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A DIMENSIONAL ANALYSIS OF THE DEPARTURE FROM NUCLEATE BOILING HEAT FLUX IN FORCED CONVECTION

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P. Griffith

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SUMMARY

As part of the current effort to produce new and improved forced convection departure from nucleate boiling (DNB) heat flux correlations that would be applicable over a wide range of water conditions, a dimensional analysis of DNB was performed. This analysis was performed by non-dimensionalizing the momentum, energy and continuity equations and applying selected boundary conditions. From this treatment, a number of dimensionless groups were formulated, and the DNB heat flux was expressed in terms of these groups as follows:

$$\frac{\phi_{DNB}}{h_{fg}\,\rho_g\,V_i} = f\left(\frac{L}{S},\ \frac{V_i\,S\,\rho_f}{\mu_f},\ \frac{V_i\,S\,\rho_g}{\mu_g},\ \frac{C\,\mu_f}{k},\ \frac{\sigma}{\rho_f\,V_i^{\,2}\,S},\ \frac{k\,\left(T_s - T_i\right)}{h_{fg}\,\rho_g\,S\,V_i},\ \frac{\rho_f}{\rho_g},\ \beta\right)$$

This relationship is applicable to various fluids over a wide range of conditions. By means of a heat balance, the continuity equation and the equation of state, ϕ_{DNB} can then be expressed in terms of local conditions plus the L/S term.

This functional relationship can be simplified if only one liquid and one pressure is considered since the fluid properties are constant and thus may be eliminated. The resulting simplified relationship is:

$$\phi_{DNB} = f\,(V,\ L,\ S,\ h)$$

(G may be substituted for V).

It should be noted that the channel geometry (characteristic length in boiling, L, and channel spacing, S) is included. Present DNB correlations of the Bettis Laboratory contain these terms except that total length rather than a characteristic boiling length is now used. (Tests and analysis are underway to demonstrate the validity of using a boiling length.)

Other similar simplified relationships are developed for the subcooled region, for the bulk boiling region, and for correlations of DNB under transient conditions.

It is concluded that:

1. For a single fluid at one pressure flowing in wide channels, DNB is a function of four independent variables. These variables could be h, G, L/S, and S.

2. When the above four variables are used to correlate DNB, any apparent inlet subcooling effect is caused by an inadequate assumption of the form of the functional relationship between these variables.

3. The mechanism of DNB in forced convection is still not understood.

4. L/S affects DNB through the distribution of phases at the DNB point. Simply specifying velocities and qualities is not sufficient; a specification of the phase distribution is also needed.

5. Further improvements in DNB correlations will probably result from better assumptions for the functional relationships and the use of larger numbers of undetermined constants. There is no reason to expect that a function of one variable alone times a function of another variable alone, etc., will suffice in correlating the data.

It is planned to use these various dimensionless groups in producing DNB correlations that would be applicable over a comparatively wide range of water conditions and pressures. It is not expected that a simplified relationship between these groups will be found. Perhaps a graphical representation will be necessary to produce the optimum correlation. (This type of representation is now used in many other fields of heat transfer and hydraulics.)

INTRODUCTION

In the course of developing the forced convection departure from nucleate boiling (DNB)* correlations from the increasing mass of DNB data, the number of factors which it has been found necessary to include has steadily increased. As more and more factors have been included, it has been considered that these factors were perhaps not independent but rather were related through one of the conservation principles (energy or mass) or some geometric condition. Further, since the generalized DNB correlations appear to be inferior to curves through the raw data for prediction purposes, the question arises, what has been omitted from the existing DNB correlations? Therefore, it would appear that a general examination of DNB correlations in enclosed channels is needed. Several Russian workers (Refs 1, 2, and 3) have included dimensional analysis in their papers, but the basis of these analyses is obscure, possibly because of the unavailability of references which are of common knowledge to all Russians working in heat transfer.

The principal correlating groups which appear in Ref 3 for pool boiling are as follows:

$$\frac{\phi_{DNB}}{h_{fg}\,\rho_g^{0.5}\left(gg_c\,\sigma\left(\rho_f-\rho_g\right)\right)^{0.25}} = f\left(\frac{\rho_f\,\sigma^{3/2}}{g^{0.5}\,\mu_f^{2}\left(\rho_f-\rho_g\right)^{0.5}}\right)$$

Other groups of lesser importance appear also. In Ref 2, subcooled forced convection DNB data is correlated using the following groups.

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* Departure from nucleate boiling (DNB) is used by the Naval Reactors Program to denote the heat flux at which heat transfer burnout occurs. See Ref 5 for a discussion of the experimental techniques used to determine this heat flux.

$$\frac{\phi_{DNB}}{h_{fg}} \sqrt{\frac{v_f}{\sigma\, g_c\, G}} = f\left(\frac{\rho_g}{\rho_f},\ \frac{h_f - h}{h_{fg}},\ \frac{V_i \sqrt{\frac{\sigma}{\rho_f - \rho_g}\left(\frac{g_c}{g}\right)}\ \rho_f}{\mu_f}\right)$$

Because the basis of these dimensional analyses is obscure, it has been decided to perform a general dimensional analysis of DNB in vertical channels and to base this analysis on the applicable differential equations. It is recognized that in formulating these equations some assumptions have to be made concerning the nature of the process they attempt to describe. These assumptions naturally follow from the character of the experimental results. Thus, this dimensional analysis does not yield any "new" results but does provide a valuable tool for organizing and comparing the results of a large number of experiments.

We shall begin by writing down the conservation principles and the necessary boundary conditions for flow with heat addition in a channel. These conditions are sufficient to determine the solution at every point in the channel. These equations and boundary conditions will then be put into dimensionless form and the nature of a complete solution will be described. A DNB criterion will then be developed and the minimum number of independent groups upon which this criterion depends will be selected. The minimum number of independent quantities will then be established for various special DNB conditions.

DERIVATION OF EQUATIONS

The Conservation Principles

On the basis of unpublished experiments performed at Massachusetts Institute of Technology and the large body of published data now in existence, it can be said that DNB is not significantly affected by either surface conditions or by surface thermal properties. It appears therefore, that DNB occurs as a result of some limiting hydrodynamic process and that this process occurs at some distance from the surface. It shall be assumed, therefore, that this is the case and that the only effect of a heat flux at a liquid-vapor interface is to cause the evolution or condensation of vapor.

Let us select the volume existing between the point of initiation of boiling and the end of the channel as the control volume.* This is illustrated in Fig. 1. In general, at each cross section, both phases exist with an interface separating them. The Navier-Stokes equations and the continuity equation must be satisfied for each of the phases. The energy equation for the liquid is also relevant as long as the liquid is subcooled. These equations are discussed below.

The steady Navier-Stokes equation, Ref 4, for an incompressible liquid in the absence of gravity in the x direction for two dimensions is,

*DNB will occur at the end of a uniformly heated channel.

$$\rho_f \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = - \frac{\partial P}{\partial x} + u_f \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) \tag{1}$$

A similar equation can be written for the y direction. Gravity is omitted because the experiments indicate that for reasonable velocities it is not an important variable. The process is steady in the sense that a turbulent flow is steady. That is, the time average velocity at a point is independent of the time for long enough time intervals. Similarly, the continuity equation for the liquid for incompressible flow is:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2}$$

This is valid whether the flow is steady or not. The energy equation for steady flow is:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = \rho_f \frac{C}{k_f} \left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) \tag{3}$$

In this equation the dissipation terms have been eliminated as insignificant. Equations similar to Eqs (1) and (2) can be written for the gaseous phase. They are the same as those for the liquid except that different properties are used. Before considering the boundary conditions in detail, it is appropriate to indicate how the solution to this problem could be obtained. To do this, it is necessary to consider briefly a more general formulation. Let us begin with the unsteady Navier-Stokes equations.

The two-dimensional unsteady Navier-Stokes and continuity equations can be reduced to a single non-linear equation in one variable. This is accomplished by defining a new variable, the stream function, which satisfies continuity, identically, and writing the Navier-Stokes equation in terms of it. Pressure is eliminated from the new Navier-Stokes equations by cross differentiation and subtraction. This procedure is described in Ref 4 in greater detail. It is sufficient to say that since the resulting equation is fourth order in the new variable, four boundary conditions are needed (two for x and two for y). The y velocity conditions are the velocity at each of the walls. The x velocity conditions are given at the inlet and consist of the velocity and its first derivative.

In principle, the resulting differential equation could be solved completely by a numerical method. Let us assume that it has been put in its finite difference form and that all initial and boundary conditions have been stated. Solution would proceed by solving for a new value for the stream function at each point for each time. The velocity components could then be obtained by performing the appropriate differentiations. The solution for a single phase would then be complete.

The solution for two phases can be accomplished in the following manner. The complete specification of the initial conditions would have to include the position of the phase boundaries. In a rigorous solution to the problem, the phase boundaries would, in general, be a function of space and time. Therefore, when solving the difference equation for the stream function at a point, it is necessary to observe which phase is at that point and to use the appropriate equation. A complete solution to the problem would consist of specifying the velocity components and the phase at each point as a function of time. Physically, there is reason to expect that the solution would not be periodic. However, if long enough time intervals are assumed, the average velocity of each phase at any point and the fraction of time that each phase occupies the point would be independent of time or the time interval. Therefore, the unsteady formulation is probably not needed and the steady formulation which is used here is sufficient. If the velocities are known everywhere the pressure derivatives can also be found. The energy equation is coupled to the Navier-Stokes equation through the evolution or condensation of vapor at an interface at which there is a flow of heat.

To summarize, it is possible to reduce the Navier-Stokes and continuity equations to a single equation in one unknown. In principle, the velocities would be determined by solving this equation by some finite difference method. The solution to the energy equation could also be accomplished in the same manner. When the velocity is known everywhere, the pressure derivatives can be obtained. Therefore, the complete solution to the original problem can be obtained and would consist of specifying average values of u, v, P, and T as functions of x and y for each phase at every point. In addition, X_s, the static quality, would also be needed to indicate what fraction of the time each phase occupies a given point.

The definition of X_s is

$$X_s = \frac{t_g \rho_g}{t_g \rho_g + t_f \rho_f} \tag{4}$$

Where t_g is the time a point is occupied by vapor and t_f is the time during which that point is occupied by the liquid. Let us now return to the statement of the boundary conditions appropriate to Eqs (1) through (3).

These four equations for the liquid ((1) for x and for y, (2) and (3)), are relations between the variables u, v, P, and T in terms of the coordinates x and y. Their solution would consist of specifying u, v, P, and T as functions of x and y. Similar variables and solutions exist for the vapor phase except that temperature is no longer a variable so that there is no energy equation. The solution for the liquid and vapor would be connected through X_s.

Boundary Conditions

Velocity boundary conditions are needed at the channel boundaries, at the entrance and at the phase boundaries. Temperature boundary conditions for the liquid are needed at the inlet and at all liquid boundaries. In addition, it is necessary to state the pressure so that physical properties of the liquid and the vapor may be evaluated. These various conditions are as follows for the liquid:

at $y = \pm S/2$

$$u = 0$$

$$v = \frac{\phi}{h_{fg}\,\rho_f}$$

$$\frac{\partial T}{\partial y} = -\frac{\phi}{k_f}$$

$$T = T_s$$

at $x = 0$

$$u = V_i$$

$$v = 0$$

$$\frac{\partial u}{\partial x} = 0$$

$$T = T_i$$

$$\frac{\partial T}{\partial x} = 0$$

$$P = P_i$$

At the triple interface

$$\beta = \text{constant}$$

$$T = T_s$$

At the liquid-vapor interface

$$\mu_g \frac{\partial v_{gt}}{\partial n} = \mu_f \frac{\partial v_{ft}}{\partial n}$$

$$P_g - P_f = \sigma \left(\frac{1}{R_1} + \frac{1}{R_2} \right)$$

$$T = T_s$$

$$\frac{k}{\rho_g h_{fg}} \left(\frac{\partial T}{\partial n} \right) = v_{fn}$$

For the vapor

at $y = \pm S/2$

$$u = 0$$

$$v = \frac{\phi}{h_{fg} \rho_V}$$

Both the temperature and the velocity boundary conditions at the wall are somewhat unreal. The vapor is not generated right at the wall and it is not uniformly generated over the entire surface. The vapor is generated at the liquid-vapor interface at a small distance from the wall. The temperature at the wall is also a little above saturation on the average. The justification for these approximations lies in the fact that the details of the heat transfer process at the wall appear to be strongly affected by the wall surface conditions while the DNB phenomenon is not. Therefore, it might be concluded that these approximate wall boundary conditions are sufficient. In essence, it is assumed that the limiting process leading to DNB occurs at some distance from the wall and that the details of the process at the wall are not important.

Non-Dimensionalizing the Equations and Boundary Conditions

The equations are non-dimensionalized by means of the dimensions and properties which appear in the formulation of the problems. Let us define dimensionless variables and coordinates as follows:

$u^* = \dfrac{u}{V_i}$ for all velocities

$x^* = \dfrac{x}{S}$ for all lengths and similar quantities

$T^* = \dfrac{T - T_i}{T_s - T_i}$ for temperature

$$P^* = \frac{P}{\rho_f V_i^2} \qquad \text{for pressure}$$

When these variables are substituted in Eqs (1) through (3), along with the similar ones for the gaseous phase, the results, when grouped, are as follows for one of the Navier-Stokes equations,

$$\left(u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} \right) = - \frac{\partial P^*}{\partial x^*} + \frac{u_f}{\rho_f V_i S} \left(\frac{\partial^2 u^*}{\partial x^{*2}} + \frac{\partial^2 v^*}{\partial y^{*2}} \right) \tag{5}$$

Similar results are obtained for the other two equations. The continuity equation becomes

$$\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} = 0 \tag{6}$$

The energy equation becomes (for all liquid properties)

$$\frac{\partial^2 T^*}{\partial x^{*2}} + \frac{\partial^2 T^*}{\partial y^{*2}} = \left(\frac{v \rho_i S}{\mu_f} \right) \left(\frac{C \mu_f}{k_f} \right) \left(u^* \frac{\partial T^*}{\partial x^*} + v^* \frac{\partial T^*}{\partial y^*} \right) \tag{7}$$

The boundary conditions for the liquid become (in the same order that they were originally presented):

at $y^* = \pm 1/2$

$$u^* = 0$$

$$v^* = \frac{\emptyset}{h_{fg} \rho_f V_i}$$

$$\frac{\partial T^*}{\partial y^*} = \frac{\emptyset S}{k \left(T_s - T_i \right)}$$

At the entrance

$$x^* = 0$$

$$u^* = 1$$

$$v^* = 0$$

$$\frac{\partial u^*}{\partial x^*} = 0$$

$$T^* = 0$$

$$\frac{\partial T^*}{\partial x^*} = 0$$

At the triple interface

$$\beta = \text{constant}$$

$$T^* = 1$$

At the liquid-vapor interface

$$\frac{\partial v^*_{gt}}{\partial n^*} = \frac{\mu_f}{\mu_g} \left(\frac{\partial v^*_{ft}}{\partial n^*} \right)$$

$$P_g^* - P_f^* = \frac{\sigma}{\rho_f V_i^2 S} \left(\frac{1}{R_1^*} + \frac{1}{R_2^*} \right)$$

$$\frac{k \left(T_s - T_i \right)}{h_{fg} \rho_g S V_i} \left(\frac{\partial T^*}{\partial n^*} \right) = v^*_n$$

The corresponding Navier-Stokes equation for the vapor phase in the x direction is as follows:

$$\left(u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} \right) = -\left(\frac{\rho_f}{\rho_g} \right) \frac{\partial P^*}{\partial x^*} + \frac{\mu_g}{V_i S \rho_g} \left(\frac{\partial^2 u^*}{\partial x^{*2}} + \frac{\partial^2 v^*}{\partial y^{*2}} \right) \qquad (8)$$

The equation for the y coordinate is similar. The continuity equation for the vapor phase is also similar so that it will not be considered further. There are no new boundary conditions at the interface viewed from the vapor side so that the only relevant boundary conditions for the vapor are:

at $y^* = \pm 1/2$

$$u^* = 0$$

$$v^* = \frac{\phi}{h_{fg} \rho_g V_i}$$

The dimensionless groups and boundary conditions in the preceding equations can be collected and are as follows:

(a) $\dfrac{V_i\ S\ \rho_f}{\mu_f}$

(b) $\dfrac{V_i\ S\ \rho_g}{\mu_g}$

(c) $\dfrac{C\ \mu_f}{k}$

(d) $\dfrac{\sigma}{\rho_f\ V_i^2\ S}$

(e) $\dfrac{\phi\ S}{k\ (T_s - T_i)}$

(f) $\dfrac{k\ (T_s - T_i)}{h_{fg}\ \rho_g\ S\ V_i}$

(g) $\dfrac{\mu_f}{\mu_g}$

(h) $\dfrac{\rho_f}{\rho_g}$

(i) $\dfrac{\phi}{h_{fg}\ \rho_g\ V_i}$

(j) $\dfrac{\phi}{h_{fg}\ \rho_f\ V_i}$

(k) β

Not all of these groups are independent. A set that is independent is as follows:

(a) $\dfrac{V_i\ S\ \rho_f}{\mu_f}$

(b) $\dfrac{V_i\ S\ \rho_g}{\mu_g}$

(c) $\dfrac{C\ \mu_f}{k}$

(d) $\dfrac{\sigma}{\rho_f\ V_i^2\ S}$

(f) $\dfrac{k\ (T_s - T_i)}{h_{fg}\ \rho_g\ S\ V_i}$

(h) $\dfrac{\rho_f}{\rho_g}$

(i) $\dfrac{\phi}{h_{fg}\ \rho_g\ V_i}$

(k) β

The complete solution to this problem would give u*, v*, P*, T*, and X_S as functions of x* and y* with the above dimensionless groups as parameters. Let us now turn our consideration to DNB.

DNB

If the entire solution to the flow problem is determined when the above dimensionless groups are fixed, then the DNB conditions must be fixed also. It is now worthwhile to consider what is meant by DNB.

A number of different definitions have been advanced, all of which are similar when compared on the basis of heat flux. At this point, the author would like to advance still another definition which is similar to the others in practice but which permits an easy interpretation in hydrodynamic terms. The definition of DNB used in this paper is different from that in common use. However, in terms of heat flux, it is very similar.

Let us imagine a patch of the surface at the most likely DNB point. Boiling is occurring at this patch. At some instant at a given heat flux, a section of the surface may look as it does in Fig. 2. The probable surface temperature profile is also shown on this figure.

When the heat flux is increased, the number of bubbles increases on this patch of surface and the heat transfer coefficient of boiling changes. Such a heat transfer coefficient can be defined as:

$$\alpha = \frac{-k \left(\frac{dT}{dy}\right)_{mean}}{\left(\frac{1}{A}\int T \; dA\right) - T_s} \qquad (9)$$

As the bubble packing on the surface changes, α will change but at some packing for given fluid properties, surface properties and contact angle, α will be at a maximum. Let X_{DNB} denote the static quality associated with such a packing and identify the maximum α point with the DNB point. X_{DNB} is a definite number. It can be shown that this is the point where the line with a slope of 1 is tangent to the ln ϕ vs ln (ΔT) curve. Let us now return to consider the nature of the solution of the problem that has been formulated in the previous discussion.

The velocities and quality at any point are determined when all the dimensionless groups are fixed. That is,

$$X_s = f \; (x^*, y^*, \text{and groups a, b, c, d, f, h, i, and k})$$

Since, with a uniform heat flux, DNB occurs at the end of the channel near the wall,

$$X_s = X_{DNB} = f \left(\frac{L}{S}, \pm \; 1/2, \text{ and groups a, b, c, d, f, h, i, and k}\right)$$

Therefore, if it is recalled that X_{DNB} is a definite number, and all the constants in the unknown functional relationship are grouped, the above equation can be solved for any one of the unknown functional groups. For example, Eq (10) can be obtained

$$\frac{\phi_{DNB}}{h_{fg}\,\rho_g\,V_i} = f\left(\frac{L}{S}, \frac{V_i\,S\,\rho_f}{\mu_f}, \frac{V_i\,S\,\rho_g}{\mu_g}, \frac{C\,\mu_f}{k}, \frac{\sigma}{\rho_f\,V_i^{\,2}\,S}, \frac{k\left|T_s - T_i\right|}{h_{fg}\,\rho_g\,S\,V_i}, \frac{\rho_f}{\rho_g}, \beta\right) \qquad (10)$$

Equation (10) expresses ϕ_{DNB} in terms of significant dimensionless groups.

Although most of the liquid and vapor properties appear in this equation, the most important variable is probably the vapor density. If one assumes that all the variations in properties are included in the $\frac{\rho_f}{\rho_g}$ term, probably only small errors will result.

It is not possible to develop, from this set of groups, the principal correlating group of Ref 3,

$$\frac{\phi_{DNB}}{h_{fg}\,\rho_g^{0.5}\left(gg_c\,\sigma\left(\rho_f-\rho_g\right)\right)^{0.25}}$$

This is because gravity has been omitted. It is felt that this should be done because the essential processes are as different as forced and natural convection. Gravity is generally omitted from the forced convection correlations, although there is a class of problems in which both forced and natural convection effects appear.

Correlation in Terms of Local Conditions

It seems more reasonable to correlate in terms of the conditions existing at the DNB point since DNB appears to be a local phenomenon. The first law, the continuity equation and the equation of state are needed to accomplish this. These equations are respectively,

$$2\,\phi\,\frac{L}{S} = G\,(h - h_i) \tag{11}$$

$$\rho V = \rho_f V_i = G \tag{12}$$

$$\rho = f\,(h) \tag{13}$$

These equations relate h and v, the enthalpy and the velocity at the DNB point, to known inlet conditions. With these equations it is possible to eliminate the inlet velocity and the inlet temperature from the groups of Eq (10). L/S remains, however, and the reason why it should appear will be explained in the next section.

General Considerations Relating to a DNB Correlation

If a correlation for a single fluid at only one pressure is desired, no variation in fluid properties exists and these fluid properties need not appear in the correlation. Then, from Eq (10), the DNB heat flux is a function of only four variables,

$$\phi_{DNB} = f\left(V_i\ \frac{L}{S},\ S,\ \left(T_s - T_i\right)\right) \tag{14}$$

An equivalent set of four variables in terms of local conditions can be obtained from Eqs (11), (12), (13), and (14),

$$\phi_{DNB} = f\ \ (V,\ L,\ S,\ h) \tag{15}$$

G could be used instead of V but it does not appear to have the physical significance that V has.

The selection of the proper four correlating variables is difficult to answer in general, but certainly the following three considerations are among the most important: (a) The simplicity of the functional relationship, (b) ease of extrapolation, and (c) ease of physical interpretation.

In the beginning of this dimensional analysis of DNB, it was decided that attention should be confined only to the patch or section of tube at which DNB occurs. It was found, however, that in order to describe completely the conditions existing at the inlet to this section, it was necessary to specify how the phases were distributed across the channel in addition to specifying the mean enthalpy and velocity. That is, X_s and the normalized velocity distribution were needed for each phase. This normalization would use the velocity calculated from Eq (12). In general, the specification of a curve involves an infinite number of parameters. Thus, a dimensional analysis with an infinite number of groups cannot be used as a device for reducing to a minimum the number of quantities which must be considered. However, when the whole flow channel is examined, the number of independent groups is reduced to the four shown in Eqs (14) and (15). Thus, the L and S of Eq (15) should be considered as quantities needed to specify the velocity and phase distributions across the channel.

The L and S effects on DNB can also be explained from the standpoint of the fluid mechanics of boiling in a channel. Imagine a channel through which two phases are flowing. For a given flow rate for each of the phases, there is one particular distribution of the two phases, i.e., static quality. If all the vapor is generated at the wall, (i.e., by heat transfer), there are other possible phase distributions each of which is dependent on the rate of steam generation. That is, the distribution depends on the throughput velocities of each phase and the normal velocity at the wall. In the next paragraph it will be shown how the normal velocity at the wall is related to the L/S effect.

Let us consider the case where L is measured from the point where bulk boiling begins and rewrite the first law, Eq (11), for the channel in the following form,

$$\frac{L}{S} = \frac{\rho V X h_{fg}}{2 \phi} \tag{16}$$

Assuming fog or homogeneous flow, the mixture density is expressed as,

$$\frac{1}{\rho} = \frac{X}{\rho_g} + \frac{(1 - X)}{\rho_f}$$

This can be written in the following form

$$\rho = \frac{\rho_f \, \rho_g}{X \, \rho_f + (1-X) \, \rho_g} \approx \frac{\rho_g}{X}$$

for the higher range of qualities.

When this value of ρ is substituted in Eq (16), the result is

$$\frac{L}{S} \approx \frac{V \, h_{fg} \, \rho_g}{2 \, \phi} \qquad (17)$$

This expression shows that the L/S is related to the ratio of the throughput velocity to the normal velocity at the wall. This ratio is probably needed to fix the phase distributions in the channel. The nature of the L/S effect on DNB is not obvious, however.

Equation (17) indicates that small values of L/S are associated with high DNB heat fluxes. However, the high DNB heat fluxes themselves mean large normal velocities at the wall so that one would expect that the concentration of liquid at the wall would be less. As a result, one would then expect a lower DNB heat flux. In addition, one finds L/S effects persisting for distances greater than our experience with single phase heat transfer would predict.

There are two sets of experimental observations which help in rationalizing these paradoxes. One is concerned with the physical nature of the L/S effect, the other with the linear extent (along the channel) of this effect. These observations do not "explain" the results but they do give a physical picture which illustrates the differences between large and small values of L/S.

References 8 and 9 report some work done at low pressure on air-water systems. Figures 91 through 96 of Ref 8 show a series of photographs taken in a simulated boiling system in which water flows in at the top and air flows in at the sides through porous bronze. Each series is taken at constant water flow rate and increasing air flow rate. On each of these pictures, lines which connect regions of constant flowing conditions can be drawn. In terms of a boiling system, these lines connect regions of constant G and h. It can be seen in these figures, especially Figs. 91, 92, and 93, that the configuration of the two phases, for the same apparent flowing conditions, is quite different. These differences can be interpreted as an L/S effect. The test section in this set of experiments is quite short, but it is all "boiling length" because the conditions of the test correspond to saturated water at the inlet. The photographs show that flows with large normal velocities tend to form gas cores at a lower quality, thereby tending to concentrate the liquid at the walls. This is consistent with our observations on DNB, but still is contrary to our instincts.

Reference 9 reports some work that was performed under conditions quite different from those of interest in reactor design. It is interesting to note, however, that the entrance effects in two phase flow without heat addition can persist for very great distances. Figure 6 of Ref 9 shows that values of L/S greater than 300 were sometimes required for fully developed two-phase flow to occur. In the case of a channel with heat addition one might expect such differences to persist indefinitely, because, as long as heat is being added no unique fully developed flow condition is being approached.

To summarize then, DNB occurs as a result of local conditions. Apparently phase distributions for the same flowing conditions are different for different values of the normal velocity at the wall, and these differences give rise to different DNB heat fluxes. These differences in normal velocities at the wall are related to L/S through the first law.

DNB Correlation for Various Special Conditions

Since the important correlating groups of Eq (15) are already in use for correlating DNB data it does not appear that any important quantity is being omitted. Therefore, it is believed that a large part of the scatter in existing DNB correlations result from the nature of the functional relationships which are used to correlate the data. There are two apparent ways in which the existing deficiencies in the correlation scheme may be improved. One way is to simply use more undetermined constants. Presumably, with enough constants and with good predictions of the proper functional forms, a superior correlation could be obtained. To the extent that one is confident of the accuracy of the data, there need not be any limit as to the number of constants, or for that matter the nature of the functional relationships.

The correlations may also be improved by maintaining the simple functional relationships and small numbers of constants but making the range of data correlated by a given set of constants much smaller so that a closer fit is possible. There are good reasons for both approaches. It appears that there are gradual changes in the importance of various parameters so that some terms can be excluded in certain regions.

Since it is believed that there are no abrupt changes in DNB conditions, it would appear that a continuous function has considerable virtue. In the next paragraphs, the important variables in each of the different regions of significance will be considered and the important correlating groups indicated. A combination of these methods is possible if one uses a simple basic equation but allows the constants in this equation to vary; this variation can be continuous provided that they are presented in the form of a side plot or equation.

(a) Correlation of Subcooled DNB Data

Photographs and density measurements indicate that when the liquid subcooling during boiling is large, the bubbles are concentrated near the wall. The flow configuration is then always the same and is very simply represented with bubbles at the wall and a generally subcooled liquid core. This situation appears to continue up until about 5% "negative quality" $\left(\frac{C\left|T - T_s\right|}{h_{fg}}\right)$. The primary correlating variables in this region are velocity and subcooling. The subcooling can be obtained from a heat balance; further, since the vapor volume is so small, the inlet velocity approximates the actual velocity.

L/S effects are also present; however, it is believed that they would appear to be of the same nature as those for a single phase in a pipe. That is, for $L/S > 30$ their effect would be minor. The large L/S effects in DNB appear to manifest themselves primarily through phase distributions. For subcooled DNB the phases are always distributed the same way. Therefore, let us say that for a single fluid and pressure and for $L/S > 30$,

$$\phi_{DNB} = f\left(V_i, \left|T_s - T\right|, S\right) \tag{18}$$

Channel spacing is probably a minor variable, as it is in the single phase heat transfer, but it does belong in the above group.

The problem becomes more complicated when one considers what parameters should be included in Eq (18) to account for variations in pressure. Groups a, c, f, h, and i all probably belong. Because their inclusion is too complicated to be practical, some simplification is desirable. When attention is confined to water, all property variations in the high pressure range are insignificant except for vapor density and surface tension. Of these, the inclusion of just one is probably sufficient because the two vary quite regularly with temperature or pressure. Density is probably the best one to choose because it is better known. Thus, for subcooled water at pressures above 300 psia it should be sufficient to consider,

$$\phi_{DNB} = f\left(V_i, \left|T_s - T\right|, S, \rho_g\right) \tag{19}$$

The author's experience is that the effect of a given value of $\left|T_s - T\right|$ on DNB is a strong function of pressure. That is, when correlating DNB data for several different pressures, it may not be possible to find a function of $\left|T_s - T\right|$ times a function of ρ_g alone that will work. A product type correlating function including both ρ_g and $\left|T_s - T\right|$ as well as, perhaps, functions of ρ_g alone and $\left|T_s - T\right|$ alone are needed. The author would suggest, for this product term, the function $\left|T_s - T\right| / \rho_g$, since a given amount of subcooling appears to have a greater effect at low pressure.

When producing a correlation for all fluids at all pressures, the problem is still more difficult. There is no longer any justification in assuming that all property variations are small so that a large number of terms must be included in the correlations. A good correlation scheme would have the primary correlating group containing the most important variables with the variations small and the functional relationships simple for the remainder of the groups. Reference 2 presents a dimensionless correlation scheme which works well on water at high pressure. It is questionable whether gravity should appear in the primary correlating variable as it does, because the equations for any forced convection process do not contain it. The properties used to make this correlation dimensionless are not very significant because the only data correlated were based on high pressure water in a relatively narrow range of pressure, (100-200 atm).

A study of the groups of Eq (10) would indicate that probably the important groups are as follows:

$$\frac{\phi_{DNB}}{h_{fg}\,\rho_g\,V_i} = f\left(\frac{V_i\,S\,\rho_f}{\mu_f},\ \frac{C\,\mu_f}{k},\ \frac{\sigma}{\rho_f\,V_i^2\,S},\ \frac{k\,\left|T_s - T_i\right|}{h_{fg}\,\rho_g\,V_i\,S},\ \frac{\rho_f}{\rho_g}\right) \qquad (20)$$

Certain groups were eliminated for physical reasons. They are the boiling length, the gas Reynolds number, and contact angle. None appeared to be of any particular significance. The above groups might be written in a physically more meaningful form but it is not obvious what that form would be. When the groups are written in terms of local conditions and in what would appear to be decreasing order of importance, there results

$$\frac{\phi_{DNB}}{h_{fg}\,\rho_g\,V_i} = f\left(\frac{k\,\left|T_s - T\right|}{h_{fg}\,\rho_g\,V_i\,S},\ \frac{\rho_f}{\rho_g},\ \frac{V_i\,S\,\rho_f}{\mu_f},\ \frac{C\,\mu_f}{k_f},\ \frac{\sigma}{\rho_f\,V_i^2\,S}\right) \qquad (21)$$

This completes the discussion of subcooled DNB correlations.

(b) Correlation of Quality DNB Data

Since the L/S or L effects in subcooled DNB are considered to be simply of thermal entrance length origin while those of the quality DNB region appear to be of a developing two phase flow origin, perhaps different correlating lengths are appropriate. The void measurements of Ref 9 show that very little volume of vapor is developed until the subcooling is less than $\frac{\phi}{4\,h_{nb}}$. That is, in terms of phase distributions, all channels are the same at subcooling greater than this. When correlating the quality DNB data, the appropriate length to use, then, would appear to be the length at which the substantial increase in void volume with length begins (at about $\frac{\phi}{4\,h_{nb}}$ subcooling). This would be almost equivalent to the length from the zero quality point for many of the data.

Therefore, the quantities which are appropriate for correlating the DNB data for a single fluid and pressure, are, from Eq (15),

$$\phi = f \left| V, L_o, S, h \right| \tag{22}$$

in which L_o is the length from the zero quality point. A consideration of the probable hydrodynamic nature of the DNB process would lead one to expect that the effect of a variation in S would not be felt at low enough qualities and for large enough tubes. The reason for this is that the limiting hydrodynamic process probably occurs at some small distance from the wall. If the thickness of the layer of liquid on the wall is sufficiently large one would not expect the thickness to be significant. In view of the probability that thickness of the liquid film on the wall, for the same enthalpy and velocity, is proportional to the channel size, one would also expect the channel size effect to disappear for large channels. The velocity term in Eq (22) is a mixture velocity which is considered to be a good indication of the velocity of both phases across the entire channel.

If we wish to extend the correlation to include a range of pressures for one fluid at least one more term is needed. Actually all properties vary somewhat, and the same fluid at a different pressure is hydrodynamically a different fluid. Nevertheless, the most important property variation is confined to the vapor density. Therefore, DNB should correlate for one fluid at a variety of pressures in the quality region with

$$\phi_{DNB} = f \left(V, L_o, S, \left| h - h_f \right|, \rho_g \right) \tag{23}$$

As long as velocity rather than G is used to correlate the data, much of the apparent pressure effect on DNB would probably be removed.

It does not appear that any very large simplifications are possible if one wishes to correlate all fluids in the quality region. However, some simplification of Eq (10) is possible. Since the heat transfer process itself does not appear to be important, the Prandtl number is probably not significant. Contact angle does not appear important. If local conditions are used, thermal conductivity will not enter. Therefore, in decreasing order of importance

$$\frac{\phi_{DNB}}{h_{fg}\,\rho_g\,V} = f \left(\frac{h - h_f}{h_{fg}}, \frac{V S \rho_f}{\mu_f}, \frac{L}{S}, \frac{\rho_f}{\rho_g}, \frac{V S \rho_g}{\mu_g}, \frac{\sigma}{\rho_f V^2 S} \right) \tag{24}$$

A word should be said about the functional relationships which hold the most promise. The liquid Reynolds number should be important primarily at low volume qualities where the liquid is the continuous phase so that a function of the type $\left(\frac{V S \rho_f}{\mu_f}\right) \left(\frac{h_g - h}{h_{fg}}\right) \left(\frac{\rho_f}{\rho_g}\right)$ would probably be the best. Conversely, one would expect that the vapor properties would only be important in the region where the vapor is the continuous phase and most of the liquid is in the form of drops. Therefore, a function of the form $\left(\frac{V S \rho_g}{\mu_g}\right)\left(\frac{h - h_f}{h_{fg}}\right)\left(\frac{\rho_f}{\rho_g}\right)$ might be good.

It would appear also that the L/S effect should be made a function of quality. The strong dependence of the "hot patch" effectiveness on enthalpy is an indication of the varying importance of "past history" or L/S effects on DNB (see Ref 5). Thus, enthalpy should be included in the functional form chosen for the L or L/S term. One possibility is $\left(\frac{h - h_f}{h_{fg}}\right)\left(\frac{L}{S}\right)$. A large number of other possibilities exist too, but this seems to be of a promising form.

(c) Correlation of Transient DNB Data

It is apparent from a consideration of the problem that more dimensionless groups are needed to correlate transient DNB data. When the Navier-Stokes equations are written in their transient form, no new dimensionless groups arise. However, the complete statement of the boundary conditions is somewhat different since some of the conditions are time-dependent. The various possible transient boundary conditions will first be stated and then they will be put into dimensionless form.

Both flow and flux can be transient. It will be assumed that before the transient starts, the flux and inlet velocity will have a steady state value of ϕ_o and V_{io} and the distributions in the channel will be those characteristic of this flux and velocity. Time will be assumed to start at the beginning of the transient. Following is a statement of the various possible types of flux and flow transients. The b's in these functions define the nature of the transient and depend on the system characteristics.

Three possible flux transients are

a. $t = 0 \qquad \phi = \phi_o$

$0 < t \leq t_o \qquad \phi = \phi_o$

$t > t_o \qquad \phi = \phi_o \dfrac{1}{1 + b_1 \left(t - t_o\right)}$

b. $t \lesseqgtr 0 \qquad \phi = \phi_o$

$t > 0 \qquad \phi = \dfrac{\phi_o}{\left(1 + b_1 t\right)}$

c. $t \lesseqgtr 0 \qquad \phi = \phi_o$

$t > 0 \qquad \phi = \phi_o e^{b_2 t}$

A possible flow transient is

$$t \leq 0 \qquad V = V_{io}$$

$$t > 0 \qquad V = \frac{V_{io}}{1 + b_3 t}$$

These equations can be non-dimensionalized by using a dimensionless time defined as follows:

$$t^* = \frac{t\, V_{io}}{L} \tag{25}$$

Length rather than channel spacing was used in this ratio since the transit time appeared to be of some physical significance and $\frac{L}{V_{io}}$ is approximately a transit time.

When the preceding transients are put in dimensionless form, the following groups result (these groups are in addition to those of Eq (10)).

$$\frac{\phi_{DNB}}{h_{fg}\, \rho_g\, V_i} = f\left(\frac{b_1 L}{V_{io}}, \frac{b_2 L}{V_{io}}, \frac{t_o V_{io}}{L}, \frac{b_3 L}{V_{io}}\right) \tag{26}$$

When there are both flow and flux transients, these groups probably interact among themselves and also with the group specifying the local enthalpy. In view of the large number of groups, the analysis of most transient DNB data is therefore very difficult. A simple physical picture might, however, help clarify the meaning of the above groups and also their interactions.

First, the terms of the type $\frac{b\, L}{V_{io}}$ are a measure of the ratio of the transit time to the transient time. One would expect that the steady-state DNB values would be appropriate for small values of this ratio. If one imagines that DNB occurs when the wall becomes dry, one would expect transient DNB effects to also be quality sensitive since the thickness of the layer of liquid on the wall is probably quality sensitive also. The thickness of this layer is probably velocity sensitive also, so that some coupling would exist there. It is desirable to use these groups in analyzing the transient data already obtained at the Bettis Atomic Power Laboratory.

CONCLUSIONS

1. For a single fluid at one pressure flowing in wide channels, DNB is a function of four independent variables. These variables could be h, G, L/S, and S.

2. When the above four variables are used to correlate DNB data, any apparent inlet subcooling effect is due to an inadequate assumption of the form of the functional relationship between these variables.

3. The mechanism of DNB is still not understood.

4. L/S affects DNB through the distribution of phases at the DNB point. Simply specifying velocities and qualities is not sufficient; a specification of phase distribution is important too.

5. Further improvements in DNB correlations will probably result from better assumptions for the functional relationships and the use of larger numbers of undetermined constants. There is no reason to expect that a function of one variable alone times a function of another variable alone, etc., will suffice in correlating the data.

NOMENCLATURE

English Letter Symbols

A	area
C	specific heat at constant pressure for the liquid
L	boiling length
L_o	length from the zero quality point in the channel at steady state
P	pressure
R_1, R_2	principal radii of curvature of liquid-vapor interface
S	channel spacing
T	temperature
V	velocity
X	quality, flowing
X_s	static quality for region near surface for DNB
b_1, b_2	constants used in specifying flux transients with units of reciprocal time
b_3	constant used in specifying flow transient with the units of reciprocal time
h	enthalpy
h_{fg}	enthalpy change from liquid to vapor
h_{nb}	heat transfer coefficient in the absence of boiling
k	liquid thermal conductivity
n	normal direction
t	time
t_o	time before flux transient starts
u	velocity component in the x direction
v	velocity component in the y direction
x	coordinate
y	coordinate
z	coordinate

Greek Letter Symbols

α	heat transfer coefficient, Eq (9)
β	contact angle measured through the liquid
ρ	density
μ	viscosity
ϕ	heat flux
ν	kinematic viscosity
σ	surface tension

Subscripts

c	core
d	delay
f	liquid
g	vapor
i	inlet
o	original
s	saturation
io	original inlet
ft	tangential in liquid
gt	tangential in vapor
fn	normal in liquid
gn	normal in vapor

Superscript

*	dimensionless variable or coordinate

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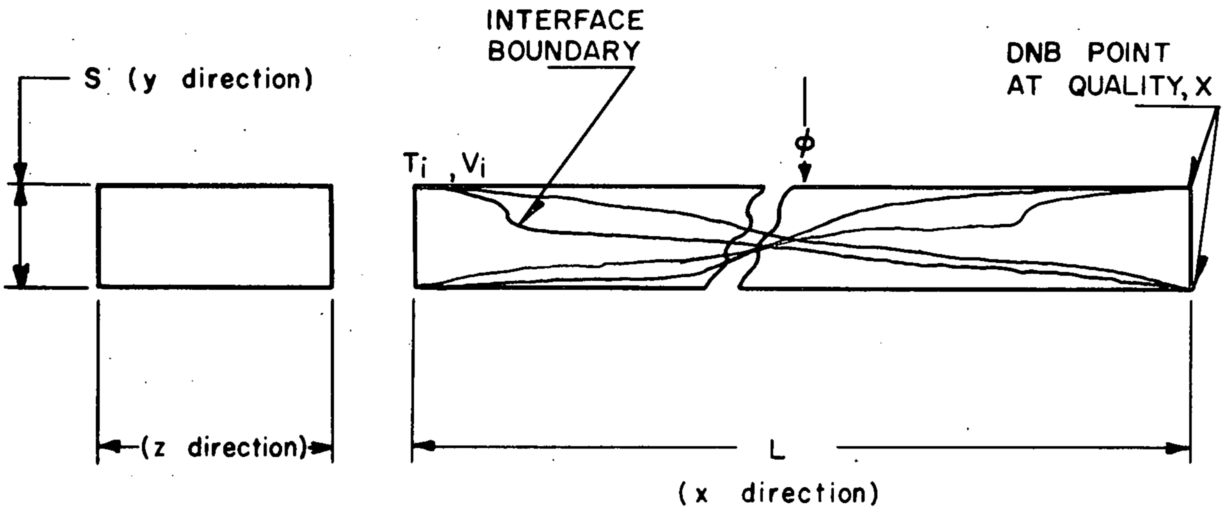

FIGURE 1
CONTROL VOLUME

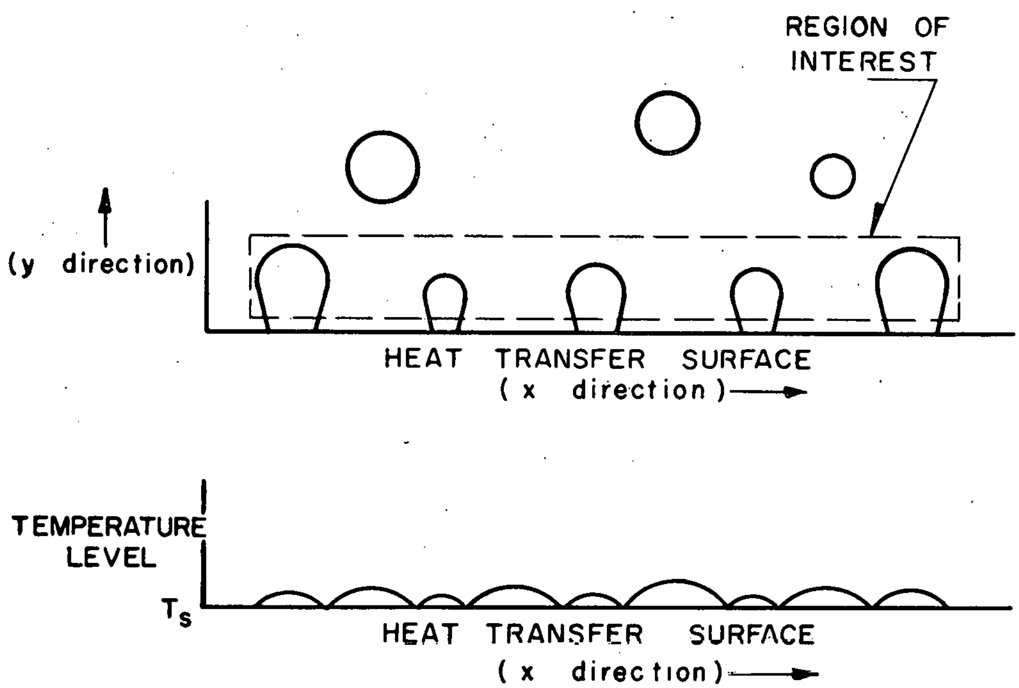

FIGURE 2
TEMPERATURE AND BUBBLE DISTRIBUTIONS

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