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BENDING STRESSES IN A PRESSURE VESSEL WITH AN INTEGRAL FLAT HEAD

by

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ABSTRACT

Formulas are derived for the bending stresses in a pressure vessel with an integral flat head using thin wall theory. These formulas are plotted in nondimensional form to facilitate computation.

LIST OF SYMBOLS

M	=	axial bending moment per unit of circumference	$\frac{in - lb}{in}$
H	=	radial shear on section perpendicular to the center line of the cylinder	$\frac{lb}{in}$
w	=	radial displacement with respect to the center line of the cylinder	in
W	=	rotation, change of angle of tangent line to middle surface	rad.
D_c	=	$\frac{Et_c^3}{12(1-\nu^2)}$	in - lb
D_p	=	$\frac{Et_p^3}{12(1-\nu^2)}$	in - lb
β^4	=	$\frac{3(1-\nu^2)}{a^2 t_c^2}$	in^{-4}
E	=	modulus of elasticity	psi
ν	=	Poisson's ratio = 0.3	
a	=	mean radius of cylinder wall	in
t_c	=	cylinder wall thickness	in
t_p	=	plate thickness	in
p	=	pressure acting normal to the middle surface of the shell	psi
σ_{c_1}	=	bending stress in the cylinder at the juncture of the cylinder and plate.	psi

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σ_{p_1} = bending stress in the plate at the juncture of the plate and the cylinder psi

σ_{p_2} = bending stress in the center of the plate psi

DISCUSSION

The data presented here were prepared to facilitate the design of thin-walled cylindrical pressure vessels with integral flat heads. Formulas and curves are given for the maximum bending stresses in the head and in the cylindrical shell.

The equations derived later in this paper are based on the solutions to the differential equations for thin plates and shells as given by Timoshenko in Reference 1. The principal assumptions are:

1. Thin wall theory for shell and plate applies; i.e., the shell and plate thicknesses are very small relative to the diameter of the cylinder.
2. The cylinder at its point of attachment to the plate has no radial displacement; i.e., the stiffness of the plate in the radial direction is assumed to be very large with respect to that of the cylinder.
3. The plate and cylinder are considered to be joined at the intersection of the center lines of the plate thickness and the cylinder wall. This assumption neglects the contribution to the bending moment which results from the eccentricity of the shear force with respect to the central plane of the plate. This is in keeping with the first assumption of thicknesses that are vey small compared to the radius.

4. The material is assumed to be elastic, homogeneous, and isotropic.

5. The cylinder is assumed to be long enough that interactions between the ends of the cylinder are negligible. This requires that the shell length be greater than about four times $\sqrt{at_c}$.*

6. Deflections of the plate are small enough that its membrane stresses can be neglected.

The limiting nominal bending stresses in the surfaces of the plate and shell are shown in Figures 2, 3 and 4 in nondimensional form. A value of 0.3 has been used for Poisson's ratio in computing data for the curves, this value being a suitable approximation for most metals. The circumferential bending stresses in the plate near the junction and the circumferential stresses in the shell are not shown, because they are smaller than those shown in the figures. Bending stresses at locations other than the junction and the center of the plate are not shown, because they also are smaller than the stresses which are shown in the figures. The most highly stressed region of the cylinder is at the junction even when the membrane stresses in the shell are superimposed upon its bending stresses.

If the pressure vessel is made of a low-ductility material or is to be subjected to many cycles of loading, it will be necessary to consider the stress concentration which will occur at the inside corner of the junction. Information on stress concentration factors and on design for cyclic loading is given in References 2, 3 and 4.

*For definitions, see Nomenclature, p. 1.

It should be emphasized that the bending stresses obtained for the cylinder-to-plate juncture are the discontinuity stresses obtained by requiring continuity of deflection and rotation between the cylinder and the plate. The total nominal axial stress in the cylinder is the sum of the axial bending stress from Figure 4 and the axial membrane stress.

In general, it can be seen from the curves that when the ratio $t_c/t_p < 0.1$, the stress in the cylinder is independent of the ratio a/t_c and approaches the value of 1.54 pa/t_c for a fixed-ended cylinder. Also, for a value of $t_c/t_p > 1.5$, the maximum bending stress in the cylinder is controlled by the moment required to give the plate a fixed edge for any given ratio a/t_c; e.g., the curves on Figure 4 are asymptotic to $0.75(a/t_c)$.

The stress in the plate for $t_c/t_p < 0.1$ approaches the value of $1.24\ p(a/t_p)^2$ for a simply supported plate when a/t_c becomes very large (>16). The maximum stress in the plate at the cylinder juncture for $t_c/t_p > 1.5$ approaches the value for a fixed edge plate and is practically independent of the the a/t_c ratio. Thus, it can be seen that no appreciable change in the magnitude of the stresses can be effected by varying the ratio of t_c/t_p outside the range of 0.1 to 1.5.

The following further generalities may be deduced from the curves:

1. For the same bending stress in the center and at the edge of the plate, the maximum bending stress is in the cylinder for $t_c/t_p = 0.85$ and $a/t_c > 4.0$;

2. For $0.85 < t_c/t_p < 1.0$, the maximum bending stress is in the cylinder.
3. For $t_c/t_p = 1.0$, the maximum bending stress in the plate is equal to the longitudinal bending stress in the cylinder.
4. For $t_c/t_p > 1.0$, the maximum bending stress is in the plate.
5. For $t_c/t_p < 0.85$, the highest stress in the plate occurs at the center.

SAMPLE PROBLEM

Find the maximum stress in a pressure vessel with an integral flat head loaded with 1000 psi internal pressure and having the following dimensions:

Cylinder I.D.	=	60 in.
Cylinder Wall Thickness	=	3 in.
Head Thickness	=	7.5 in.

The ratios for entering the curves are:

$$a/t_c = \frac{30 + 1.5}{3} = 10.5$$

$$t_c/t_p = \frac{3}{7.5} = 0.4$$

From Figs. 2, 3, 4

$$\sigma_{p_2} = 0.92\ p\ (a/t_p)^2$$

$$\sigma_{p_1} = 0.32\ p\ (a/t_p)^2$$

$$\sigma_{\sigma_1} = 3.8\ pa/t_c$$

Since

$$a/t_p = \frac{30 + 1.5}{7.5} = 4.2$$

The maximum stress in the head is

$$\sigma_{p_2} = 0.92 \times 1000 \times (4.2)^2$$

$$= 16200 \text{ psi}$$

The maximum stress in the cylinder is the bending stress plus the axial membrane stress, i.e.

$$\sigma = \sigma_{c_1} + \frac{pa}{2t_c} = (3.8 + 0.5)\,\frac{pa}{t_c}$$

$$= 4.3\,\frac{pa}{t_c}$$

$$= 4.3 \times 1000 \times 10.5$$

σ = 45200 psi, total axial stress in the cylinder

REFERENCES

1. Timoshenko, S., Theory of Plates and Shells, 1st Ed., McGraw-Hill Book Co., Inc., New York, 1940.

2. Peterson, R. E., Stress Concentration Design Factors, John Wiley and Sons, Inc., New York, 1953.

3. Heywood, R. B., Designing by Photoelasticity, Chapman and Hall Ltd., London, 1952.

4. Bureau of Ships, Department of the Navy; PB151987, Tentative Structural Design Basis for Reactor Pressure Vessels and Directly Associated Components, December 1, 1958, US Dept. of Commerce, Office of Technical Services, Washington, DC.

LIST OF ILLUSTRATIONS

APPENDIX

BENDING STRESSES IN A PRESSURE VESSEL WITH AN INTEGRAL FLAT HEAD

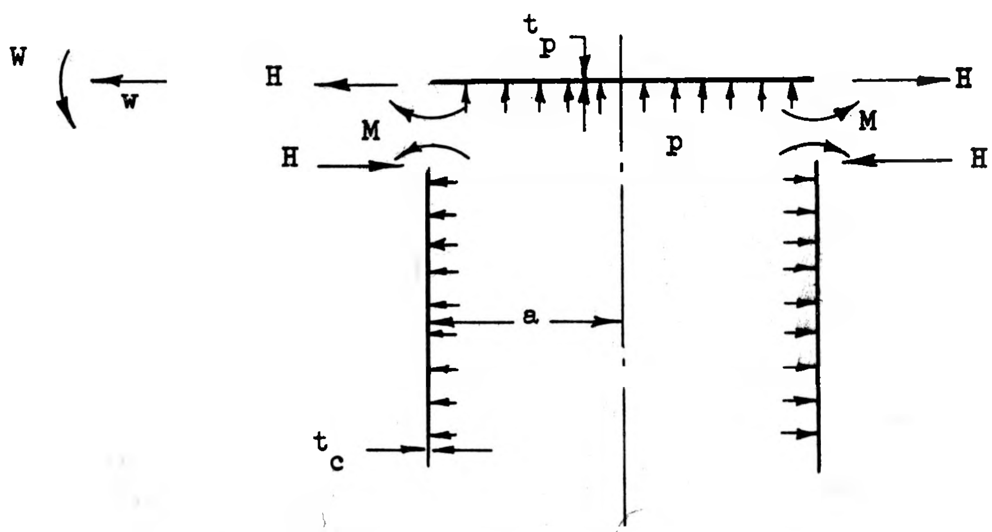

FIGURE 1. ANALYTICAL MODEL

Since the structure is continuous, the slope and deflection of the cylinder must match that of the plate at their juncture. The resistance of the plate to deformation in the radial direction is much higher than the resistance of the cylinder; therefore, the cylinder displacement, at the juncture with the plate, is assumed to be zero.

Therefore, the equations for continuity are

$$w_c = 0 \qquad (1)$$

$$W_c = W_p \, . \qquad (2)$$

For the cylinder, these expressions are (see Timoshenko[1], pp. 389-407)

$$W_c = \frac{-H}{2\beta^2 D_c} + \frac{M}{\beta D_c} \qquad (3)$$

$$w_c = \frac{-H}{2\beta^3 D_c} + \frac{M}{2\beta^2 D_c} + \frac{pa^2}{Et_c}\left(1-\frac{\nu}{2}\right) \qquad (4)$$

and for the plate (see Timoshenko[1], pp. 55-62),

$$W_p = \frac{a}{(1+\nu)D_p}\left(\frac{pa^2}{8}\right) - \frac{Ma}{(1+\nu)D_p} \qquad (5)$$

where

$$\beta = \sqrt[4]{\frac{3(1-\nu^2)}{a^2 t_c^2}} \qquad (6)$$

$$D_c = \frac{Et_c^3}{12(1-\nu^2)} \qquad (7)$$

$$D_p = \frac{Et_p^3}{12(1-\nu^2)} \qquad (8)$$

The following equation for M is obtained by substituting equations (3) through (8) into equations (1) and (2):

$$M = pat_c \left[\frac{0.257 + 0.247 \sqrt{(a/t_c)^3}\,(t_c/t_p)^3}{1 + 1.98 \sqrt{a/t_c}\,(t_c/t_p)^3} \right] \qquad (9)$$

The bending stress in the cylinder is given by:

$$\sigma_c = \frac{6M}{t_c^2} \qquad (10)$$

Therefore the nondimensional bending stress in the cylinder is:

$$\frac{\sigma_c}{pa/t_c} = \frac{1.54 + 1.48 \sqrt{(a/t_c)^3}\,(t_c/t_p)^3}{1 + 1.98 \sqrt{a/t_c}\,(t_c/t_p)^3} \qquad (11)$$

The bending stress in the center of the plate is given by:

$$\sigma_{p_1} = 1.24\, p(a/t_p)^2 - \frac{6M}{t_p^2} \qquad (12)$$

(see Timoshenko[1], pp. 61-62)

The nondimensional bending stress at the center of the plate is given by:

$$\frac{\sigma_{p_1}}{p(a/t_p)^2} = 1.24 - \frac{1}{a/t_c}\left[\frac{\sigma_c}{pa/t_c}\right] \qquad (13)$$

The radial bending stress at the edge of the plate is given by:

$$\sigma_{p_2} = \frac{6M}{t_p^2} \qquad (14)$$

The nondimensional stress is:

$$\frac{\sigma_{p_2}}{p(a/t_p)^2} = \frac{1}{a/t_c}\left[\frac{\sigma_c}{pa/t_c}\right] \qquad (15)$$

BENDING STRESS IN THE CENTER OF THE PLATE

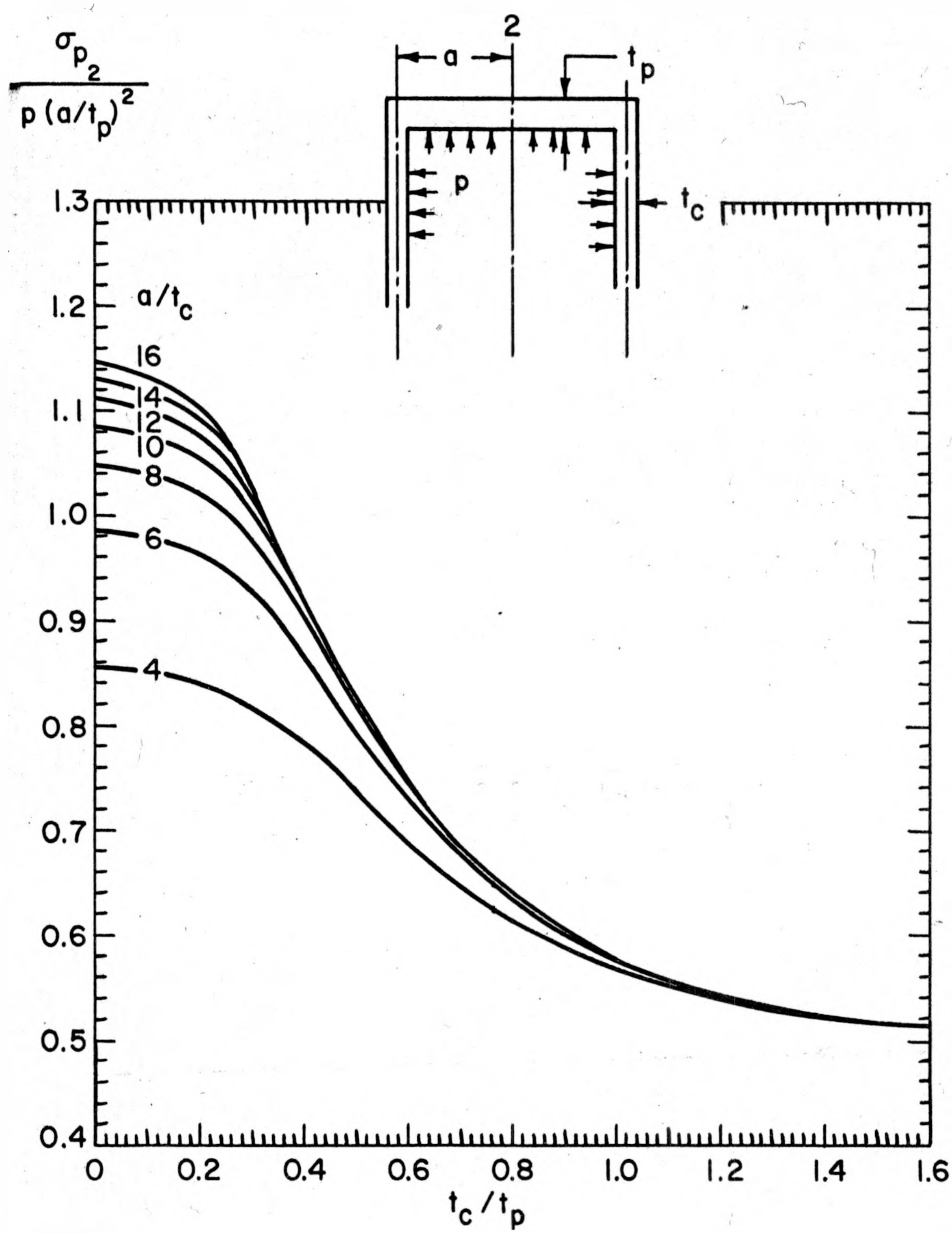

FIGURE 2.

BENDING STRESS IN THE PLATE AT THE CYLINDER JUNCTURE

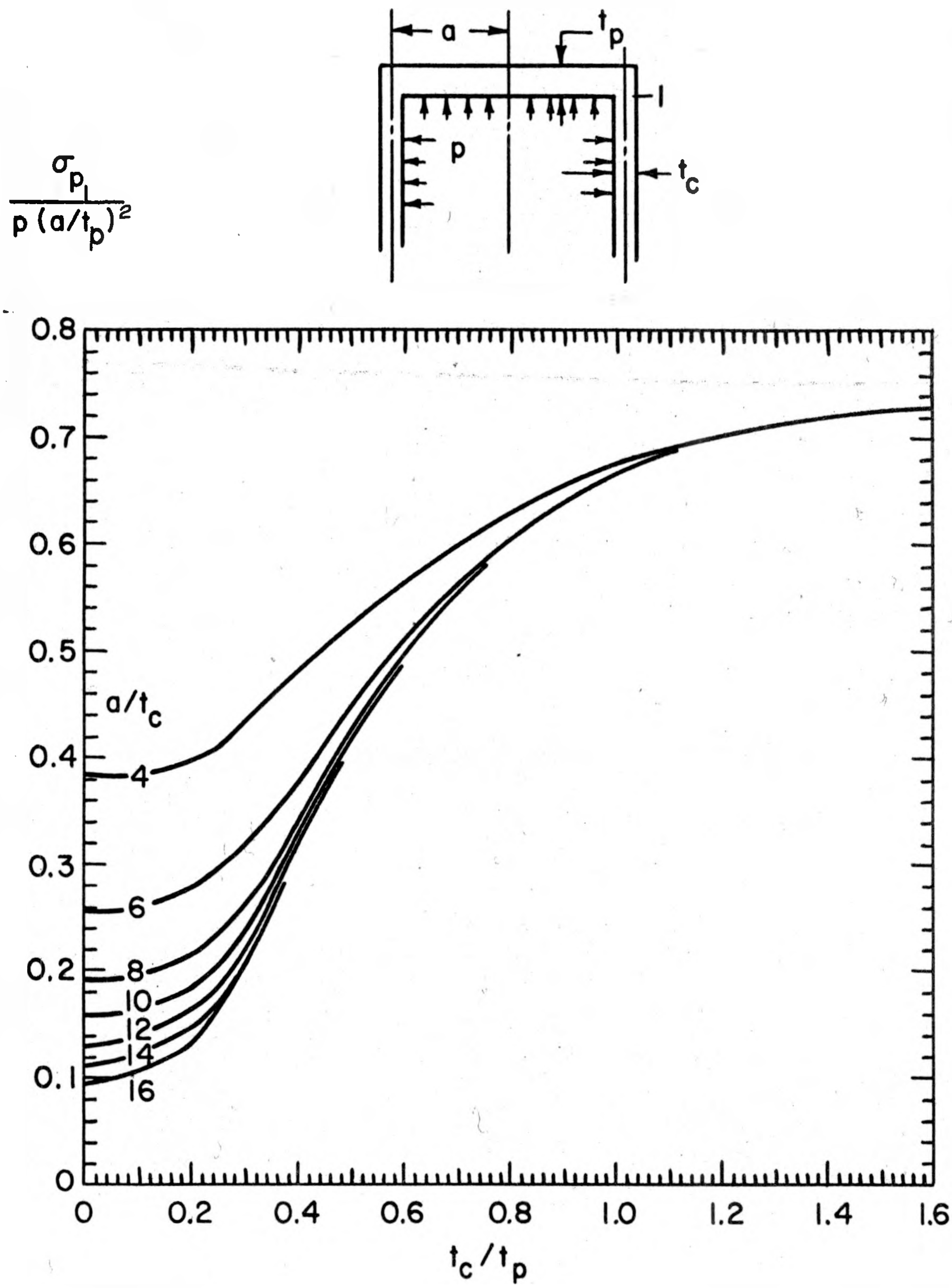

FIGURE 3.

BENDING STRESS IN THE CYLINDER AT THE HEAD JUNCTURE

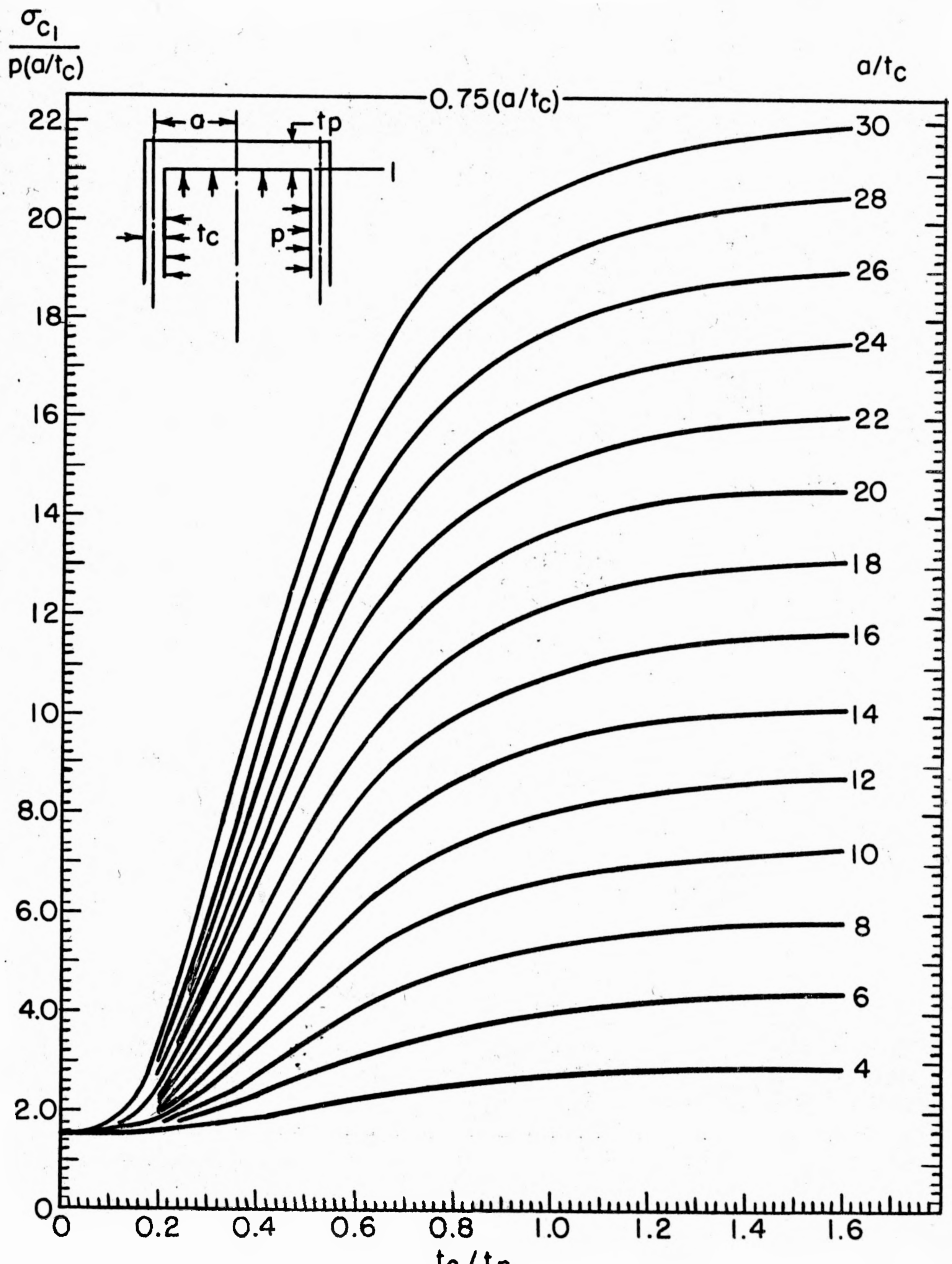

FIGURE 4.