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SPIN ZERO EXCHANGE MODEL OF WEAK INTERACTIONS

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ABSTRACT

A renormalizable model in which weak interactions are mediated by spin-zero exchange rather than by spin-one exchange is studied. The lowest order diagrams for μ and β decay are box diagrams. Neutrino cross sections off leptons and hadrons are calculated for the effective charged and neutral currents of the model. Weak corrections to $e^+e^- \to \mu^+\mu^-$ and higher order contributions to μ decay are also calculated. Neutral current effects are predicted to be small for neutrinos on lepton targets and large for $e^+e^- \to \mu^+\mu^-$; their strength is fixed by ν-hadron scattering. In particular, there is a sizeable suppression of $\nu_\mu e$ scattering and a large enhancement of the asymmetry in $e^+e^- \to \mu^+\mu^-$. The only troublesome prediction is a parameter-free value of one for the ratio $\sigma(\nu_\mu + N \to \nu_\mu + X) / \sigma(\bar{\nu}_\mu + N \to \bar{\nu}_\mu + X)$. On the whole, however, the model provides a sensible, renormalizable alternative to the gauge theories.

I. INTRODUCTION

The study of weak interactions has blossomed in the past five years, on the theoretical as well as on the experimental side. Of key importance has been the development of a unified theory of weak and electromagnetic interactions[1] with the attractive feature of renormalizability.[2] This has led to a formidable array of calculations and a great deal of model building[3] to obtain agreement with experiment while operating within the framework of these gauge models. A basic ingredient has been the GIM[4] cancellation mechanism based on an SU(4) symmetry[5] for hadrons.

It seems likely that many of these ideas are correct, even though all the components of a complete theory of the weak interactions may not yet be in hand. A good dose of skepticism, however, is probably healthy and with that in mind we turn to the predictions of an alternative renormalizalbe model of weak interactions. Our purpose is not to propose adoption of the alternative model, but rather to show that there is still a good deal of flexibility in developing a theoretical scheme to fit weak interaction experiments and to demonstrate within the context of the specific model how improved experimental results will resolve ambiguities. The model[6] we consider is one in which the weak interactions are mediated by spin zero bosons. Earlier

versions of this model required a large number of as yet undiscovered particles to appear in the weak interaction Lagrangian. One of us (G.S.) recently found[7] that by using the GIM[4] mechanism, and by allowing the coupling of the spin zero bosons mediating the weak interaction to be fairly large, considerable simplifications would occur.

This paper will explore in some detail both the theoretical framework and the experimental predictions of the model of reference 7. We will place particular emphasis on contrasting our results with those of gauge models.

In section II we introduce the model and calculate the lowest order diagram contributing to μ decay, in this case a box diagram in which two spin zero mesons are exchanged. We then calculate some higher order diagrams and discuss the limits placed on the scalar meson's coupling to hadrons and leptons by universality, the muon g-2, etc. In section III we consider the model's predictions for purely leptonic scattering processes, namely $\nu_\mu + e^- \to \nu_\mu + e^-$, $\nu_e + e^- \to \nu_e + e^-$ and $e^+ + e^- \to \mu^+ + \mu^-$. The first of these reactions is of particular interest, as it is forbidden in order G (the Fermi coupling) in this model, but allowed in the Weinberg-Salam[1] model. Weak effects in the third reaction should soon be measured and these may be particularly large in the present model. In section IV we analyze neutrino hadron scattering; the neutral current in our model has an isovector vector part and an isoscalar axial vector part. We analyze the consequences of this form of neutral current. Since the coupling constant of the scalar boson to hadrons is quite large, renormalization effects may be appreciable. We examine these with particular regard to universality in section V. In section VI we conclude by reviewing the more important experimental predictions. Finally we present two appendices. In the first some of the details of the higher order calculations of section II are given and in the second a model with manifest universality is displayed.

II. LIMITS ON THE COUPLING CONSTANT

The interaction Lagrangian is[7]

$$\mathcal{L} = -if\Big\{\sum_{\ell=e,\mu}[\bar{L}_\ell(1-\gamma_5)\ell B^0 + \bar{L}_\ell(1-\gamma_5)\nu_\ell B^-]$$

$$+ [\bar{n}_c(1-\gamma_5)\mathcal{P} + \bar{\lambda}_c(1-\gamma_5)\mathcal{P}']B^- \qquad (2.1)$$

$$+ [\bar{n}_c(1-\gamma_5)n_c + \bar{\lambda}_c(1-\gamma_5)\lambda_c]B^0\Big\} + \text{h.c.}$$

where the values of ℓ are the usual leptons, e and μ, while ν_ℓ are the usual neutrinos. L_ℓ are two massive, charged leptons, and B^0, $\overline{B^0}$, and $B^\pm$ are the scalar mesons which mediate the interaction. The interactions with baryons are given in terms of the quarks $\mathcal{P}$, n, λ, $\mathcal{P}'$ where $n_c = n \cos\theta + \lambda \sin\theta$ and $\lambda_c = -n \sin\theta + \lambda\cos\theta$.

The lowest order diagrams for μ decay and β decay are shown in Fig. 1. If the masses of the scalar mesons are much larger than those of the heavy leptons and $m_{B^+} - m_{B^0} < m_{B^0}$ or m_{B^+} then these box diagrams reduce to an effective V-A interaction

$$H_{eff} = \left(\frac{f^2}{4\pi}\right)^2 \frac{1}{m_B^2}\, \bar{e}\gamma^\alpha(1-\gamma_5)\nu_e\bar{\nu}_\mu\gamma_\alpha(1-\gamma_5)\mu \qquad (2.2)$$

Therefore we identify

$$\left(\frac{f^2}{4\pi}\right)^2 \frac{1}{m_B^2} = \frac{G}{\sqrt{2}} \qquad (2.3)$$

where G is the usual Fermi coupling constant.

Of course (2.3) is only valid if: (a) the mass of the charged scalar (m_+) is equal to the mass of the neutral scalar (m_o) and they are much larger than the masses of the heavy leptons of muon type (M_μ) and of electron type (M_e); and (b) $f^2/4\pi$ is small enough that the lowest order diagram is a good approximation. We are also assuming that the coupling of the charged scalar, f_+, is equal to the coupling of the neutral scalar, f_o. This assumption can be relaxed in a trivial manner by introducing a factor ε where $f_+^2 = \varepsilon f_o^2$, and in later sections we will do this. For the time being, however, let us take $f_+ = f_o$ and consider (a) and (b) in turn.

If we calculate the diagrams in Fig. 1 more carefully we have (still to just first order in M^2/m^2)

$$\frac{G}{\sqrt{2}} = \left(\frac{f^2}{4\pi}\right)^2 \left\{ \frac{1}{m_+^2 - m_o^2} \ln \frac{m_+^2}{m_o^2} + \frac{M_e^2 + M_\mu^2}{m_+^2} \frac{1}{m_+^2 - m_o^2} \ln \frac{m_+^2}{m_o^2} - \frac{M_e^2 + M_\mu^2}{m_+^2 m_o^2} \ln \frac{m_o^2}{M_e^2} + \frac{M_\mu^4}{m_+^2 m_o^2} \frac{1}{M^2 - M_e^2} \ln \frac{M_\mu^2}{M_e^2} \right\} \qquad (2.4)$$

as the effective coupling constant for μ decay while β decay is the same with M_μ replaced by the quark mass. Now we can compare the couplings for μ and β decay and use universality to put a lower bound on $f^2/4\pi$. The least restrictive lower bound comes about when $m_+ = m_o = m$ and $M_\mu = M_e = M$ for μ decay. Then

$$\frac{G_\beta}{\sqrt{2}} = \left(\frac{f^2}{4\pi}\right)^2 \frac{1}{m^2}\left[1 + \frac{M^2}{m^2} - \frac{M^2}{m^2} \ln \frac{m^2}{M^2}\right] \qquad (2.5a)$$

$$\frac{G_\mu}{\sqrt{2}} = \left(\frac{f^2}{4\pi}\right)^2 \frac{1}{m^2}\left[1 + \frac{3M^2}{m^2} - \frac{2M^2}{m^2} \ln \frac{m^2}{M^2}\right] \qquad (2.5b)$$

where we have assumed M^2 is much larger than the square of the quark mass. Thus

$$\frac{G_\beta}{\sqrt{2}} - \frac{G_\mu}{\sqrt{2}} = \left(\frac{f^2}{4\pi}\right)^2 \frac{1}{m^2} \frac{M^2}{m^2}\left(\ln \frac{m^2}{M^2} - 2\right) \qquad (2.6)$$

If we now require that there be no more than a 2% difference (for example) between the coupling constants then we must have $m \geq 12$ M. If we restrict M to be larger than 5 GeV then m must be larger than 60 GeV and, from (2.3), $f^2/4\pi$ must be greater than 0.17. If we allow a 5% difference in the coupling constants then $m \geq 27$ GeV and $f^2/4\pi \geq 0.08$. These last numbers seem to be reasonable values to take as absolute lower bounds. Notice that even if higher order diagrams contribute significantly they will not change this estimate of the lower bound on $f^2/4\pi$ since they will not affect the difference in (6) as long as $M^2 << m^2$.

As we have said, 0.08 is the least restrictive lower bound on $f^2/4\pi$. In the following sections of the paper we will use

$$\frac{G_F}{\sqrt{2}} = \left(\frac{f^2}{4\pi}\right)^2 \frac{1}{m_+^2 - m_o^2} \ln \frac{m_+^2}{m_o^2} \tag{2.7}$$

with $m_+ \geq m_o$ and $m_o \geq 27$ GeV. If $m_+ > m_o$, then (2.6) requires $f^2/4\pi$ to be larger than 0.08. Fig. 2 shows the minimum value of $f^2/4\pi$ as a function of R where $R \equiv m_+^2/m_o^2$. Since (7) is symmetric in m_+ and m_o we only need to consider $R \geq 1$. In most of the calculations which follow we will have three unknowns, $f^2/4\pi$, m_o^2, and R. We will generally use (7) to eliminate m_o^2 and give the results in terms of $f^2/4\pi$ and R, remembering the lower bounds on $f^2/4\pi$ shown in Fig. 2.

Now consider the question (b) of how big $f^2/4\pi$ can be if we are allowed to calculate perturbatively. To make the calculations simple, we will set $m_+ = m_o$ although this may result in some error. We can estimate the relative magnitude of higher order graphs by performing the following count.

a) each vertex has an f

b) every closed loop has $\frac{1}{(2\pi)^4}$

c) each four-dimensional integration gives π^2

d) each vertex has a factor of $(1-\gamma_5)$; these are commuted until they stand next to each other and this gives a power of 2 times $(1-\gamma_5)$ since $(1-\gamma_5)^2 = 2(1-\gamma_5)$

e) there are a number of diagrams in a given order, say N,

f) after the four-dimensional integrations are done we are left with a multiple integral, I, over Feynman parameters.

Our experience is that these combine to give, for graphs of order 2n,

$$\left(\frac{f^2}{4\pi}\right)^n \frac{1}{m^2} \left(\frac{1}{\pi}\right)^{n-2} [I_1 + \cdots + I_N] \tag{2.8}$$

where I is the integral for a given graph. (We only consider graphs which go as $1/m^2$.) Some of the I's will be of order one and, therefore, if $f^2/4\pi$ is of order one, the only suppression in higher orders is the $(1/\pi)^{n-2}$ factor.

As an example of the above we have calculated the contribution to μ decay of order f^6. The graphs are shown in Fig. 3. After renormalization subtractions are made, the scalar self energy contribution is zero while the leptonic self energy diagrams give

$$-\frac{1}{3\pi}(21 - 2\pi^2)\left(\frac{f^2}{4\pi}\right)^3 \frac{1}{m^2} \bar{e}\gamma^\alpha(1-\gamma_5)\nu_e\bar{\nu}_\mu\gamma_\alpha(1-\gamma_5)\mu \tag{2.9}$$

There are no vertex corrections in this order if the B^0 particle is not self conjugate. The sixth order contribution, (2.9), is less than 10% of (2.3) if $f^2/4\pi$ is less than one, but this may be accidentally small because of the $(21 - 2\pi^2)$ factor.

The self energy corrections seem small so we have also evaluated the contribution to muon decay of order f^8 shown in Fig. 4. The result is

$$\left(\frac{f^2}{4\pi}\right)^4 \frac{1}{m^2}\frac{1}{\pi^2}(1.68 \pm .76)\bar{e}\gamma^\alpha(1-\gamma_5)\nu_e\bar{\nu}_\mu\gamma_\alpha(1-\gamma_5)\mu \qquad (2.10)$$

where the error arises because some of the integrals were done numerically. Combining (2.9) and (2.10) we see that, through 8th order,

$$\frac{G_F}{\sqrt{2}} = \left(\frac{f^2}{4\pi}\right)^2 \frac{1}{m^2}\left[1 - .134\left(\frac{f^2}{4\pi}\right) + (.170 \pm .077)\left(\frac{f^2}{4\pi}\right)^2\right] \qquad (2.11)$$

Therefore, lowest order perturbation theory would seem to be a very sensible procedure if $f^2/4\pi \leq 1$. In this range the contribution from 6th and 8th order is a maximum of -2.5% but the higher order contribution grows rapidly when $f^2/4\pi$ becomes larger than one.

Details of these calculations are given in Appendix A.

III. LEPTONIC SCATTERING PROCESSES

In this section we discuss scattering processes in which only leptons are involved. The contents of this section are in part contained in a shorter article written by two of us[8] (D.A.D. and V.L.T.); promised details are given here. Four calculations are described; two are processes ($\nu_\mu e$ and $\nu_e e$ scattering) currently being measured, while two are combinations of $e^+e^- \to \mu^+\mu^-$ amplitudes that should be measured at SPEAR and PEP in due course.

A. $e + \nu_\mu \to e + \nu_\mu$

This process is forbidden[7] in lowest (fourth) order because of the form of (2.1). It is allowed in sixth order, however, and also in order e^2f^2 (where e is the electric charge) because of the neutrino charge radius. These diagrams are shown in Fig. 5. The value of the two diagrams of order f^6 is

$$\left(\frac{f^2}{4\pi}\right)^3 \frac{1}{\pi} I(m_+^2,m_0^2)\bar{e}\gamma^\alpha(1-\gamma_5)e\bar{\nu}_\mu\gamma_\alpha(1-\gamma_5)\nu_\mu \qquad (3.1)$$

where

$$I(m_+^2,m_0^2) = \int_0^1 dx\int_0^1 dy\int_0^1 dz\int_0^1 dw\, \frac{y(1-x)(1-w)z^2(4-3z)}{m_+^2 zwx + m_0^2 y(1-z)} + (m_+^2 \leftrightarrow m_0^2) \qquad (3.2)$$

This integral is elementary but tedious.

The neutrino charge radius is proportional to the momentum transfer, q^2, but that factor is cancelled by the photon propagator leaving the matrix element equal to

$$\left(\frac{f^2}{4\pi}\right)\frac{1}{m_+^2}\left\{1 + 4\int_0^1 dy(1-y)y \ln \frac{M_\mu^2 - q^2 y(1-y)}{m_+^2}\right\}$$

$$\times\ \bar{e}\gamma^\alpha e\bar{\nu}_\mu\gamma_\alpha(1-\gamma_5)\nu_\mu \qquad (3.3)$$

where α is the fine structure constant. In the lab system q^2 can be written in terms of the kinetic energy of the final electron T as

$$q^2 = -2m_e T \qquad (3.4)$$

For reasonable values of m and the heavy lepton mass, M, we can neglect the q^2 term in (3.3).

If we write the total matrix element as

$$M = \frac{G_F}{\sqrt{2}}\bar{\nu}\gamma^\alpha(1-\gamma_5)\nu\bar{e}\gamma_\alpha(C_V' - C_A'\gamma_5)e \qquad (3.5)$$

the cross section in the lab frame is

$$\frac{d\sigma}{dT} = \frac{G_F^2}{2\pi}m_e\left[(C_V' - C_A')^2 + (C_V' + C_A')^2\left(1 - \frac{T}{\omega}\right)^2 - (C_V'^2 - C_A'^2)\frac{m_e T}{\omega}\right] \qquad (3.6)$$

where ω is the neutrino energy.

Using (2.7) to write the answers in terms of R we have

$$C_A' = \frac{f^2}{4\pi}\frac{1}{\pi}\frac{R-1}{\ln R}I(R) \qquad (3.7a)$$

$$C_V' = \frac{f^2}{4\pi}\frac{1}{\pi}\frac{R-1}{\ln R}I(R)$$

$$+\ \alpha\frac{1}{f^2/4\pi}\left[1 - \frac{2}{3}\ln\frac{m_+^2}{M_\mu^2}\right]\frac{R-1}{R\ln R} \qquad (3.7b)$$

where I(R) comes from (3.2) and is equal to

$$I(R) = \frac{1}{2} + \frac{1}{2R} + \frac{1}{2}\ln R - \frac{1}{2R}\ln R$$

$$+ \frac{R-1}{2R^2}(R^2+1)\ln R\ln\left(1 - \frac{1}{R}\right)$$

$$+ \frac{R-1}{4R^2}\ln^2 R + \frac{R-1}{2R^2}\frac{\pi^2}{3}$$

$$- \frac{R-1}{2R^2}(R^2+1)\sum_{n=1}^{\infty}\frac{1}{n^2}\left(\frac{1}{R}\right)^n \qquad (3.8)$$

Notice that $I(R=1) = 1$.

C_A' and the first term in C_V', which comes from the f^6 contribution, are symmetric in $m_+ \leftrightarrow m_o$ (i.e., $R \rightarrow \frac{1}{R}$) but the neutrino charge radius depends only on m_+. Therefore, the second term in C_V' is asymmetric as $R \rightarrow \frac{1}{R}$ and becomes large as R gets small ($m_o > m_+$). A plot of C_V' and C_A' is given in

Fi 6 for various values of R and ranges of $f^2/4\pi$. We see that C_V' and C_A' are probably quite small although for extreme values of R they could be as large as 0.5 in magnitude. Therefore, we cannot draw any definite conclusions except that although the cross section is not zero it is probably smaller than is predicted by the Weinberg-Salam theory,[1,9] where the process is of order G_F. In section IV we will estimate R and find R > 1.

B. $\nu_e + e \to \nu_e + e$

This process is allowed in order f^4. The relevant graph is shown in Fig. 7. This has the same form as μ decay; in particular it has the exchange of one neutral and one charged scalar. Therefore the matrix element simply reduces to the V-A form with no dependence on the relative size of m_+ and m_0. If we perform a Fierz transformation and write the matrix element as

$$M = \frac{G_F}{\sqrt{2}}\,\bar{\nu}_e\gamma^\alpha(1-\gamma_5)\nu_e\bar{e}\gamma_\alpha(C_V - C_A\gamma_5)e \tag{3.9}$$

then we have $C_V = C_A = 1$. The cross section, in terms of C_V and C_A, is given by (3.6). The point $C_V = C_A = 1$ is well within the experimentally allowed[10] region in C_V - C_A space. The Weinberg-Salam[1] theory predicts $1/2 \leq C_V \leq 5/2$ and $C_A = 1/2$.

There will also be graphs of order e^2f^2 like the photon

graphs in Fig. 5. Based on our calculations for $\nu_\mu + e \to \nu_\mu + e$ these should not change C_V and C_A by more than ~20%.

C. $e^+ + e^- \to \mu^+\mu^-$

Thus far we have seen that there is no neutral current effect in $\nu_e e$ scattering and that $\nu_\mu e$ scattering has a neutral current effect only in order f^6. For $e^+e^- \to \mu^+\mu^-$ however there is a neutral current effect in order f^4. The diagram is shown in Fig. 8; this graph gives a weak matrix element

$$M_{weak} = -\left(\frac{f^2}{4\pi}\right)^2 \frac{1}{m_0^2}\,\bar{\mu}\gamma^\alpha(1-\gamma_5)\mu\bar{e}\gamma_\alpha(1-\gamma_5)e \tag{3.10}$$

This gives a significant contribution to the cross section only through the cross term with the one photon exchange matrix element

$$M_\gamma = \frac{e^2}{4E^2}\,\bar{\mu}\gamma^\alpha\mu\bar{e}\gamma_\alpha e \tag{3.11}$$

where E is the c. of m. energy of one of the initial particles. The weak neutral current can then be observed by looking for terms in the cross section that are asymmetric in scattering angle or helicity.[11] These effects can be separated from the similar effects due to two photon intermediate states as discussed in Ref. 12.

Consider electron and position beams with equal and opposite polarizations, s, perpendicular to the direction of motion. The differential cross section due to one-photon exchange is

$$\frac{d\sigma^{(o)}}{d\Omega} = \frac{\alpha^2}{16E^2} W_o \tag{3.12}$$

where

$$W_o = 1 + z^2 - s^2(1-z^2)\cos 2\phi \tag{3.13}$$

The scattering angles are ϕ and θ with $z = \cos\theta$.

The total cross section may then be written as

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma^{(o)}}{d\Omega}(1+\delta) \tag{3.14}$$

where δ contains all the higher order effects. If we call δ_z the part of δ that comes from the cross term between eqns. (3.10) and (3.11) and is odd in cos θ then (using (2.7)) we have

$$\delta_z = -\frac{8\sqrt{2}\, G_F}{e^2}\,\frac{R-1}{\ln R}\,\frac{zE^2}{W_o} \tag{3.15}$$

Notice that if R = 1 this is exactly twice as large as the same quantity in the Weinberg theory.[1,11,12] This means that if s^2 is close to unity the asymmetry in the cross section

$$\frac{d\sigma(\theta) - d\sigma(\pi-\theta)}{d\sigma(\theta) + d\sigma(\pi-\theta)} \tag{3.16}$$

will be 2% if R is 1. If R is larger than one, δ_z will be even larger (almost 8% if R = 10) while if R is less than one δ_z will be smaller than 2%, but it is still bigger than the Weinberg[1] theory if $R \geq 1/5$. In section IV we will see that $(R-1)/\ln R \simeq 6$ is a reasonable estimate which gives a dramatically large value for the asymmetry.

The second way neutral currents may manifest themselves in this process is through a non-zero polarization for the final particles. If the polarization of the final μ^- is called h, then we define the polarization from the square of the matrix element as

$$P = \frac{|M|^2_{h=+1} - |M|^2_{h=-1}}{|M|^2_{h=+1} + |M|^2_{h=-1}} \tag{3.17}$$

Using (3.10) and (3.11) we find

$$P = \frac{4\sqrt{2}\, G_F E^2}{e^2}\,\frac{R-1}{\ln R}\left[1 + \frac{2z}{W_o}\right] \tag{3.18}$$

At $\phi = 0, \pi$ and

$$z = \left[\frac{1-s^2}{1+s^2}\right]^{1/2} \tag{3.19}$$

P has a maximum

$$P_{max} = \frac{4\sqrt{2}\, G_F E^2}{e^2}\,\frac{R-1}{\ln R}\,[1 + (1-s^4)^{-1/2}] \tag{3.20}$$

This can be compared with the prediction in the Weinberg theory

$$P_{Weinberg}\Big|_{max} = \frac{2\sqrt{2}\, G_F E^2}{e^2}\,[3\sin^2\theta_W - \cos^2\theta_W][1 + (1-s^4)^{-1/2}] \quad (3.21)$$

where θ_W is the Weinberg angle. For $s^2 = .924$, $E = 3.5$ GeV, and $R = 1$, (3.20) gives

$$P_{max} = 3.1\% \quad (3.22)$$

This value is much larger than the value predicted by the Weinberg theory, given current estimates of θ_W. If the estimate $(R-1)/\ln R \approx 6$ of the next section is correct, the muon polarization is also dramatically large in this model.

The parameters of a weak interaction model are also constrained by the experimental limits on the weak correction to the muon's magnetic moment. In the present model the weak correction comes from the diagram in Fig. 9. Its contribution to $(g-2)/2$ is[13]

$$a_\mu^W = \frac{3f^2}{8\pi^2}\frac{m_\mu^2}{m_o^2}\int_0^1 \frac{dx\; x^2(1-x)}{1 - x + \frac{M^2}{m_o^2}x - \frac{m_\mu^2}{m_o^2}(1-x)x} \quad (3.23)$$

where m_μ is the muon mass and, as before, M_μ and m_o are the masses of the heavy lepton and the neutral scalar. Since, as

we saw in section II, universality requires M_μ^2/M_o^2 to be small we have

$$a_\mu^W = \frac{f^2}{8\pi^2}\frac{m_\mu^2}{m_o^2} = \frac{1}{2\pi}\frac{G_F}{\sqrt{2}}\, m_\mu^2\, \frac{R-1}{\ln R}\,\frac{1}{(f^2/4\pi)} \quad (3.24)$$

where the second equality comes from using (2.7).

The experimental bounds on the weak correction are[14]

$$a_\mu^W = (2.8 \pm 3.1) \times 10^{-7} \quad (3.25)$$

As long as $f^2/4\pi$ is larger than the lower bounds derived in section II the weak correction of (3.24) is smaller than the present experimental bound.

IV. NEUTRINO-HADRON INTERACTIONS

The effective lowest order weak Lagrangian for inclusive neutrino scattering with a muon in the final state, $\nu_\mu + N \to \mu + X$, is the same as in ordinary weak interaction theories

$$\mathcal{L}\binom{\text{charged}}{\text{current}} = \frac{G_F}{\sqrt{2}} [\bar{\mu}\gamma^\alpha(1-\gamma_5)\nu_\mu][\bar{P}\gamma_\alpha(1-\gamma_5)n] \qquad (4.1)$$

where we have set the Cabibbo angle, θ_c, equal to zero, and P and n are quarks.

The counterpart for muonless events is

$$\mathcal{L}\binom{\text{neutral}}{\text{current}} = \frac{G_F}{\sqrt{2}} [\bar{\nu}\gamma^\alpha(1-\gamma_5)\nu] \qquad (4.2)$$
$$\times\ [\bar{P}\gamma_\alpha(a(1-\gamma_5) + b(1+\gamma_5))P + \bar{n}\gamma_\alpha(c(1-\gamma_5) + d(1+\gamma_5))n]$$

assuming the couplings to be V - A or V + A. For example in the Weinberg-Salam model[1] we obtain, with θ_W the Weinberg angle,

$$a = \frac{1}{2} - \frac{2}{3}\sin^2\theta_W \qquad c = -\frac{1}{2} + \frac{1}{3}\sin^2\theta_W$$
$$b = -\frac{2}{3}\sin^2\theta_W \qquad d = \frac{1}{3}\sin^2\theta_W \qquad (4.3)$$

In our model,[7] $\mathcal{L}^{(\text{neutral})}_{\text{eff.}}$ is generated by the two box

diagrams in Fig. 10, which lead to

$$\mathcal{L}^{\text{neutral current}}_{\text{eff.}} = \frac{G_F}{\sqrt{2}} \{\bar{\nu}\gamma^\gamma(1-\gamma_5)\nu\}\{\bar{n}\gamma_\alpha(1+\gamma_5)n - \bar{P}\gamma_\alpha(1-\gamma_5)P\} \qquad (4.4)$$

i.e., a = -1 and d = 1 with b and c equal zero. Using (4.1) and (4.4) one can calculate in the usual way[15] the ratio of muonless to muonfull ν and $\bar{\nu}$ induced events, obtaining,

$$R_\nu = \frac{\sigma(\nu_\mu + N \to \nu_\mu + X)}{\sigma(\nu_\mu + N \to \mu^- + X)} = \frac{4}{3} \qquad (4.5)$$

$$R_{\bar{\nu}} = \frac{\sigma(\bar{\nu}_\mu + N \to \bar{\nu}_\mu + X)}{\sigma(\bar{\nu}_\mu + N \to \mu^+ + X)} = 4 \qquad (4.6)$$

where N is a target with equal number of P and n quarks and X means we sum over all allowed final states.

The values of R_ν and $R_{\bar{\nu}}$ so obtained are too large to agree with experiment[16] so a suppression factor must be introduced. The easiest way to do this without affecting universality is to multiply all $B^\pm$ couplings by a factor $\epsilon^{1/2} < 1$ leaving B^0 couplings unchanged as mentioned in section II. Since $\nu + N \to \mu + X$ proceeds by a B^+, B^0 exchange and $\nu + N \to \nu + X$ by B^+, B^- exchange we find

$$R_\nu = \frac{4}{3}\epsilon^2 \qquad R_{\bar{\nu}} = 4\epsilon^2 \qquad (4.7)$$

and ε may be adjusted to experiment. A similar effect could also be obtained by having the $B^{\pm}$ mass be larger than the B^{o} mass in such way as to decrease the effective coupling when two charged B's are exchanged. We will discuss this in detail at the end of this section.

The ratio

$$Q = \frac{\sigma(\nu_\mu + N \to \nu_\mu + X)}{\sigma(\bar{\nu}_\mu + N \to \bar{\nu}_\mu + X)} = 1 \qquad (4.8)$$

is however independent of ε (or m_+^2/m_o^2) and hence appears to be a good test of the model. de Rujula et al.[16] quote a value of Q = 0.53 ± 0.15, in disagreement with (4.8), but we believe it is premature to rule out the model on this basis.

We can also calculate the ratio of elastic neutrino proton scattering to the charge exchange reaction[17]

$$S = \frac{d\sigma(\nu + P \to \nu + P)/dq^2}{d\sigma(\nu + P \to \mu + N)/dq^2} \qquad (4.9)$$

If we assume the ratios of form factors are independent of q^2, then the cross section ratio is

$$S = .4\varepsilon^2(1 + (g_A^o)^2) \qquad (4.10)$$

where g_A^o is the form factor at $q^2 = 0$ for the proton matrix element of the isoscalar, axial vector current in (4.4).

This result is effectively the cross section ratio at $q^2 = 0$ and, as Sakurai and Urrutia have shown,[18] there are large corrections away from $q^2 = 0$.

A third process that we can calculate is $\nu + p \to \nu + p + \pi^o$. Adler[19] has given a detailed treatment of this in the (3.3) resonance region and Lee,[20] using Adler's results, calculated the ratio

$$R = \frac{\sigma(\nu + p \to \nu + p + \pi^o) + \sigma(\nu + N \to \nu + N + \pi^o)}{2\sigma(\nu + N \to \mu^- + p + \pi^o)} \qquad (4.11)$$

in the Weinberg-Salam model. He found

$$R \geq 0.6 \qquad (4.12)$$

It is easy to take his calculation over to our case and we find, in our model,

$$R \geq 0.76\,\varepsilon^2 \qquad (4.13)$$

Here corrections[21] must be made for the nuclear interactions within the target.

A more interesting conclusion can be drawn by observing that the effective hadronic neutral current, in (4.4), can be rewritten as

$$J_\alpha^{(\Delta Q=0)} = -[(\bar{P}\gamma_\alpha P - \bar{n}\gamma_\alpha n) - (\bar{P}\gamma_\alpha\gamma_5 P + \bar{n}\gamma_\alpha\gamma_5 n)] \qquad (4.14)$$

i.e., the spacial vector current is pure isovector while the axial vector current is isoscalar. This implies that $\nu + N \rightarrow \nu + \Delta \rightarrow \nu + N + \pi^o$ proceeds only through the vector current and hence may be compared directly to electroproduction $e + N \rightarrow e + \Delta \rightarrow e + N + \pi^o$. In the region of the Δ, we have

$$\frac{\sigma(\nu + N \rightarrow \nu + N + \pi^o)}{\sigma(e + N \rightarrow e + N + \pi^o)} = G_F^2\, \epsilon^2 \left(\frac{q^2}{e^2}\right)^2 \qquad (4.15)$$

where q is the difference in the momenta of the final and initial leptons. This is an exact result with no structure corrections.

Finally, we note that the vector current does not lead to coherent scattering of neutrinos off massive nuclei since it is an isovector current and an isoscalar vector current is required to obtain this effect.[22]

A second means of generating apparent neutral current effects at high incident neutrino energies exists in the model, namely the production and subsequent decay of real heavy leptons (L),

$$\nu_\mu + N \rightarrow L_\mu^- + X \qquad (4.16)$$
$$\hookrightarrow \begin{cases} \mu^- + \text{hadrons} \\ \nu_\mu + \text{hadrons} \end{cases}$$

Since the branching ratio, neglecting the muon mass, is

$$\frac{\Gamma_{L_\mu^- \rightarrow \nu_\mu + X}}{\Gamma_{L_\mu^- \rightarrow \mu^- + X}} = \epsilon^2 \qquad (4.17)$$

the ratios R_ν, $R_{\bar{\nu}}$ and Q given in (4.7) and (4.8) remain unchanged though the cross sections all increase as the incident neutrino energy crosses the L production threshold. The present mass limits on heavy lepton production[23] are not directly applicable since they are relevant to a neutrino producing an L^+, not an L^-, but it is clear that the mechanism of (4.15) will be important unless the L mass is very large. This will be particularly true since L production is proportional to f^2/m^2 not G_F and $f^2 >> (f^2/4\pi)^2$ for $f^2 << 4\pi$.

Since the exchange mechanism that leads to L production is S - P rather than V - A, the differential cross section for e.g. $\nu_\mu + N \rightarrow \mu^- + X$ will change its angular distribution[24] as well when we cross the L threshold.

Rather than scaling the charged to neutral couplings by ϵ we could take the mass of the charged B to be larger than the mass of the neutral B. In terms of

$$R = \frac{m_+^2}{m_o^2}$$

as defined in section II, we would then replace ϵ by

$$\varepsilon \to \frac{R-1}{R \ln R}$$

(To be completely general we could consider both $\varepsilon \neq 1$ and $R \neq 1$, but the net effect would be somewhere between the $R = 1$, $\varepsilon \neq 1$ and $\varepsilon = 1$, $R \neq 1$ extremes.)

The need to suppress the hadronic processes of this section has a profound effect on the leptonic processes of section III. The asymmetry and polarization in $e^+e^- \to \mu^+\mu^-$ are enhanced by $1/\varepsilon$ (if $R = 1$) or by the $(R-1)/\ln R$ factor shown in (3.15) and (3.20) (if $\varepsilon = 1$). If R_ν and $R_{\bar{\nu}}$ in (4.7) need to be suppressed by a factor of 10 as indicated by Ref. 16, then the $e^+e^- \to \mu^+\mu^-$ effects are enhanced by a factor between $\sqrt{10}$ ($R = 1$) and 19/3 ($\varepsilon = 1$). This is the large enhancement refered to in Sec. III. At the same time $\nu_\mu e$ scattering is not significantly enhanced as we can see by the R dependence of (3.7) as shown in Fig. 6.

V. RENORMALIZATION AND UNIVERSALITY

Since $f^2/4\pi$ is of order one, we might expect large parity violating effects due to diagrams in which a single quark emits and then re-absorbs a B particle. These appear as wave function and mass renormalizations, and the parity violating part may be transformed away[25] by suitable renormalization. Our concern, however, is that, in the process of doing so, we may destroy universality, namely the equality of G_μ and G_β, because of the fact that hadrons and leptons do not appear symmetrically in the original interaction, i.e., there is no hadron analogue of L_e, L_μ. (See however Appendix B, in which a model with manifest universality is displayed by introducing the hadron analogues of L_e, L_μ.)

To examine this question in more detail, let us consider a fermion field ψ coupled to a spin one gluon, A_μ, with coupling constant g. The Lagrangian for the ψ field is

$$\mathcal{L}_\psi = \bar{\psi}\{i\gamma^\alpha(\partial_\alpha - igA_\alpha) - M\}\psi \qquad (5.1)$$

If we also allow ψ to couple by a $f(1\pm\gamma_5)$ coupling to a spin zero boson, fermion self energy diagrams due to this coupling and their iterations modify $\mathcal{L}$ to

$$\mathcal{L}' = \bar{\psi}\{i\gamma^\alpha(\partial_\alpha - igA_\alpha)(a + b\gamma_5) - M\}\psi \qquad (5.2)$$

with a-1 and b being power series in f^2. By defining

$$\chi = \sqrt{a + b\gamma_5}\,\psi \tag{5.3}$$

we can write a new Hamiltonian for the system in terms of the χ field in which the only effect of the diagram will be to rescale the mass by $1/\sqrt{a^2 - b^2}$

$$\bar{\psi}M\psi = \chi^+ \frac{1}{\sqrt{a + b\gamma_5}} \gamma_o M \frac{1}{\sqrt{a + b\gamma_5}} \chi = \frac{\bar{\chi}M\chi}{\sqrt{a^2 - b^2}} \tag{5.4}$$

Allowing ψ to have several components, each one of which we renormalize, we find that the effective non-diagonal coupling to the spin zero boson changes by e.g.

$$f\bar{\psi}_1(1-\gamma_5)\psi_2 = f\bar{\chi}_1 \frac{1}{\sqrt{a_1 - b_1\gamma_5}} (1-\gamma_5) \frac{1}{\sqrt{a_2 + b_2\gamma_5}} \chi_2$$

$$= f\bar{\chi}_1(1-\gamma_5) \frac{1}{\sqrt{(a_1 - b_1\gamma_5)(a_2 + b_2\gamma_5)}} \chi_2 \tag{5.5}$$

Since $(1 \pm \gamma_5)$ are projection operators, the form of the weak coupling will be unchanged; its magnitude will however be altered.

We now compare the basic box diagrams for β decay and for μ decay, as calculated however with the lepton and hadron fields renormalized as indicated in (5.3). Universality requires that

$$(1-\gamma_5) \frac{1}{(a_L - b_L\gamma_5)(a_\mu + b_\mu\gamma_5)} = \frac{1-\gamma_5}{\sqrt{a_n^2 - b_n^2}} \frac{1}{\sqrt{(a_p + b_p\gamma_5)(a_n - b_n\gamma_5)}} \tag{5.6}$$

or, using the projection properties of $(1 \pm \gamma_5)$,

$$(a_L + b_L)^2(a_\mu - b_\mu)^2 = (a_n + b_n)(a_p - b_p)(a_n + b_n)^2 \tag{5.7}$$

where of course $a_\mu = a_\nu$ and b_μ equals b_ν.

The fermion one loop self energy diagram for, say, a proton of momentum k, equals $\xi(1-\gamma_5)$ with ξ being a logarithmically divergent integral. Neglecting terms of order k^2/m_B^2 because of the largeness of the B mass, ξ is only a function of Λ^2/m_B^2, where Λ is a cutoff. Calculating the corresponding contribution for the other hadrons and leptons, we find

$$\begin{array}{llll} a_\mu = a_\nu = 1+\xi & a_L = 1+2\xi & a_n = 1+3\xi & a_p = 1+\xi \\ b_\mu = b_\nu = -\xi & b_L = 2\xi & b_n = \xi & b_p = -\xi \end{array} \tag{5.8}$$

Substitution of these values into (5.7) shows that the two sides are equal and that universality holds. Furthermore, suppose B^+ and B^0 have different coupling strengths f_1 and f_2, leading to different ξ's which we call ξ_1 and ξ_2. We then have

$$a_\mu = 1+\xi_1 \qquad a_\nu = 1+\xi_2 \qquad a_L = 1+\xi_1+\xi_2$$

$$b_\mu = -\xi_1 \qquad b_\nu = -\xi_2 \qquad b_L = \xi_1+\xi_2 \tag{5.9}$$

$$a_p = 1+\xi_2 \qquad a_n = 1+2\xi_1+\xi_2$$

$$b_p = -\xi_2 \qquad b_n = \xi_2$$

Universality holds, as one can verify by substituting the above values into (5.7). What we have proved is that a class of diagrams, namely the one loop and iterated one loop wave function and mass renormalization diagrams, do not alter the validity of universality in the model.

Since, with a little bit of work, one can see that there are no one loop vertex corrections, Fig. 3 of Ref. 7 vanishes. This then completes the proof that there are no corrections of order $f^2/4\pi$ to the universality statement $G_B = G_\mu$ of (2.6). Two loop vertex corrections to both hadron and lepton vertices exist as displayed in Fig. 4C and we have not established that $(f^2/4\pi)^2$ and higher order corrections to universality do not exist in the model.

Finally we would like to comment on the possibility of strong interaction corrections on the hadron line altering universality. Imagine the strong interactions to be mediated by spin one gluons; external hadrons exchanging such a gluon

would not affect universality since the exchange would be in the nature of a strong correction to an effective CVC interaction. What would cause trouble is a diagram such as the one of Fig. 11. This diagram is of order $g^2\, G_F/4\pi \to G$ where g is the gluon coupling constant and apparently leads to a violation of universality. We must recognize however, that the significant contributions to the box diagram come from internal momenta of order m_B. This is true because the box diagram has two B meson propagators so that low momentum contributions go as $(M^2/m_B^2)(1/m_B^2)$ and it is momenta of order m_B which give the dominant contribution, $\sim 1/m_B^2$. Hence, in Fig. 11, the gluon is coupled inside the box to an n quark with momentum $\sim m_B$. This suggests that g should be replaced by an effective coupling constant, which may in fact be very small if asymptotic freedom[26] holds for gluon quark coupling. If this were correct universality would still hold.

VI. CONCLUSIONS

The most obvious important tests of the scalar exchange model in the form given here (Eq. (2.1)) and in Ref. 7 are:

(a) $Q = \sigma(\nu_\mu + N \to \nu_\mu + X)/\sigma(\bar{\nu}_\mu + N \to \bar{\nu}_\mu + X) = 1$
(b) $\sigma(\nu + N \to \nu + \Delta)/\sigma(e + N \to e + \Delta) = (Gq^2/e^2)^2\varepsilon^2$
(c) the large asymmetry and muon polarization in $e^+e^- \to \mu^+\mu^-$
(d) $C_V', C_A' << 1$ in $\nu_\mu + e \to \nu_\mu + e$
$C_V = C_A = 1$ in $\nu_e + e \to \nu_e + e$

(a) and (b) follow from the fact that the neutral current in this model has only vector $I = 1$ and axial vector $I = 0$ parts. The resulting absence of vector, axial vector interference (after averaging over isospin) make the prediction (a) of equal ν and $\bar{\nu}$ neutral inclusive scattering model independent.[15] Deviations from equality could not be remedied by varying parameters in the Lagrangian (2.1), i.e., taking ε (or R) different than one. A different form would be required.

The large effect (c) results from the data of references 16 ($R_{\bar{\nu}} = .4$) which indicates suppression of charged B exchange relative to neutral B exchange by a factor of $\varepsilon^2 \simeq 1/10$ or $R \simeq 20$. Since $e^+e^- \to \mu^+\mu^-$ proceeds by $2B^0$ exchange this has the effect of enhancing the predictions for δ_z and P by approximately a factor of 3 to 6 (depending on whether we take

$\varepsilon \neq 1$ or $R \neq 1$) to a maximum of 6% to 12% for δ_z and 10% to 20% for P. These values are, of course, dramatically larger than those of the Weinberg-Salam theory.

At the same time the $R_\nu < 1$ data does not give significant enhancement of $\nu_\mu e$ scattering (at least in lowest non-vanishing order) which should therefore be much smaller than the Weinberg-Salam minimum ($C_A' = 1/2$, $C_V' = 0$).

The previous sections contained other results but these four seem the most likely to provide critical tests of the model in the near future. It would not be surprising if the Lagrangian of (2.1) should fail one of these tests and require modification. At the present time, however, it gives sensible predictions and demonstrates the possibility of viable alternative approaches to the weak interactions.

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Appendix A. Higher Order Terms in Mu Decay

To simplify the calculation of the higher order diagrams we will set $m_+ = m_o$ and M (the mass of the heavy leptons) equal to zero. We know from the discussion of universality that M/m must be small. Setting the masses of the charged and neutral scalars equal will result in some error, however, if they are indeed very different.

Once the renormalization subtractions have been made the scalar self energy is zero (really of order M/m which we set to zero) in order f^2 and there are no f^4 scalar self energy diagrams. There are also no vertex corrections in order f^3 if the B^o particle is not self conjugate. This considerably reduces the number of diagrams.

The leptonic self energy is non-zero after the substractions have been made so the diagrams of Fig. 3 contribute to order f^6. Each heavy lepton self energy graph contains both an electron-B^o and a neutrino-B^- intermediate state. The total 6th order correction is

$$\left(\frac{f^2}{4\pi}\right)^3 \frac{1}{m^2} \frac{4}{\pi} I \; \bar{\nu}_\mu \gamma^\alpha (1-\gamma_5) \mu \; \bar{e} \; \gamma_\alpha (1-\gamma_5) \nu_e \qquad \text{(A1)}$$

where

$$I \equiv \int_0^1 dx \int_0^1 dy \int_0^1 dz \; \frac{xyz(4 - 5x - 2z + 2xz)}{z(1-x) + y(1-z)} \qquad \text{(A2)}$$

This integral can be done analytically; the result is

$$I = -\frac{1}{12}(21 - 2\pi^2) \qquad \text{(A3)}$$

Thus the 6th order correction is very small for $f^2/4\pi$ less than, or equal to, one.

In order f^8 there are five types of corrections to mu decay; the ladder diagrams shown in Fig. 4a, the ladder with crossed rungs as in Fig. 4b, the ladder in the crossed channel with the lepton or quark box in the middle as in Fig. 4c, the vertex correction of Fig. 4d, and the leptonic self energy graphs of Fig. 4e. The ladder graphs of Fig. 4a and 4b are finite and given by

$$\left(\frac{f^2}{4\pi}\right)^4 \frac{1}{m^2}\frac{2}{\pi^2}(J_o + J_1)\, \bar{\nu}_\mu \gamma^\alpha (1-\gamma_5)\mu\, \bar{e}\, \gamma_\alpha (1-\gamma_5)\nu_e \qquad \text{(A4)}$$

where

$$J_o \equiv \int_o^1 dx \int_o^1 dy \int_o^1 dz \int_o^1 dw \int_o^1 dv\, \frac{N(x,y,z,w,v)}{D(x,y,z,w,v)} \qquad \text{(A5)}$$

with

$$N(x,y,z,w,v) = xyzw[xz + 4xv(1-z) + 2v(1-x)(1-z)(1-v)] \qquad \text{(A6a)}$$

$$D(x,y,z,w,v) = xzv(1-x)(1-w) + xv(1-x)(1-z) + xz(1-y)(1-z)(1-v) + z(1-x)(1-z)(1-v) \qquad \text{(A6b)}$$

The integrand of (A4) has integrable singularities at the end points. We evaluated the w and y integrals analytically and the final three integrals numerically. The result is

$$J_o = 1.30 \pm .11 \qquad \text{(A7)}$$

where the error is an estimate based on how fast the integral converges as we keep doubling the number of points starting at 3 points per integral and ending with 129 points per integral.

J_1 is a seven dimensional integral over Feynman parameters. Two of the integrals can be done analytically leaving

$$J_1 = \int_o^1 dx \int_o^1 dy \int_o^1 du \int_o^1 dv \int_o^1 dz\, N(x,y,u,v,z)G(x,y,u,v,z) \qquad \text{(A8)}$$

where, if we define

$$\delta = vx(1-x) + x(1-y)(1-v)(1-x+xy) - ux(1-x)(1-y+vy)^2 \qquad \text{(A9a)}$$

$$\alpha_1 = u(1-y+vy) \qquad \text{(A9b)}$$

$$\alpha_2 = 1-\alpha_1 \qquad \text{(A9c)}$$

$$\alpha_3 = x(1-y) + (1-x)\alpha_1 \qquad \text{(A9d)}$$

$$\alpha_4 = 1-x+xy-\alpha_1(1-x) \qquad \text{(A9e)}$$

then

$$N(x,y,u,v,z) = x^2yuv + \frac{z}{\delta} x^3(1-x)yv\alpha_1\alpha_2$$

$$+ 6xyu^2v(1-v)(1-x) + \frac{2}{\delta^2} z^2x^3y(1-v)(1-x)\alpha_1\alpha_2\alpha_3\alpha_4 v$$

$$+ \frac{z}{\delta} x^2yvu(1-v)[\alpha_3\alpha_4 + 2\alpha_2\alpha_4(1-x) - 2\alpha_2\alpha_3(1-x)$$

$$- 2\alpha_1\alpha_4(1-x) + 2\alpha_1\alpha_3(1-x) + \alpha_1\alpha_2(1-x)^2] \tag{A10}$$

$G(x,y,u,v,z)$ is defined as

$$\frac{1}{D}\left\{\frac{D+F}{E} \ln \frac{D+E+F}{D+F} + \ln \frac{D+E+F}{E+F} - \frac{F}{E} \ln \frac{E+F}{F}\right\} \tag{A11}$$

where

$$D = -vxz(1-x) \tag{A12a}$$

$$E = -xyz(1-v) \tag{A12b}$$

$$F = vx(1-x) + z(1-v)(1-x+xy) + x(1-y)(1-v)(1-z)(1-x+xy)$$

$$- ux(1-x)(1-z)(1-y+vy)^2 \tag{A12c}$$

At 17 points per integral J_1 has the value $1.02 \pm .09$.

The fermion loop in the cross channel (Fig. 4c) diverges

and a subtraction must be made (a renormalization of the B^4 coupling constant). After this subtraction the diagram has the value

$$-\left(\frac{f^2}{4\pi}\right)^4 \frac{1}{m^2} \frac{4}{\pi^2} (J_2 + J_3)\ \bar{\nu}_\mu \gamma^\alpha(1-\gamma_5)\mu\ \bar{e}\ \gamma_\alpha(1-\gamma_5)\nu_e \tag{A13}$$

where we have included two different lepton and two different quark loops. If we define

$$a_1 \equiv y(1-xy) \tag{A14a}$$

$$a_2 \equiv (1-y)(1-x+xy) \tag{A14b}$$

$$a_3 \equiv xy(1-y) \tag{A14c}$$

$$\beta \equiv a_3^2(1-z)^2 - a_2a_1(1-z) \tag{A14d}$$

$$D_1 = \beta v - zwa_1^2(1-v) \tag{A14e}$$

$$D_2 = D_1(w=1) \tag{A14f}$$

then

$$J_2 = \int_0^1 dx \int_0^1 dy \int_0^1 dz \int_0^1 dv\ 4xyzv(1-z)\ \frac{1}{\beta D_2}$$

$$\cdot \{(1-x)[y\beta - 2a_1a_3(1-y)(1-z)(1-v) - 4ya_3^2(1-z)^2(1-v)] - a_3\beta\} \tag{A15}$$

$$J_3 = \int_0^1 dx \int_0^1 dy \int_0^1 dz \int_0^1 dw \int_0^1 dv \; 2wvz^2 a_1 \frac{(-1)}{\beta^2 D_1}$$

$$\cdot \{\beta x a_1 [\beta + 4a_3^2(1-z)(1-v)] + 6\beta(1-x)(1-z)^2(1-v)a_3^2$$

$$+ 12(1-x)(1-z)^4(1-v)^2 a_3^4\} \qquad \text{(A16)}$$

Again the integrands have only integrable end point singularities and the integrals were done numerically using 17 points for each variable in J_3 and 33 points for each variable in J_2. The result is

$$J_2 = -.19 \qquad \text{(A17a)}$$

$$J_3 = +0.17 \qquad \text{(A17b)}$$

The sum of $J_2 + J_3$ is small and did not vary appreciably as we varied the number of points per integral over 3, 5, 9, 17. Thus the sum has a small error

$$J_2 + J_3 = -.02 \pm .05 \qquad \text{(A18)}$$

The vertex correction of Fig. 4d, after subtraction, is also large, in part because all four vertices must be corrected. It is given by

$$-\left(\frac{f^2}{4\pi}\right)^4 \frac{1}{m^2} \frac{16}{\pi^2} J_4 \; \bar{\nu}_\mu \gamma^\alpha (1-\gamma_5)\mu \; \bar{e} \; \gamma_\alpha (1-\gamma_5)\nu_e \qquad \text{(A19)}$$

J_4 is a seven dimensional integral over Feynman parameters. Two of the integrals can be done analytically leaving five to be done numerically. Define

$$b_1 \equiv u(1-v) + \frac{1-x}{1-xy} uv \qquad \text{(A20a)}$$

$$b_2 \equiv u(1-v) + \frac{1-x}{y(1-xy)} uv \qquad \text{(A20b)}$$

$$b_3 \equiv 1 - u + \frac{zuv}{1-xy} \qquad \text{(A20c)}$$

$$D \equiv b_1^2 - b_2 + b_3 \qquad \text{(A20d)}$$

Then the five dimensional integral is

$$J_4 = \int_0^1 dx \int_0^1 dy \int_0^1 dz \int_0^1 du \int_0^1 dv \; \frac{xu}{1-xy}$$

$$\cdot \Big\{ \frac{1}{4} [1-b_1)(-1+3x+2b_1-3b_1xy) + uv(-2x-3b_1(1-xy) + 6b_1x(1-b_1y))$$

$$+ \frac{uv}{y}(1-x) - \frac{uv}{1-xy} x(1-x)(1-3b_1y)]R_1 + \left[\frac{3}{2} xy(1-uv)-1\right] R_2$$

$$+ \frac{1}{2} \frac{uv}{y(1-xy)} \frac{1}{b_2 - b_1^2} R_3$$

$$\cdot [(1-b_1)(1-b_1y)(1-x-b_1xy)(1-x-b_1+b_1xy)] \Big\} \qquad \text{(A21)}$$

where

$$R_1 \equiv \frac{1}{D} + \frac{b_3}{D^2} \ln\left(\frac{b_2 - b_1^2}{b_3}\right) \qquad \text{(A22a)}$$

$$R_2 \equiv \frac{b_1^2 - b_2}{D} \ln\left(\frac{b_2 - b_1^2}{b_3}\right) - \ln b_3 \qquad \text{(A22b)}$$

$$R_3 \equiv \frac{1}{2D} - \frac{b_3}{D^2} + \frac{b_3(b_1^2 - b_2)}{D^3} \ln \frac{b_2 - b_1^2}{b_3} \qquad \text{(A22c)}$$

Notice that R_1, R_2, R_3 all have finite limits when $b_2 - b_1^2 = b_3$. J_4 converges nicely to the value .19 ± .01 as the number of points per integral is increased.

The total of the three self energy graphs, Fig. 4e, is

$$\left(\frac{f^2}{4\pi}\right)^4 \frac{1}{m^2} \frac{4}{\pi^2} J_5 \, \bar{\nu}_\mu \gamma^\alpha (1-\gamma_5)\mu \; \bar{e} \; \gamma_\alpha (1-\gamma_5)\nu_e \qquad \text{(A23)}$$

where J_5 is a six dimensional integral. Two of the integrals can be done analytically. If we define

$$C_1 \equiv uv(1-x)(1-z) \qquad \text{(A24a)}$$

$$C_2 \equiv u(1-v)(1-z) \qquad \text{(A24b)}$$

$$C_3 \equiv (1-u)(1-x) \qquad \text{(A24c)}$$

then

$$J_5 = \int_0^1 dx \int_0^1 dz \int_0^1 du \int_0^1 dv \, \frac{3}{2} xzu^2 v$$

$$\cdot \left[(2-3x)(2-3z) + 6u(1-v)(2-3z)(1-x) + 12C_2C_3\right]$$

$$\cdot \left\{ \frac{1}{6C_2C_3}\left[C_1 + 2C_2 + 2C_3\right] \right.$$

$$+ \frac{1}{6C_3^2}\left[C_2 - 3C_1 - 3C_2\right] \ln\left(\frac{C_1 + C_2 + C_3}{C_1 + C_2}\right)$$

$$- \frac{1}{6C_2^2}\left[3C_1 + 2C_3\right] \ln\left(\frac{C_1 + C_2 + C_3}{C_1 + C_3}\right)$$

$$\left. + \frac{C_1^3}{6C_2^2C_3^2} \ln\left(\frac{C_1(C_1 + C_2 + C_3)}{(C_1 + C_2)(C_1 + C_3)}\right)\right\} \qquad \text{(A25)}$$

Notice that the { } bracket is finite unless all of C_1, C_2, C_3 are zero.

J_5 has the value .001 ± .001.

Now the sum of (A4), (A13), (A19), and (A23) give the result (2.10) in section II.

Appendix B. A Model with Manifest Universality

We display here a weak interaction Lagrangian in which, except for hadron-lepton mass differences, μ decay and β decay are equal in strength to all order in $f^2/4\pi$. This may also be true in the model of (2.1), but we have not been able to prove it as of yet.

The model requires an SU(2) group under which the four quarks and the four known leptons transform as doublets:

$$\vec{N} = \begin{pmatrix} P \\ n \end{pmatrix} \qquad \vec{N}' = \begin{pmatrix} c \\ s \end{pmatrix}$$
$$\vec{\mu} = \begin{pmatrix} \mu^+ \\ \bar{\nu}_\mu \end{pmatrix} \qquad \vec{e} = \begin{pmatrix} e^+ \\ \bar{\nu}_e \end{pmatrix} \tag{B1}$$

In addition we have a doublet of heavy spin zero muons $\vec{B} = \begin{pmatrix} B^+ \\ B^o \end{pmatrix}$ and four SU(2) singlets, two of them, L_e^o and L_μ^o, leptons and the other two, F^o and $F^{o'}$, baryons. The weak interaction Lagrangian has the form

$$L_{int.} = -if(\vec{B}[\vec{\bar{e}}(1+\gamma_5)L_e^o + \vec{\bar{\mu}}(1+\gamma_5)L_\mu^o + \vec{\bar{N}}(1+\gamma_5)F^o + \vec{\bar{N}}'(1+\gamma_5)F^{o'}]) \tag{B2}$$

which is obviously SU(2) invariant and has full hadron lepton symmetry. The Cabibbo angle may now be introduced by the usual GIM[4] mixing of n and s. The hadron-lepton symmetry is sufficient to insure universality to all orders of $f^2/4\pi$

except for mass difference effects.

Neutral current strangeness changing, i.e., semi-leptonic $\Delta S = 1$, $\Delta Q = 0$ terms have an effective Hamiltonian of the form

$$H_{eff.} = \tau G_F(\bar{s}\gamma^\alpha(1-\gamma_5)n)(\bar{\nu}\gamma_\alpha(1-\gamma_5)\nu) \tag{B3}$$

with τ proportional to $\frac{m_F^2 - m_{F'}^2}{m^2}$.

If m_F equals $m_{F'}$, a higher order diagram gives

$$\tau \sim \frac{m_C^2 - m_P^2}{m^2}\left(\frac{f^2}{4\pi}\right)^2 \tag{B4}$$

An additional consequence of this model is that to lowest order, the amplitudes for elastic neutrino - P quark scattering vanish so neutrinos only scatter off n quarks in nucleons.

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FIGURE CAPTIONS

FIGURE 1 Lowest order diagrams for μ and β decay.

FIGURE 2 The minimum value of $f^2/4\pi$ vs. R where $R = \frac{m_+^2}{m_o^2}$ and we require $m_o > 27$ GeV. If $m_o > m_+$ then this figure is still correct with $R = \frac{m_o^2}{m_+^2}$ and $m_+ > 27$ GeV. Thus we only need to consider $R \gtrsim 1.0$.

FIGURE 3 Sixth order corrections to μ decay. There are no vertex corrections or scalar self energy corrections in this order.

FIGURE 4 The eighth order corrections to mu decay.

FIGURE 5 The lowest order contributions to $\nu_\mu + e \rightarrow \nu_\mu + e$.

FIGURE 6 The vector and axial-vector coupling constants for the reaction $\nu_\mu + e \rightarrow \nu_\mu + e$ for various values of the ratio $R = \frac{m_+^2}{m_o^2}$. The curves (a) - (e) have R = 0.01, 0.1, 1.0, 10.0, and 100. For a given curve $f^2/4\pi$ ranges from its minimum value, given by Figure 2, at the lower left end of the curve, to 1.0 at the upper right end of the curve. The factor $\ln \frac{m_+^2}{M_\mu^2}$ in the neutrino charge radius was set equal to 4.0.

FIGURE 7 The lowest order contribution to $\nu_e + e \rightarrow \nu_e + e$.

FIGURE 8 Lowest order weak contribution to $e^+ + e^- \rightarrow \mu^+ + \mu^-$

FIGURE 9 Weak correction to the muon magnetic moment.

FIGURE 10 Diagrams which generate the effective neutral current.

FIGURE 11 A possible source of strong interaction corrections to universality. The wavy line is a vector gluon.

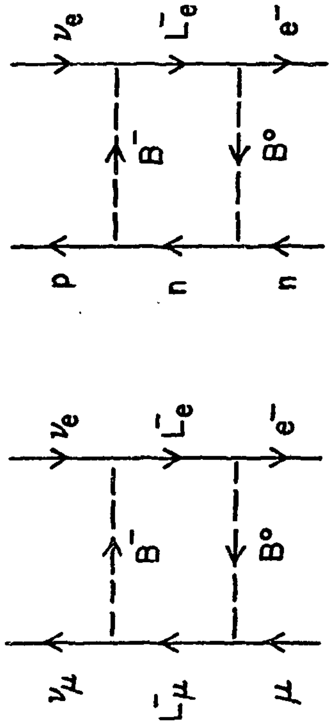
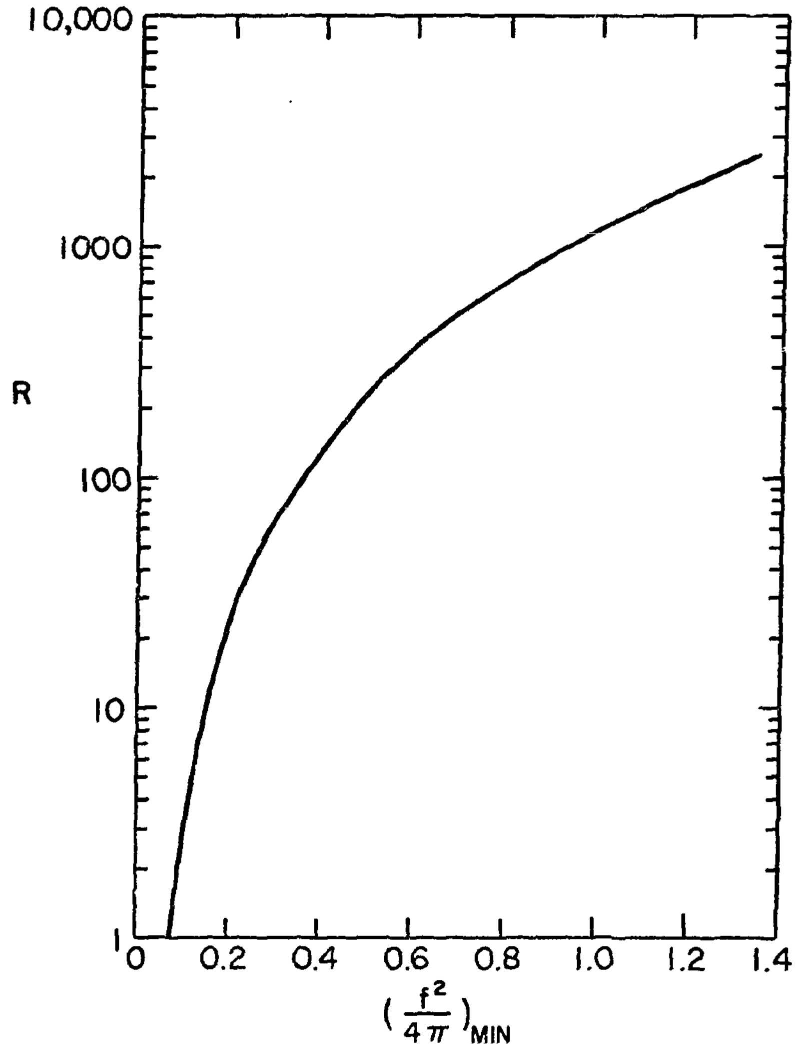


Figure 1



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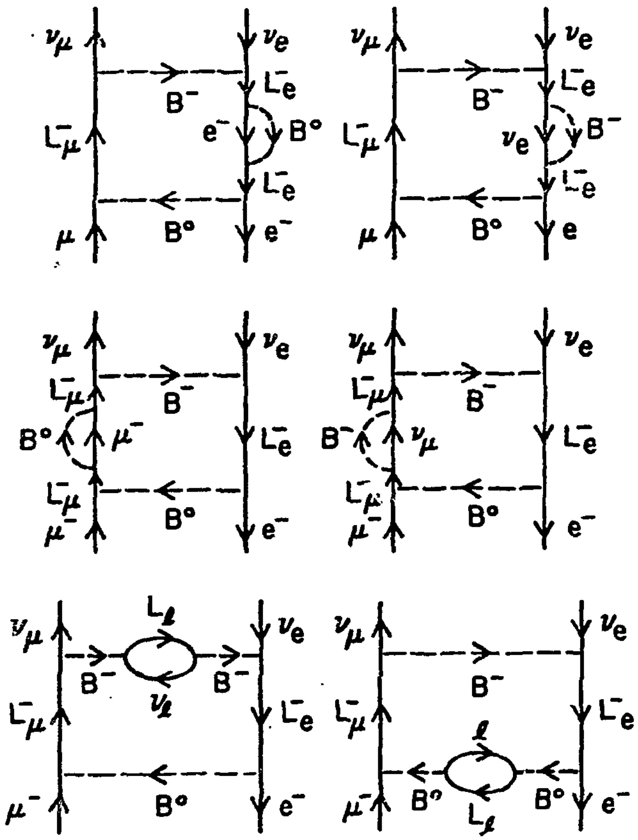
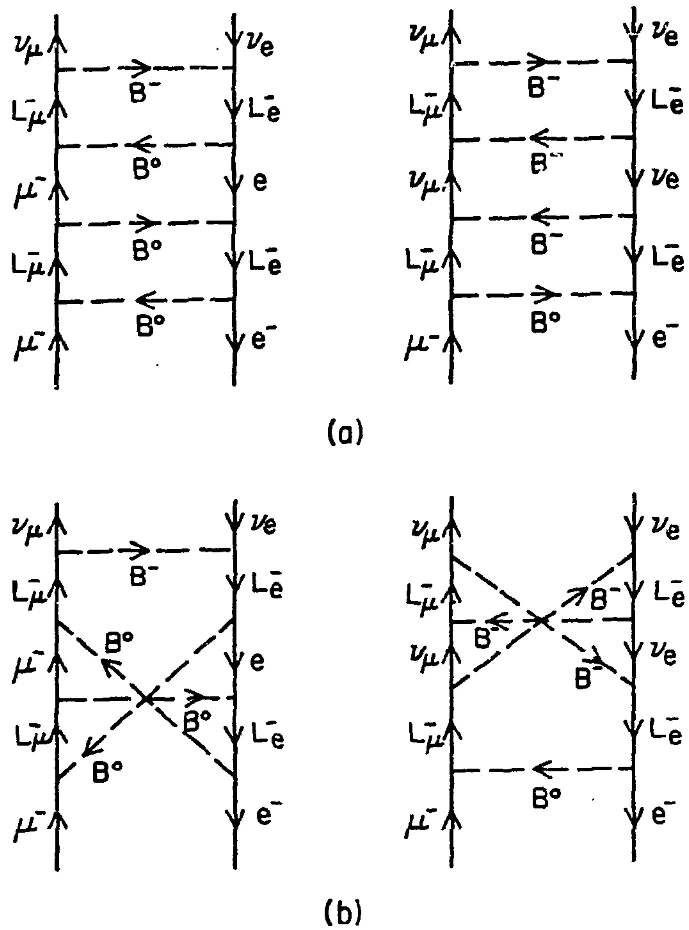
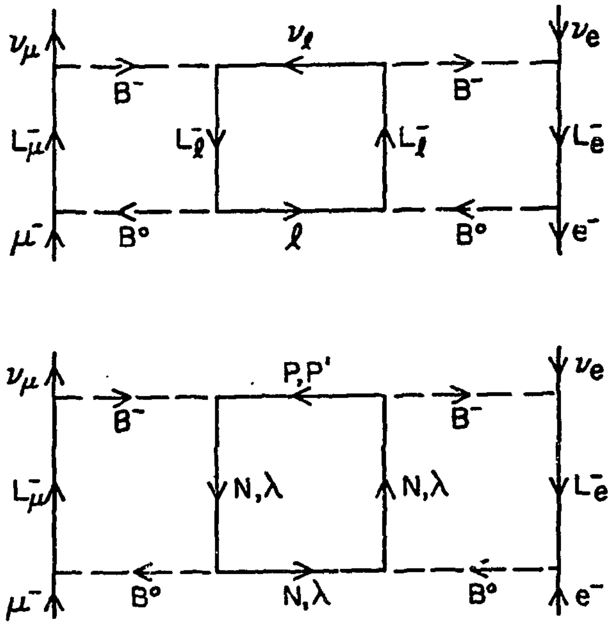


Figure 3

Figure 4

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e⁻
νμ
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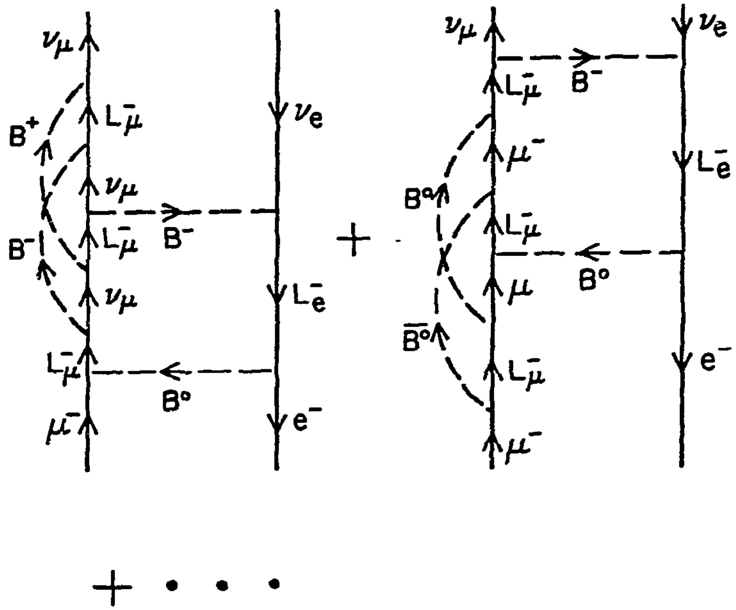
(c)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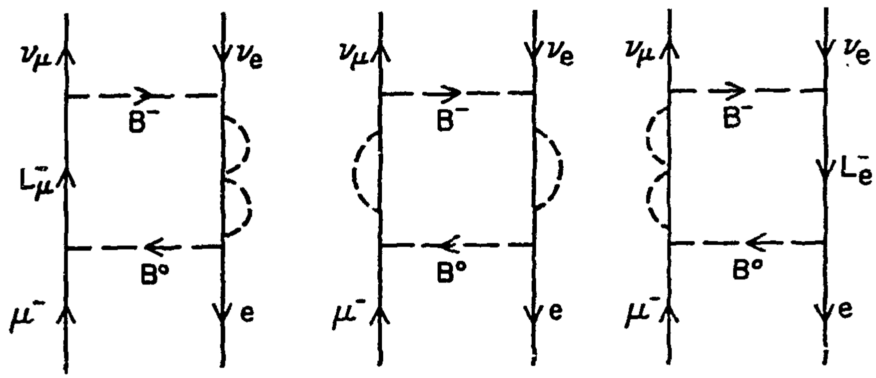
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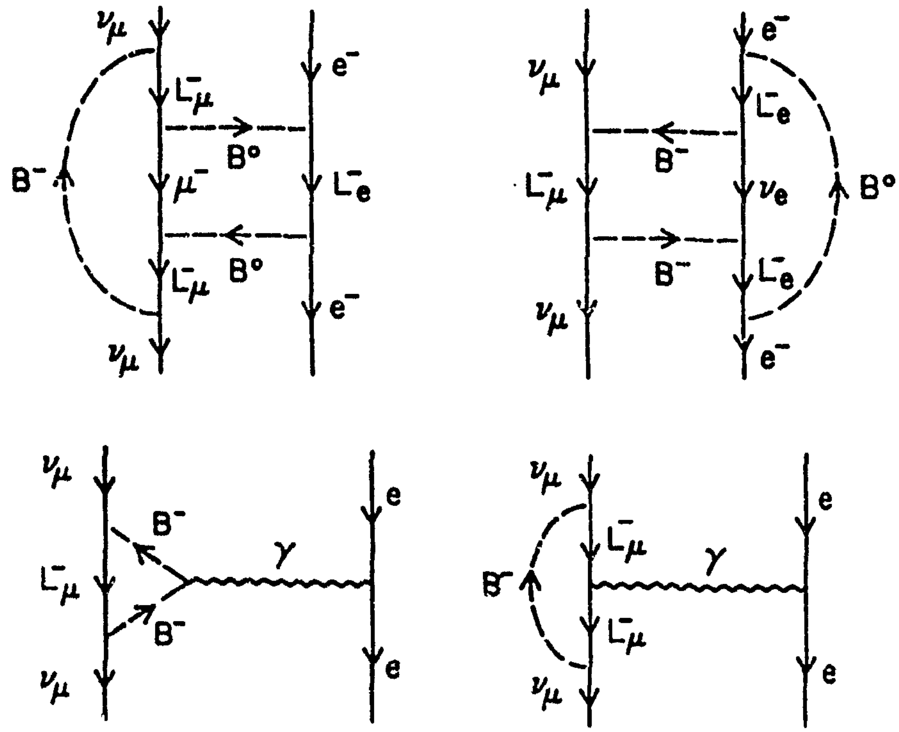

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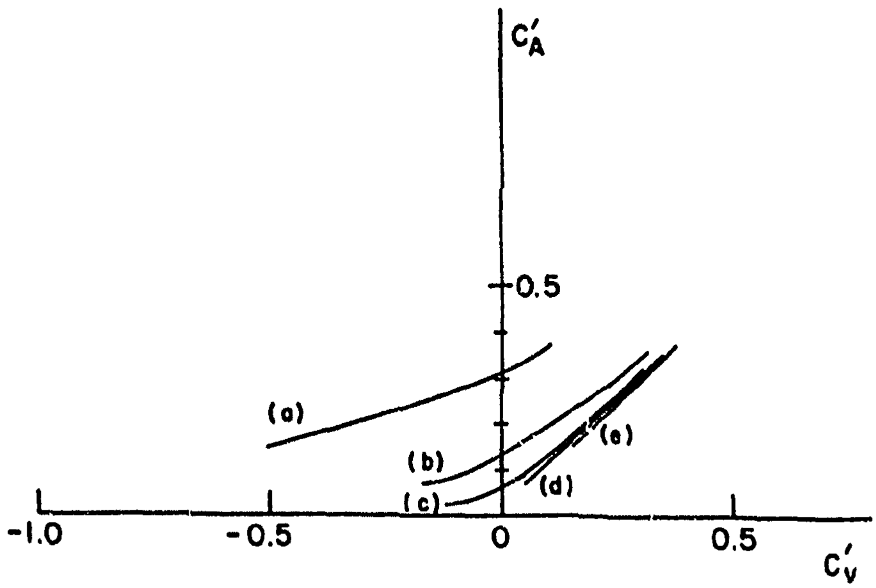
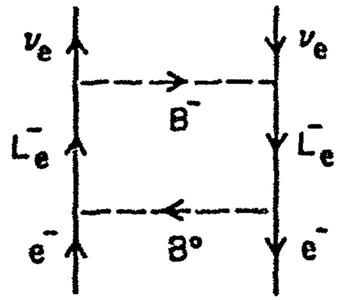


Figure 6

Figure 7



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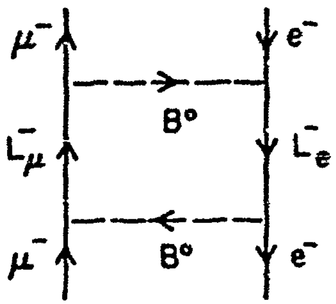


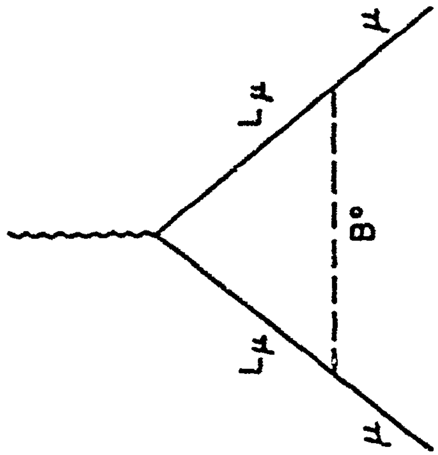
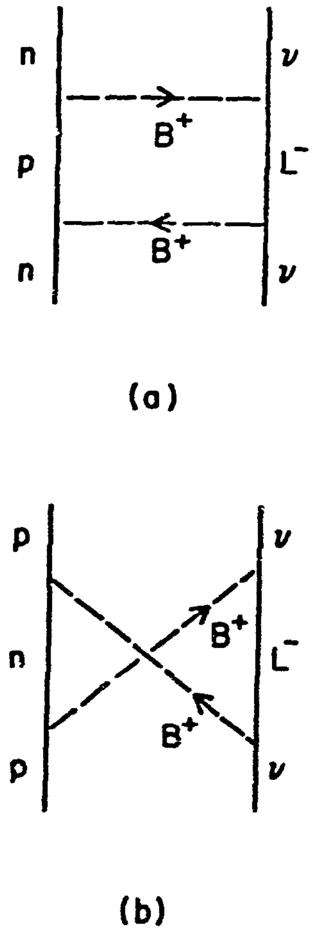


Figure 9

Figure 10



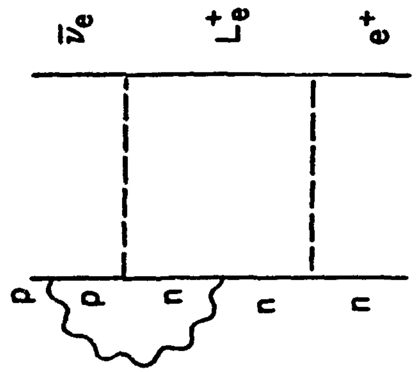


Figure 11