THE UNIVERSITY OF TEXAS AT AUSTIN

Center for Particle Theory

ORO 236

Magnetic Monopoles and Distorted Gauge Symmetry*

J. P. Hsu
Center for Particle Theory
The University of Texas at Austin
Austin, Texas 78712

We show that magnetic monopoles occur in massive vector boson theory with an intrinsic symmetry breakdown while still possessing distorted gauge symmetry. The exact static spherically symmetric solutions for the monopole are obtained. Their meaning is discussed on the basis of gauge symmetry.

*Work supported in part by the U.S. Atomic Energy Commission.

Recently, it has been shown that magnetic monopoles occur among the solutions in unified gauge theories (with spontaneous symmetry breakdown) in which the electromagnetic group U(1) is a subgroup of a larger group with a compact covering group, e.g. SU(2). An approximate solution for a magnetic monopole has been obtained.[1] This is interesting in view of its connection with the fundamental problem of charge quantization and the observation by C. N. Yang that gauge theories with compact gauge group provide for the necessary quantization.[2]

We wish to point out that magnetic monopoles also exist in vector boson theories with intrinsic symmetry breakdown rather than spontaneously broken symmetry.[3] Furthermore, we obtain exact static spherically symmetric solutions for the magnetic monopole. Their meaning is understood in terms of the electromagnetic field on the basis of symmetry. In the theory, there is no quartic potential of scalar fields and the vector boson mass M is due to an intrinsic symmetry breakdown. Nevertheless, the Lagrangian is strictly invariant under the distorted gauge transformation which reduces to the usual gauge transformation when $M = 0$. The quantum field theory with such Lagrangian is renormalizable by standard power counting. There is a general formal proof of unitarity and gauge independence which has been substantiated by explict calculations at the 2-loop level.[3,4]

Let us consider the following Lagrangian density involving massive vector fields $B^a_\mu(x)$ and massless scalar fields $\phi^a(x)$:

$$\mathcal{L} = -\frac{1}{4} B^a_{\mu\nu} B_a^{\mu\nu} + \frac{1}{2}(\partial_\mu \Phi_a + e\varepsilon^{abc} B^b_\mu \phi^c)(\partial^\mu \Phi_a + e\varepsilon_{adf} B^\mu_d \phi^f)$$
$$+ v^c M \varepsilon_{abc} B^b_\mu (\partial^\mu \Phi_a + e\varepsilon_{adf} B^\mu_d \phi^f)$$
$$+ \frac{1}{2} M^2 B^b_\mu B^\mu_b - \frac{1}{2} M^2 v^f B^f_\mu v^c B^\mu_c , \tag{1}$$

$$B^a_{\mu\nu} = \partial_\mu B^a_\nu - \partial_\nu B^a_\mu + e\varepsilon^{abc} B^b_\mu B^c_\nu , \quad \Phi_a = \phi_a + v_a M/e, \tag{2}$$

where v_a is a unit isovector, $v_a^2 = 1$. The Lagrangian (1) is invariant under the SU(2) distorted gauge transformation,

$$B^a_\mu \to B^a_\mu - \varepsilon^{abc}\omega^b(x) B^c_\mu + \partial_\mu \omega^a(x)/e ,$$
$$\phi^a \to \phi^a - \varepsilon^{abc}\omega^b(x)(\phi^c + v^c M/e). \tag{3}$$

If one chooses $v_a = (0,0,1)$,[3] two vector fields B_1^μ and B_2^μ have the same mass M, while B_3^μ is massless and can be identified with the photon field $A^\mu = B_3^\mu$.

We are interested in the static spherically symmetric solution of the classical field equations. So, we look for the particle-like solution of the form:[1]

$$\phi^a(\vec{r},t) = v^a \phi(r),$$
$$B^a_\mu(\vec{r},t) = \begin{cases} \varepsilon_{\mu ab} v_b B(r) , & \mu = 1,2,3. \\ 0 , & \mu = 0. \end{cases} \tag{4}$$
$$v^a = (x/r,\ y/r,\ z/r), \quad r^2 = x^2 + y^2 + z^2,$$

where we take the position of the magnetic monopole as the

origin of the coordinate system. In this case, the Lagrangian $L = \int d^3 r\, \mathcal{L}(B^a_\mu(\vec{r}), \phi^b(\vec{r}))$ can be written as

$$L = \int 4\pi r^2 dr \Big[-\left(\frac{dB}{dr}\right)^2 - 2\frac{B}{r}\frac{dB}{dr} - 3\frac{B^2}{r^2} - 2e\frac{B^3}{r} - \frac{1}{2}e^2 B^4$$
$$- M^2 B^2 - \frac{1}{2}\left(\frac{d\phi}{dr}\right)^2 - \left(\frac{1}{r^2} + \frac{2eB}{r}\right)\left(\phi + \frac{M}{e}\right)^2 - B^2(e^2\phi^2 + 2e\phi M)\Big]. \tag{5}$$

This Lagrangian leads to the following equations for $B(r)$ and $\phi(r)$:

$$\frac{d}{dr}\left(r^2 \frac{dB}{dr}\right) - 2B - 3erB^2 - e^2 r^2 B^3 - M^2 r^2 B$$
$$- er(\phi + M/e)^2 - r^2 B(e^2\phi^2 + 2eM\phi) = 0, \tag{6}$$
$$\frac{d}{dr}\left(r^2 \frac{d\phi}{dr}\right) - 2(1 + 2erB)\left(\phi + \frac{M}{e}\right) - r^2 B^2(2e^2\phi + 2eM) = 0.$$

With the help of the dimensionless quantity $C(r) \equiv 1 + erB(r)$, equations in (6) become

$$r^2 \frac{d^2 C}{dr^2} + C - C^3 - e^2 r^2 C(\phi + M/e)^2 = 0, \tag{7.1}$$

$$\frac{d}{dr} r^2 \frac{d\phi}{dr} - 2C^2(\phi + M/e) = 0. \tag{7.2}$$

By inspection, the particular solution to (7.1), $C(r) = 0$, leads to the nontrivial special solution to equation (6),

$$B(r) = -1/(er), \tag{8}$$

and leads to the following equation for $\phi(r)$

$$\frac{d}{dr} r^2 \frac{d\phi}{dr} = 0. \tag{9}$$

It follows from (4), (8) and (9) that

$$\phi^a = r^a \phi_0 / r^2, \quad B^a_\mu = -\varepsilon_{\mu ab} r^b/(er^2), \quad \phi_0 = \text{constant}, \tag{10}$$

which vanish as $r \to \infty$ and diverge as $r \to 0$.

We also have the nontrivial special solution to eq. (7) which is finite at $r = 0$:

$$C(r) = R/\sin R\,, \qquad R = \alpha r\,,$$
$$\phi(r) = -M/e + \alpha(R\cos R - \sin R)/(eR\sin R). \tag{11}$$

This is found by trial and error and luck. Unfortunately, this solution is undefined as $r \to \infty$ if α is real. However, if α is imaginary, i.e. $\alpha = i\sigma$ with σ real, we have the well-behaved solution to eq. (7):

$$\phi^a(r) = -Mr^a/(er) + (\alpha r^a/er)(S\cosh S - \sinh S)/(S\sinh S),$$
$$B^a_\mu(r) = \varepsilon_{\mu ab} r^b (S - \sinh S)/(er^2 \sinh S), \qquad S = \sigma r, \tag{12}$$

which can be easily verified.

The total energy E of the stationary system is given by

$$E = -L \tag{13}$$

which is divergent for the solutions (10) and (11). But the solution (12) leads to a finite energy:

$$E = \frac{-4\pi}{e^2}\int_0^\infty dr\, \frac{d}{dr}\left[\frac{\sigma S^2\cosh S}{(\sinh S)^3} - \frac{\sigma\cosh S}{\sinh S} - \frac{\sigma S}{(\sinh S)^2} + \frac{\sigma}{S}\right]$$
$$= 4\pi|\sigma|/e^2. \tag{14}$$

The parameter σ has the dimension of mass, yet it cannot be determined dynamically.

If one formally follows the method in Ref.1, one has to express the Lagrangian (1) in terms of the fields B^a_μ and $\Phi^a \equiv \phi^a + v^a M/e$. One would end up with the approximate solutions:

$$B^a_\mu(\vec{r},t) \approx -\varepsilon_{\mu ab} r^b/(er^2),$$
$$\Phi^a(\vec{r},t) \approx v^a M/e = r^a M/(er), \tag{15}$$

for the region far from the origin. The physically observable electromagnetic field tensor $F_{\mu\nu}$ is defined by[1]

$$F_{\mu\nu} = \frac{1}{|\Phi|}\Phi^a B^a_{\mu\nu} - \frac{1}{e|\Phi|^3}\varepsilon_{abc}\Phi_a(D_\mu\Phi_b)(D_\nu\Phi_c), \tag{16}$$
$$D_\mu\Phi_b \equiv \partial_\mu\Phi_b + e\,\varepsilon_{bcd}B^c_\mu\Phi^d,$$

which is gauge invariant and will yield the usual definition $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, $A^\mu = B^\mu_3$, when Φ^a lies along the z direction everywhere. We may remark that if one applies the technique in Ref.1 to evaluate E in (13), i.e., using trial functions and adjusting their parameters to find the extreme of the Lagrangian density in (5) by computer calculations, one would obtain $E \approx M/e^2$.[5] This result corresponds to the natural choice $\sigma = M$ in (14). We stress that $\sigma = M$ is not a dynamical consequence *of the theory*. Because of this, it is not quite clear that the theory really predicts the existence of a magnetic monopole with the mass $M_m = E \approx M/e^2$, although it is certainly suggestive.

We observe that the ratio $\Phi^a/|\Phi|$ does not involve $\phi(r)$, whose structure is determined by the dynamical equation (7.2), and hence $\Phi^a/|\Phi|$ really has nothing to do with dynamics. Furthermore, in the local (distorted or undistorted) isospin gauge invariant theory, we can always introduce a unit isovector v^a, e.g. $v^a = r^a/r$, to construct the observable electromagnetic field tensor $\bar{F}_{\mu\nu}$ (for understanding the meaning of the solutions):

$$\bar{F}_{\mu\nu} = v^a B^a_{\mu\nu} - e^{-1}\varepsilon^{abc}v^a(D_\mu v^b)(D_\nu v^c)\,, \qquad (v^a)^2 = 1\,, \tag{17}$$

which is gauge invariant and $\bar{F}_{\mu\nu}$ becomes $\partial_\mu A_\nu - \partial_\nu A_\mu$ when $v^a = (0,0,1)$. We note that $\bar{F}_{\mu\nu}$ has little to do with the dynamics of gauge theory. Rather, it is a theoretical construction on the basis of local gauge symmetry. In this sense, $\bar{F}_{\mu\nu}$ and its consequences are consequent on symmetry rather than dynamics. The magnetic field H_a is related to $\bar{F}_{\mu\nu}$ by the relation

$$\bar{F}_{\mu\nu} = \varepsilon_{\mu\nu a} H^a \quad , \qquad \mu,\nu,a = 1,2,3. \tag{18}$$

All three solutions (10), (11) and (12) lead to the same $\bar{F}_{\mu\nu}$:

$$\bar{F}_{\mu\nu} = -\varepsilon_{\mu\nu a} r^a/(er^3). \tag{19}$$

Both the definitions (17) and (16) lead to the result

$$H^a = -r^a/(er^3). \tag{20}$$

This shows the presence of a stable magnetic monopole at $\vec{r} = 0$ with a total flux $-4\pi/e$. It satisfies Schwinger's condition eg=1.[6,1]

From the above discussions, we see that one can construct the theory of magnetic monopoles without introducing the Dirac string.[7] We have obtained the exact static spherically solutions of stable magnetic monopole in massive vector boson theory with distorted gauge symmetry. The energy of the system can be finite. The definition (17) for $\bar{F}_{\mu\nu}$ holds for <u>any</u> <u>local</u> isospin gauge invariant theory with or without the Higgs scalars.[8] Although the monopole and the conserved magnetic charge[9] have little to do with the dynamics in (1), we may regard them as consequences of the local isospin gauge symmetry.

The author wishes to thank his colleagues for discussions.

References

1. G. 't Hooft, Nucl. Phys. B79, 276 (1974).
2. C. N. Yang, Phys. Rev. D1, 2360 (1970).
3. J.P. Hsu, Lett. Nuovo Cimento, 12, 503 (1974)
4. J.P. Hsu, Lett. Nuovo Cimento, 11, 525 and 12, 128 (1974); J. P. Hsu and J. A. Underwood, to be published in Phys. Rev..
5. This is related to the result $C(\beta)$=finite when $\beta = 0$ in Ref.1. We thank 't Hooft for communicating the result, $C(\beta=0)$ = finite, to us.
6. J. Schwinger, Phys. Rev. 144, 1087 (1966).
7. P.A.M. Dirac, Proc. Roy. Soc. (London) A133, 60 (1934); Phys. Rev. 74, 817 (1948).
8. J. P. Hsu, CPT preprint (1975).
9. The conserved electric and magnetic currents, j^e_μ and j^m_μ, are related to $\bar{F}_{\mu\nu}$ by $j^e_\mu = \partial^\nu \bar{F}_{\mu\nu}$ and $j^m_\mu = \varepsilon_{\mu\nu\alpha\beta}\partial^\nu \bar{F}^{\alpha\beta}/2$ respectively.