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Saturation of the Parametric Decay Instability Near the Lower Hybrid Frequency

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ABSTRACT

The nonlinear evolution and saturation of parametrically excited lower hybrid waves are studied numerically by solving the corresponding weak turbulence wave kinetic equation. The saturation level and the effective collision frequency ν_f are obtained as functions of pump power.

A number of experiments have observed the parametric decay of an externally imposed electric field oscillating at a frequency ω_o close to the lower hybrid frequency ω_{lh}.[1] This decay leads to rapid absorption of energy by plasma waves which ultimately heat the plasma through Landau damping and collisional damping. Since radio frequency power is readily available at frequencies corresponding to the lower hybrid frequency for typical thermonuclear plasmas, this process provides an interesting possibility to heat plasmas to fusion temperatures.

It is necessary to follow the time evolution of the lower hybrid decay waves to the eventual saturated state in order to predict the rate of energy absorption, total ion and electron heating, and the presence or absence of high energy tails. It is the primary purpose of this paper to study this process.

It has been shown that only two wave decays are allowed for hot equal temperature plasmas.[2] These are decay into lower hybrid waves and either electrostatic ion cyclotron waves or quasimodes. This latter process is nonlinear Landau damping on electrons (NELD) which dominates the more familiar damping on ions when the ratio of parallel to perpendicular wavenumber is less than the square root of the mass ratio of ions to electrons. We treat the saturation of the latter instability here even though it has a higher threshold in general than the former decay instability because NELD occupies a larger region of phase space if $\omega_o < \sqrt{2}\, \omega_{lh}$ and is affected

less by inhomogeneity than decay into electrostatic ion cyclotron waves.

The pump electric field $E_O(t)$ is taken to be spatially homogeneous, circularly polarized, and perpendicular to the static magnetic field $B_O = B_O e_z$, i.e.,

$$E_O(t) = E_O(e_x \cos\omega_O t - e_y \sin\omega_O t) , \qquad (1)$$

which corresponds to a fast (whistler) wave pump. Consideration has been restricted to weak pumps so that the maximum parametric growth rate γ_p^m is proportional to the minimum lower hybrid wave damping decrement γ_e^c but is much less than the beat wave damping, i.e.,

$$1/8 \ (\pi/2e)^{1/2} \ (u_O^2/c_s^2) \ (\omega_{1h}^2/\omega_O^2) \ (\omega_k/\gamma_e^c) \gtrsim 1 , \qquad (2)$$

where $u_O = E_O/B_O$, $C_s = (T_e/m_i)^{1/2}$, $\omega_{1h} = \omega_{pi}/(1 + \omega_{pe}^2/\omega_{ce}^2)^{1/2}$, and $\omega_k = \omega_{1h}(1 + m_i k_{\parallel}^2/m_e k^2 + 3k^2 v_i^2/\omega_{1h}^2)^{1/2}$ is the lower hybrid wave frequency including ion but not electron pressure. Here, $k_{\parallel}$ is the component of the wavevector parallel to B_O . The minimum damping decrement is determined by electron ion collisions, i.e., $\gamma_e^c = \nu_e$, where ν_e is the collisional damping of lower hybrid waves.

The weak turbulence wave kinetic equation governing the evolution of the parametrically excited lower hybrid waves has been derived elsewhere[3,4] and is given below.

$$(1/2)(\partial/\partial t)N(\underset{\sim}{k},\theta) = (\gamma^o_{\underset{\sim}{k}} - \gamma^D_{\underset{\sim}{k}} - \nu_e + \gamma^{NL}_{\underset{\sim}{k}})N(\underset{\sim}{k},\theta) + S(\underset{\sim}{k},\theta) \quad , \quad (3)$$

where

$$N(\underset{\sim}{k},\theta) = 2\pi\varepsilon_o(1 + \omega^2_{pe}/\omega^2_{ce})|\underset{\sim}{E}(\underset{\sim}{k},\theta)|^2/\omega_{\underset{\sim}{k}} \quad , \quad (4)$$

$$\gamma^o_{\underset{\sim}{k}} = \frac{1}{8}\pi^{1/2}\omega_{\underset{\sim}{k}}\left(\frac{\omega_{lh}\underset{\sim}{E}_o}{\omega_o B_o C_s}\right)^2 \beta(\underset{\sim}{k},\underset{\sim}{k}_o) \quad , \quad (5)$$

$$\gamma^{NL}_{\underset{\sim}{k}} = \frac{1}{4}\pi^{1/2}\left(\frac{\omega_{lh}}{B_o C_s}\right)^2\left(1 + \frac{\omega^2_{pe}}{\omega^2_{ce}}\right)^{-1}\int_o^\infty dk' \; k'^2$$

$$\int_{-\pi/2}^{\pi/2} d\theta' \; \sin\theta' \; \beta(\underset{\sim}{k},\underset{\sim}{k}')N(\underset{\sim}{k}',\theta') \quad , \quad (6)$$

$$\beta(\underset{\sim}{k},\underset{\sim}{k}') = \frac{\omega_{\underset{\sim}{k}'} - \omega_{\underset{\sim}{k}}}{(2)^{1/2}|k_\parallel - k'_\parallel|v_e}\exp\left(-\frac{(\omega_{\underset{\sim}{k}} - \omega_{\underset{\sim}{k}'})^2}{2(k_\parallel - k'_\parallel)^2 v_e^2}\right) \quad . \quad (7)$$

In the above equations, $k_\parallel = k\cos\theta$, $\omega_{\underset{\sim}{k}}$ is the lower hybrid wave frequency, $\gamma^o_{\underset{\sim}{k}}$ is the linear parametric growth rate for the dipole ($\underset{\sim}{k}_o = 0$) pump given in Eq. (1), $\gamma^d_{\underset{\sim}{k}}$ is the sum of ion and electron Landau damping, and $\gamma^{nl}_{\underset{\sim}{k}}$ is the nonlinear damping (growth) due to NELD. We take $\underset{\sim}{k}_o = 0$ and $\omega_{\underset{\sim}{ko}} = \omega_o$. We assumed that $\omega_{ci} \lesssim |\omega_{\underset{\sim}{k}} - \omega_{\underset{\sim}{k}'}| < |\underset{\sim}{k} - \underset{\sim}{k}'|C_s$ in obtaining Eq. (7).

Equation (3) has been averaged over the azimuthal angle ψ in $\underset{\sim}{k}$ space. The number density of lower hybrid waves or plasmon number N_k is assumed to be independent of ψ and symmetric about the $\theta = \pi/2$ plane. The last term in Eq. (3), S_k, is the spontaneous emission which we set equal to a small constant. In some cases the value of S_k has been varied by an order of magnitude without altering the results essentially. It is easy to show from Eq. (7) that the wave energy density $\varepsilon_k = |\omega_k| N_k$ flows in time from high to low frequency modes. We also see from Eq. (7) that $\beta(\underset{\sim}{k}, \underset{\sim}{k}')$ is antisymmetric with respect to the interchange of $\underset{\sim}{k}$ and $\underset{\sim}{k}'$. As a consequence, the total number of plasmons and the total wave energy are conserved by NELD. Moreover, Kruer and Valeo[5] have shown for an equation equivalent to Eq. (3) that a steady state solution of Eq. (3) is accessible and unique and for which the supply of energy by the pump is balanced by the loss of energy to particles.

Equation (3) is solved numerically by dividing wavenumber space into a discrete number of modes with a number of modes represented in the characteristic widths of the matrix element $\Delta k/k \sim \eta/(kC_s/\omega_{LH})$ and $\Delta\eta \sim kC_s/\omega_{LH}$ where $\eta = (m_i/m_e)^{1/2} \cos\theta$. The integral in Eq. (3) is replaced by a sum such that the volume element in $\underset{\sim}{k}$ space is a constant. Wavenumber space is divided into a rectangular grid such that $\eta < 1$ and $k_{min} < k < k_{max}$. The maximum value of k, k_{max}, is chosen such that there are no growing modes with this wavenumber. The

minimum value of k is determined by k_{max} and the k division. A

total of 300 modes with 20 divisions of η and 15 divisions of

k were used to integrate Eq. (3). The time step was chosen to

be small compared to the growth time of the instability.

The evolution of the normalized field energy density

$W_k = |E_k|^2/E_o^2$ was followed in time for values of pump intensity

corresponding to 1.3 , 1.5 , 1.7 , 2.0 , and 3.0 times threshold

intensity. In all cases we set $T_e = T_i$, $\omega_{pe} = \omega_{ce}$, and

$\omega_o = \omega_{pi} = \sqrt{2}\,\omega_{LH}$. Linearly unstable modes exponentiate in

time and transfer energy to lower frequency modes. The total

decay wave energy saturates when the number of plasmons created

by the pump equals the number of plasmons dissipated by Landau

and collisional damping. Since the frequency of the lower

hybrid mode and the Landau damping are increasing functions of

k , the nonlinear saturated state will be characterized by a

concentration of energy at small k values. Because the energy

density at small k is dissipated primarily by collisions, the

main body electrons are heated.

Figure 1 shows the time evolution of the total decay wave

energy $W = \sum_k |E_k|^2/2E_o^2$ for a pump intensity 1.3 times threshold.

There is an initial overshoot followed by an oscillatory approach

to a steady state value. This is a general feature of the time

evolution of the spectrum. In Fig. 2, the two dimensional

saturated energy density W_k is shown for a pump intensity 1.3

times threshold. The energy is concentrated in the linearly

unstable region where the decay wave energy is barely large
enough to exceed the threshold for growth of small k and small η
modes, i.e., lower frequency modes. As the pump intensity is
raised, additional modes with lower frequency are destabilized by
the linearly unstable modes. At pump intensity 2.0 times thresh-
old, shown in Fig. 3, the decay wave energy density is enhanced
even near the lowest frequency mode in the system. The energy
density for a pump 3.0 times threshold is shown in Fig. 4. The
intensity of the lowest frequency mode in the system has been
enhanced several orders of magnitude to a level comparable with
the linearly unstable modes. This mode cannot decay and thus
condensation of energy appears likely in the lowest frequency
mode for a pump only a few times threshold. Another feature of
higher pump intensities is the concentration of plasmons in
small wavenumbers k for all values of η because the frequency
is a decreasing function of k as well as η .

Once the energy in the lowest frequency mode is comparable
to the energy in higher frequency modes, this saturation mechan-
ism breaks down. We can estimate the pump intensity for which
this occurs as follows. Summing over all modes and using the
antisymmetric properties of the matrix element $\beta(k, k')$ we find
from Eq. (3)

$$\frac{\partial}{\partial t} N_T = \sum_k 2\gamma_o(k, \Phi) N(k, \Phi) - \nu_e N_T ,$$ (8)

where N_T is the total number of plasmons. We define an average growth rate $\bar{\gamma}_o$ such that $\bar{\gamma}_o N_1 \equiv \sum_k 2\gamma_o(k,\Phi)N(k,\Phi)$ where N_1 is the total number of linearly unstable modes. An upper limit on the ratio of N_T to N_1 can be estimated by the ratio of the total phase space, A_T, occupied by modes with enhanced intensity to the phase space A, occupied by N_1. Defining a steady state by setting $\partial N_T/\partial t = 0$ and using Eq. (8), we find that $\bar{\gamma}_o/\nu_e < N_T/N_1 < A_T/A_1$. The ratio A_T/A_1 is ~ 2-3 and thus only very weak pumps saturate according to weak turbulence theory. In Fig. 5, the total field energy at saturation W normalized to the pump amplitude squared is plotted versus E_o^2/E_{th}^2 where E_{th} is the threshold pump amplitude. The results indicate an approximately linear proportionality, i.e.,

$W = \sum_k |E_k|^2/2E_o^2 \simeq 0.2\, E_o^2/E_{th}^2$. The effective collision frequency is defined by

$$\frac{1}{2}\nu_f\epsilon_o <|E_o(t)|^2> = \nu_f\,\epsilon_o\,E_o^2$$

$$\simeq \nu_e \int d^3k\,\omega_k N_k$$

$$= \left(1 + \frac{\omega_{pe}^2}{\omega_{ce}^2}\right)\nu_e\,(\epsilon_o E_o^2)\sum_k |E_k|^2/2E_o^2.$$

Hence, we find that the effective collision frequency is proportional to E_o^2, i.e.,

$$\nu_f = 0.8 \ \nu_e E_o^2 / E_{th}^2 \ . \tag{9}$$

The linear dependence of ν_f on the pump power is characteristic of weak turbulence saturation mechanisms. Following arguments similar to Galeev and Sagdeev,[6] we can estimate the effective collision frequency by setting ν_f equal to the maximum parametric growth rate γ_k^o . That is, we assume that the instability will stop growing when the damping decrement on the fastest growing mode is raised to the level such that the pump is at threshold again. This estimate agrees with the numerical scaling law given in Eq. (9). Within the limits of weak turbulence theory, the predicted heating rate is not enhanced much above classical collisional absorption because $(E_o/E_{th})^2 \lesssim 3$. Moreover, at saturation, the decay wave energy is concentrated in long wavelength waves which are damped by electron-ion collisions. This leads to main body electron heating.

ACKNOWLEDGMENTS

REFERENCES

[1] W. M. Hooke and S. Bernabei, Phys. Rev. Lett. $\underline{29}$, 1218 (1972); T. K. Chu, S. Bernabei, and R. W. Motley, Phys. Rev. Lett. $\underline{31}$, 211 (1973); R. P. H. Chang and M. Porkolab, Phys. Rev. Lett. $\underline{32}$, 1227 (1974).

[2] R. L. Berger and F. W. Perkins, Princeton University Report No. PPL-MATT-1975 (to be published in Phys. Fluids).

[3] A. Hasegawa and L. Chen, (to be published in Phys. Fluids Oct. 1975).

[4] A. Rogister, Phys. Rev. Lett. $\underline{34}$, 80 (1975).

[5] W. L. Kruer and E. J. Valeo, Phys. Fluids $\underline{16}$, 675 (1973).

[6] A. A. Galeev and R. Z. Sagdeev, Nuclear Fusion $\underline{13}$, 603 (1973).
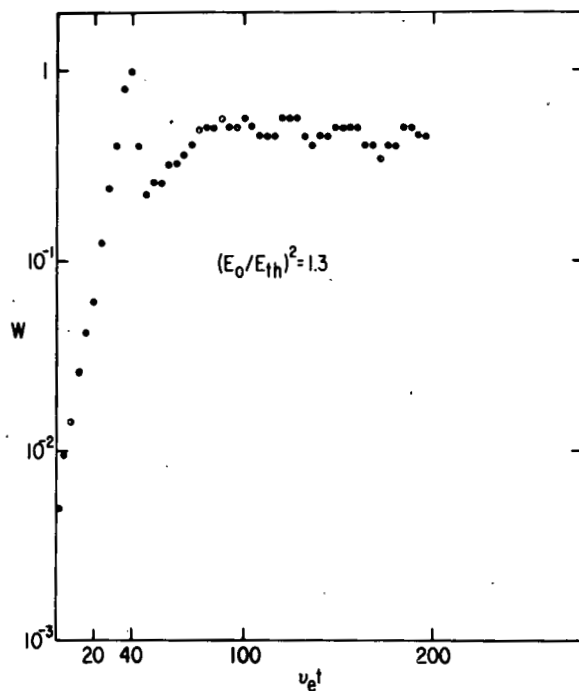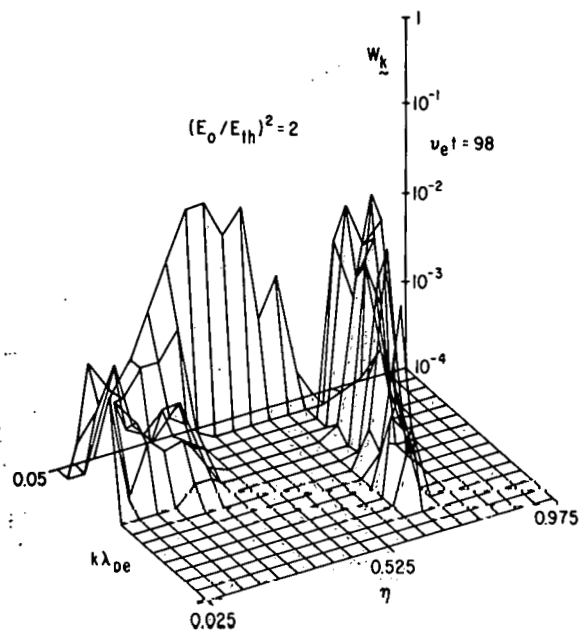
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Fig. 1. Plot of the total field energy in decay modes normalized to the pump amplitude squared versus time in units of inverse electron-ion collision frequency for a pump power 1.3 times threshold power.
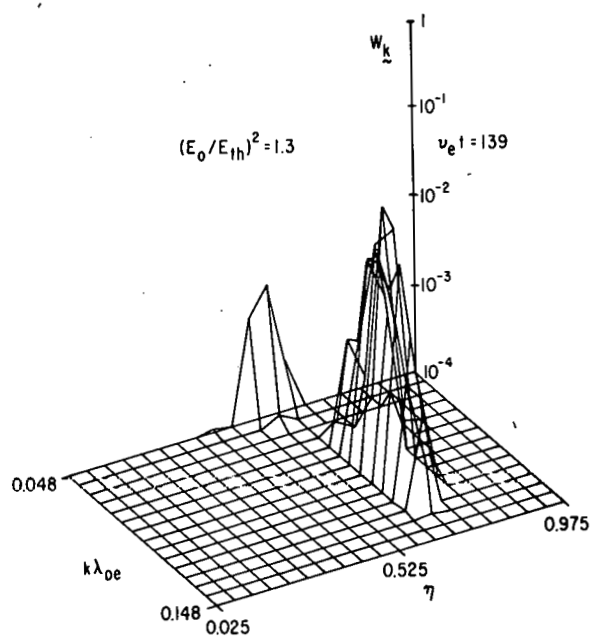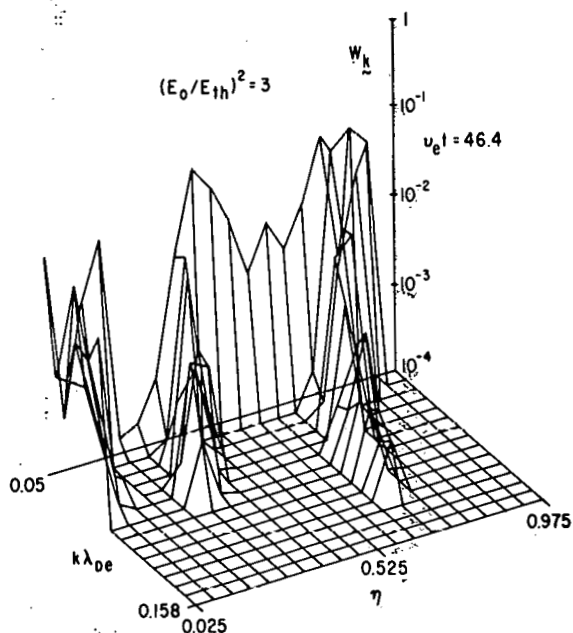

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Fig. 2. The normalized field energy density $W(k,\theta)$ at time $\nu_{ei}t = 139$ after saturation (see Fig. 1) for a pump power 1.3 times threshold plotted as a function of $\eta = (m_i/m_e)^{1/2} k_\parallel/k$ and $k\lambda_{De}$.
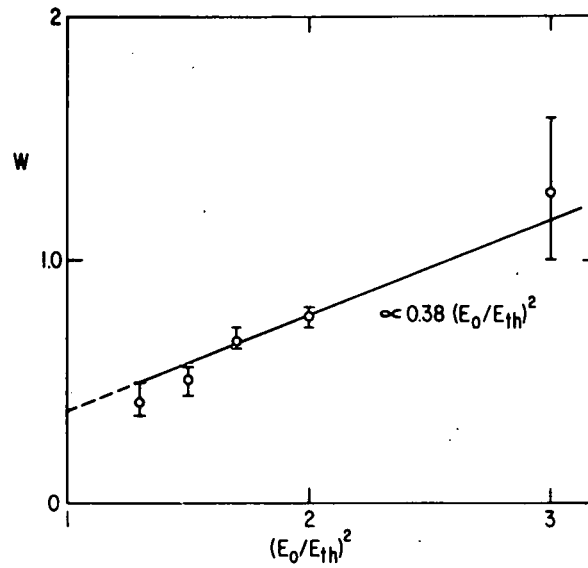
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Fig. 3. The saturated field energy density $W(k,\theta)$ at time $\nu_{ei}t = 98$ for a pump power 2.0 times threshold plotted as a function of $\eta = (m_i/m_e)^{1/2}k_{||}/k$ and $k\lambda_{De}$.

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Fig. 4. The saturated field energy density W_k at time $\nu_{ei}t = 46.4$ for a pump power 3 times threshold plotted as a function of $\eta = (m_i/m_e)^{1/2}k_{||}/k$ and $k\lambda_{De}$.

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Fig. 5. The total field energy $W = \sum_k W_k$ normalized by the pump amplitude squared plotted versus the pump power normalized to threshold power. (Note: No cases were run for $(E_0/E_{th})^2 < 1.3$ and the initial energy lies far below the intersection of the dotted line with the ordinate.)