ORO-3992-230

CONSEQUENCES OF UNITARITY AND ANALYTICITY FOR WEAK INTERACTIONS

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*Work supported in part by the Energy Research and Development Administration under Contract AT(40-1)-3992.

†Work supported in part by the National Science Foundation.

**Permanent Address: Virginia Polytechnic Institute and State University.

Submitted to Conference on Gauge Theories and Modern Field Theory, Northeastern University.

ABSTRACT

We consider the implications of the fact that weak two body scattering amplitudes, when calculated to all orders in any renormalizable model, should exhibit unitary, analytic, "Mandelstam" behaviour. We discuss, from this point of view, the existence of neutral currents, the suppression of $\nu_\mu e$ elastic scattering, and the possibility of lepton hadron resonances. We review briefly the predictions for lepton-lepton scattering of two models in which Mandelstam behaviour is explicitly realized.

Conf-750955-1

BNL 20433

I. Introduction

A few years ago Applequist and Bjorken[1] made an observation which may be paraphrased as follows: let us suppose that we are given a theory of weak interactions and that we use it to calculate the lepton-lepton scattering amplitude A(s,t) to all orders in the weak coupling constant. The scattering amplitude A(s,t) should satisfy our usual notions about unitarity and analyticity; it would be very surprising if any decent (renormalizable) theory could manage to fail such basic requirements.[2] Thus, Applequist and Bjorken expect that it is reasonable to write a Mandelstam representation for the resulting lepton-lepton scattering amplitude; the double spectral function must be small for low s and t because the weak interactions are weak at low energies. However, the double spectral function may well rise considerably toward the "unitary limit" energy of $\sqrt{s} \approx 300$ GeV where the low energy V-A approximation must fail. In some theories, the Weinberg-Salam model for example,[3] there will be resonance poles ($W^{\pm}$, Z, etc.) in one or more of the s, t, u channels.

One can make a further observation based on mankind's experience with Mandelstam representation: <u>No matter what particles the theory starts with, the final A(s,t) can not be large in one channel only</u>. If one begins with just t channel poles, unitarity will generate large s-t and t-u double spectral functions (in field theory language higher order

corrections, say t-channel ladders, will add up) and there will be s and/or u channel scattering of roughly the same size as the original t-channel scattering.

We give, in section II, two illustrative examples from experiment of Mandelstam behaviour in lepton-lepton scattering, review briefly in sections III and IV two weak interaction theories that exhibit Mandelstam behavior, in section V summarize the predictions for lepton-lepton scattering in these two models, and finally in section VI make a conjecture about lepton-hadron scattering.

II. Two Examples of Mandelstam Behavior

Consider $\nu_e e$ scattering (left handed e's only). We know that at low energies the u channel process $\bar{\nu}_e e$ is present and can be well described by a charged W pole. We therefore expect that (to the same approximation) there must be either an s channel pole (dilepton resonance[4]) or a t channel pole (the Z coupling to the neutral current). These poles could be real if the W meson exists; or if the W pole is just an effective low-energy approximation the amplitude could just behave at low energies as though an s or t channel pole existed. Thus one could say that neutral currents are required by analyticity and unitarity. These requirements can be summarized by saying that the weak interactions, like the strong interactions, should be dual.

Consider, on the other hand, $\nu_\mu e$ scattering. We know the scattering is well described by the absence of a u-channel pole (because $\nu_\mu \neq \nu_e$). We therefore expect that either (a) neutral currents don't contribute or (b) there are s-channel dilepton resonances. The data at present seems to indicate a suppression of $\nu_\mu e$ scattering as expected from (a).

These examples, while rather casual and qualitative, are supported by experience with many systems and theories: in Q.E.D., for example, photon poles in one channel are always accompanied by poles or bound states in another.

III. A Dual Gauge Theory

Take a model with two weak isospins, F-spin and W-spin,[5] and consider the scattering of two (left-handed, zero mass) quartets

e		$W_3 = 1/2, \quad F_3 = 1/2$
ν_e	with	$W_3 = -1/2, F_3 = 1/2$
μ		$W_3 = 1/2, \quad F_3 = -1/2$
ν_μ		$W_3 = -1/2, F_3 = -1/2$

as quantum number assignments. Assume for a moment degenerate t and u poles with spin 1 and ask for the quantum numbers they must have in order that (i) each amplitude $A_{W,F}$ (with (W,F) = (0,0), (0,1), (1,0), or (1,1)) have Mandelstam behavior and (ii) the amplitude be "universal" with the effective coupling to $\nu_e e$ equal to that for $\nu_\mu \mu$. The answer is that the vector particles must have (W,F) = (0,0), (0,1), and (1,0) only; that is they form a representation of SU(2) x SU(2) x U(1).[6]

From this hint one may construct a "dual" SBGT[7] based on SU(2) x SU(2) x U(1) with the leptons in a left-handed quartet and two right handed singlets, e_R and μ_R. In Ref. 7, we choose scalar doublets to split the vector masses and scalar quartets to give e and μ non-zero masses. The quartets are necessary to give duality in the scattering of (s-channel) right-handed leptons with left-handed leptons (here crossing connects t-channel vectors with u-channel scalars). In section V we give the predictions for lepton-lepton scattering. We refer to this model as the DTY model.[7]

IV. Scalar Model of Weak Interaction

A second model with Mandelstam behaviour is the Scalar Model of Weak Interaction (SMWI) of Kummer and Segre[8] in which V-A exchange is replaced by the exchange of pairs of heavy scalar bosons (mass m) interacting with $\bar{\ell}(1-\gamma_5)L$ where ℓ is a normal lepton and L is a heavy lepton (with mass $M << m$).

Renormalizability is assured by the boson being scalar: an effective V-A interaction results from using the identity

$$(1-\gamma_5)(\not p-m)^{-1}(1+\gamma_5) = \frac{2\not p(1+\gamma_5)}{p^2 - m^2}$$

One sees from the structure of the model that Mandelstam behaviour for lepton-lepton scattering is built in from the beginning in that the amplitude is given by box diagrams. ν_e scattering vanishes in fourth order (but is allowed in higher order).

V. Experimental Predictions

[illegible] evaluated in both of the above models the processes $\nu_e e \to \nu_e e$ and $\nu_\mu e \to \nu_\mu e$, the asymmetry in the cross-section of $e^+e^- \to \mu^+\mu^-$ and the polarization of one of the muons in $e^+e^- \to \mu^+\mu^-$.[7,9]

The matrix element for each of the first two processes can be written as

$$M = \frac{G}{\sqrt{2}} \bar{u}_\nu \gamma^\alpha (1-\gamma_5) u_\nu \bar{u}_e \gamma_\alpha (C_V - C_A\gamma_5) u_e$$

and parameterized by the values of C_V and C_A (C'_V and C'_A for the ν_μ process). For comparison the processes have also been evaluated in the Weinberg-Salam model.

For $\nu_e e \to \nu_e e$ we have

DTY Model:

$$C_V = \frac{1}{8} + \frac{13}{12} \sin^2\alpha + \frac{3\sqrt{2}}{8} \frac{e^2}{Gm^2} x_{21}$$

$$C_A = \frac{1}{4} + \frac{1}{2} \sin^2 - \frac{3\sqrt{2}}{8} \frac{e^2}{Gm^2} x_{21}$$

where α is a "Weinberg" angle and x_{ij} are Higg's particles.

SMWI:

$$C_V = 1$$
$$C_A = 1$$

Weinberg-Salam Model:

$$C_V = 1/2 + 2 \sin^2\theta_W$$
$$C_A = 1/2$$

where θ_W is the Weinberg angle.

For $\nu_\mu e \to \nu_\mu e$ we have

DTY Model:

$$C'_V = \frac{1}{8} - \frac{1}{3} \sin^2\alpha - \frac{3\sqrt{2}}{8} \frac{e^2}{Gm^2} x_{22}$$

$$C'_A = \frac{1}{8} + \frac{3\sqrt{2}}{8} \frac{e^2}{Gm^2} x_{22}$$

SMWI:

$$C'_V = C'_A + \frac{e^2}{f^2} \frac{R-1}{R\ell nR} \left[1 - \frac{2}{3} \ell n \frac{m^2}{M^2}\right]$$

$$C'_A = \frac{f^2}{4\pi^2} \frac{R-1}{\ell nR} \quad I(R)$$

where f is the lepton heavy-lepton scalar boson coupling, R is the ratio of the mass of the charged scalar to the mass of the heavy lepton, and I(R) is a known function of R with I(1) = 1. m/M is the ratio of the charged scalar mass to the heavy-lepton mass.

Weinberg Salam Model:

$$C'_V = 1/2 - 2\sin^2\theta_W$$

$$C'_A = 1/2$$

The weak contribution to $e^+e^- \to \mu^+\mu^-$ gives terms in the cross section which are asymmetric in the scattering angle, i.e. the quantity

$$D = \frac{d\sigma(\theta) - d\sigma(\pi-\theta)}{d\sigma(\theta) + d\sigma(\pi-\theta)}$$

is non-zero; it also gives non-zero values for the polarization of the final μ^-

$$P \equiv \frac{d\sigma|_{h=+1} - d\sigma|_{h=-1}}{d\sigma|_{h=+1} + d\sigma|_{h=-1}}$$

(h is the helicity of the μ^-).

These quantities are most conveniently written in terms of the cosine of the polar scattering angle, z, the c. of m. energy of one beam, E, and a function W_o of z, the azimuthal scattering

angle ϕ, and the polarization of the initial beam, s,

$$W_o = 1 + z^2 - s^2(1-z^2)\cos 2\phi$$

For the symmetry D we have

DTY Model:

$$D = -\frac{z}{W_o}\frac{2\sqrt{2}\,GE^2}{e^2}\left[1 + 3\cos^2\alpha - 3\frac{e^2}{\sqrt{2}\,G}\left(\frac{1}{m^2_{x_{12}}} + \frac{1}{m^2_{\phi_{11}}}\right)\right]$$

where ϕ_{ij} are also Higg's particles.

SMWI:

$$D = -\frac{z}{W_o}\frac{8\sqrt{2}\,GE^2}{e^2}\frac{R-1}{\ell nR}$$

Weinberg-Salam Model:

$$D = -\frac{z}{W_o}\frac{4\sqrt{2}\,GE^2}{e^2}$$

For the polarization P we have

DTY Model:

$$P = \frac{\sqrt{2}\, GE^2}{e^2}\left[\frac{1}{3} - \frac{7}{3}\cos^2\alpha + \frac{2z}{W_o}(1-5\cos^2\alpha)\right] + 3E^2\left(\frac{2z}{W_o} - 1\right)\left(\frac{1}{m^2_{\chi_{11}}} - \frac{1}{m^2_{\phi_{11}}}\right)$$

SMWI:

$$P = \frac{4\sqrt{2}\, GE^2}{e^2}\,\frac{R-1}{\ln R}\left[1 + \frac{2z}{W_o}\right]$$

Weinberg Salam Model:

$$P = -\frac{2\sqrt{2}\, GE^2}{e^2}\left[1 + \frac{2z}{W_o}\right](3\sin^2\theta_w - \cos^2\theta_w)$$

These equations show that while $\nu_e e$ scattering is appreciable in all three models $\nu_\mu e$ scattering may be much smaller in the DTY or SMWI than it is allowed to be in the Weinberg-Salam model. In both models C'_V can be zero and C'_A, although it cannot be zero, can be small. This suppression of $\nu_\mu e$ scattering is the quantitative expression of our argument in Sec. II. D and P can have almost any values in the DTY model ranging from zero to something large if the masses of the Higg's scalars are small. For convenient values of the scattering angles the Weinberg- Salam theory predicts D and P to be of the order of 1%. In the SMWI D and P are at least twice as large as this and, depending on the value of R, may he much larger.

VI. Lepton Hadron Scattering

Finally let us consider lepton hadron scattering. The amplitude A(s,t) for

$$\bar{\nu} + p \rightarrow e^+ + n$$

has a t-channel pole, W, (or at least an effective pole). There is also a u-channel pole, the hydrogen atom.

The related process

$$\bar{\nu} + p \rightarrow \bar{\nu} + p$$

has an effective t-channel pole, the neutral current (z) exchange. Mandelstam behaviour or duality leads us to expect that effective s-channel or u-channel poles must also occur in this amplitude. Such poles will be non-electromagnetic, lepton hadron resonances.[10] They should occur at masses of the order of the W mass. They do not give rise to any non-conservation of baryon number.[11] In unitifed theories of strong, electromagnetic, and weak interactions[12] they should arise from the continuation in g^2 of the hadron-hadron scattering amplitude (or the lepton-lepton scattering amplitude) into that for lepton-hadron scattering.

With the usual intermediate vector boson still unobserved, the νp resonances must, of course, be viewed with caution. On the other hand we think the idea of checking weak interaction theories for Mandelstam (or dual) behaviour could be a useful tool in choosing among them and in considering higher order corrections.

Acknowledgements

We should like to thank Gino Segre and James Young in collaboration with whom the calculations discussed here were performed. We are also grateful to Lincoln Wolfenstein for several helpful conversations.

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ERRATA

Reference 8 should include, of course, the recent paper by Gino Segre that revitalized interest in scalar models of weak interactions;

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