APAE Memo 126

APPR-1 Burnout Calculat ons

Contract No. AT (30-3)-278

Issued April 10, 1958

Alco Products, Inc.
Post Office Box 414
Schenectady, N. Y.

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Abstract

A general non-uniform burnup program has been developed to determine the lifetime of the APPR-1. The calculation is preformed using two one dimensional multiregion burnout calculations. The approach to the problem, equations and derivation of burnout equations are presented. The results have been plotted and compared with the rod bank position as measured at Fort Belvoir. On the basis of these calculations the expected total energy release of the APPR-1 is 13 MWYRS.

1. Introduction

A general non-uniform burnup program has been developed to determine the expected lifetime of the APPR-1, both Core I and Core II. The calculation is performed using two one dimensional calculations; the core is divided into radial and axial regions then burned out in each direction for small time intervals. The radial and axial burnups are then connected by the total energy release.

The non uniform calculation involves the following IBM-650 programs:

Muft III (1)* prepares fast few group constants by calculating the multigroup Fourier transform of the slowing down distribution. This program takes core number densities as input, finds the fast flux, and appropriately weights the fast constants, $\nu\Sigma^f$, Σ^a, P, γ and D_f.

P-3 (2) solves the one velocity transport equation to the third spherical harmonic approximation for plate type elements and gives the flux distribution both perpendicular and parallel to the plates. The output includes the following core properties: Σ^a_{th}, $\nu\Sigma^f_{th}$, D_{th}, L^2 and a thermal shielding factor g'. (The actual output is $\frac{\bar{\Phi}_{plate}}{\bar{\Phi}_{active}}$ and $\frac{\bar{\Phi}_{active}}{\bar{\Phi}_{core}}$, normalized to $\bar{\Phi}_{core} = 1$. g' is then the product of the two outputs). Both the Muft III and P-3 codes will be discussed in more detail in the ZPE II analysis to be published by B. J. Byrne.

* Numbers in parentheses refer to references listed at the end of the report.

Valprod and Windowshade (3), (4) are two group flux distribut on programs which treat respectively the rad al and the axial problem. The general two group equations, including fast fission and absorption are:

$$-D_f \nabla^2 \Phi_f(\vec{r}) + [\Sigma_r - (1-P) K_f \Sigma_r] \Phi_f(\vec{r}) = K_{th} \Sigma^a_{th} \Phi_{th}(\vec{r})$$

$$-D_{th} \nabla^2 \Phi_{th}(\vec{r}) + \Sigma^a_{th} \Phi_{th}(\vec{r}) = P \Sigma_r \Phi_f(\vec{r})$$

The output of these codes includes thermal and fast fluxes and K_{eff}. A uniform poison can be put into the Windowshade to represent the rod bank; the program will adjust the bank to criticality.

A new code, Nub I, has been written to calculate the burnout distribution and to prepare input for Valprod and Windowshade. The equat ons programmed are derived in the back of this report. In Nub I the curves for the core constants have been fitted as functions of burnout in the following manner:

First order polynomial - D_{th}, D_f, ν, P, thermal shielding factor g', and percent thermal fissions β.

Second order polynomial - $\nu\Sigma^f$, Σ^a_{th}.

Two Second order polynomials - K_f (for regions of different concavity).

Nub I takes as input the average thermal flux and the ratio of the fast to thermal flux per region. A program is being written to normalize and average the Valprod and Windowshade fluxes.

There are two general approaches to the non-uniform calculation: (1) one step in which the initial flux distribution is used throughout core life and (2) multistep in which a new distribution is found for ea h time step. WAPD-MR-50 and APAE 11 both ndicate that the error in the one step approach relative to the multistep is small. However, it is suspected that the error may

be a function of the initial position of the control rods, i. e. which parts of the core burnout first and most rapidly. This point will be discussed in more detail later.

The two approaches will be discussed in detail and all pertinent equations presented. The report also includes results of a calculation made on the APPR-1 and all core constants used.

2. Multistep

I. Run Muft III's to prepare fast constants (γ, p, D_F, K_f) as functions of burnout. The input to the Muft III should include the fuel burnout, depletion of burnable poisons, and equivalent boron accumulation due to fission products. Plot curves of the fast constants versus burnup fraction.

II. Run the P-3 program to prepare thermal constants (D_{th}, $\nu\Sigma^F$, Σ^a_{th}, and thermal self shielding factor g). Again the input should include fuel burnout, burnable poisons, and fission products, but not xenon which will be treated separately. Plot the thermal constants against burnup.

III. Using the above fast and thermal constants, including the xenon cross-section, calculate K_{eff} for the equivalent bare reactor for various burnout points by the modified two group formulation. Calculate and plot the percent thermal fissions as a function of burnup.

IV. Run a radial Valprod with the initial fuel, poison and xenon concentrations to get plots of the thermal and fast fluxes. Divide the core into radial regions and find the average thermal flux and average fast flux in the core and in each region. From the power output of the reactor, determine the real

average thermal flux in neuts/cm^2-sec and get a normalizing factor for the Valprod results.

V. Burn out each region of the core for a specified time interval. Calculate Σ^a_{xe} for each region; get thermal and fast properties for each region from graphs plotted in steps I and II. Run Valprod for second time interval with core divided into radial regions, each with different properties.

VI. Repeat steps IV and V until the core has been burned out.

VII. Find the energy release for each time interval by calculating the total number of U-235 atoms fissioned and multiplying by the energy release per fission. Note that the energy release is not the power times time because in reality the fluxes do not remain constant with time as has been assumed.

VIII. Plot K_{eff} for uniform burnout (modified two group model) and for radial non-uniform burnout (Valprod output includes K_{eff}) against energy release. The difference in the two curves is ΔK for radial non-uniform burn-up.

IX. Run a Windowshade for the hot dirty zero burnout case. For this Windowshade calculation, it is necessary to know a value of Σ^p for the bank, i. e. the results of some bank worth measurements. The Windowshade program should not iterate to $K_{eff} = 1.000$ but to $K_{eff} = 1.000 \pm \Delta K$ where ΔK includes the correction for rad al non-uniform burnout and any other corrections thought necessary.

X. Obtain axial fast and thermal flux distributions, divide core into axial regions, normalize and proceed as in the radial case. The main difference from the radial calculation is that the Windowshade will be iterating to a K which

includes the radial non-uniform correction and the final result will be a plot of control rod bank position versus energy release. If a value of $\Delta\rho$ for axial non-uniform burnout is desired, the Windowshade program may be run with rods out.

3. One Step Calculation

In the one step approximation, the flux distribution is assumed constant throughout lifetime. The average radial and axial fluxes in each region obtained from the initial Valprod and Windowshade are used for all burnout calculations. However, a new real thermal flux must be calculated at the beginning of each time interval since $\bar{\phi}^{th}_{core}$ is proportional to β and $1/\Sigma^f$, both of which change with burnout.

4. Results

A non-uniform one shot calculation has been made for the APPR-1. From ZPE experiments, the reactivity of the cold clean core with rods out and with rods in can be found. (Numbers are tabulated in Appendix III.) Also available from Fort Belvoir are rod bank positions for the cold clean and hot equilibrium xenon cases. The value of Σ^p_{Cold} found from the ZPE experiments inserted in the Windowshade calculation resulted in an absorber position 0.91 inches above the measured bank position (the bank position measured at Ft. Belvoir is to the top of the control rod fuel element). In the reactor there is a 1.3 inch gap between the bottom of the absorber and the top of the control rod fuel, where the bank position is measured (See Fig. 1). There is some uncertainty as to where the effective boundary between the absorber and the fuel occurs.

Therefore, the 0.91 inches was used as a constant correction throughout the calculation to find bank position.

Radial Valprods where run hot and cold with a central rod (water, absorber, core, and reflector regions) to determine the cold to hot change in Σ^{ρ} because no hot reactivity measurements were available. The result was that at temperature, the rod worth is 13 per cent* greater than at 68°F The values of Σ^{ρ} used were .08824 at 68°F and .07756 at 440°F.

Another correction, called the model correction, was found by running the hot Windowshade at points near the expected bank position. At this point, a Δ K was found that resulted in a calculation which matched the experimental results at beginning of life. This correction was held constant with burnout.

The results of a one shot calculation on the APPR-1 are shown in Fig. 2. The bank positions as measured at Fort Belvoir are also shown and extrapolated to total bank removal. The difference in the two curves may be due to one shot calculation, i.e. in this case the bank was initially inserted about 65 per cent of the way into the core. Thus the initial flux distribution was peaked in the bottom sections of the core and this part of the core was burned out more rapidly than the center and top. A multistep calculation should burn out the center, which is worth more than the edges, quicker and therefore the rods should rise faster than in the one step approximation.

In APAE II, F. B. Fairbanks predicted a non-uniform to uniform lifetime ratio of 0.78. The present analysis gives a non-uniform to uniform ratio of 0.85. Again this difference may be due to the fact that the rod bank started above the center of the core and thus burned out the important center region more quickly.

* See Page 9

On the basis of Fig. 2 we feel that the best estimate for the APPR-1 core lifetime is 13 MWYR. Fig. 3 shows the uniform energy release and the effect of radial non-uniformity. Figs. 4 and 5 are the radial and axial non-uniform fuel distributions and Figs. 6 and 7 the flux distributions used in the calculations.

* (Page 7) The rod worth at temperature was found to be 13 per cent greater than at 68° F. if rod worth is defined as,

$$\rho_{rod\ out} - \rho_{rod\ in} = \frac{K_{out} - K_{in}}{K_{out}\ K_{in}}$$

The worth is about 6 per cent greater if the worth is defined as,

$$\rho = \frac{K_{out} - K_{in}}{K_{out}}$$

Equations for Non-uniform Burnout.

The Roman numerals refer to the sections in the description of the progam.

I. Uniform U-235 concentration

$$N^{25} = N_o^{25}(1-B)$$

where B is the fraction burnout.

Uniform B-10 burnout $N^{B-10} = N_o^{B-10}[B]^g + N^{B-10}_{equivalent\ F.P}$

$$g = \frac{\frac{\sigma^{a-B}_{th}}{\sigma^{a}_{th}} B(1+\alpha_{th}) + \frac{\sigma^{B-10}_{f}}{\sigma^{a}_{f}}(1-B)(1+\alpha_f)}{B(1+\alpha_{th}) + (1-B)(1+\alpha_f)}$$

(It will be noted that g is not constant with burnout, however, calculations have shown that assuming it to be constant amounts to less than a tenth of a per cent error in ρ .)

$$N^{B-10}_{equivalent\ FP} = \psi \frac{6.02 \times 10^{23}}{10} \times Initial\ kg.\ U\text{-}235$$

where ψ can be read from a curve in KAPL 1501. For machine calculations, the curve may be fitted by the following equations

$$\psi = 2.971 \times 10^{-3} + 6.166 B - 1.460 \times 10^{2} B + 1.675 \times 10^{3} B^3 - 7.000 \times 10^{3} B^4 \qquad B \leq .10$$

$$\psi = 8.864 \times 10^{-2} + 5.679 \times 10^{-1} B - 1.089 B^2 + 1.664 B^3 - 9.678 \times 10^{-1} B^4 \qquad B > .10$$

II. Uniform xenon cross section

$$\Sigma^{a}_{xe} = \frac{\alpha \sigma^{a}_{xe}(\gamma_1+\gamma_2)\frac{\delta P}{V}}{\lambda_1 + \sigma^{a}_{xe}\frac{\delta \beta P}{V\Sigma^{F}_{o}(1-B)}}$$

where:

α = 1.09 = xenon non-uniform factor

σ_{Xe}^{a} = 1.82×10^{-18} cm^2 = xenon thermal absorption cross section

γ_1 = .059 = Fractional fission yield of iodine

γ_2 = .003 = Fractional fissional yield of xenon

λ_1 = 2.09×10^{-5} sec^{-1} = Decay constant of iodine

P = 10^7 watts = Reactor power

δ = 3.24×10^{10} fissions/watt sec.

V = 1.39×10^5 cm^3 = core volume

B = Fraction burnout

III. Modified two group multiplication factor.

$$K_{eff} = \frac{K_{th} P}{(1+L^2B^2)(1+\tau B^2)} + \frac{K_f(1-P)}{(1+\tau B^2)}$$

Per cent thermal fissions

$$\beta = \frac{K_{th} P}{(1+L^2B^2)(1+\tau B^2) K_{eff}}$$

$$= \frac{K_{th} P}{K_{th} P + K_f(1-P)(1+L^2B^2)}$$

IV. The average flux per region from Valprod can be calculated by

$$\bar{\Phi}'_{region} = \frac{\sum_{r_1}^{r_2} \Phi(r) r \Delta r}{\sum_{r_1}^{r_2} r \Delta r}$$

where Δr can be taken arbitrarily small; r is the radius at the midpoint of Δ r and r_1 and r_2 are the limits of the region. For $\bar{\Phi}'_{core}$, r_1 = 0 and r_2 = R, the outer radius of the core.

The real average thermal flux in the core

$$\bar{\Phi}^{th}_{real} = \frac{\delta \beta P}{V \Sigma^F}$$

the normalizing constant is then

$$K = \frac{\bar{\Phi}^{th}_{core}}{\bar{\Phi}'^{th}_{core}}$$

where the prime indicates the valprod results.

V. The burnout equations are as follows (all derived in Appendix I).

$$N^{25}(t) = N^{25}(t_0)\, e^{-A(t-t_0)}$$

$$A = \sigma_F^a \bar{\Phi}_F + g' \sigma_{th}^a \bar{\Phi}_{th}$$
$$= K \bar{\Phi}'_{th} \left[\sigma_F^a \frac{\bar{\Phi}'_F}{\bar{\Phi}'_{th}} + g' \sigma_{th}^a \right]$$

$$N^{B\text{-}10}(t) = N^{B\text{-}10}(t_0) \left[\frac{N^{25}(t)}{N^{25}(t_0)} \right]^g$$

$$g = \frac{\frac{\sigma^a_{B\text{-}10}}{\sigma^a_{th}} \beta(1+\alpha_{th}) + g' \frac{\sigma^a_{B\text{-}10}}{\sigma^a_F}(1-\beta)(1+\alpha_F)}{\beta(1+\alpha_{th}) + (1-\beta)(1+\alpha_F)}$$

$$X(t) = \frac{(\gamma_1+\gamma_2)\, N^{25}(t) \left[\sigma^F \bar{\Phi}_F + g' \sigma^F_{th} \bar{\Phi}_{th} \right]}{\lambda_1 + \sigma^a_{xe} \bar{\Phi}_{th}}$$

VII. To determine the energy release, in MWYRS, at the end of each time interval,

$$E\,(MWYRS) = \left[\frac{\beta}{1+\alpha_{th}} + \frac{1-\beta}{1+\alpha_F} \right] \frac{N_-^{25}}{10.7 \times 10^{24}}$$

N_-^{25} is the total U-235 atoms burned $N_-^{25} = \sum_i N_{0i}^{25} B_i Vol_i$

where the i summation is over all regions.

IX. To find a value of Σ^P for the rod bank, one needs K_{eff} with the rods out and K_{eff} with the rods in the core.

$$K_{eff} = \frac{K_{th} P}{(1+\tau B^2)(1+L^2 B^2)} + \frac{K_f (1-P)}{1+\tau B^2}$$

$$= K_{th} G + H$$

and $$K'_{eff} = K'_{th} G + H$$

which assumes that the change in Σ^a takes into account any changes that might occur in the other parameters. Thus one can solve for K'_{th} and Σ^p in the following manner.

$$\frac{K_{th}}{K'_{th}} = \frac{\Sigma^a + \Sigma^p}{\Sigma^a}$$

Appendix II

NOMENCLATURE

B		Fraction burnout
B^2	Cm^{-2}	Buckling
D_f	Cm	Fast diffusion coefficient
D_{th}	Cm	Thermal diffusion coefficient
g'		Thermal shielding factor
I (t)	atom/cm^3	Iodine atom concentration
K_f		Fast multiplication factor
K_{th}		Thermal multiplication factor
L	Cm	Diffusion length
N^{25}	atoms/cm^3	U-235 atom concentration
N^{B-10}	atoms/cm^3	Boron 10 atom concentration
p		Resonance escape probability
P	MW	Reactor power
t	Sec	Time
V	Cm^3	Core volume
X (t)	atoms/cm^3	Xenon atom concentration
α		Xenon non uniform factor
α_f		Fast U-235 capture to fission ratio
α_{th}		Thermal U-235 capture to fission ratio
β		Fraction thermal fissions
γ_1		Iodine fraction fission yield

γ_2		Xenon fraction fission yield
λ_1	sec^{-1}	Iodine decay constant
λ_2	sec^{-1}	Xenon decay constant
$\nu\Sigma^F$	neuts/cm	Neutrons per fission times macroscopic fission cross section
ρ		Percent reactivity $\frac{K_{eff}-1}{K_{eff}}$
σ_{th}^{a-B}	cm^2	Boron-10 thermal absorption cross section
σ_F^{a-B}	cm^2	Boron-10 fast absorption cross section
σ_{th}^{a}	cm^2	U-235 thermal absorption cross section
σ_F^{a}	cm^2	U-235 fast absoprtion cross section
σ_{th}^{f}	cm^2	U-235 thermal fission cross section
σ_F^{f}	cm^2	U-235 fast fission cross section
σ_{xe}^{a}	cm^2	Xenon thermal absorption cross section
Σ^a	cm^{-1}	Macroscopic absorption cross section
Σ_{xe}^{a}	cm^{-1}	Macroscopic xenon absorption cross section
Σ^p	cm^{-1}	Rod bank equivalent poison cross section
Σ_a^{sub}	cm^{-1}	Cross section representing effect of substituting 7 control rod elements for 7 fixed elements
τ	cm^2	Fermi age
$\bar{\Phi}_{th}$	neuts/cm^2-sec	Thermal flux
$\bar{\Phi}_f$	neuts/cm^2-sec	Fast flux
$\bar{\bar{\Phi}}_{real}^{th}$	neuts/cm^2-sec	Real average thermal flux in core
$\bar{\bar{\Phi}}_{region}$		Average region flux from Valprod or Windowshade
$\bar{\bar{\Phi}}_{core}$		Average core flux from Valprod or Windowshade

Appendix III

APPR-1 Core Constants

Number Densities for Muft III

Zero Burnout - Fixed Elements

	440°F atoms/cm^3	68°F atoms/cm^3
H	$.044532 \times 10^{24}$	$.053226 \times 10^{24}$
SS	$.014653 \times 10^{24}$	$.014653 \times 10^{24}$
O	$.023211 \times 10^{24}$	$.027477 \times 10^{24}$
C	$.000047 \times 10^{24}$	$.000047 \times 10^{24}$
B*	$.000045946 \times 10^{24}$	$.000045946 \times 10^{24}$
U-235	$.0004242 \times 10^{24}$	$.0004242 \times 10^{24}$

* This is natural boron and not B-10 i.e. $N^{B-10} = .188N^B$

P-3 Input

Fixed elements - Zero Burnout

Dimensions: 1 = 0.038100 cm

m = 0.207256 cm

p = 3.175000 cm

q = 3.730625 cm

Region cross sections: 440°F

	Plate	Channel	Dead
Σ^a	1.296257	0.011052	0.057069
Σ^s	0.762252	1.728751	1.456698
Σ^{tr}	2.048792	1.258399	1.185148
$\upsilon\Sigma^f$	2.171908	0.000000	0.000000

Region Cross Sections: 68°F

	Plate	Channel	Dead
Σ^a	1.713152	0.017012	0.075372
Σ^s	0.762252	2.424332	1.926505
Σ^{tr}	2.465638	1.940469	1.660122
$\nu\Sigma^f$	2.888019	0.0000000	0.000000

Relative Sources:

S_{active} = 0.816170

S_{dead} = 0.675409

Unshielded Microscopic Cross Sections (barns)

	68°F (0.0331 ev)*	440°F (0.0549 ev)*
σ_{th}^{a-B}	3088.48	2398.13
σ_F^{a-B}	83.62	84.68
σ_{tr}^{a}	460.41	385.54
σ_F^{a}	19.29	18.98
σ_{tr}^{f}	390.18	326.73
σ_f^{f}	13.59	13.34
σ_{xe}^{a}	2.26×10^6	1.82×10^6

$1+\alpha_{th}$ = 1.18

$1+\alpha_f$ = 1.42

*These energies correspond to a hardened neutron temperature.

Core Constants From Muft III
and P-3

Fixed Elements

	440°F 0% Burnout	440°F 20% Burnout	440°F 40% Burnout	68°F 0% Burnou
$\nu\Sigma^F$	0.317149	0.258304	0.196621	0.405789
$\Sigma^{a\ *}_{th\text{-}core}$	0.207138	0.168625	0.135681	0.266863
P	0.729024	0.769590	0.806508	0.747790
η	42.9631	43.5384	44.1006	32.3788
K_f	1.280967	1.246522	1.147493	1.286817
Σ^o_{xe}	0.006272	0.005525	0.004610	-------
Σ^o_{sub}	0.006131	0.004905	0.003679	0.007846
D_f	1.491955	1.495531	1.499421	1.321185
D_{th}	0.245022	0.252206	0.258685	0.170538
g'	0.933348	0.950216	0.964403	0.898091
K_{th}(fixed elements)	1.486101	1.488526	1.410885	1.520589
K_{th} (Mixed elements)	1.444681	1.442595	1.365708	1.477159

* Does not include Σ^o_{xe} or Σ^o_{sub}

Reflector Properties (Pure Water)

	440°F	68°F
D_{th}	0.264886	0.172304
D_f	1.893739	1.596015
Σ^{o}_{th}	0.011052	0.016960
γ	47.5034	33.6945
P	0.988482	0.987116

Buckling (cm^{-2})

	440°F	68°F
B^2 radial	.004465	.004895
B^2 axial	.002129	.002259
B^2 total	.006594	.007154

Thermal Flux radial max/avg = 1.477

Thermal Flux axial max/avg = 1.894

Maximum Fuel Burnup at 13 MWYR $\frac{N_o - N^{25}(t)}{N_o} = .654$

Initial Reactivity (%)

	45 Fixed Elements	38 Fixed Elements 7 Control Rod Elements
68°F		
Calculated	15.47	13.54
Measured	17.22	15.35
Measured		-3.87 (rods in)
440°F		
Calculated (clean)	11.82	9.81
Calculated (Eq Xe)	9.80	7.85
Measured (Eq Xe)		8.1*

* Based on comparison of window shade calculation with bank position measured at Fort Belvoir.

Derivation of Burnup Equations

V. The fuel burnup equation is

$$\frac{dN^{25}}{dt} = N^{25}(t)\left[\sigma_f^a \Phi_f + g'\sigma_{th}^a \Phi_{th}\right]$$

Let

$$A = \left[\sigma_F^a \Phi_f + g'\sigma_{th}^a \Phi_{th}\right] = K\Phi_{th}'\left[\sigma_c^a \frac{\Phi_f'}{\Phi_{th}'} + g'\sigma_{th}^a\right]$$

and assume A to be constant with time (this will not introduce serious error if the time intervals are taken sufficiently small, i. e. as small as possible with regard to the amount of time available to run many Valprods and Windowshades). The solution to the burnout equation is then

$$N^{25}(t) = N^{25}(t_0)\, e^{-A(t-t_0)}$$

The boron burnup equation is

$$\frac{dN^{B\text{-}10}}{dt} = N^{B\text{-}10}(t)\left[\sigma_f^{a\text{-}B\text{-}10}\Phi_f + g'\sigma_{th}^{a\,B\text{-}10}\Phi_{th}\right]$$

which can be solved in the same manner as the uranium burnout, i. e.

$$N^{B\text{-}10}(t) = N^{B\text{-}10}(t_0)\, e^{-A'(t-t_0)}$$

where

$$A' = K\Phi_{th}'\left[\sigma_{th}^{a\,B\text{-}10}\frac{\Phi_f'}{\Phi_{th}'} + g'\sigma_{th}^{a\,B\text{-}10}\right]$$

The boron burnout can also be coupled directly to the uranium burnout in the following manner.

$$\frac{dN^{B\text{-}10}}{dt} = N^{B\text{-}10}(t)\left[\sigma_F^{a\,B\text{-}10}\Phi_f + g'\sigma_{th}^{a\,B\text{-}10}\Phi_{th}\right]$$

$$\Phi_{th} = \frac{\delta\beta P}{V\Sigma_{th}^F} = \frac{\delta\beta P(1+\alpha_{th})}{VN^{25}(t)\sigma_{th}^a} \quad ; \quad \Phi_f = \frac{\delta(1-\beta)P}{V\Sigma_f^F} = \frac{\delta(1-\beta)P(1+\alpha_f)}{VN^{25}(t)\sigma_f^a}$$

$$\frac{dN^{B\text{-}10}}{N^{B\text{-}10}(t)} = \left[\frac{\sigma_{+h}^{a\,B}\beta(1+\alpha_{+h})}{\sigma_{+h}^{a}} + \frac{g'\sigma_f^a(1-\beta)(1+\alpha_f)}{\sigma_f^a} \right] \frac{\delta P\, dt}{V N^{25}(t)}$$

but $N^{25}(t) = N^{25}(t_o)\, e^{-At}$ $\qquad t_o = 0$

$$\cong N^{25}(t_o)\left[1 - At\right]$$

$$\cong N^{25}(t_o)\left[1 - (\sigma_f^a \Phi_f + g'\sigma_{+h}^a \Phi_{+h})t\right]$$

$$\cong N^{25}(t_o) - \frac{\delta P N^{25}(t_o)}{V N^{25}(t)}\left[(1-\beta)(1+\alpha_f) + g'\beta(1+\alpha_{+h})\right]t$$

Approximate $N^{25}(t_o) \cong N^{25}(t)$ good for small time intervals

$$N^{25}(t) = N^{25}(t_o) - \frac{\delta P}{V}\left[(1-\beta)(1+\alpha_f) + g'\beta(1+\alpha_{+h})\right]t$$

$$\frac{dN^{B\text{-}10}}{N^{B\text{-}10}(t)} = \frac{\left[\frac{\sigma_{+h}^{a\,B}\beta(1+\alpha_{+h})}{\sigma_{+h}^{a}} + \frac{g'\sigma_f^a(1-\beta)(1+\alpha_f)}{\sigma_f^a} \right] \frac{\delta P}{V} dt}{N^{25}(t_o) - \frac{\delta P}{V}\left[(1-\beta)(1+\alpha_f) + g'\beta(1+\alpha_{+h})\right]t}$$

which has the solution

$$\frac{N^{B\text{-}10}(t)}{N^{B\text{-}10}(t_o)} = \left[\frac{N^{25}(t)}{N^{25}(t_o)}\right]^g$$

where $$g = \frac{\frac{\sigma_{+h}^{a\,B}}{\sigma_{+h}^{a}}\beta(1+\alpha_{+h}) + g'\frac{\sigma_f^{a\,B}}{\sigma_f^a}(1-\beta)(1+\alpha_f)}{\beta(1+\alpha_{+h}) + (1-\beta)(1+\alpha_f)}$$

This accounts for the burnout of burnable boron poison built into the core.

To find the xenon cross-section, one needs the solution of the following equations

$$\frac{dI(t)}{dt} = -I(t)\lambda_1 + \gamma_1 N^{25}(t)\left[\sigma_f^f \Phi_f + g'\sigma_{+h}^f \Phi_{+h}\right]$$

and $$\frac{dX(t)}{dt} = -X(t)\lambda_2 - \sigma_{xe}^a \Phi_{+h} X(t) + \gamma_2 N^{25}(t)\left[\sigma_f^f \Phi_f + g'\sigma_{+h}^f \Phi_{+h}\right] + \lambda_1 I(t)$$

The solution of the xenon equation is

$$X(t) = \frac{M N^{25}(t_0)}{C-A}\left[\gamma_2 + \frac{\lambda_1 \gamma_2}{\lambda_1 - A}\right]\left[e^{-A(t-t_0)} - e^{-C(t-t_0)}\right] + \frac{\lambda_1 I(t_0)}{C-\lambda_1}\left[e^{-\lambda_1(t-t_0)} - e^{-C(t-t_0)}\right] + X(t_0)e^{-C(t-t_0)}$$

where

$$M = \bar{\Phi}_{th}\left[\frac{\sigma_f^0}{1+\alpha_f}\frac{\bar{\Phi}_f}{\bar{\Phi}_{th}} + g'\frac{\sigma_{th}^a}{1+\alpha_{th}}\right]$$

$$C = \lambda_2 + \sigma_{Xe}^a \bar{\Phi}_{th}$$

A sample calculation for the APPR-1 core and for a time interval of one-fourth year ($.7875 \times 10^7$ sec) resulted in the following magnitudes and simplifications.

$A \cong 10^{-9}$ $\quad A\Delta t \cong 10^{-2}$

$C \cong 10^{-5}$ $\quad C\Delta t \cong 10^{2}$

$\lambda_1 \cong 10^{-5}$ $\quad \lambda_1 \Delta t \cong 10^{2}$

$M \cong 10^{-9}$

$\therefore$ $C - A \cong C$

$\lambda_1 - A \cong \lambda_1$ $\quad$ So $\gamma_2 + \frac{\lambda_1 \gamma_1}{\lambda_1 - A} \cong \gamma_1 + \gamma_2$

$e^{-C(t-t_0)} \cong e^{-10^2} \cong 0$

$e^{-\lambda_1(t-t_0)} \cong e^{-10^2} \cong 0$

So $$X(t) = \frac{(\gamma_1 + \gamma_2) M N^{25}(t_0) e^{-A(t-t_0)}}{C}$$

$$= \frac{(\gamma_1 + \gamma_2) N^{25}(t)\left[\sigma^f \bar{\Phi}_f + g' \sigma_{th}^f \bar{\Phi}_{th}\right]}{\lambda_1 + \sigma_{Xe}^a \bar{\Phi}_{th}}$$

5. References

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