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REMARKS ON PROPERTIES OF EVEN-EVEN NUCLEI RELATED TO THE ASYMMETRIC ROTOR THEORY OF DAVYDOV AND FILIPPOV

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ABSTRACT

Some remarks concerning the relation of the predictions of the Davydov and Filippov asymmetric rotor theory to the experimental data are pointed out.

The different ratios of the energy of the excited states as a function of the gamma parameter are discussed, according to other authors. The reduced electric-quadrupole transition ratios as a function of the gamma parameter are also discussed, according to other authors.

A brief summary of the nuclei to be studied which do not fall in the general picture predicted by the mentioned theory is given.

REMARKS ON PROPERTIES OF EVEN-EVEN NUCLEI RELATED TO THE ASYMMETRIC ROTOR THEORY OF DAVYDOV AND FILIPPOV*

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I. INTRODUCTION

In the past year a new collective theory of the behavior of nuclei has been developed by Davydov and Filippov[1] (DF) taking into account possible violations of the axial symmetry of the nucleus.[2] This violation affects the rotational spectrum of the axial even nucleus, and some new rotational states with total angular momenta of 2, 3, 4,...appear.

The operator corresponding to the rotational energy of the nucleus has the form

$$H = \sum_{\lambda=1}^{3} \frac{D J_{\lambda}^{2}}{2 \sin^{2} (\gamma - \frac{2\pi}{3} \lambda)}$$

where $D = \hbar^2/4B\beta^2$, and γ varies from 0 to $\pi/3$ and determines the deviation from axial symmetry. Here K is no longer a good quantum number. For $\gamma = 0$ the energy spectrum of an even-even nucleus is the same as that of the axially-symmetric one. The transition from the second excited state (2+) is allowed as a result of the violation of the oscillator approximation. A recent paper of DeMille <u>et al.</u> shows that the D F treatment can account for a large number of nuclear energy levels.[3]

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II. ELECTRIC QUADRUPOLE MOMENTS

The operator for the nuclear electric quadrupole moment is given by

$$Q_{2\mu} = (4\pi/5)^{1/2} \sum_{i=1}^{Z} r_i^2 \, Y_{2\mu} \, (\theta_i, \phi_i).$$

For $\gamma = 0$ one obtains a prolate ellipsoid of resolution with three axes of symmetry. For $\gamma = \pi/3$ one obtains an oblate ellipsoid of revolution.

For even-even nuclei the mean value of the quadrupole moment in the ground state is zero. In the first excited state of spin 2+, the mean value of the quadrupole moment is

$$Q_1 = -Q_o \, \frac{6 \cos (3\gamma)}{7 \quad 9-8 \sin^2 (3\gamma)}$$

where $Q_o = 3ZR^2\beta \, (5\,\pi)^{-1/2}$. In the 2+ second excited state (2') the mean value of the quadrupole moment is

$$Q_2 = -Q_1.$$

III. RATIO OF REDUCED TRANSITION PROBABILITIES FOR ELECTRIC-QUADRUPOLE AND MAGNETIC-DIPOLE RADIATIONS

We can write the ratio of the reduced transition probabilities for electirc-quadrupole and magnetic-dipole radiations as

$$\frac{B(E2;E' \rightarrow 2)}{B(M1;2' \rightarrow 2)} = \frac{80}{7} \cdot P,$$

where $P = (e_r Z R_o^2/g_R \mu_o)^2$. We define $|\delta|^2 = T(E2)/T(M1)$, then according to this theory, we get

$$|\delta|^2 = (0.21 \, k^2/80P),$$

where $k = (E_2-E_1)/\hbar c$, and

$$|\delta|^2 = \frac{2}{3} E_\gamma^{\,2} \, Z^2 \, A^{4/3} \times 10^{-10}$$

where E_γ is expressed in kev.

IV. ENERGY-LEVEL RATIOS

The ratio of the energy of the 2+ second excited state $\mathcal{E}_{2'}$ to the energy of the first excited state $\mathcal{E}_2$ is expressed by

$$R_{2'} = \frac{\mathcal{E}_{2'}}{\mathcal{E}_2} = \frac{3 + [9 - 8\sin^2(3\gamma)]^{1/2}}{3 - [9 - 8\sin^2(3\gamma)]^{1/2}}$$

For $\gamma < 21.5^\circ$ the 2' level should lie above the 4+, whereas for $\gamma > 21.5^\circ$ it should lie below this level. If we define $R_4 = \mathcal{E}_4/\mathcal{E}_2$ and $R_6 = \mathcal{E}_6/\mathcal{E}_2$, their values for $\gamma = 0^\circ$ and 30° are given in Table I.

Table I

Values of R_4 and R_6 for $\gamma = 0^\circ$ and 30°

γ (°)	R_4	R_6
0°	10/3	7
30°	8/3	5

Table II deals with values of R_i for different values of γ.[4]

Table II

Values of R_i for different values of γ[a]

γ (°)	$R_{2'}$	R_4	R_6	R_8
0	∞	3.3	7.00	12.00
8	25.16	3.329	6.978	11.93
10	15.04	3.324	6.943	11.82
13	9.254	3.301	6.829	11.48
15	6.854	3.272	6.695	11.10
20	3.732	3.117	6.060	9.844
25	2.408	2.836	5.372	8.602
30	2.000	2.666	5.000	8.000

[a] Values obtained from reference 4.

If we make $\sin^2(3\gamma) = y$, the $R_{2'}$ ratio becomes

$$R_{2'} = \frac{9 - 4y + 3\,(9 - 8y)^{1/2}}{4y}$$

V. RATIOS FOR REDUCED ELECTRIC-QUADRUPOLE TRANSITION PROBABILITIES

The asymmetric-rotor theory gives the following expressions for the ratios of the electric-quadrupole transition probabilities:

$$R_I = \frac{B(E2;\ 2' \rightarrow 2)}{B(E2;\ 2' \rightarrow 0)} = \frac{20}{7}\ \frac{y}{9 - 8y - (3 - 2y)\,(9 - 8y)^{1/2}}$$

$$R_{II} = \frac{B(E2;\ 2' \rightarrow 0)}{B(E2;\ 2 \rightarrow 0)} = \frac{(9 - 8y)^{1/2} - 3 + 2y}{(9 - 8y)^{1/2} + 3 - 2y}$$

$$R_{III} = \frac{B(E2;\ 2' \rightarrow 2)}{B(E2;\ 2 \rightarrow 0)} = \frac{20}{\ }\ \frac{y\,(9 - 8y)^{1/2}}{9 - 8y + (3 - 2y)\,(9 - 8y)^{1/2}} .$$

VI. RELATION BETWEEN THEORETICAL PREDICTIONS AND EXPERIMENTAL DATA

The Bohr-Mottelson (BM) model gives the following expression for the energy levels, taking into account vibrational-rotational interactions:

$$E_I = a\,I\,(I + 1) - b\,[I\,(I + 1)]^2 ,$$

where <u>a</u> and <u>b</u> are constants.

This theory predicts the following behavior for R_8 and R_6 as a function of R_4:

$$R_8 = -\frac{312}{7} + \frac{594}{35}\ R_4$$

$$R_6 = -11 + \frac{27}{5}\ R_4 .$$

Mallmann plotted the experimental ratios R_6 and R_8 as a function of R_4 (see Fig. 1) and found the following behavior:[4]

(1) <u>R_6 as a function of R_4</u>:

(a) For $3.27 < R_4 < 3.33$ the experimental values agree with BM predictions, with the exceptions of Hf^{178}, U^{232}, U^{236}, and Pu^{240}.

(b) For $R_4 < 3.27$ the experimental values depart more and more from BM model predictions. When DF theory with vibration-rotation interaction is introduced, the experimental ratios fall in the theoretical curve down to $R_4 = 8/3$, which is the least value of R_4 accounted for by the theory. Values for Ti^{48} and Sn^{120} fall outside the empirical curve.

(c) If one plots an empirical curve of R_6, for R_4 values from one up to 10/3, one can see that there is not any discontinuity in going from "vibrational" to "rotational" nuclei, This is an indication that the behavior of R_6 with respect to R_4 is independent of the "type" of the nucleus. The DF cannot predict the behavior of R_6 in this wide range, because it does not hold for values of $R_4 < 8/3$.

The behavior of R_4, R_6, R_8, and R_4, as a function of R_2 has recently been studied by Mallmann and Kerman.[5] Figure 2 shows the mentioned ratios as a function of R_2. The following results are shown:

(2) <u>R_4, R_6, R_8, and R_2, as a function of R_2</u>:

(a) Points (a) and (b) pointed out in case (1) also hold in this case. The exceptions are W^{182} and W^{184} and U^{232}.

(b) For nuclei with $8/3 < R_4 < 3.27$, all ratios are in fairly good agreement with DF predictions (plus vibrational-rotational interactions). The following exceptions are pointed out: Sm^{152}, Gd^{154}, and Os^{186}.

(c) Again, for values of R_4 less than 8/3, the DF theory cannot explain the mentioned ratios as a function of R_2.

We conclude that the DF-model predictions agree in a larger scale than those of the BM model, but cannot account for the behavior of all types of nuclei because of the limited range of variations of R_4. Also the empirical curves shown in Fig. 1 demonstrate that there are no discontinuities indicating a change of "type" of nuclei.

The mixing ratios measured in transitions $2' \to 2 \to 0$ in even-even nuclei from A = 56 to A = 198 were also compared with DF predictions. Malik, Potnis, and Mandeville made this study, plotting the E2/M1 ratios as a function of $Z^2A^{4/3}$, and found a fairly good agreement.[6] Values for Hg^{198}, Zr^{92}, and Fe^{56} fall far from the theoretical curve (see Fig. 3).

A survey of the available data for the gamma branching of the second 2+ level (2') was recently made by Van Patter.[7] Together with Coulomb excitation data, three ratios R_I, R_{II}, and R_{III} of reduced E2 transition probabilities have been calculated and compared with the predictions of various theories.

As was shown before, DF theory gives expressions for $\mathcal{E}_{2'}/\mathcal{E}_2$ and R_I, R_{II}, and R_{III}, as a function of γ. It is equivalent, in comparing with the experiment, to study the variation of the mentioned ratios as a function of $\mathcal{E}_{2'}/\mathcal{E}_2$, instead of γ. The extreme values of $\mathcal{E}_{2'}/\mathcal{E}_2$, R_I, R_{II}, and R_{III} for $\gamma = 0^\circ$ and $\gamma = 30^\circ$ are shown in Table III.

Table III

Values of y, R_2, R_I, R_{II}, and R_{III} for γ = 0 and 30 degrees

γ (°)	y	R_2	R_I	R_{II}	R_{III}
0	0	∞	0	0	0
30	1	2	∞	0	10/7

Van Patter plotted the experimental ratios R_I, R_{II}, and R_{III} as a function of R_2 and found the following behavior:

(1) R_I as a function of R_2

About 50 values of R_I have been calculated and plotted by Van Patter as a function of R_2 (see Fig. 4).[7] The following comments can be made:

(a) Qualitatively, DF theory gives the general trend of the variation of experimental B(E2) ratios. The agreement seems to be better in the region of strong deformation.

(b) Nuclei near a closed shell such as ${}_{26}Fe^{56}_{30}$, ${}_{40}Zr^{92}_{52}$, and ${}_{58}Ce^{140}_{82}$ fall below the DF theoretical curve.

(c) Definite departures from DF predictions exist, particularly in the medium-weight nuclei with $R_2 < 3$.

(2) R_{II} as a function of R_2

In this case also, DF predicts a good qualitative agreement with the experimental data. Again, nuclei close to closed shells fall outside the DF theoretical curve (see Fig. 5).

(3) R_{III} as a function of R_2

Here we can repeat what was said in case (2). The only theory that provides quantitative predictions for this ratio is DF. For values of R_2 comprised between 2 and 2.5, the theoretical results are somewhat model-independent and all of them agree with experiment within a factor of 4 (see Fig. 6).

As a general conclusion, we can say that the agreement with predictions of the DF theory is fairly good (as shown in Figs. 1 through 6), especially in the so-called rotational region and in a wider range than that of the BM theory. Definite departures from theoretical DF predictions exist in the vibrational region.[8]

VII. CONCLUSIONS

The descriptions in Section VI reveal that some experimental review of the excited levels and B(E2)-transition probabilities in even-even nuclei is needed. More information is also needed to verify that DF predictions are better for strongly deformed nuclei.

From the results discussed in Section VI we propose to review the experimental work in the following nuclei:

(a) Zirconium-92

The nucleus Zr^{92} should be studied more carefully. One does not know with certainty that the second excited level is 2+. Also in both E2/M1 ratios and B(E2) ratios, it falls far from theoretical rotational or vibrational predictions. It seems to conform to single-particle estimations. The study of this nuclide is particularly interesting because its excited levels are fed from Nb^{92} and Nb^{92m} (by electron capture) and from Y^{92} (by β^- decay).

(b) Osmium-186

The systematics on the energy ratios for excited states indicate that the energy of the 6+ level in Os^{186} is not well assigned so far. This nucleus is also interesting because the feeding of its excited states from two different parents, namely, Re^{186} (by β^- decay) and Ir^{186} (by electron capture and β^+ decay).

(c) Samarium-152 and Gadolinium-154

Figure 2 shows that Sm^{152} and Gd^{154} fall outside the theoretical curve predicted by DF. This indicates that a more careful study of these nuclei is needed. In addition, in both nuclides a 2+ level will very likely be found near the 4+ second excited state.

(d) Titanium-48 and Tin-120

These two nuclei fall outside the experimental curve, as is shown in Fig. 1. This is probably because both the 6+ and 4+ levels are not well assigned, or because these nuclei have some intrinsic property that makes their behavior different from the general trend. A careful study of their excited levels is clearly indicated.

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FOOTNOTES

1. A. S. Davydov and G. F. Filippov, Nucl. Phys. 8, 237 (1958).
2. Further similar work has recently been done by D. A. Daikin, J. Exptl. Theoret. Phys. 8, 365 (1959) and B. T. Geilikman, J. Exptl. Theoret. Phys. 8, 690 (1959).
3. DeMille, Kavanagh, Moore, Weaver, and White, Can. J. Phys. 37, 1036 (1959).
4. C. A. Mallmann, Phys. Rev. Lett. 2, 507 (1959).
5. C. A. Mallmann and A. K. Kerman, Argonne National Laboratory, Lemont, Illinois, private communication.
6. Malik, Potnis, and Mandeville, Bull. Am. Phys. Soc. 4, 233 (1959).
7. D. M. Van Patter, Bull. Am. Phys. Soc. 4, 233 (1959) and Nucl. Phys. 14, 42 (1960).
8. The experimental curve of R_6 and R_8 as a function of R_4 and the R_{III} ratio as a function of R_2 indicate that there are no sharp breaks in going from vibrational to rotational nuclei. This implies that the mentioned behavior can be considered as model-independent.

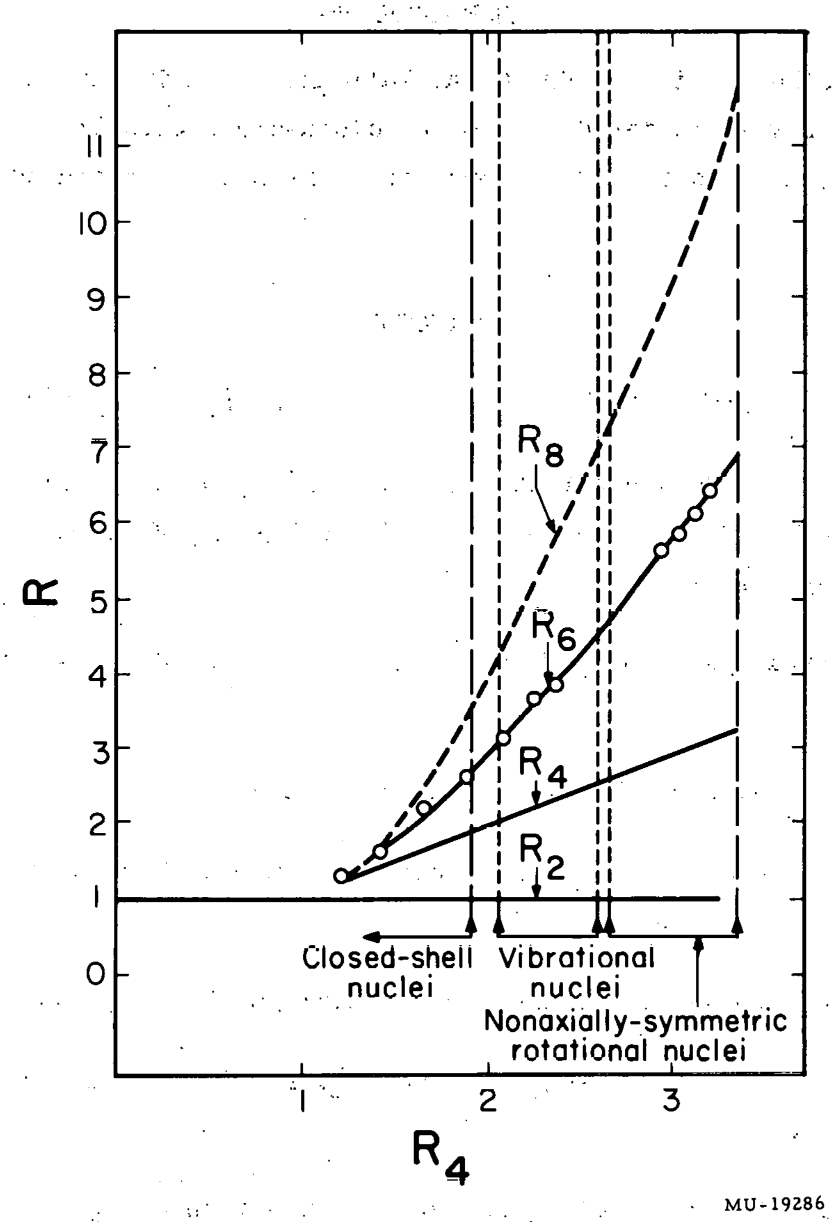
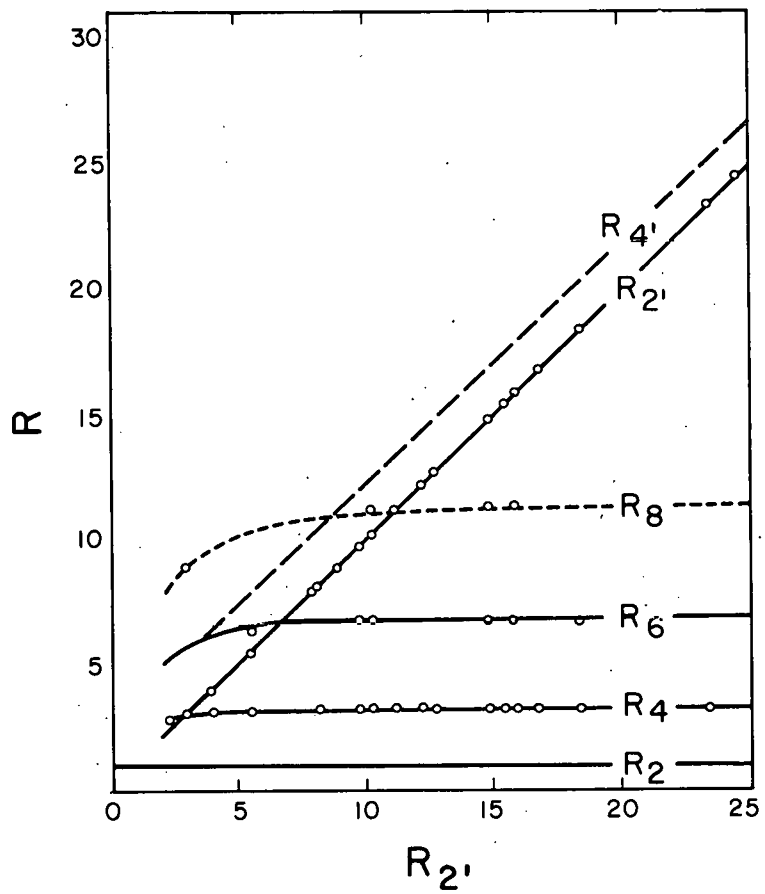
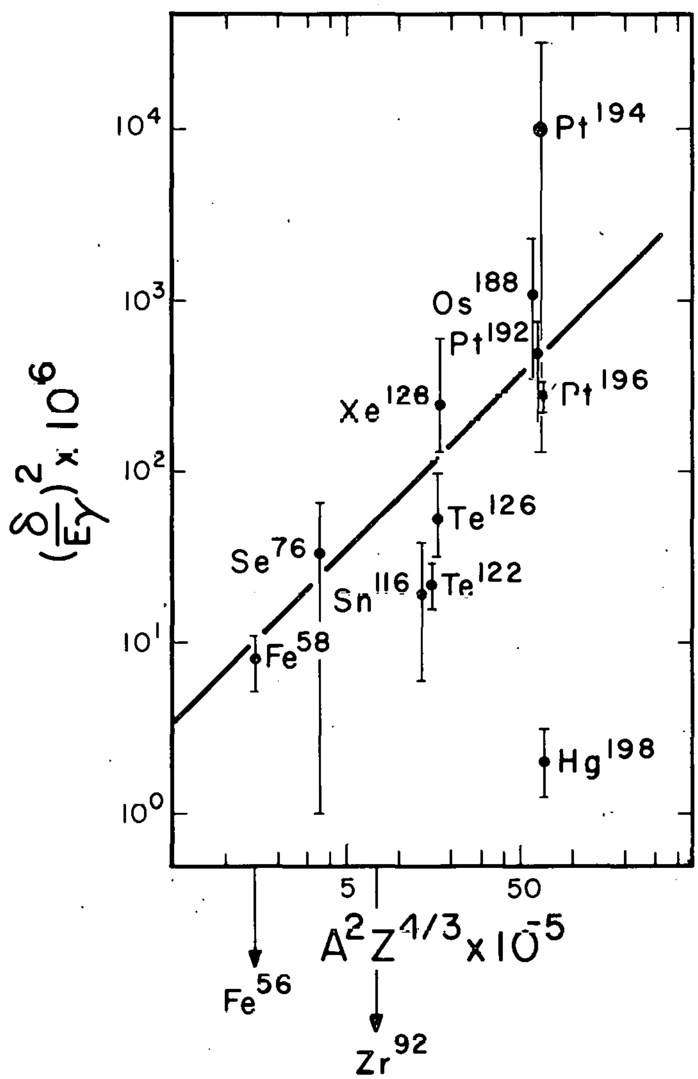


Fig. 1. Relative energy ratios of excited states in even-even nuclei as a function of R_4 (simplified version from C. A. Mallmann, reference 4).



MU-19287

Fig. 2. Relative energy ratios of excited states in even-even nuclei as a function of $R_{2'}$ (simplified version of C. A. Mallmann and A. K. Kerman, reference 5).



MU-19289

Fig. 3. Mixing ratios in 2' → 2 transitions in even-even nuclei, as a function of $A^2Z^{4/3}$, according to Malik, Potnis, and Mandeville, reference 6.

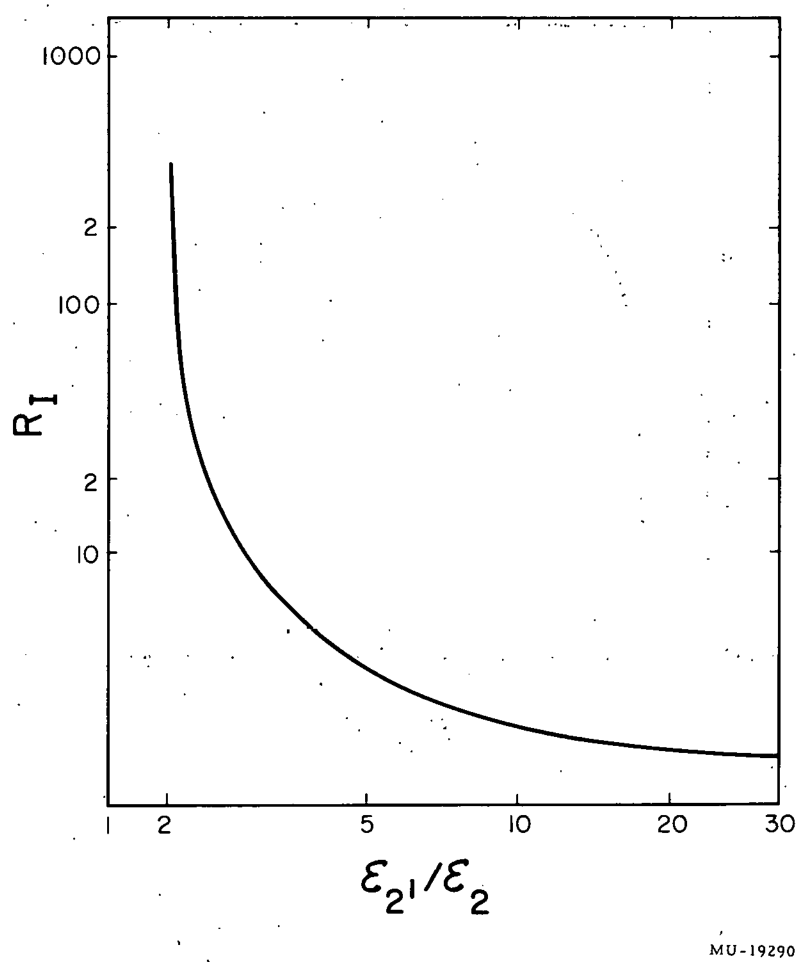


Fig. 4. Ratio of R_I as a function of $\mathcal{E}_{2'}/\mathcal{E}_2$, in even-even nuclei (simplified version from D. M. Van Papper, reference 7).

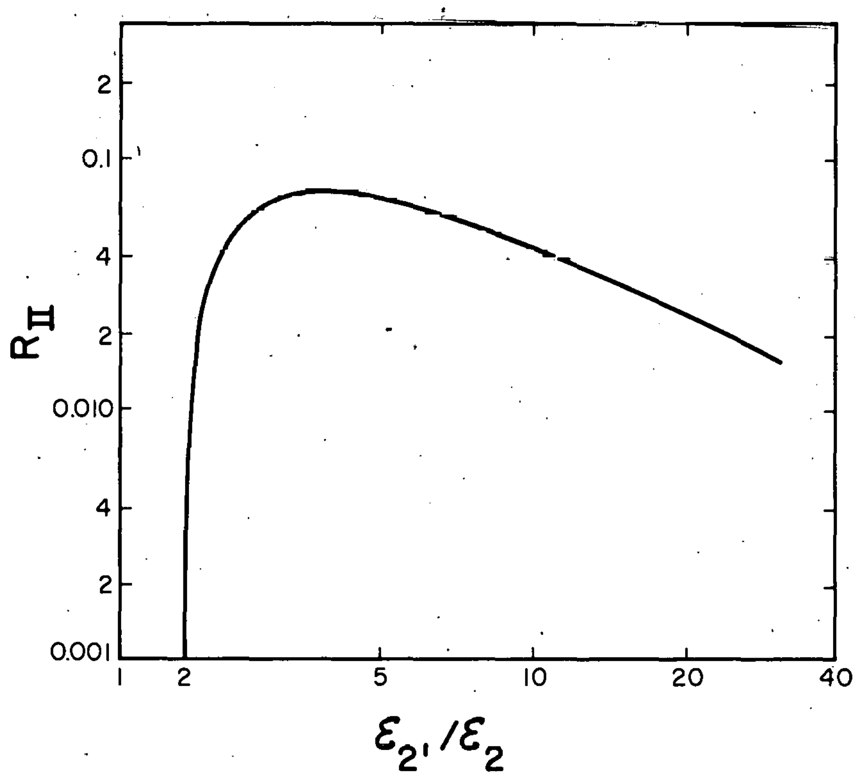


Fig. 5. Ratio of R_{II} as a function of $\mathcal{E}_{2'}/\mathcal{E}_2$, in even-even nuclei (simplified version from D. M. Van Patter, reference 7).

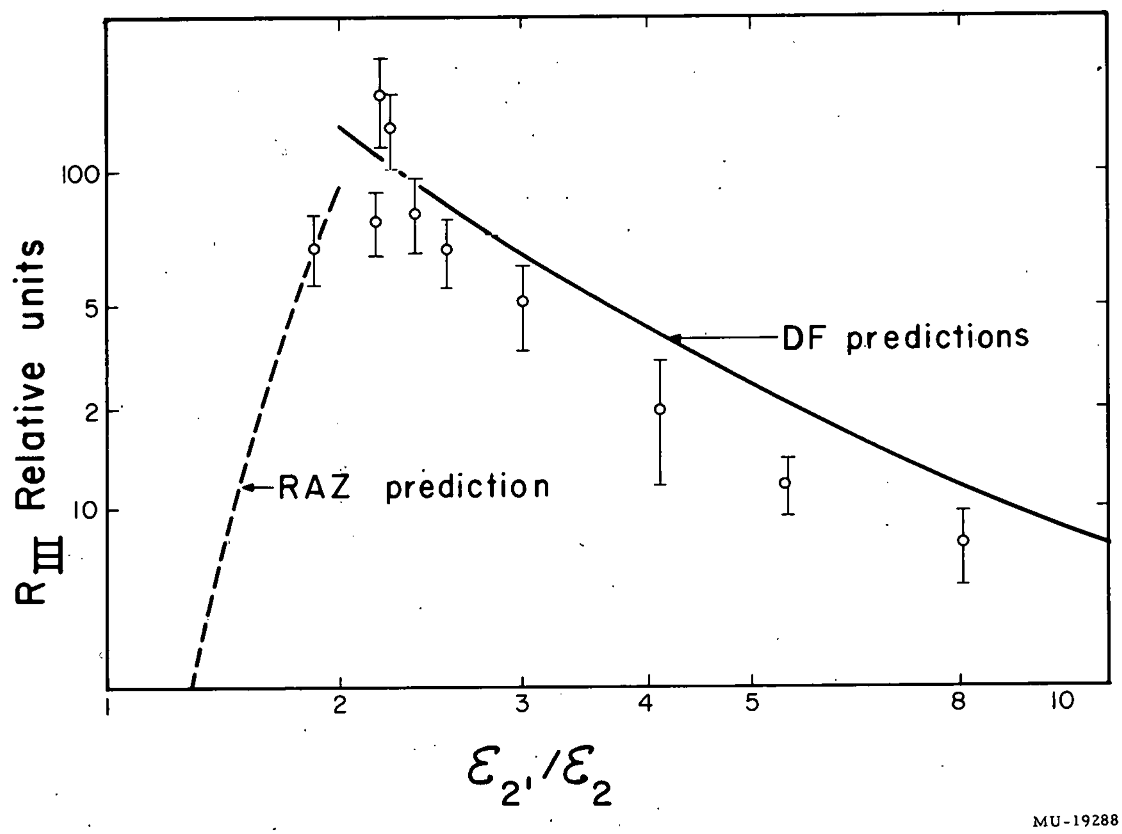


Fig. 6. Ratio of R_{III} as a function of $\mathcal{E}_{2'}/\mathcal{E}_2$, in even-even nuclei (simplified version from D. M. Van Patter, reference 7).