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ANOMALOUS TRANSPORT THEORY FOR THE REVERSED FIELD PINCH

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Abstract

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Physically motivated transport models with predictive capabilities and significance beyond the reversed field pinch (RFP) are presented. It is shown that the ambipolar constrained electron heat loss observed in MST can be quantitatively modeled by taking account of the clumping in parallel streaming electrons and the resultant self-consistent interaction with collective modes; that the discrete dynamo process is a relaxation oscillation whose dependence on the tearing instability and profile relaxation physics leads to amplitude and period scaling predictions consistent with experiment; that the Lundquist number scaling in relaxed plasmas driven by magnetic turbulence has a weak $S^{-1/4}$ scaling; and that radial E×B shear flow can lead to large reductions in the edge particle flux with little change in the heat flux, as observed in the RFP and tokamak.

1. INTRODUCTION

Recent measurements in the reversed field pinch (RFP) have provided new information crucial to the development of predictive models of anomalous transport for both core and edge plasmas with significance beyond the RFP. Measurements in the Madison Symmetric Torus (MST) have demonstrated that anomalous heat loss inside the reversal layer is caused by the low frequency global tearing modes resonant at the few lowest order rational surfaces in the core [1]. The heat loss just inside the reversal layer is convective (despite the presence of a temperature gradient). Its magnitude is well matched by a Rechester-Rosenbluth diffusivity, *but with the ion thermal velocity for the streaming factor*. Deep in the core, the magnitude of the heat loss appears to be consistent with a standard Rechester-Rosenbluth diffusivity with the electron thermal velocity for the streaming factor [2]. This observation poses a challenge to theoretical modeling because magnetic fluctuation-induced heat transport is not conventionally subject to ambipolar constraints [3]. The established importance of tearing mode fluctuations to core confinement also points to the need for a reconsideration of the Lundquist number scaling issue [4] and the desirability of modeling the nonstationary tearing mode activity known as the discrete dynamo process [5-7]. In the edge, application of an externally driven shear flow is observed to reduce particle transport with little or no change in the heat loss [8]. Noting that heat and particle losses are sensitive to phase angles between different fluctuations, this observation indicates the need for a theoretical portrayal of the role of shear flow on transport phase angles.

This paper presents theoretical work that models the plasma behavior underlying these and related observations. Specifically, this paper: 1) describes a theory for ambipolar constrained magnetic fluctuation-induced electron heat transport that accounts for resonant energy and momentum exchanges between aggregated electrons moving along the stochastic magnetic field and the Landau damped plasma shielding response; 2) models the discrete dynamo process as a relaxation oscillation and predicts amplitude and period scalings consistent with experiment; 3) incorporates dynamo physics to predict that magnetic turbulence has a weak Lundquist number scaling with an exponent of less that 1/4; and 4) demonstrates quantitatively how radial electric field shear can suppress the particle flux with little change in the heat flux, as observed in the edge of the RFP and tokamak.

2. AMBIPOLAR CONSTRAINTS

The magnetic fluctuation-induced electron thermal heat flux in the edge of MST is convective [1] and ambipolar constrained [2], despite the presence of a temperature gradient. The ambipolar constraint is evidenced in the magnitude of the thermal diffusivity, which requires the ion thermal velocity to enter the product with the magnetic diffusivity. (The standard Rechester-Rosenbluth flux estimate with electron thermal diffusivity overestimates the edge flux by more than an order of magnitude.) This measurement is constructed from the correlated product of the field aligned heat flux with the radial component of the perturbed field. The cross power spectrum for this quantity peaks at 10 kHz and is zero above 20 kHz, indicating that core resonant tearing modes are driving the transport, even near the inside of the reversal layer.

Ambipolar constraints in the transport fluxes derived from higher order moments of the electron distribution (such as the heat flux) are a signature of the dissipative electron-ion coupling process associated with the ballistic motion of electron clumps [9]. Clumps are aggregates of parallel streaming electrons. Under the exponential separation of stochastic magnetic field lines, electrons on closely neighboring field lines remain correlated far longer than those on more remotely spaced field lines. Sufficiently close electrons remain correlated for a time that exceeds collective plasma responses. As such correlated electrons move through the plasma, they generate a wake by emitting into the collective modes of the plasma shielding response. Damping of the wake on the ion distribution regulates the rate of turbulent energy exchanges between fluctuations and thereby introduces the ambipolar constraint [2].

The radial electron heat flux is an energy moment of the transport correlation associated with E×B motion (electrostatic component) and the radial component of electron motion along the perturbed field $\delta\mathbf{b} = \nabla A_{\|}\times\mathbf{b}_o$. Thus, the heat flux is given by

$$Q_e = \mathrm{Re}\int d^3v\,(m_e v^2/2)\,\Sigma_k\,(ic/B_o)\mathbf{k}\times\mathbf{b}_o\cdot\mathbf{r}\langle(\phi - v_{\|}A_{\|}/c)\,h_e\rangle. \quad (1)$$

The perturbed electron distribution function h_e has a coherent response, representing the electron contribution to the collective normal mode response, and an incoherent response, representing the correlated electrons that move along the field. These two components of the electron distribution are linked through quasineutrality and Ampere's Law, which together describe the shielding of the correlated electrons by the collective mode response. Including both components of h_e in Eq. (1) and imposing quasineutrality and Ampere's Law constraints leads to an electron heat flux expression given by

$$Q_e = v_i(\delta b/B_o)^2 k_o^{-1} L_n^{-1}[\omega/k_o v_i]^2(4\pi)^{-1/2}(1-\Delta k_{\|}^2/4k_o^2)^{-2}, \quad (2)$$

where δb is the spectrum averaged magnetic fluctuation level, k_o is the value of parallel wavenumber at which the spectrum of transport causing fluctuations peaks, and $\Delta k_{\|}$ is the width of the spectrum in parallel wavenumber. This result is appropriate for adiabatic ions ($\omega/k_{\|} \leq v_i = \sqrt{T_i/m_i}$), and for a spectrum in which transport-causing fluctuations are resonant at a distant rational surface, while locally resonant fluctuations make no contribution to transport (k_o-$\Delta k_{\|} > 0$). Both conditions apply to the MST edge. There, the frequency spectrum of the heat flux unambiguously identifies the transport producing fluctuations as tearing modes resonant at interior surfaces corresponding to m=1 and n=5-8. Furthermore, $\omega/k_o \approx v_i$, and the above expression becomes a Rechester-Rosenbluth flux, but with ion thermal velocity in place of the electron thermal velocity. Its magnitude is consistent with the measured flux.

In regions where transport is caused by locally resonant fluctuations, such as the core of the RFP, the parallel wavenumber spectrum peaks at zero and Eq. (2) is no longer valid. The correct expression in this regime is given by

$$Q_e = v_e(\delta b/B_o)^2 \Delta k_{\|}^{-1} L_T^{-1}(4\pi)^{-1/2}. \quad (3)$$

The core heat flux has no ambipolar constraint, but follows the standard Rechester-Rosenbluth form with electron thermal velocity as the streaming factor. Equations (2) and (3) are consistent with experimental observations. As noted, the measured edge heat flux is consistent in its magnitude with an ambipolar constraint in the form of an ion thermal velocity streaming factor. In contrast, the transient response of the central temperature through a sawteeth cycle indicates the possibility that core heat transport is consistent with an electron thermal velocity streaming factor.

The above model is also consistent with observations in the CCT tokamak, where magnetic transport at low order surfaces satisfies a Rechester-Rosenbluth diffusivity, but falls to much lower values away from the surface.

The ambipolar constrained edge flux of heat represented in Eq. (2), and a related flux of field-aligned momentum [10] have important implications. The exchange of momentum and energy from electrons to ions means there is an anomalous ion heating process and an anomalous ion current in the direction of bulk electron motion. Both of these features are observed in MST and warrant further investigation.

3. NONSTATIONARY RFP DYNAMO

Reversed field pinch experiments exhibit relaxation phenomena which sustain the magnetic configuration longer than should be allowed by electrical resisitvity. This effect is usually attributed to the MHD dynamo. A novel feature of the dynamo is that the relaxations often occur as a discrete and nearly periodic set of sawtoothy bursts [4-6].

A simple analytic model for the RFP sawtooth has been developed [11]. The sawtooth cycle is viewed as a relaxation oscillation describing the competition between the dynamo producing tearing instabilities which try to keep the RFP in a relaxed state and the diffusion properties of the equilibrium which reacts to a driving toroidal electric field which peaks the current profile. As such, the RFP is described as a dynamical system with two reduced degrees of freedom. The basic model used in the calculation is resistive MHD, where all physical quantities are written as the sum of an initial time independent equilibrium, a slowly evolving "equilibrium" quantity and a fluctuation associated with tearing instabilities. The two degrees of freedom are the average value of the current gradient $\Delta\lambda$, which measures how close to a relaxed state the system becomes ($\Delta\lambda =$ 0 represents the Taylor state), and the tearing mode induced MHD dynamo W. Coupled dynamical equations for $\Delta\lambda$ and W are derived and given by

$$\frac{d\Delta\lambda}{dt} = 1 - W - \varepsilon_1 \Delta\lambda \ , \tag{4}$$

$$\frac{dW}{dt} = \Lambda(\Delta\lambda - 1)W\,\Phi(W) + \varepsilon_2 W(W-1)(1-\sigma W) \ , \tag{5}$$

where time is normalized to a fraction of the resisitive diffusion time associated with how long the initial equilibrium takes to diffuse to a tearing unstable current profile. The first equation describes the evolution of the average current gradient where the terms on the right side are the driving electric field, the dynamo and resisitive diffusion. The second equation describes the evolution of the dynamo. The first term indicates the excitation of tearing instabilities when a critical current gradient is reached and $\Phi(W)$ describes the quasilinear and nonlinear modifications to the tearing modes. The last term is associated with an inductive back reaction associated with the fact the equilibrium is evolving as described in Eq. (4) [12]. Formally, the last terms in each equation are small.

We seek solutions of Eqs. (4) and (5) by assuming $\varepsilon_1, \varepsilon_2 << 1$. To lowest order, the system has an integral of motion which parameterizes a family of solutions. The solution, $\Delta\lambda = 1$, $W = 1$ is an elliptic fixed point under the modest assumption $\Phi(1) > 0$. Typically, the cyclic solutions are characterized by two phases. In the slow ramp phase, W remains smaller than unity and $\Delta\lambda$ grows in time indicating a peaking of the normalized current profile. In the sawtooth crash phase, W grows rapidly when $\Delta\lambda$ gets sufficiently larger than unity. The rapid rise in the dynamo term then causes a flattening of the current profile which removes the free energy source for the tearing fluctuations. An extension of the model to include 1-D radial profile evolution via an anomalous diffusivity and nonuniform radial transport of current will allow for examination of current profile relation in the RFP as a self-organized criticality.

By including the small order ε terms, the integral of motion is broken. The solution then asymptotes to a stable limit cycle solution. The model predicts that the sawtooth cycle and amplitude increase with plasma current and Lundquist number, both of which appear to be qualitatively consistent with MST observations.

4. LUNDQUIST NUMBER SCALING

The link between thermal transport and tearing mode fluctuations calls for a reconsideration of the Lundquist number scaling of magnetic turbulence in the RFP. This scaling quantifies how turbulence in the RFP, and other relaxed plasmas such as the spheromak, is affected by plasma resistivity (the Lundquist number is the ratio of resistive diffusion time to an Alfvén time). Because the linear growth rate of tearing modes decreases with decreasing resistivity according to the familiar fractional power [13], it has long been assumed that the magnitude of turbulence in the RFP should also decrease as the plasma becomes less resistive. While experiments of OHTE indicated a Lundquist number scaling of $\delta b \propto S^{-1/2}$, where δb is the root-mean-square magnetic fluctuation level and S is the Lundquist number, subsequent numerical and experimental work have generally suggested a weaker scaling [4, 14-16].

To investigate this issue, three-dimensional, nonlinear MHD simulations with Lundquist number ranging from 2.5×10^3 to 4×10^4 have been completed. These simulations were carried out with a pseudo-spectral algorithm, and hyper-dissipation terms were utilized in the pressureless MHD equations to suppress numerical aliasing [17]. The results indicate a weak Lundquist number scaling: $\delta b \propto S^{-0.18}$. It is important to understand mode interactions that contribute to this result. In particular, *m*=0 modes, whose fluctuation level increases with S in this range, have a significant impact on the scaling. This behavior is attributed in part to enhanced nonlinear coupling as dissipation is reduced and in part to changes in the parallel current profile. The parallel current gradient at the *m*=0 resonance surface increases with S due to an increasingly more effective MHD dynamo. This effect is expected to eventually saturate at larger S, leading to a decreasing m=0 fluctuation level and a stronger overall scaling.

The temporal behavior of the simulations also changes as S is increased. Oscillations in the reversal parameter become more regular, acquiring a sawtooth character at the largest values of S studied. Similar but less dramatic observations have been reported earlier [18]. Here, the nonlinear coupling between internally resonant *m*=1 modes and *m*=0 modes plays an important role. The events leading to a sawtooth crash are consistent with a three phase description of quasiperiodic oscillations at modest S [19]. The three phases (1. interior m=1 modes gain energy, 2. nonlinear transfer of energy to m=1 modes that are resonant near the reversal surface, and 3. mean poloidal current drive) produce enhanced current inside the reversal surface. However, at large S, there is an added phase of mean current redistribution from *m*=0 modes, which subsequently suppresses these modes. When they are small, the second phase of the dynamo is impeded until the *m*=1 modes are large enough to nonlinearly drive the *m*=0 modes back to a significant level.

The scaling of magnetic turbulence with S has also been examined analytically from a theoretical framework that links the fluctuation spectrum to the dynamo through the cascade properties of three dimensional MHD and also accounts for the role of discrete dynamo events [20]. For a discrete (sawtoothing) dynamo, the time averaged root-mean-square magnetic fluctuation level scales as $\delta b \propto S^0$, but it can be argued that the averaged flux surface destroying magnetic fluctuations scale as $\delta b_{br} \propto S^{-1/2}$. For a continuous dynamo, with a steady state saturated turbulence spectrum, the magnetic field perturbations scale as $\delta b \propto S^{-1/4}$.

5. RADIAL ELECTRIC FIELD SHEAR AND EDGE TRANSPORT

Suppression of turbulent fluctuations by sheared E×B flow [21] has emerged as a robust paradigm that is thought to explain key features of confinement improvements in H and VH modes and in the enhanced reversed shear or negative central shear discharges. A challenge for this paradigm is to explain detailed transport measurements and measurements of phase angles, whose dependence on the shear flow can be puzzling or counterintuitive. One such set of experiments involves the creation of a sheared radial electric field in the CCT tokamak and in MST by an emitting probe [22] or by electron current injectors [8]. In these experiments there is a marked decrease in particle transport with little or no change in the heat flux. These results are difficult to explain with the standard treatments of shear suppression, which only account for amplitude reductions by shear flow, but do not address the effect of shear flow on the phase angles that contribute to the transport fluxes.

We present a calculation of heat and particle fluxes that accounts for shear flow effects on both phase angle and amplitudes [23]. This calculation is relevant to edge fluctuations in machines such as CCT and MST where the observations were made, and utilizes an edge turbulence model based on resistive interchange turbulence. An important feature of this model is the effect of parallel thermal conductivity on temperature fluctuations. Though the instability in the present calculation is the resistive interchange mode, the results are generally representative of classes of instabilities driven by density and vorticity dynamics with temperature playing a secondary role.

The model consists of evolution equations for vorticity, density, and temperature, with E×B advection of each field the dominant nonlinear process. An equilibrium poloidal flow with radial shear is also included in the advective term. Field line bending provides localization of fluctuations in the neighborhood of the rational surface, i.e., the linear mode width is set primarily by the balance of the field line bending term and the time derivative of vorticity. Shear induces a shift of the eigenmode structure away from the rational surface and generally enhances the effect of field line bending. The instability is driven by curvature and the pressure gradient. Due to the magnitude of parallel thermal conduction in the edge of Ohmic discharges, the density gradient is assumed to provide the free energy and temperature fluctuations are treated as a passive scalar. The nonlinearities are renormalized in the usual fashion, yielding three equations, each of which has an amplitude dependent turbulent diffusivity. A nonlinear eigenmode calculation solves the renormalized equations, yielding the magnitude of the diffusivity (and hence the magnitude of the saturated turbulence), the mode width at finite amplitude, and the magnitude of the shift of the eigenmode structure induced by the flow shear [24].

From this solution, particle and heat transport fluxes are constructed from the correlations of density and temperature fluctuations with the electrostatic potential. These correlations depend on amplitude and phase angle, both of which in turn depend on the flow shear, dissipation, and the nonlinear diffusivity. The latter quantity itself depends on shear flow through the shift of the eigenmode structure. The particle and heat fluxes are given respectively by $\Gamma = \sum_k |v_k||n_k|\cos\delta_n$ and $Q = \sum_k |v_k||T_k|\cos\delta_T$ where v_k, n_k, and T_k are Fourier amplitudes for fluctuations in the flow, density and temperature, and δ_n and δ_T are the phases for the particle and heat fluxes. In a weak shear regime, the phase angle factors $\cos\delta_n$ and $\cos\delta_T$ are given by

$$\cos\delta_n = 1 - \frac{[0.2k_y^2 V_E'^2 W^2/\gamma^2]}{[1-0.9k_y^2 V_E'^2 W^2/\gamma^2]^2} \tag{6}$$

$$\cos\delta_T = 1 - \frac{[2k_y V_E' W]^2}{[\chi_{\|e} k_y^2 W^2/n_o L_s^2]^2} \tag{7}$$

where V_E' is the shear in the E×B flow, W and γ are the mode width and growth rate of the resistive interchange mode for zero shear, $\chi_{\|e}$ is the parallel electron thermal conductivity, and the phase angle cosines are normalized to their values in the absence of a shear flow. From these expressions, it is evident that for strong parallel thermal conduction, shear flow has a weak effect on the phase angle factor for the heat flux. In contrast, the phase angle factor for the particle flux is highly sensitive to the turbulent diffusivity, which in turn is sensitive to the shear flow. For parameters consistent with the edge of CCT and MST, a level of shear flow sufficient to make $\cos\delta_n$ equal to zero, produces a change in $\cos\delta_T$ of only a few percent. When combined with the amplitude factors, both fluxes decrease with flow shear, but the variation in the particle flux is much stronger due to the variation of its phase factor. This demonstrates that the dependence of the phase angle on flow shear is an important aspect of shear flow suppression. Moreover, it is apparent that with phase angle effects, different transport fluxes need not respond identically to a sheared E×B flow. In particular, a dissipative processes with a shorter time scale than the shear decorrelation time can make a flux whose phase angle depends on that dissipative process relatively impervious to the effect of shear flow.

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References

[1] FIKSEL, G., et al., Phys. Rev. Lett. **72** (1994) 1028.
[2] TERRY, P.W., et al., Phys. Plasmas **3** (1996) 1999.
[3] THOUL, A.A., SIMILON, P.L., SUDAN, R.N., Phys. Plasmas **1** (1994) 601.
[4] LAHAYE, R.J., et al., Phys. Fluids **27** (1984) 2576.
[5] WATT, R.G., NEBEL, R.A., Phys. Fluids **26** (1983) 1168.
[6] HOKIN, S., et al, Phys. Fluids B **3** (1991) 2241.
[7] DEN HARTOG, D.J., et al, Phys. Plasmas **2** (1995) 2281.
[8] CRAIG, D., et al., Bull. Amer. Phys. Soc. **41** (1996) poster 2S.01.
[9] TERRY, P.W., DIAMOND, P.H., HAHM, T.S., Phys. Rev. Lett. **57** (1986) 1899.
[10] TERRY, P.W., DIAMOND, P.H., Phys. Fluids B **2** (1990) 1128.
[11] HEGNA, C.C., et al, "Theory of Discrete Dynamo Activity in Laboratory Plasmas: RFP Sawteeth," 1996 Sherwood Theory Conference.
[12] THAYAGARAJA, A., HAAS, F.A., Plasma Phys. Control. Fusion **37** (1995) 415.
[13] FURTH, H.P., KILLEEN, J., ROSENBLUTH, M.N., Phys. Fluids **6** (1963) 459.
[14] Z.G. AN, et al., in Plasma Physics and Controlled Nuclear Fusion Research 1986 (IAEA, Vienna, 1987) Vol. 2, p. 663.
[15] SARFF, J.S., et al., Bull. Amer. Phys. Soc. **41** (1996) poster 2S.14.
[16] CAPPELLO, S., BISKAMP, D., Nuc. Fusion **36** (1996) 571.
[17] SOVINEC, C.R., SCHNACK, D.D., Sherwood Fusion Theory Conference, Dallas, 1994.
[18] CAPPELLO, S., BISKAMP D., International Conference on Plasma Physics, Foz do Iguacu, 1994.
[19] HO, Y.L., CRADDOCK, G.G., Phys. Fluids B **3** (1991) 721.
[20] MATTOR, N., Phys. Plasmas **3** (1996) 1578.
[21] BIGLARI, H., DIAMOND, P.H., TERRY, P.W., Phys. Fluids B **2** (1990) 1.
[22] TYNAN, G., et al., Plasma Phys. Control. Fusion **38** (1996) 1301.
[23] TERRY, P.W., et al., 9th Transport Task Force Workshop, Philadelphia, 1996.
[24] WARE, A.S., et al., Plasma Phys. Control. Fusion **38** (1996) 1343.

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