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SUBJECT: A Parametric Study of a Gas Cooled Reactor

TO: Listed Distribution

FROM: L. G. Epel

ABSTRACT

The results of a parametric study on a gas cooled reactor are reported on herein. The system considered was a helium cooled, UO_2 fueled arrangement with the fuel assemblies consisting of clusters of long cylindrical elements, each element covered by a stainless steel jacket. The axial power distribution was assumed to be a "chopped cosine" having an axial peak-to-average power of 1.32.

The independent variables used were:

1. gas inlet temperature
2. gas outlet temperature
3. maximum permissible fuel rod surface temperature
4. fuel rod diameter
5. number of rods per cluster
6. heat output per foot of rod at the center of the reactor
7. pressure level of gas coolant
8. length of coolant channel

These eight quantities determine uniquely the three parameter sought once the coolant's physical properties are assumed. The three parameters of interest in this study are:

a. diameter of coolant channel
b. pressure drop through core
c. pumping power expended

The analysis is presented for the central channel.

NOTICE

DISCLAIMER

INTRODUCTION

The one thing which ultimately limits the power density a reactor is capable of achieving is heat transfer capability. If system temperatures are stipulated, the cooling problem will dictate the heat fluxes which are feasible and hence the power which may be withdrawn for any power expended in pumping the coolant.

Once physics calculations set core parameters such as lattice pitch, core dimensions, and various volume ratios, the optimum operating conditions from a standpoint of power removal and power required to remove heat must be ascertained. After a fuel element configuration has been decided upon, there are any number of variables which will determine these quantities.

Under the assumptions of the present study, eight independent parameters were used as input data and three quantities were computed. It is obvious that the number of calculations necessary with so many variables, each encompassing a reasonable range will lead to a large number of cases before the entire field can be surveyed. The use of the IBM 704 digital computer simplified the problem of doing a large number of numerical calculations, some of which were of the trial and error type. However, the presentation of the output as functions of the input variables required some further thought. To make the results easy to assimilate and to make obvious the drawbacks in certain combinations of independent input parameters, the carpet plots which follow were decided upon. Study of these plots should point out ranges of operation which are reasonable and regions which are impractical.

To make the charts easier to use, all quantities are in the units most likely to be used in their description rather than in consistent dimensions. The dimensions used throughout this report for all quantities are given in the table of nomenclature which follows (except as otherwise noted explicitly).

It should be emphasized here that the purpose of the present study was not to determine an optimum operating point but rather to show which regions were the most fertile ground for a more concentrated exploration. The main value of the results shown on the accompanying charts is that they make possible a rapid evaluation of any combination of primary parameters. Two stumbling blocks in such an evaluation, namely the necessary convective heat transfer coefficient for a given inlet, outlet, and maximum surface temperature and the channel diameter required for this heat transfer coefficient have been calculated and charted.

NOMENCLATURE

Symbol	Quantity	Units
A	cross-sectional flow area	ft^2
c_p	specific heat of coolant	Btu/lb ^oF
d	fuel rod diameter	inches
D	channel diameter	inches
De	equivalent diameter	inches
f	friction factor	-
F	heat flux factor	Btu/hr ^oF
g	acceleration due to gravity	ft/sec^2
h	heat transfer coefficient	Btu/hr ft^2 ^oF
k	thermal conductivity of coolant	Btu/hr ft ^oF
L	length of channel	feet
n	number of rods per channel	-
p	pressure level of coolant	psia
Δp	pressure drop of coolant	psi
P	pumping power	horsepower
Pr	Prandtl number	-
q	linear heat flux	Btu/hr ft. of rod
R	gas constant	ft lb/lb ^oR
Re	Reynold's number	-
t in	coolant inlet temperature	^oF
t out	coolant outlet temperature	^oF
t_s	rod surface temperature	^oF
T in	coolant inlet temperature	^oR
T out	coolant outlet temperature	^oR
ΔT	temperature difference between rod surface and coolant	^oF
δT	temperature difference between coolant and the coolant inlet temperature	^oF
V	velocity of coolant	ft/sec
x	position in channel	feet
x_1	distance to inlet of active core	feet
x_0	extrapolated core length	feet

NOMENCLATURE (continued)

Symbol	Quantity	Units
Z	defined in equation 20	-
μ	coolant viscosity	lb/ft sec
ρ	coolant density	lb/ft^3

Subscripts

e	entrance, expansion and contraction, exit	
f	frictional	
m	signifies maximum	

DISCUSSION

The determination of pressure drop and pumping power hinges on knowledge of the channel diameter once the eight primary quantities are fixed. The channel diameter in turn is related to the maximum surface-to-gas temperature difference to be found in the channel by means of the heat transfer coefficient. This maximum ΔT is shown as a function of maximum surface temperature, inlet temperature, and outlet temperature in Fig. I. It is found from the following relations.

$$\Delta T \text{ at } x = \Delta T_m \sin \frac{x}{x_o}\pi \qquad (1)$$

$$\delta T \text{ to } x = \frac{\delta T_m \left(\cos \frac{x_1}{x_o}\pi - \cos \frac{x}{x_o}\pi \right)}{2 \cos \frac{x_1}{x_o}\pi} \qquad (2)$$

Adding Eqs. (1) and (2) and adding t_{in} to this sum yields the fuel rod surface temperature as a function of position in the reactor. By differentiating this resulting expression with respect to x and setting the derivative equal to zero, the position of maximum surface temperature is found to be

$$\frac{x}{x_o}\pi = \arctan\left[- \frac{2\Delta T_m \cos \frac{x_1}{x_o}\pi}{\delta T_m} \right] \qquad (3)$$

where $\frac{x}{x_o}\pi$ is restricted to the second quadrant because of the physical situation.

Inserting the result of Eq. (3) into the expression for surface temperatures yields the maximum surface temperature.

$$t_{sm} = \Delta T_m \sin \arctan \left[\frac{2 \Delta T_m \cos \frac{x_1}{x_o} \pi}{\delta T_m} \right]$$

$$+ \delta T_m \frac{\cos \frac{x_1}{x_o} \pi + \cos \arctan \left[\frac{2 \Delta T_m \cos \frac{x_1}{x_o} \pi}{\delta T_m} \right]}{2 \cos \frac{x_1}{x_o} \pi} + t_{in} \qquad (4)$$

The ratio $\frac{x_1}{x_o}$ (the "chopped cosine" cut-off point) is determined from

$$\text{Axial Peak-to-Average Power Ratio} = \frac{\left(1 - 2 \frac{x_1}{x_o}\right) \pi}{2 \cos \frac{x_1}{x_o} \pi} \qquad (5)$$

For the special case under consideration, then, equation 4 became

$$t_{sm} = \Delta T_m \sin \arctan \left(1.90 \frac{\Delta T_m}{\delta T_m} \right)$$

$$+ \delta T_m \frac{.951 + \cos \arctan \left(1.90 \frac{\Delta T_m}{\delta T_m} \right)}{1.90} + t_{in} \qquad (6)$$

ΔT_m is shown in Fig. I for various inlet, outlet, and maximum surface temperatures.

To find what heat transfer coefficient is necessary to maintain the ΔT_m calculated in Eq. (6) above, the convection heat transfer equation was employed. The result is

$$h = 3.82 \frac{q_m}{d \Delta T_m} \qquad (7)$$

where all quantities are in the units given in the nomenclature. This equation is represented in Fig. II for convenient reference.

Knowing the necessary value of heat transfer coefficient, the channel diameter is found from a modified Dittus-Boelter equation in conjunction with a heat balance on the coolant.

$$\frac{hDe/12}{k} = .0184 \left(\frac{\rho VDe/12}{\mu}\right)^{.8} Pr^{.4} \qquad (8)$$

$$\frac{1}{1.32} q_m Ln = \rho 3600 V \frac{\pi}{576} (D^2 - nd^2) c_p \delta T_m \qquad (9)$$

Taking $k = .145$, $\mu = 23.05 \times 10^{-6}$, and $c_p = 1.24$, combining and extricating D from the result as much as possible yields

$$\left(\frac{D^2 - nd^2}{D + nd}\right)^5 = \frac{2.70}{h^5} \left(\frac{q_m Ln}{\delta T_m}\right)^4 \qquad (10)$$

Equation (10) was solved for D for various values of n, d, h, and the parameter $F = \frac{q_m Ln}{\delta T_m}$ and the results are presented in Figs. IIIa to IIIf, inclusive.

Reference to Figs. III shows that for reasonable channel sizes a large number of combinations of primary independent variables is out of the question. From this point on, channel diameters in excess of 10 in. were not considered.

The next quantity of interest was the pressure drop. It consists of two separate components; one is the pressure drop due to friction including hangers and spacers, the other is the drop due to entrance, expansion and contraction between bundles, and exit losses. The friction pressure drop was assumed to follow the relation

$$\Delta p_f = f \frac{L}{12 De} \frac{\rho V^2}{2g} \qquad (11)$$

where $f = .23\ Re^{-.19}$ as per Eq. 5 on page 34 of ORNL 2676 which was obtained for a seven rod bundle. The pressure drop due to entrance, expansion and contraction, and exit losses was assumed to be equal to the density of the helium times one-half, one, and one velocity head, respectively.

$$\Delta p_e = \frac{1}{144}\left[\left.\frac{\rho V^2}{4g}\right]\text{inlet} + \left.\frac{\rho V^2}{2g}\right]\text{average} + \left.\frac{\rho V^2}{2g}\right]\text{exit}\right] \qquad (12)$$

Now in Eqs. (11) and (12), it is necessary to know ρV^2. This comes from Eq. (9) together with the perfect gas law, which for helium states that

$$\rho = .373\ \frac{p}{T} \qquad (13)$$

$$\text{where } T = \frac{T_{in} + T_{out}}{2}$$

Eq. (9) yields

$$\rho V = .0311\left(\frac{q_m\ Ln}{\delta T_m}\right)\frac{1}{\left(D^2 - nd^2\right)} \qquad (14)$$

From Eqs. (13) and (14), it is easy to see that

$$\frac{\rho V^2}{2g} = 4.029 \times 10^{-5}\left(\frac{q_m\ Ln}{\delta T_m}\right)^2\frac{1}{\left(D^2 - nd^2\right)^2}\ \frac{T}{p} \qquad (15)$$

Finally then, Eqs. (11) and (12) are

$$\frac{\Delta p_f}{\frac{LT}{p}\ F^{1.81}} = 3.15 \times 10^{-7}\ \frac{(D + nd)^{1.19}}{\left(D^2 - nd^2\right)^3} \qquad (16)$$

$$\frac{\Delta pe}{\frac{2T_{in} + 3T_{out}}{p} F^2} = 1.40 \times 10^{-7} \frac{1}{(D^2 - nd^2)^2} \qquad (17)$$

The results of Eq. (16) are shown in Fig. IV with $\frac{\Delta p_f}{\frac{LT}{p} F^{1.81}}$ listed along the vertical scale. Eq. (17) is plotted in Fig. V with $\frac{\Delta pe}{\frac{2T_{in} - 3T_{out}}{p} F^2}$ as the dependent variable. The sum of the two pressure drops multiplied by the volume rate of coolant flow yields the pumping power required.

From Eqs. (9) and (13) the volume rate of flow is

$$VA = 4.55 \times 10^{-4} \frac{(q_m\ Ln)}{(\delta T_m)} \frac{T}{p} \qquad (18)$$

The horsepower needed for pumping then is

$$P = 1.19 \times 10^{-4} \Delta p \frac{(q_m\ Ln)}{(\delta T_m)} \frac{T}{p} \qquad (19)$$

The quantity $\frac{P}{\Delta p}$ has been plotted in Fig. VI with $\frac{(q_m\ Ln)}{\delta T_m}$, T, and p as independent variables.

One final comment seems in order at this point. Since it seems reasonable to expect most of the "F" values of interest to lie in the range between 1000 and 50,000, this range was investigated more closely with regard to the necessary channel diameters for given values of n, d, h, and F. This came, as before, from Eq. (10) which was rewritten as follows to facilitate plotting of the results.

$$\frac{D^2 - nd^2}{(D+nd)^{.2}} = \frac{1.22}{h} F^{.8} \equiv Z \qquad (20)$$

The dimensional quantity Z is shown in Fig. VIIa for various values of F and h. Fig. VIIb then yields D accurately as a function of n, d, and Z.

The extension of the central channel analysis, as presented on the previous pages, to the reactor as a whole follows fairly simply.

If one starts with the premise that all channels are to yield helium at the same outlet temperature (and of course have a uniform inlet temperature) then orificing is necessary. The orificing will control the mass rate of flow so that the amount of gas in each channel is proportional to the integrated heat flux in that channel. It is easy to see from Eq. 2 that the gas temperature structure through the reactor will be exactly the same in any channel. The heat transfer coefficient will be nearly proportional to the mass flow rate (Eq. 8) and hence ΔT_m will be almost the same for off-center channels as for the central channel (actually the ΔT_m will be slightly less than in the central channel because the heat transfer coefficient decreases a little less rapidly than the mass flow).

If r represents the radial peak-to-average power ratio, it is obvious that the total power out of the reactor in Btu/hr is:

$$\frac{.757\ q_m L n}{r} \text{ (number of channels)}$$

and the total pumping power is from Eq. (19)

$$P_{total} = 1.19 \times 10^{-4}\ \Delta p\ \frac{(q_m L n)}{\delta T_m} \cdot \frac{T}{p} \quad \frac{(\text{Number of channels})}{r}$$

A good operating point for a reactor system can be ascertained by plotting the <u>net</u> power output against the <u>reactor</u> power output and operating slightly to the left of the peak of such a representation. This is left as a method to be utilized by the reader for the design of his choice.

Illustrative Example on Use of Charts:

Suppose a reactor utilizing helium as the coolant with an axial peak-to-average power ratio of 1.32 has the following characteristics:

1.	gas inlet temperature	= 500°F
2.	gas outlet temperature	= 1200°F
3.	maximum permissible fuel rod temperature	= 1400°F
4.	fuel rod diameter	= 3/4 inches
5.	number of rods per cluster	= 13
6.	heat output per foot of rod at the center of the reactor	= 30,000 But/hr ft of rod
7.	pressure level of gas coolant	= 700 psia
8.	length of coolant channel	= 25 feet

Find:

a. the diameter of the coolant channel
b. the pressure drop through the core
c. pumping power expended

Procedure:

From Fig. I for $t_s = 1400^\circ F$, $t_{in} = 500^\circ F$ and $t_{out} = 1200^\circ F$, the maximum surface-to-gas temperature difference is seen to be 408°F.

Reference to Fig. II shows that the h necessary to achieve a 408°F ΔT_m for $q_m = 30{,}000$ with 3/4 in. rods is 375 Btu/hr ft² °F.

From the definition of F, one finds that it is numerically equal to 13,929 in this instance. Since this is less than 50,000, use of Figs. VII rather than Figs. III is implied. From Fig. VIIa with an h = 375 and an F = 13,929, one finds Z = 6.5. Then Fig. VIIb yields a channel diameter of 4.3 inches for a Z = 6.5, n = 13, and d = 3/4 inches. (The scale in the upper left hand corner of Fig. VIIb may be cut out and used to interpolate in Z.)

Turning to Fig. IV, the friction pressure drop parameter is seen to be 5.7×10^{-9} for a d = 3/4 inches, n = 13, and D = 4.3 inches.

Similarly Fig. V yields the entrance, expansion and contraction, and exit pressure drop parameter as 1.05×10^{-9}. Hence the pressure drops are

respectively, $\Delta p_f = 5.7 \times 10^{-9} \dfrac{LT}{p} F^{1.81} = 8.40$ psi

$$\Delta p_e = 1.05 \times 10^{-9} \frac{2T_{in} + 3T_{out}}{p} F^2 = 2.01 \text{ psi},$$

for a total of 10.41 psi.

The pumping power for the central channel then is found with the aid of Fig. VI. The average operating temperature is $1310^{\circ}R$, and for an $F = 13,929$ and a 700 psia pressure level, interpolation between the $1260^{\circ}R$ and $1360^{\circ}R$ planes yields $\frac{P}{\Delta p} = 2.85$. (In certain regions on these charts, the dependent variable is very sensitive to a slight change in independent parameters. In such cases better accuracy is obtained by using the equations from which the charts were drawn. For example, Eq. 19 yields, $\frac{P}{\Delta p} = 3.10$, a much better value.)

Using $P/\Delta p = 3.10$, the pumping power for the central channel is seen to be $P = 3.10\ \Delta p = 32.3$ horsepower.

For a 1500 channel reactor with a radial peak-to-average power ratio of 1.4 then, the total pumping power required would be

$$\frac{32.3 \times 1500}{1.4} = 34,600 \text{ horsepower.}$$

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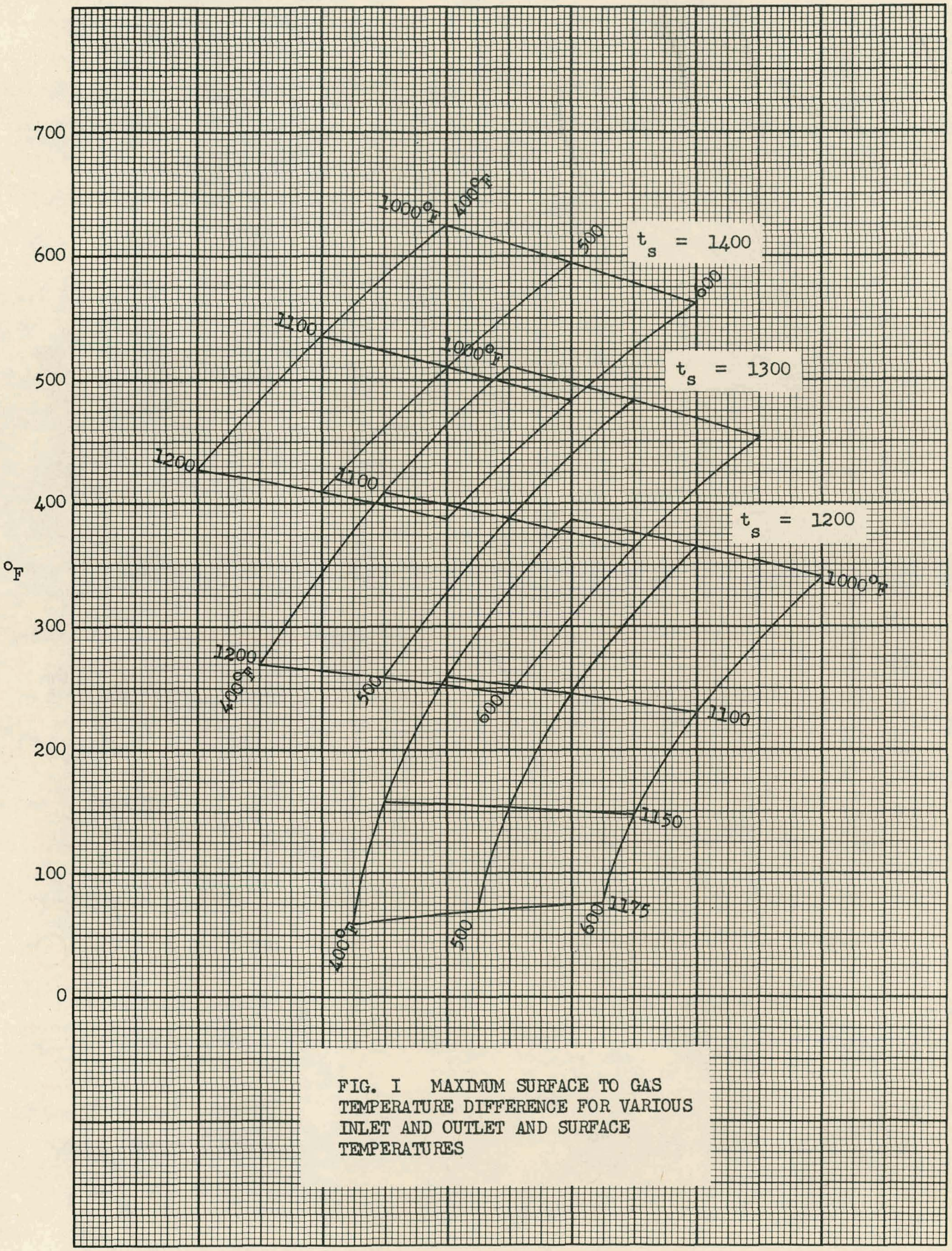

FIG. I MAXIMUM SURFACE TO GAS TEMPERATURE DIFFERENCE FOR VARIOUS INLET AND OUTLET AND SURFACE TEMPERATURES

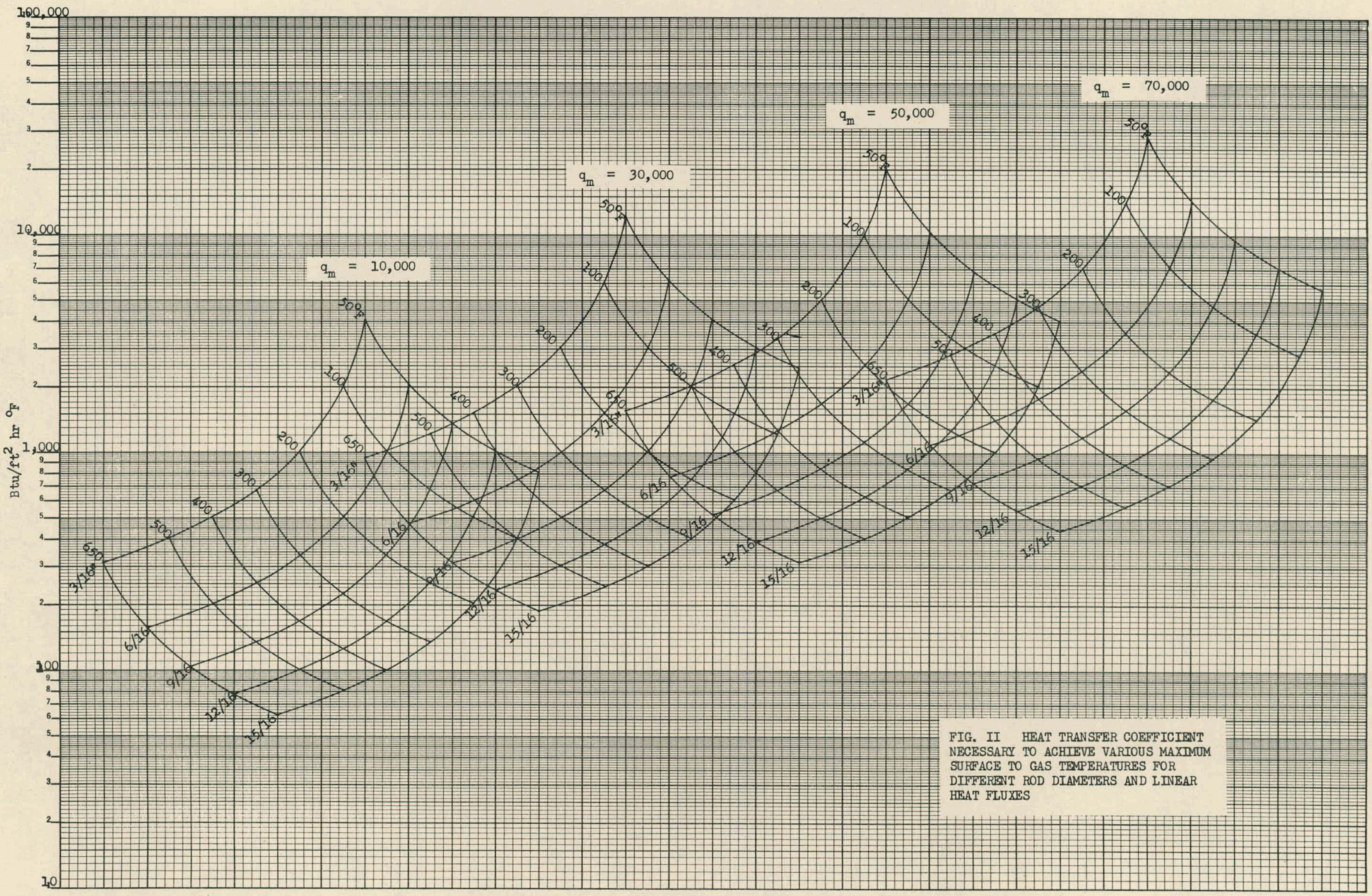

FIG. II HEAT TRANSFER COEFFICIENT NECESSARY TO ACHIEVE VARIOUS MAXIMUM SURFACE TO GAS TEMPERATURES FOR DIFFERENT ROD DIAMETERS AND LINEAR HEAT FLUXES

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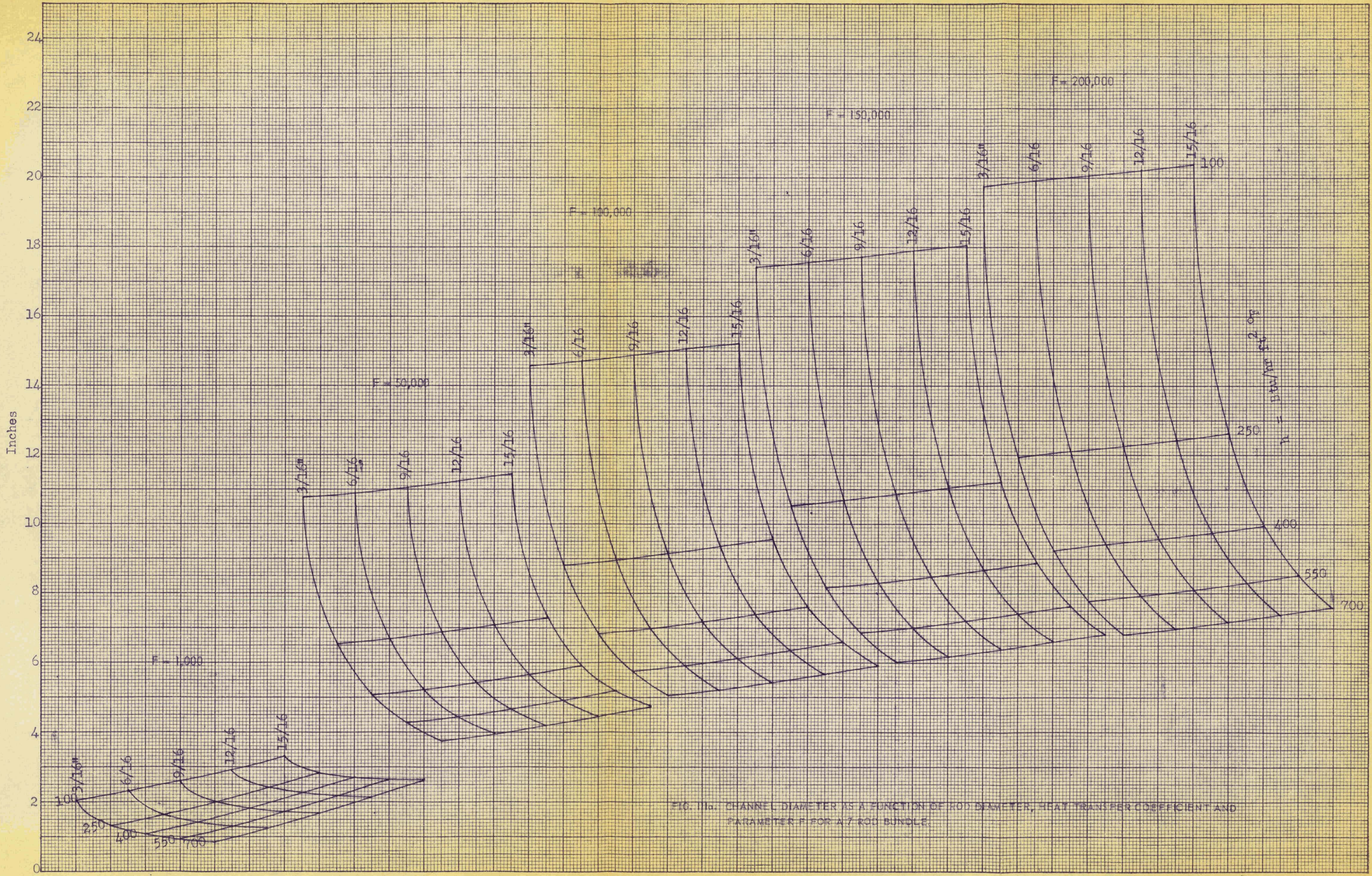

FIG. IIIa. CHANNEL DIAMETER AS A FUNCTION OF ROD DIAMETER, HEAT TRANSFER COEFFICIENT AND PARAMETER F FOR A 7 ROD BUNDLE.

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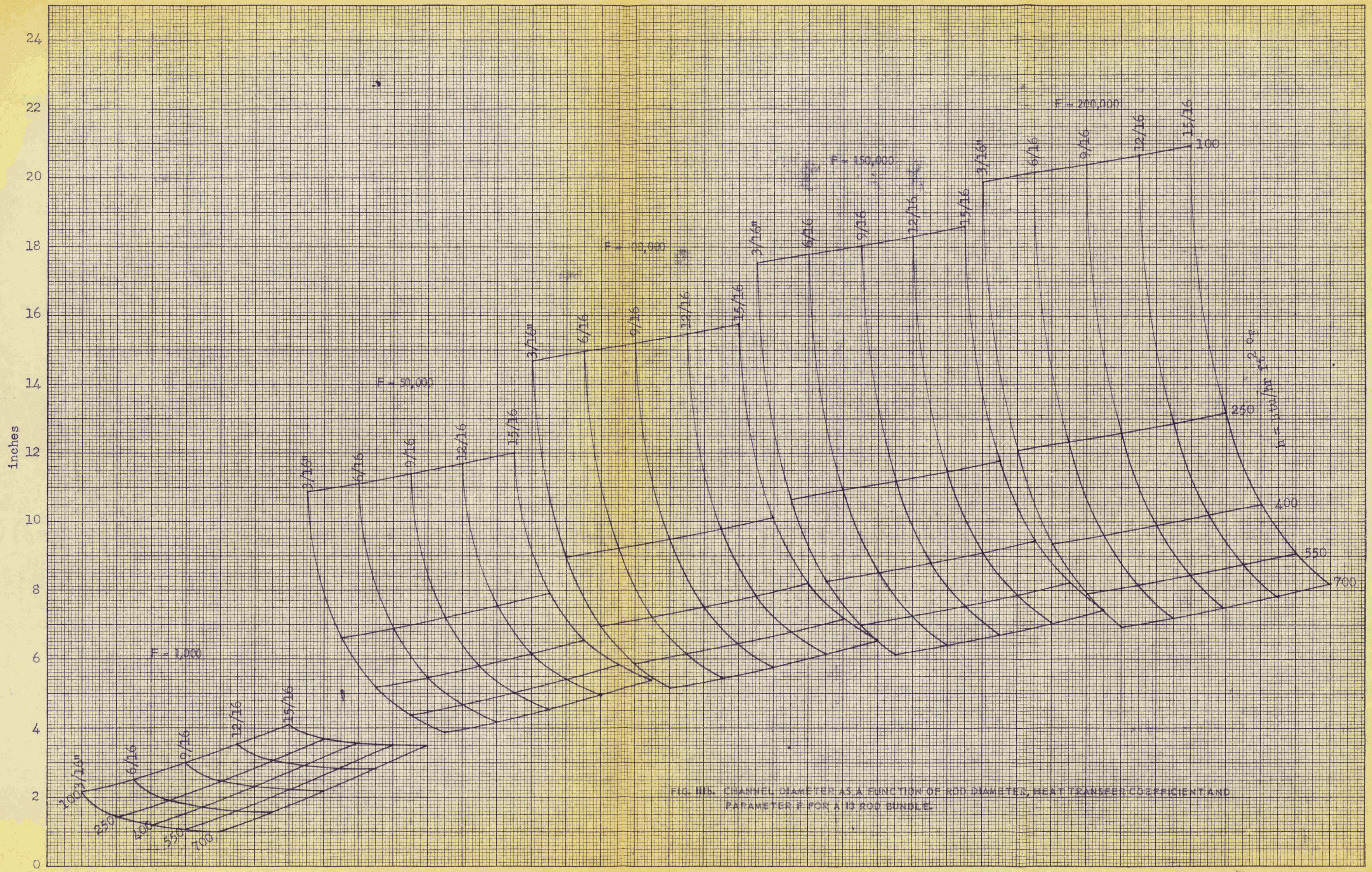

FIG. IIIb. CHANNEL DIAMETER AS A FUNCTION OF ROD DIAMETER, HEAT TRANSFER COEFFICIENT AND PARAMETER F FOR A 13 ROD BUNDLE.

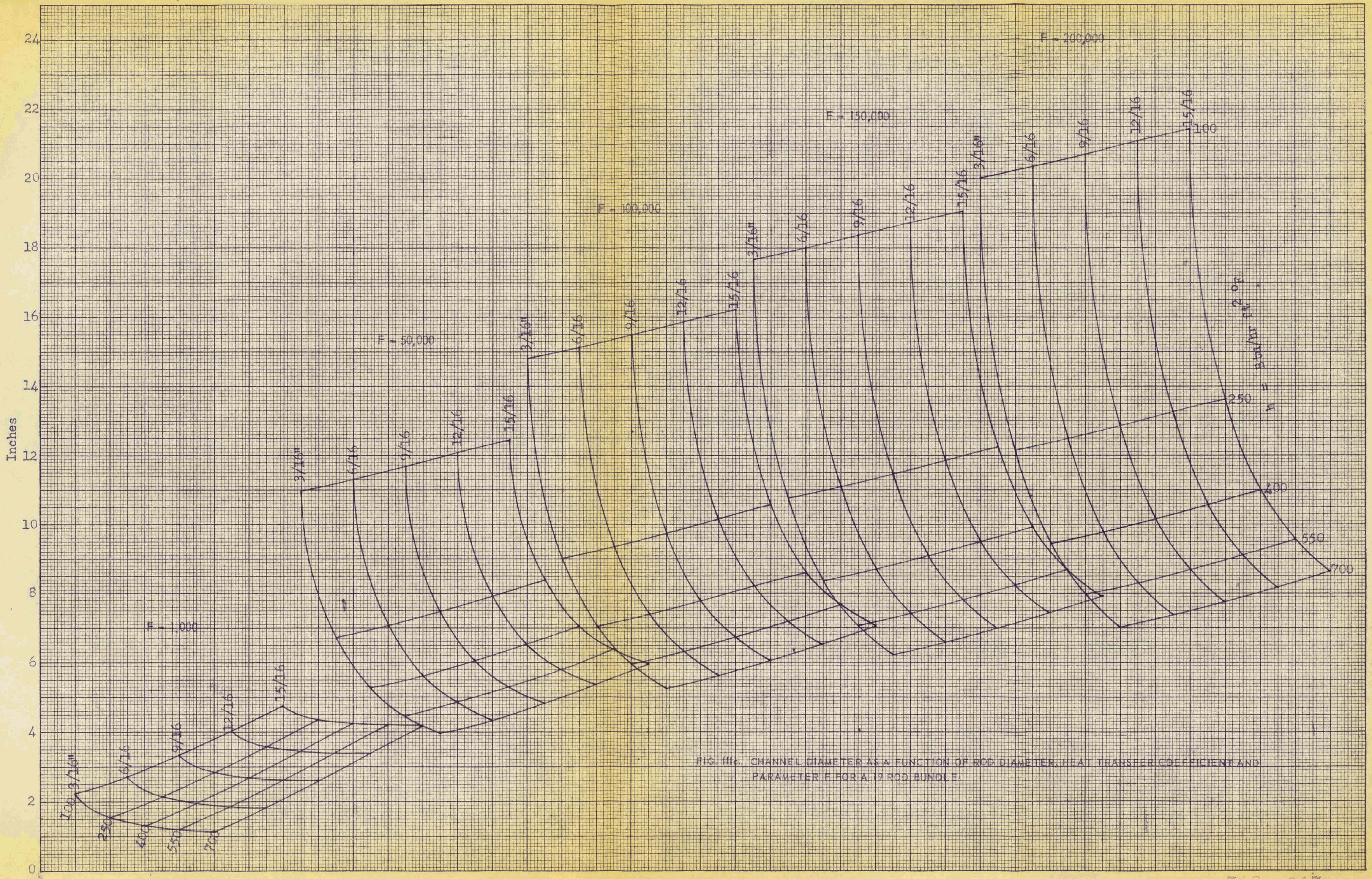

FIG. IIIc. CHANNEL DIAMETER AS A FUNCTION OF ROD DIAMETER, HEAT TRANSFER COEFFICIENT AND PARAMETER F FOR A 19 ROD BUNDLE.

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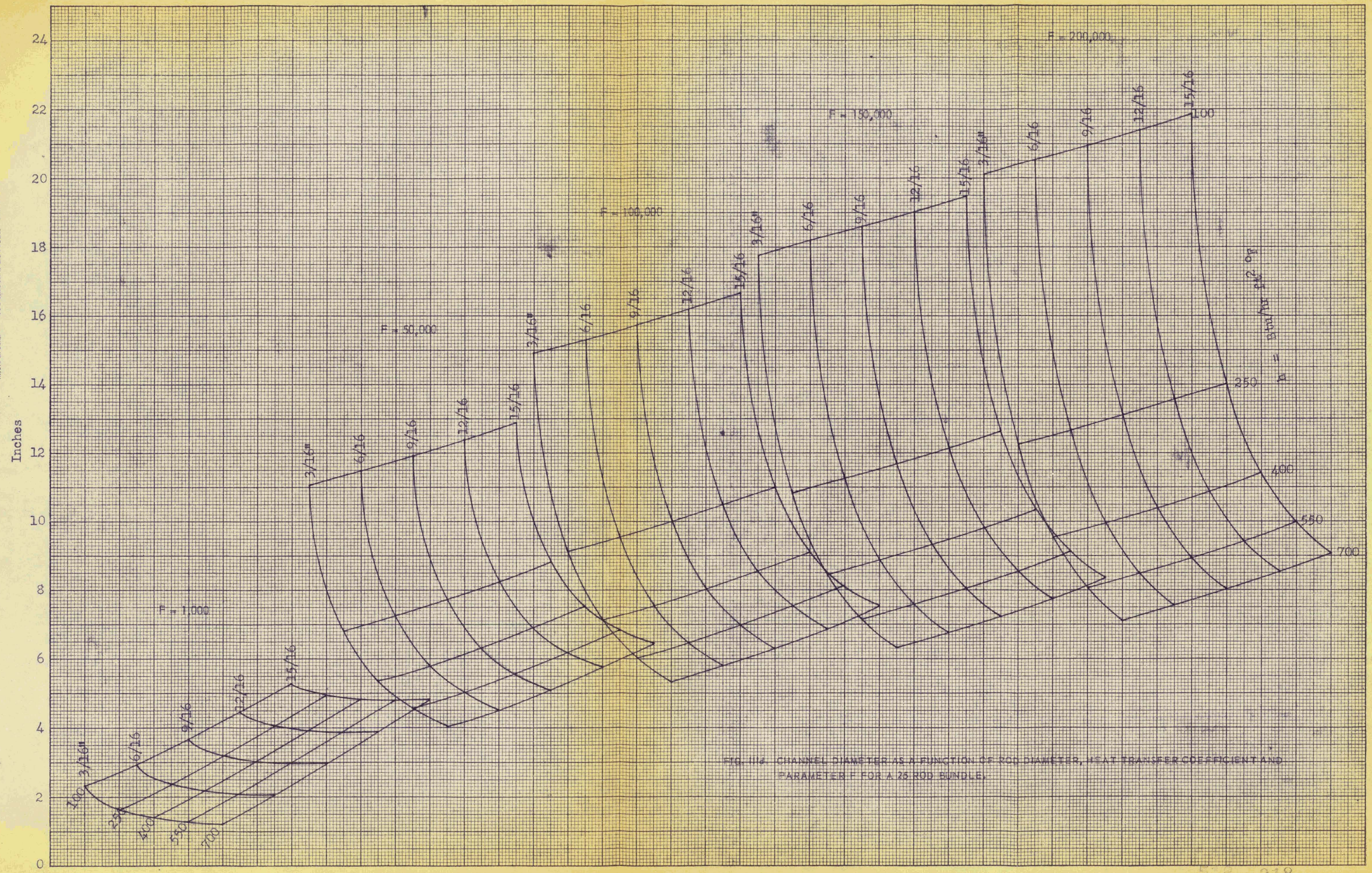

FIG. 11d. CHANNEL DIAMETER AS A FUNCTION OF ROD DIAMETER, HEAT TRANSFER COEFFICIENT AND PARAMETER F FOR A 25 ROD BUNDLE.

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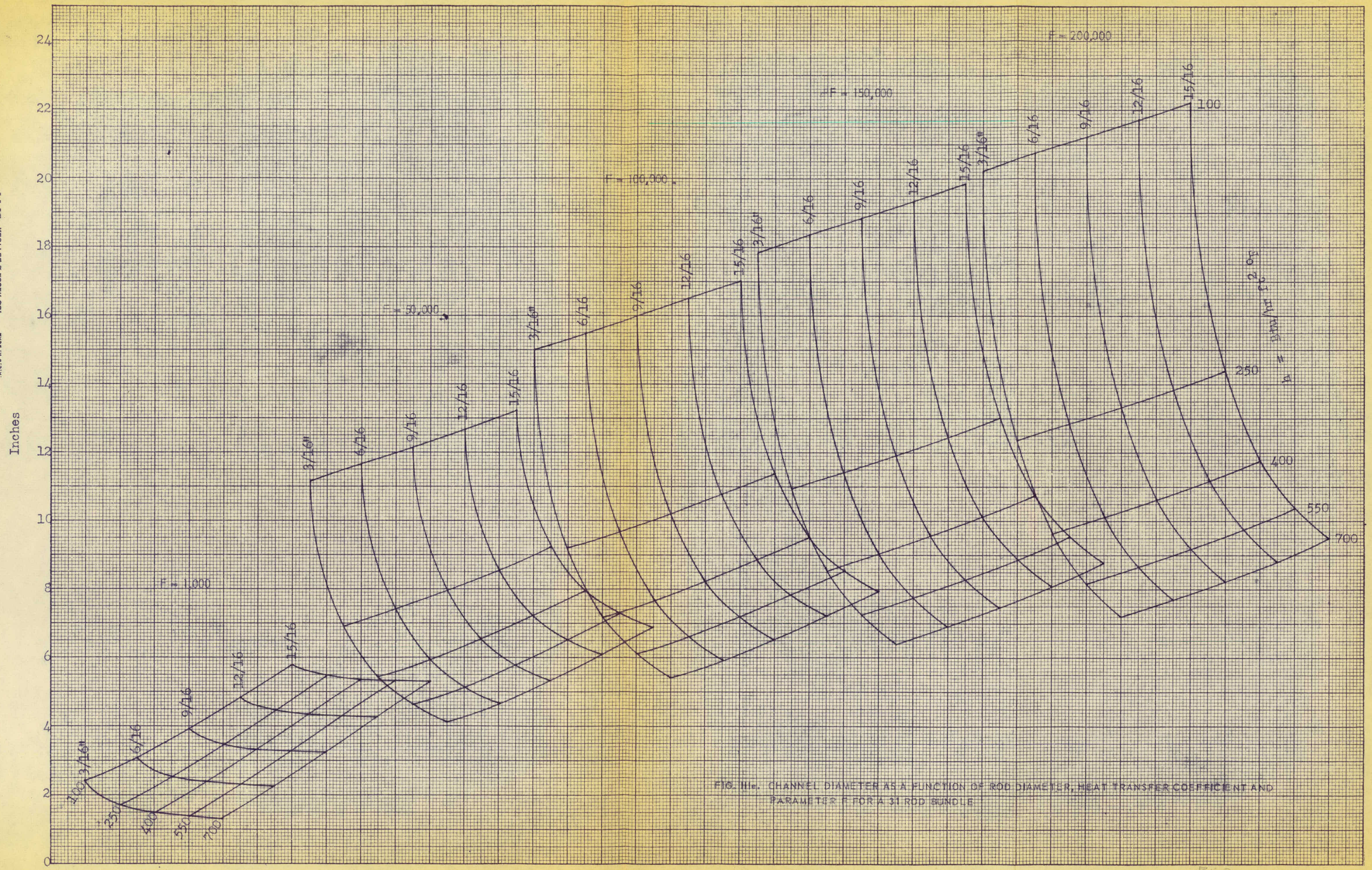

FIG. 11c. CHANNEL DIAMETER AS A FUNCTION OF ROD DIAMETER, HEAT TRANSFER COEFFICIENT AND PARAMETER F FOR A 31 ROD BUNDLE

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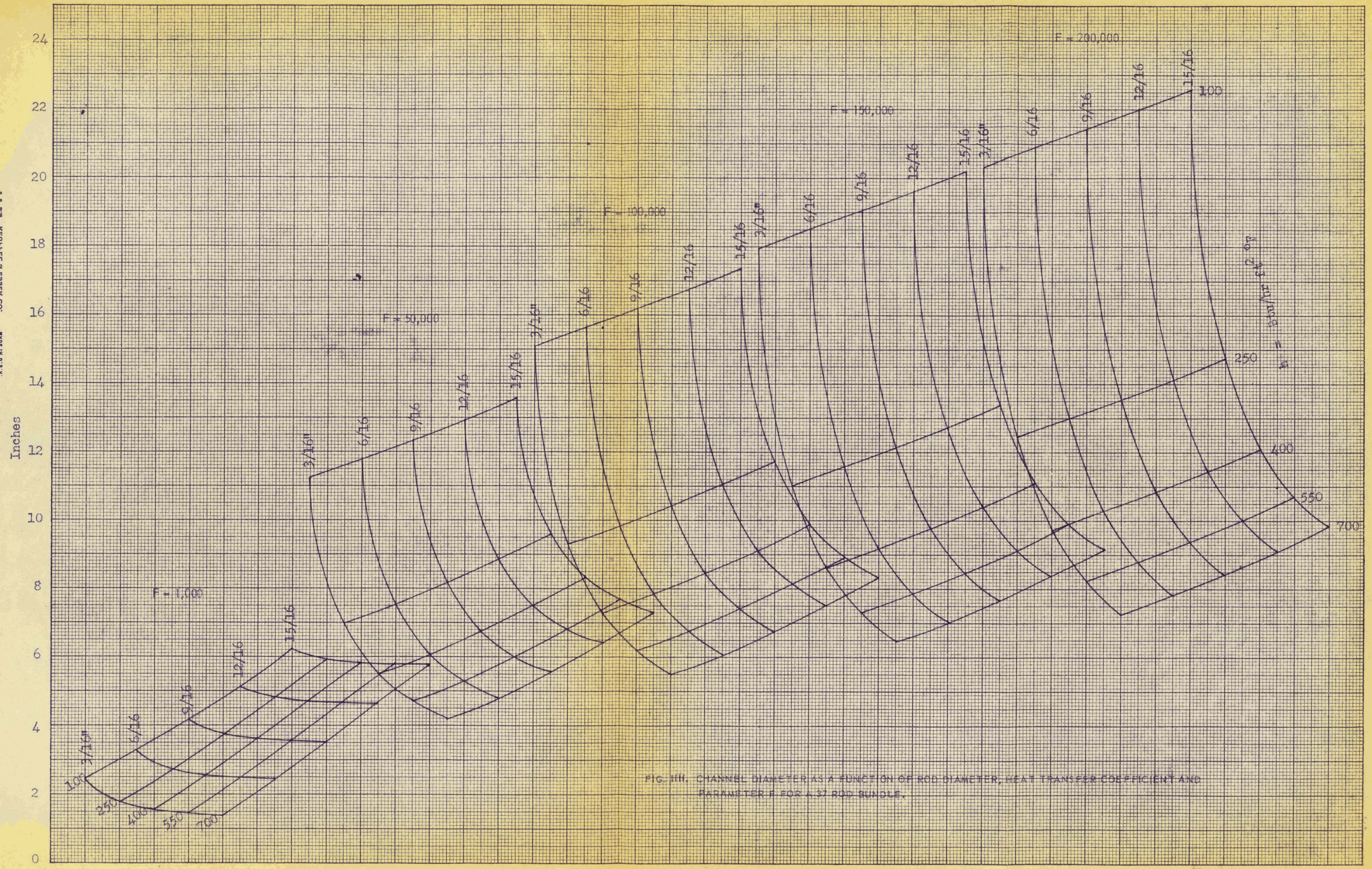

FIG. IIII. CHANNEL DIAMETER AS A FUNCTION OF ROD DIAMETER, HEAT TRANSFER COEFFICIENT AND PARAMETER F FOR A 37 ROD BUNDLE.

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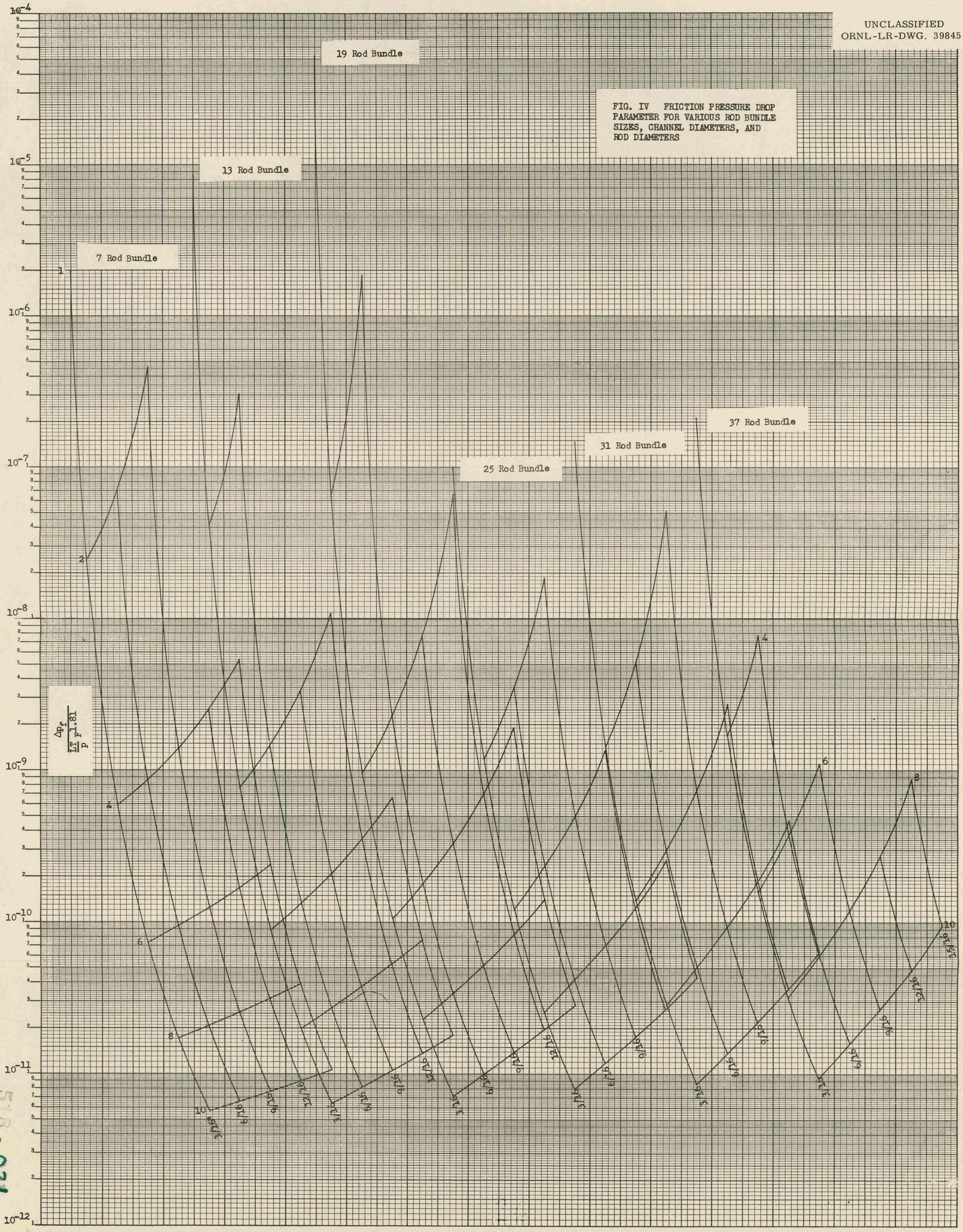

FIG. IV FRICTION PRESSURE DROP PARAMETER FOR VARIOUS ROD BUNDLE SIZES, CHANNEL DIAMETERS, AND ROD DIAMETERS

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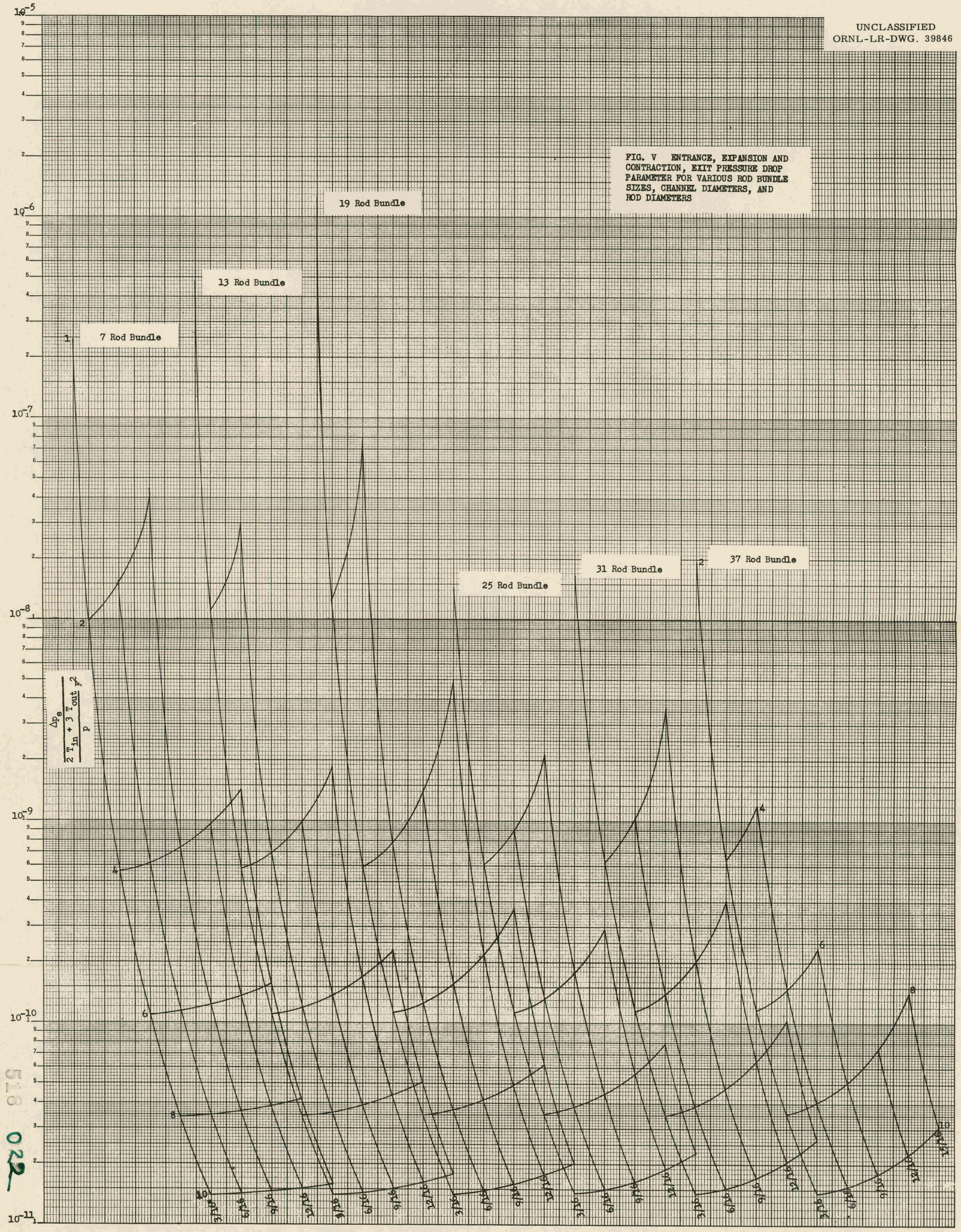

FIG. V ENTRANCE, EXPANSION AND CONTRACTION, EXIT PRESSURE DROP PARAMETER FOR VARIOUS ROD BUNDLE SIZES, CHANNEL DIAMETERS, AND ROD DIAMETERS

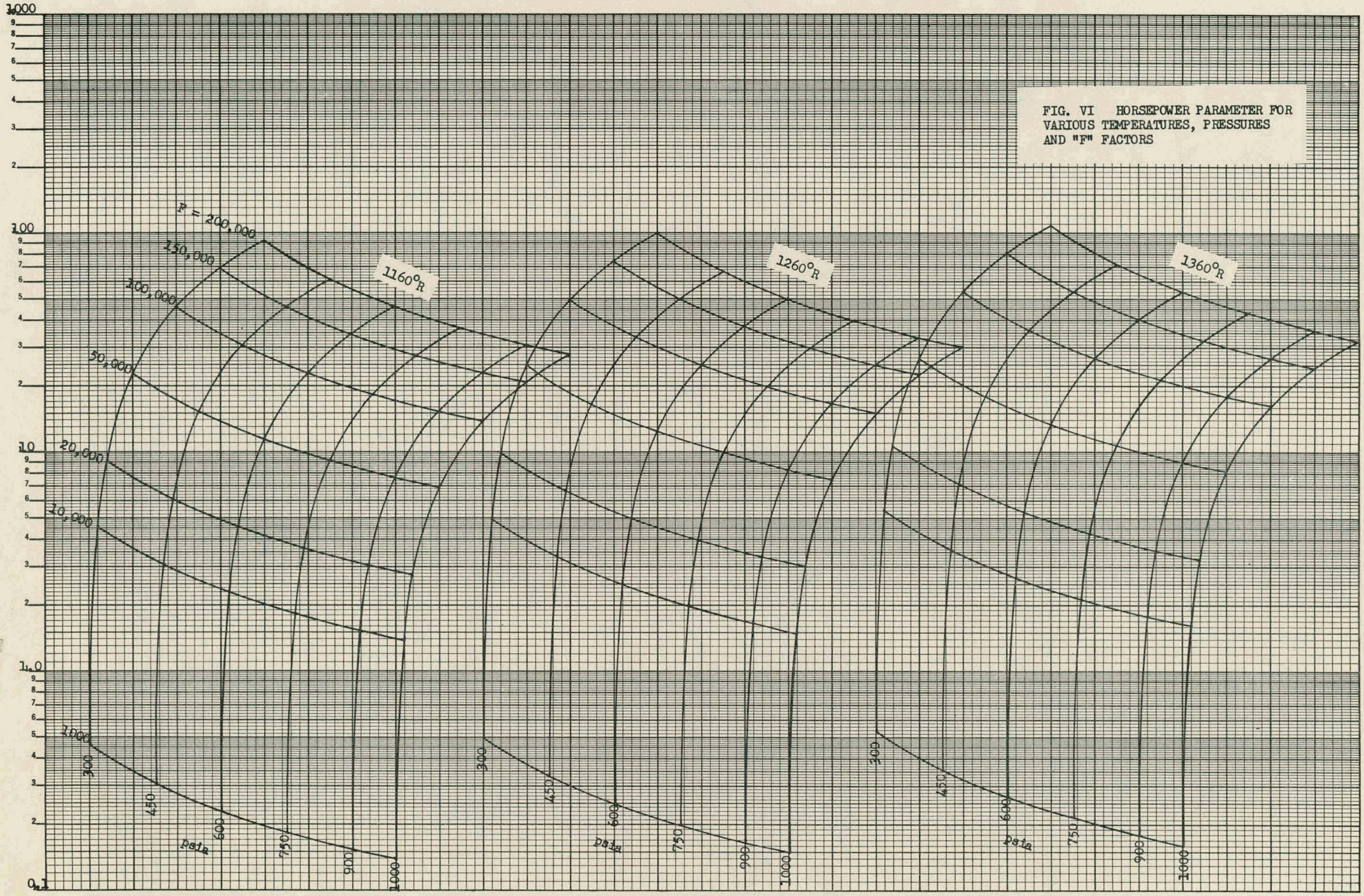

FIG. VI HORSEPOWER PARAMETER FOR VARIOUS TEMPERATURES, PRESSURES AND "F" FACTORS

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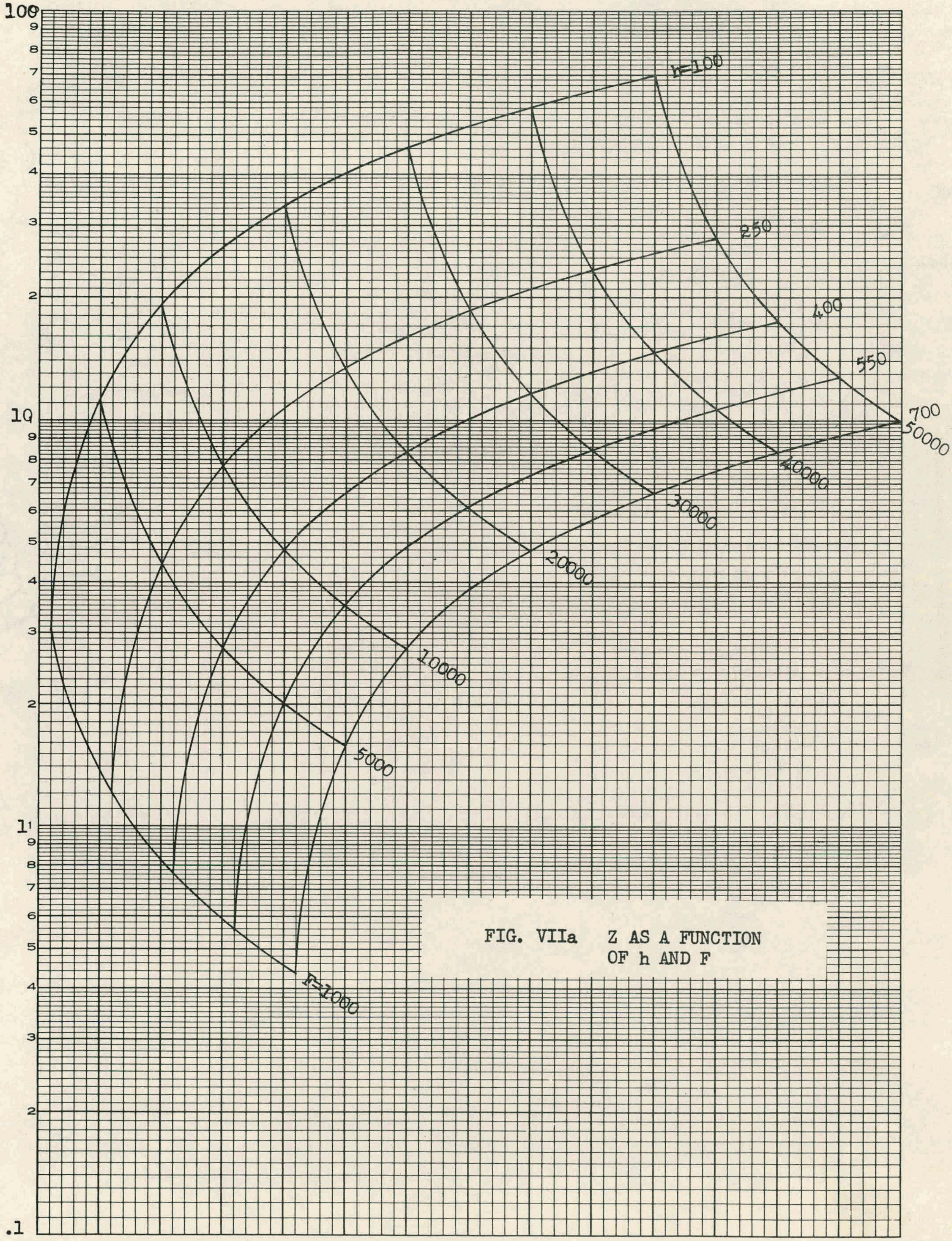

FIG. VIIa Z AS A FUNCTION OF h AND F

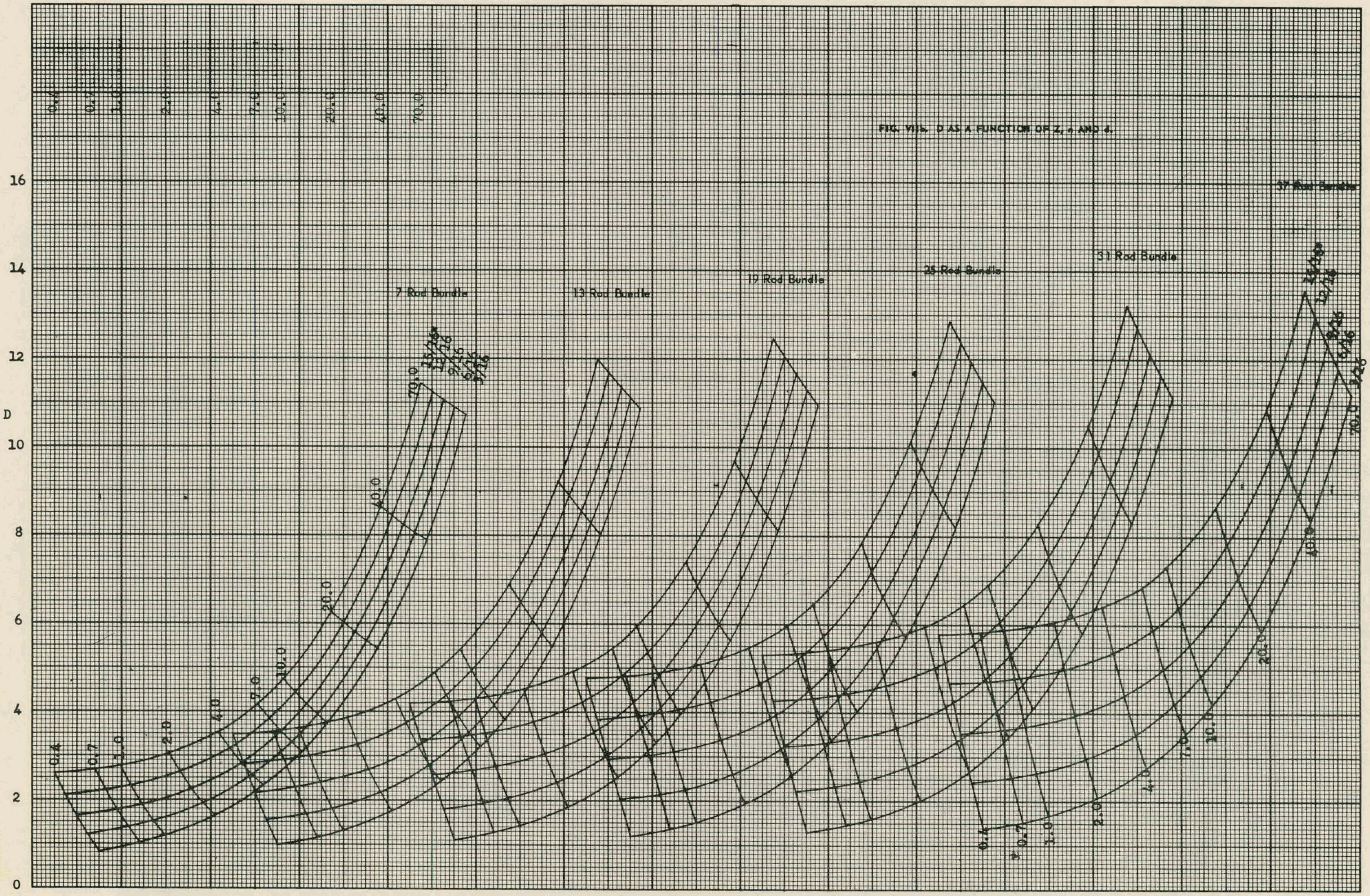

FIG. VIIb. D AS A FUNCTION OF Z, n AND d.

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