimated reaction rate.

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