NYO-2707
REACTORS—POWER

FEASIBILITY STUDY ON CENTRIFUGE ENRICHMENT OF THE BY-PASS STREAM OF THE PEBBLE BED REACTOR

By
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November 1959

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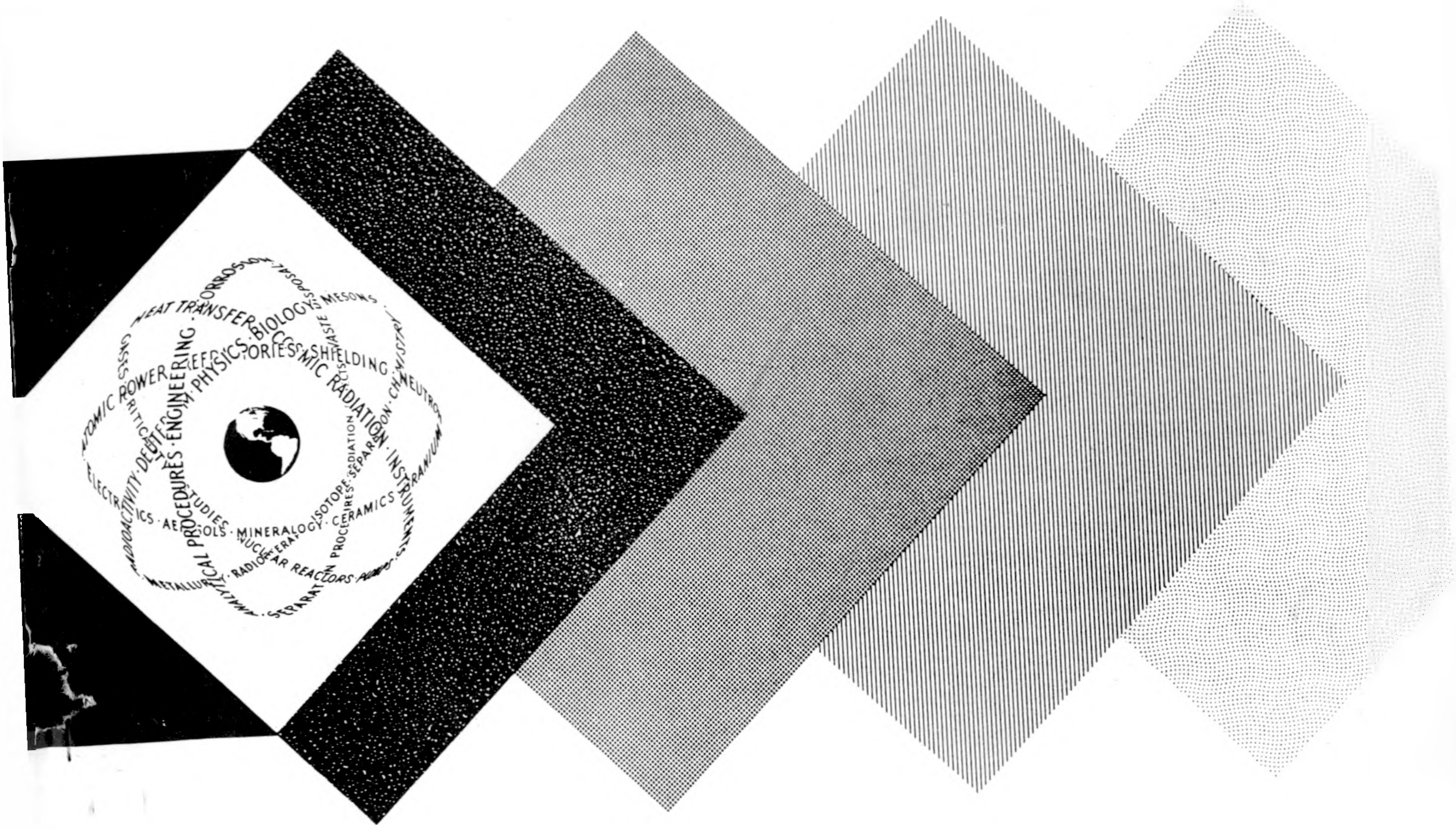

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CENTRIFUGE ENRICHMENT OF THE BY-PASS STREAM OF THE PEBBLE BED REACTOR

BY

H. M. Parker
and
John Mowat

A final report prepared for
Sanderson and Porter
Under AEC Contract AT-(30-1)-2207
S & P Sub 1965-3

November 1959

Division of Astronautics and Engineering Physics
Research Laboratories for the Engineering Sciences
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Charlottesville, Virginia

PREFACE

This report is the result of a feasibility study undertaken by RLES as a subcontractor to Sanderson and Porter under their prime contract AT-(30-1)-2207. The theoretical study which was initiated on 1 July 1959 had as its prime purpose to investigate the possibility of using an ultracentrifuge to concentrate the fission products produced in a pebble bed reactor into a rather small by-pass stream. The results of this study, which are not at all encouraging, are presented herein.

During the course of the study an alternative approach was suggested involving the introduction of a cloud of extremely small solid particles into the gas stream to serve as adsorbers or nucleii for the attachment of the fission products. The removal of such solid particles would be an entirely different process from that of gas separation. This problem will be considered as part of a separate report to be issued shortly.

Acknowledgement is due Drs. K. H. Quasebarth, J. L. Meem, and A. R. Kuhlthau for their assistance on this project.

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TABLE OF SYMBOLS AND UNITS

L = Flow entering enrichment device (moles-sec^{-1})
P = Product or by-pass flow leaving enrichment device (moles-sec^{-1})
W = Waste flow leaving enrichment device (moles-sec^{-1})
N = Mole fraction of impurity or impurities in loop (non-dimensional)
N' = Mole fraction of impurity or of impurities in by-pass flow (non-dimensional)
α = Enrichment factor (non-dimensional)
Δu = Separative power associated with N, N' and P $\left(\frac{\text{moles separative work}}{\text{sec}}\right)$
D = Diffusion coefficient, heavy gas into light (cm^2 - sec^{-1})
P = Pressure in centrifuge (dynes-cm^{-2})
R = Universal gas constant $\left(8.3144 \times 10^7 \ \frac{\text{ergs}}{°\text{K mole}}\right)$
T = Temperature of gas in centrifuge (°K)
M_2 = Mass of heavy isotope (amu)
M_1 = Mass of light isotope (amu)
= Peripheral velocity of centrifuge (cm-sec^{-1})
Z = Total length of centrifuge (meters)
C = Total quantity of helium in loop including that in centrifuge (moles)
λ = Radioactive decay constant (sec^{-1})
$\bar{\lambda}$ = Average decay constant for mixture of isotopes (sec^{-1})
B = Creation rate by fission of an isotope or isotopes (moles-sec^{-1})
A = Activity of an isotope or isotopes (moles-sec^{-1})
θ = "cut" P to L ratio (non-dimensional)
V' = Elementary value function (non-dimensional)
q = Fraction of impurity removed from stream, L (non-dimensional)
N'' = Mole fraction of impurity or impurities in waste flow (non-dimensional)

SECTION I

INTRODUCTION

Further developments of the gas cooled PBR, as envisioned in Sanderson and Porter Report NYO 2373 will entail higher operating temperatures and thinly clad or unclad fuel elements, i.e., most of the long lived, volatile fission products will escape into the helium stream. Thus direct maintenance of the gas loop equipment (external to the reactor proper) becomes questionable unless the radiation level is reduced by the removal of fission products from the gas stream. The removal of poison products (notably Xe_{135}) from the gas loop is also an important consideration in reactor economics. Reactor engineers at Sanderson and Porter, faced with the economics of gas stream purification even at the present stage of PBR development, have begun to consider methods of enriching the fraction of the flow which is by-passed and purified. Their preliminary analysis indicates that, essentially because of the large difference in molecular weights of helium and the contaminants, the centrifuge should be considered as a device for contaminant concentration.

This report presents the results of a feasibility study of centrifugation as a method of concentrating the contaminants in the PBR gas stream. The general results may be applied to a wide range of similar separation or purification problems. However, specific numerical results are given for the 125 eMW PBR described in NYO 2373.

The analysis begins with the application of isotope separation theory to derive formulas for the total length of centrifuge needed to attain a given enrichment of a given by-pass flow. Then certain of the formulas for equilibrium activity are shown written in terms of the separation theory parameters.

The combined formulas from these two sections allow the calculation of the length of centrifuge vs. the activity reduction for different reactors and by-pass flows. Particular results are presented for the 125 eMW reactor mentioned above; a discussion of centrifuge development and a rough cost estimate follow, and finally, some general conclusions are drawn.

SECTION II

APPLICATION OF ISOTOPE SEPARATION THEORY

Figure 1 is a diagram which illustrates the position of the enrichment device in the main loop and allows a comparison of isotope separation terminology with the terminology of NYO 2373. For example, the "product flow" is the S and P "by-pass flow" except for units. In this report the flows L, P, and W are always in units of moles-sec^{-1} of working substance, (in practice - helium). If N is the mole fraction of an impurity in the feed and N', in the product, the ratio

$$\alpha = \frac{N' (1-N)}{N (1-N')} \tag{1}$$

is called the enrichment factor of the device. If N and N' are sufficiently small, (1) becomes

$$\alpha = \frac{N'}{N} \tag{2}$$

Since mole fractions of the most plentiful contaminants in the gas stream of a PBR will be less than 10^{-6} with very small un-enriched by-pass streams, this report will always use an analysis based on the small N approximation. Another more serious approximation is the use of a simplified analysis which eliminates the flow, L, from consideration.

Consider the entire gas system (reactor, heat exchangers, enrichment devices, and separators) as being a large box containing gas (helium) with mole fraction, N, of impurity. The enrichment device removes P moles/sec (product flow) of mole fraction N', and a replacement flow equal to P moles/sec of pure helium is added to the box. (See Figure 2.) The parameters P, N, and N' determine a quantity called the "separative power" which is proportional to the size of the enrichment device required to perform the separation specified by P, N, and N'. Following Cohen[1], the expression for the separative power is

$$\Delta u = P[(2N'-1) \ln \alpha + (N'-N)\frac{1-2N}{N(1-N)}] \tag{3}$$

---

[1] Cohen, Karl, The Theory of Isotope Separation, McGraw-Hill (1951).

where the mole fraction in the box, N, has been chosen for the reference mole fraction N_o in Cohen's expression. In the small N approximation (3) becomes

$$\Delta u = P[\alpha - 1 - \ln \alpha] \quad . \tag{4}$$

P, the only dimensional quantity in the right members of (3) and (4), is a mass flow (moles/sec); yet Δu is a quantity of different and considerable significance as evidenced by the units attached, moles separative work/sec. The most satisfying interpretation of Δu is that it is a measure of the magnitude of the separation job specified by the parameters P, N, and N', and thus of the size of separating plant required to achieve it.

In writing equation (3) for the separative power, a term corresponding to the waste stream from the enrichment device has been neglected. This term is positive and for reasonable arrangements is only a few percent of (3). Thus (3) and hence (4) underestimate by a few percent the magnitude of the specified separation job. An exact analysis is summarized in Appendix I.

For the present purpose it is assumed that the enrichment device is composed of an array (in general a cascade) of centrifuges. The pertinent result of centrifuge theory (as presented, for example, by Cohen in reference 1) is that the maximum theoretical separative power of an array of gas centrifuges at constant operating conditions (peripheral speed, temperature working substance) is proportional to their total length and independent of their diameter, specifically

$$\Delta u_{\text{max theo.}} = \frac{DP}{RT}\left[\frac{(M_2 - M_1)V^2}{2RT}\right]^2 50\ \pi Z \text{ moles separative work/sec} \tag{5}$$

In using this expression to relate separative power and total length of centrifuge, two fundamental assumptions are made, namely:

- Each centrifuge unit is operating at maximum theoretical separative power, a condition that cannot be achieved. A brief discussion of efficiencies to be expected is given in Appendix II.
- No separative power is lost in the cascade connection, i.e., gas streams of different mole fractions are not mixed within the enrichment device. One can expect a very small loss of separative power due to violation of this assumption.

Thus the use of (4), a slight underestimate of the magnitude of the separation job, and (5), a considerable over-estimate of the separative power to be obtained from Z meters of centrifuge, results in an optimistic estimate of the length of centrifuge required to perform the separation job specified by P, N, and N' .

Combining (4) and (5) yields

$$Z = \left\{\frac{DP}{RT}\left[\frac{(M_2 - M_1)V^2}{2RT}\right]^2 50\,\pi\right\}^{-1} P[\alpha - 1 - \ln \alpha] \quad \text{meters} \qquad (6)$$

The insertion of the following values is considered reasonably appropriate for the PBR gas purification problem,

$T = 500°F - 533°K$

$V = 4 \times 10^4$ cm/sec

$M_2 = 104$ (intermediate between light and heavy fission product groups)

$M_1 = 4$

$\frac{DP}{RT} = 4.73 \times 10^{-5} \frac{\text{mole}}{\text{cm. sec}}$ (from AIP Handbook for HeA mixtures)

gives

$$Z = 41.3\,P[\alpha - 1 - \ln \alpha] \quad \text{meters} \qquad (7)$$

$$Z = 41.3\,\alpha P[1 - \frac{1}{\alpha}(1 + \ln \alpha)] \quad \text{meters} \qquad (7a)$$

SECTION III

ACTIVITY LEVEL AND PURIFICATION

While for different aspects of the purification problem the total activity, specific activity, or mole fraction of impurity might be the most important parameter, this report will always consider only the total activity reduction brought about by purification. Since the contents of the centrifuge must be considered in evaluating the parameter C, and since this inclusion effects total activity reduction and specific activity reduction in different ways, this may make some difference on occasion although not much for reasonably short centrifuge lengths. No attempt is made to do the highly complicated analysis which is necessary to predict the exact effect of purification in a specific PBR. Rather, two simplified approaches are taken which give limited results and which show some of the important effects of the purification process.

A. The $\bar{\lambda}$ Approach

Consider a box containing C moles of gas, mostly helium, but also small amounts of many radioactive substances. At any instant, the total activity in the box may be written as

$$A = \lambda_1 C N_1 + \lambda_2 C N_2 + \dots + \lambda_j C N_j + \dots = C \Sigma_j \lambda_j N_j$$

where λ_j is the decay constant of the j^{th} radioactive substance and N_j is the mole fraction, at that instant, of the j^{th} substance. One may define an average decay constant, $\bar{\lambda}$, by the expression

$$A = \bar{\lambda} C N$$

where $N = \Sigma_j N_j$, total impurity mole fraction. Thus $\bar{\lambda}$ is

$$\bar{\lambda} = \frac{\Sigma_j \lambda_j N_j}{\Sigma_j N_j}$$

Obviously, in a non-equilibrium situation, the N_j's varying with time, $\bar{\lambda}$, will not be constant. In an equilibrium state a given isotope (for our problem and in the most general case) is being produced by fission, being produced by decay of a parent isotope, being removed by decay, and being removed by purification. Thus the equilibrium mole fraction of the isotope is determined by the balancing of its production and removal rates. It is easy to see that a change in the gross purification rate affects each isotope differently, changes their equilibrium mole fractions differently and hence changes $\bar{\lambda}$. In general, one expects $\bar{\lambda}$ to be a function of the particular total equilibrium state.

Nevertheless, the average decay-constant approach enables one to make easily a rough approximation of the effect of purification on level of activity. One considers the total impurities to be a single isotope, the decay constant of which is $\bar{\lambda}$. Then at equilibrium

$$B = \bar{\lambda} CN + \alpha PN \tag{8}$$

where B is the rate at which the hypothetical isotope is being added to the gas, $\bar{\lambda}CN$ is the rate at which it is disappearing by decay, and αPN is the rate at which it is being removed by purification. Then the equilibrium activity is

$$A = \frac{B}{1 + \frac{\alpha P}{\bar{\lambda} C}} = \bar{\lambda} CN \quad \text{moles/sec.} \tag{9}$$

Certainly formula (9) is valid if the value of $\bar{\lambda}$ used is appropriate for the particular equilibrium state; i.e., for the particular αP for a given B and C. Further, the use of (9) with a constant $\bar{\lambda}$ to exhibit the variation of activity with purification rate can only give approximate results, the error of which has not been estimated since a reasonably accurate analysis of how $\bar{\lambda}$ varies with αP is beyond the scope of this study. Thus, results based on formula (9) should be viewed with caution.

B. The Single Series Approach (The 135 Problem)

In this approach one considers each decay series separately. For example, the purification effect on the 135 series may be determined easily if one makes the following assumptions:

- No Te_{135} escapes from the fuel elements.
- All I_{135} escapes immediately to the stream and no Xe_{135} is created directly by fission. (Actually, about 6% of the primary 135 creation is Xe_{135}.)
- Cs_{135} deposits on the walls of the loop as soon as it is created thereby removing itself from the stream.

These assumptions may be summed up by the statement that only Iodine is created and that the activity which is to be reduced by purification comes only from I_{135} and Xe_{135}. Then, if B_{135} is the I_{135} creation rate

$$A_I = \frac{B_{135}}{1 + \frac{\alpha P}{\lambda_I C}} \tag{10}$$

is the equilibrium activity of I_{135} and

$$A_{Xe} = \frac{B_{135}}{(1 + \frac{\alpha P}{\lambda_I C})\,(1 + \frac{\alpha P}{\lambda_{Xe} C})} \tag{11}$$

is the equilibrium activity of Xe_{135}. It is interesting to observe the "double cut-down effect" on Xe_{135} which arises from the fact that Xe_{135} is removed in two ways: One, by direct removal; and, two, by removal of I_{135} which is then not present to decay to Xe_{135}. This effect is evident in equation (11) which shows that Xe_{135} activity and quantity are cut down roughly twice as many orders as are I_{135} activity and quantity (since λ_I and λ_{Xe} are roughly equal). Combining (10) and (11) one finds the total equilibrium activity of the series

$$A_{tot} = \frac{B_{135}}{1 + \frac{\alpha P}{\lambda_I C}} + \frac{B_{135}}{(1 + \frac{\alpha P}{\lambda_I C})(1 + \frac{\alpha P}{\lambda_{Xe} C})} \tag{12}$$

Depending on whether the total 135 activity or Xe_{135} "poisoning" is the concern, equation (12) or (11) should be used. Due to its long half-life, Cs_{135} never comes to equilibrium and therefore its equilibrium activity is of little significance.

In contrast to the uncertainty in the $\overline{\lambda}$ approach noted above, the assumptions on which the single series approach is based are considered quite sound. Thus, a considerable amount of confidence can be attached to calculations of I_{135} and Xe_{135} activities from equations (10), (11) and (12).

C. αP vs. α and P

It is an important fact that activity reduction and total length of centrifuge depend upon α and P, the parameters of the purification problem, in fundamentally different ways. Since αPN is the removal rate (moles/sec) of the impurity at equilibrium, the reduction in activity depends only on the product, αP. The total length of centrifuge required to do the separation (enrichment) job, on the other hand, depends upon α and P separately (equations (6) and (7)). This corresponds to the physical fact that any possible reduction in activity can be achieved by increasing P, the by-pass flow, with no enrichment device ($\alpha = 1.0$). Of course, the design criterion will be how may the desired αP be most economically obtained?

SECTION IV

NUMERICAL RESULTS

The results of the last two sections will be used in this section to obtain numerical results for the 125 eMW PBR described in Sanderson and Porter NYO 2373. The lengths obtained should be approximately doubled if centrifuge efficiency is to be taken into account. (See Appendix II.) The reactor data used are:

Capacity of the loop - 2070 lbs (2.35×10^5 moles) helium
Primary loop flow rate - 378 lbs/sec (4.28×10^4 moles/sec)
Average decay constant - $\bar{\lambda} = 2.68 \times 10^{-5}$ sec^{-1} (The precise conditions for which $\bar{\lambda}$ is calculated are not clear.)
Decay constant for $I_{135} = 2.87 \times 10^{-5}$ sec^{-1}
Decay constant for $Xe_{135} = 2.09 \times 10^{-5}$ sec^{-1}

Tables I and II are calculated from equations (7), (7a), and (9); i.e., using the average decay constant concept. The results are similar to those of Figure 5-4 of NYO 2373, except that this report adds centrifuge length data. The activity remaining is expressed as a percent of the equilibrium activity which would obtain with no purification. A literal interpretation of these two tables must be viewed with caution due to the uncertainty in the variation of $\bar{\lambda}$ with degree of purification.

Table I shows the decrease in activity with increase in Z at constant P for a few selected by-pass flows. It shows, for example, that a by-pass flow of 5 lbs/sec lowers activity two orders and that 156,500 meters of centrifuge will concentrate impurities in this by-pass enough to reduce the activity an additional order.

Table II shows combinations of by-pass flows and total centrifuge lengths to attain a few given activity reductions. For example, the last row shows combinations which reduce activity three orders. 55.5 lbs sec^{-1} of by-pass flow and no centrifuge, 53.9 lbs sec^{-1} of by-pass and 100 meters of centrifuge, or 40.8 lbs sec^{-1} of by-pass and 10,000 meters of centrifuge, all achieve the same three order reduction. One can immediately see in this particular case that adding 100 meters of centrifuge would lower the amount of by-pass flow required by 1.6 lbs/sec, and that adding 10,000 meters would lower it by 14.7 lbs/ sec;

TABLE I

$\bar{\lambda}$ Approach. Centrifuge Length, Z, vs % of No Purification Equilibrium Activity (NPEA)

	P = 0.5 lb/sec = 56.7 mole/sec		P = 1 lb/sec = 113.5 mole/sec		P = 2.5 lb/sec = 284 mole/sec		P = 5 lb/sec = 567 mole/sec		P = 10 lb/sec = 1135 mole/sec	
α	Z Meters	% NPEA	Z Meters	% NPEA	Z Meters	% NPEA	Z Meters	% NPEA	Z Meters	% NPEA
1	0	10	0	5.25	0	2.17	0	1.1	0	.552
1.5	223	6.9	444	3.54	1,115	1.46	2,220	.734	4,440	.368
2	720	5.25	1,433	2.70	3,600	1.10	7,170	.552	14,130	.276
2.5			2,740	2.17						
4					22,300	.552				
5	5,600	2.17	11,160	1.1	28,000	.442	55,800	.22	111,600	.111
10	15,700	1.10	31,300	.552	78,600	.220	156,500	.111	313,000	.0554
20	37,400	.544								
100	238,000[1]	.111								

[1] Exact formula

TABLE II

$\bar{\lambda}$ Approach. By-pass Flow P with Given Centrifuge Length to Attain Given Reduction

% of No Purification Equilibrium Activity	P lb/sec with no centrifuge	P lb/sec with 100 Meters	P lb/sec with 200 Meters	P lb/sec with 500 Meters	P lb/sec with 1000 Meters	P lb/sec with 2000 Meters	P lb/sec with 10,000 Meters	P lb/sec with 100,000 Meters
10	0.5	.361	0.308	0.211	0.102	0.017		
2.17	2.5	2.19	2.07	1.81	1.54	1.19	0.084	
1.0	5.49	5.05	4.87	4.45	4.22	3.48	1.42	
0.1	55.5	63.9	53.4		51.0		40.8	14.5

i.e., if one knew cost figures and compared the cost of expanding a by-pass flow purification plant from 40.8 to 55.5 lbs/sec with the cost of a 10,000 meter centrifuge plant, there would be firm basis for a decision between two such alternatives.

Part of the data of Table I has been plotted in Figure 3, and is essentially identical to Figure 5-4 of NYO2373. The enrichment factor, α, used in this report is identical to the fractional concentration, c, used in NYO 2373.

Tables III and IV present numerical results calculated from equations (7), (7a), (10), (11), and (12); i.e., using the 135 series approach to the purification problem. Again, the activity remaining is expressed as a percent of the equilibrium activity which would result with no purification. Since the assumptions on which this approach to the purification problem is based are considered to be very sound, much more confidence is placed in these numbers than in those resulting from the first approach. Table III shows the decrease in I_{135} and Xe_{135} activity with increasing centrifuge length at constant P for a few by-pass flows, P. Table IV shows combinations of by-pass flows and centrifuge lengths to attain a few given activity reductions. The "double cut-down effect" is shown nicely in both tables. For example, Table IV shows that a purification which reduced I_{135} activity to 1.17% (two orders) of its equilibrium value reduces Xe_{135} to 0.01% (four orders) of its equilibrium value. Of course, the two equilibrium values are equal.

TABLE III

135 Decay Series

Centrifuge Lengths, Z vs % of No Purification Equilibrium Activity (NPEA)

	P = 0.1 lb/sec = 11.35 moles/sec			P = 0.5 lb/sec = 56.7 moles/sec		
		% NPEA			% NPEA	
α	Meters	I_{135}	Xe_{135}	Meters	I_{135}	Xe_{135}
1.0	0	37.3	11.3	0	10.6	0.846
1.5	44.3	28.4	6.35	222	7.34	0.401
2.0	144	22.9	4.06	720	5.60	0.233
2.5						
4				3,780	2.88	0.061
5	1,120	10.6	0.846	5,600	2.32	0.0396
10	3,140	5.60	0.233	15,700	1.17	.0101
20	7,520	2.88	0.061			

	P = 1.0 lb sec = 113.5 moles sec			P = 5 lb sec = 567 moles sec		
		% NPEA			% NPEA	
1.0	0	5.60	0.233	0	1.17	.0101
1.5				2,220	0.787	.00443
2.0	1,440	2.88	0.061	7,200	0.59	.00254
2.5	2,740	2.32	0.0396			
4						
5	11,200	1.17	.0101	56,000	0.237	.00041
10	31,400	0.59	.00254			
20						

TABLE IV

135 Decay Series. By-pass Flow, P, with Given Centrifuge Length to Attain Given Reduction

% of No Purification Equilibrium Activity			P in lb/sec with Z of					
I_{135}	Xe_{135}	Total	0 Meters	100 Meters	200 Meters	500 Meters	1000 Meters	10,000 Meters
34.4	10	22.2	0.113	0.0512	0.0306	0.00123		
16.2	2	9.1	0.309	0.1975	0.1615	0.0912	0.0276	
11.5	1	6.3	0.458	0.328	0.275	0.182	0.0948	
3.68	0.1	1.85	1.555	1.295	1.216	1.018	0.815	
1.17	0.01	0.59	5.02	4.56	4.32	3.99	3.66	1.18

SECTION V

CENTRIFUGE COST ESTIMATES

In order to settle completely the enrichment feasibility question one needs to know cost figures per meter of centrifuge, both for the original investment and for operating costs. These latter would include figures for heat loss from centrifuge surface and plant pumping power. Unfortunately, none of these figures can be given accurately since the centrifugation art is at present not sufficiently far advanced. No large scale centrifuge separation plants have ever been built or operated and, indeed, only one type of ultracentrifuge (the Zippe centrifuge) is ready for production engineering. Furthermore, not only the total length, but the diameter and the unit length of the centrifuges come into the cost picture and these can only be decided on exactly by a very detailed analysis. However, once diameter and unit length are fixed, the cost will be nearly proportional to the total length for reasonably long lengths. An estimate of $4,000 per meter investment cost and $2,000 per meter per year operating cost would probably be of the right order of magnitude.

SECTION VI

CONCLUSIONS

No clear-cut conclusion about the feasibility of centrifuge enrichment may be drawn immediately from this report, since detailed economic information in three important regions is not known: (1) Centrifuge plant cost, (2) By-pass purification cost, (3) Upper limits for purification costs from the standpoint of reactor economics. Nonetheless, the situation may be looked at in two ways which, taken together, give the definite impression that centrifuge enrichment is feasible only as a last resort.

(1) Total length figures required for the various separations turn out to be much greater than expected. Certainly, it would be impossible to house the required length of centrifuge in the immediate vicinity of the primary loop. This means that a mere enlargement of the shielded volume about the primary loop would not be sufficient, but that a separate structure would be required to house the enrichment plant. A large centrifuge enrichment plant would seriously violate the requirement of minimizing contact between radioactivity material and equipment containing moving parts. Ultracentrifuges run near the limits of material strength and bearing design and require a good deal of maintenance.

(2) The order of magnitude of the operating cost estimate in the last section may be used in conjunction with Tables II or IV to calculate the cost of reducing by-pass flow to a tolerable level. Using Table II, suppose that a three order reduction in activity was necessary, but that only 40 lbs/sec of by-pass flow could be tolerated, as opposed to the 55.5 lbs/sec of un-enriched flow required. Table II shows that the necessary concentration could be attained with 10,000 meters of centrifuge and an approximate operating cost of $20,000,000 per year. The question arises, is the 27% reduction in by-pass flow worth $20,000,000 per year? On the face of it, it seems doubtful, since this is about 1.8 cents per kwhr.

One gets the final impression that centrifuge enrichment does not seem to be a promising answer to reactor purification problems if convenience and low cost are criterions of a "promising" answer. The centrifuge idea should probably be dropped for now; at least until other, more promising methods of purification have been investigated.

APPENDIX I

EXACT ANALYSIS OF SEPARATIVE POWER

In the report it was assumed that the separative work required was independent of the flow, L. This is a good approximation for L >> P, but is not true in general. Instead,

$$\Delta u = PV'(N') + WV'(N'') - LV'(N) \tag{13}$$

where

$$V'(N) = (2N-1)\ \ln\frac{N}{1-N} \tag{14}$$

V'(N) is known as the elementary value function.[2] Formula (13) takes into account the values of feed, waste, and product separately (see Figure 1) instead of assuming that the value of waste and feed are the same as does (3).

Defining the "cut", $\theta = \frac{P}{L}$, using the relations

$$W = (1-\theta)L \tag{15}$$

and

$$N'' = \frac{1-\alpha\,\theta}{1-\theta}\,N \tag{16}$$

and assuming N small, (13) becomes

$$\Delta u = L\ln\left[\left(\frac{\theta}{q}\right)^{\theta}\left(\frac{1-\theta}{1-q}\right)^{1-\theta}\right] \tag{17}$$

where

$$q = \alpha\,\theta \tag{18}$$

the fraction of impurity removed from the stream.

Equation (17) may be expanded in powers of $\frac{1}{L}$:

$$\Delta u = P(\alpha - 1 - \ln\alpha) + \frac{P^2(\alpha - 1)^2}{L} + 0L^{-2} + \ldots. \tag{19}$$

(19) shows the weak L dependence for large L and gives the main error term for (4). Using (5) and the values given on page 4 one obtains

$$\frac{Z}{L} = 41.2 \ln\left[\left(\frac{\theta}{q}\right)^{\theta} \left(\frac{1-\theta}{1-q}\right)^{1-\theta}\right] \tag{20}$$

as the number of meters of centrifuge required per unit flow to effect the separation defined by θ and q. Figure 4 is a graph containing several plots of $\frac{Z}{L}$ vs q for a number of fixed cuts (θ). To use the curves one must know:

- L, the total flow in moles-sec^{-1}
- q desired
- θ desired.

Example: Suppose the total flow, L = 378 lbs/sec; P = 1 lb/sec; and α = 10. This is:

L = 42,900 moles/sec
θ = 0.00265
q = 0.0265

Looking on the curve for θ = 0.003 opposite q = 0.0265, one finds 0.72. Multiplying by 42,900 moles/sec, one finds that 30,900 meters of centrifuge are needed. The figure calculated according to (9) is 29,400 meters if one uses an α and P corresponding to θ = 0.003, q = 0.0265.

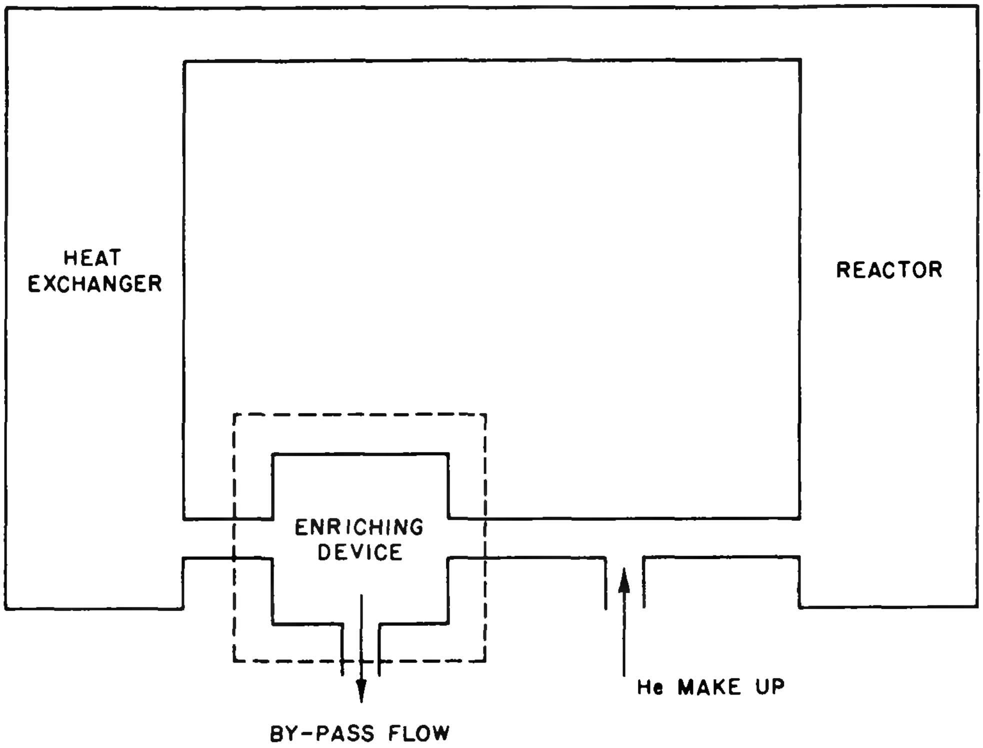
APPENDIX II

CENTRIFUGE EFFICIENCY

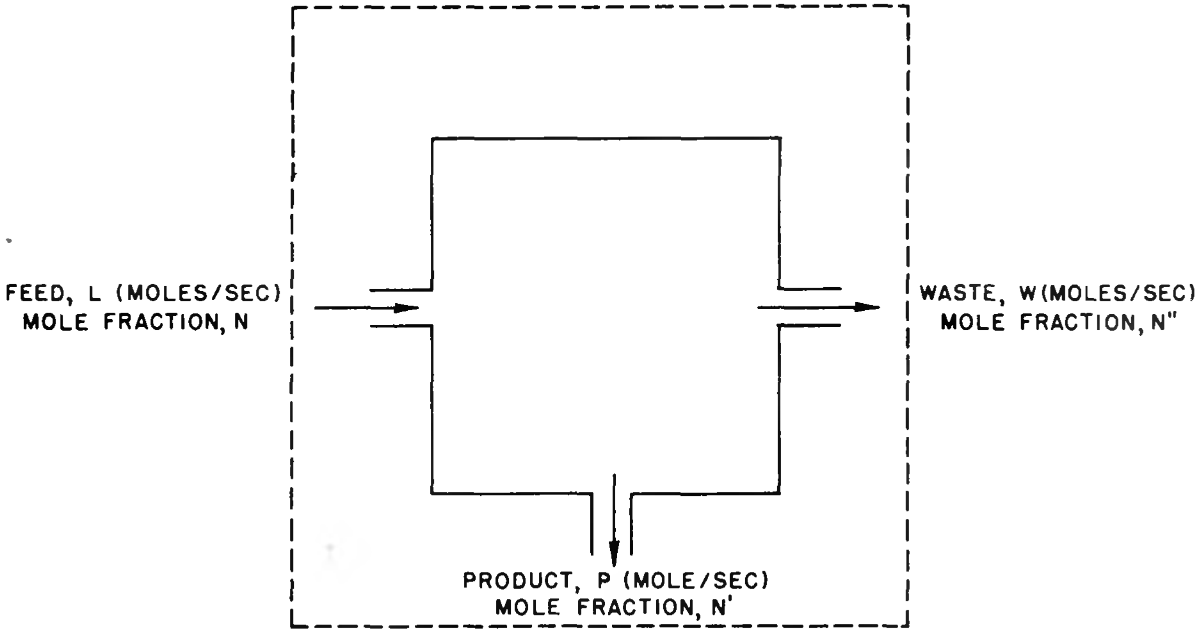
Equation (5) is an expression for the theoretical maximum separative power of a centrifuge which is not attainable in practice. The practical limitations of the two general types of gaseous centrifuge operation, concurrent or flow-through and countercurrent, may be discussed separately. In both types the detailed internal flow pattern influences the efficiency attainable and theoretical analyses are simple only for quite unrealistic flow patterns. Thus, while quantitative efficiencies cannot be stated with confidence, approximate efficiencies to be expected, as well as general factors and trends, may be given.[1]

The concurrent centrifuge may be operated in either of two ways, double entry or single entry. In the former method, the feed to a unit consists of two gas streams of different and optimally chosen mole fractions. In the latter, the feed is at a single mole fraction (one stream, in principle). The separative power per unit length is a sensitive function of the radial gradient of the mole fraction. The double entry concurrent centrifuge allows, in principle, a larger average separative power per unit length (by a factor of the order of 2) than the single entry concurrent centrifuge at the price of a somewhat more complicated cascading arrangement. All concurrent machines suffer from the requirement that longer units, desirable to reduce the total number of units, are achieved without drastic reduction in average separative power only by increasing the flow rate, a quantity which is already excessively large in comparison with that required in countercurrent centrifuges. Thus, direct separative efficiency is costly (in the combination of flow and number of units required) in concurrent centrifuges. While one could guess that 25 to 50% of the theoretical maximum separative power might be achievable, the cost of the separative power compares even less favorably with countercurrent machines. Workers in the field have abandoned concurrent operation in favor of the countercurrent machines.

For countercurrent operation of the centrifuge the radial gradient of the mole fraction can be maintained reasonably constant and close to optimum over quite long lengths. The efficiency depends primarily on the magnitude of the countercurrent flow and the internal flow pattern and is, at not too high peripheral speeds, essentially independent of the unit length. The lengths are limited primarily by purely mechanical considerations. Also of considerable importance is that the total flow required for countercurrent operation is about an order of magnitude less than for concurrent operation. It seems reasonable to assume that efficiencies of 50-60% can be obtained in a countercurrent centrifuge operating at 400 m/sec.



A. LOOP CONFIGURATION



B. FLOW DETAIL IN ENRICHER UNIT

Fig. 1—Problem Schematic

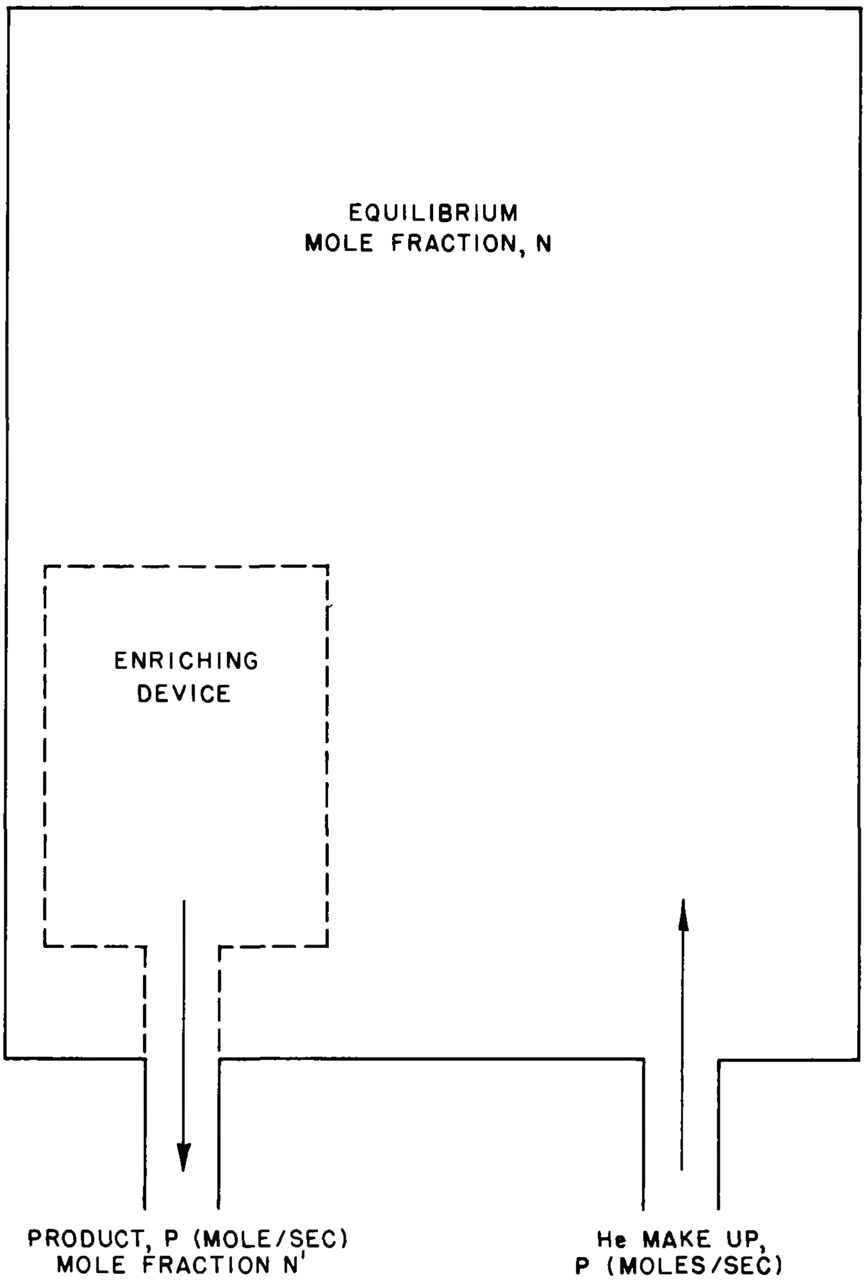


Fig. 2—Schematic for Approximate Method of Attack

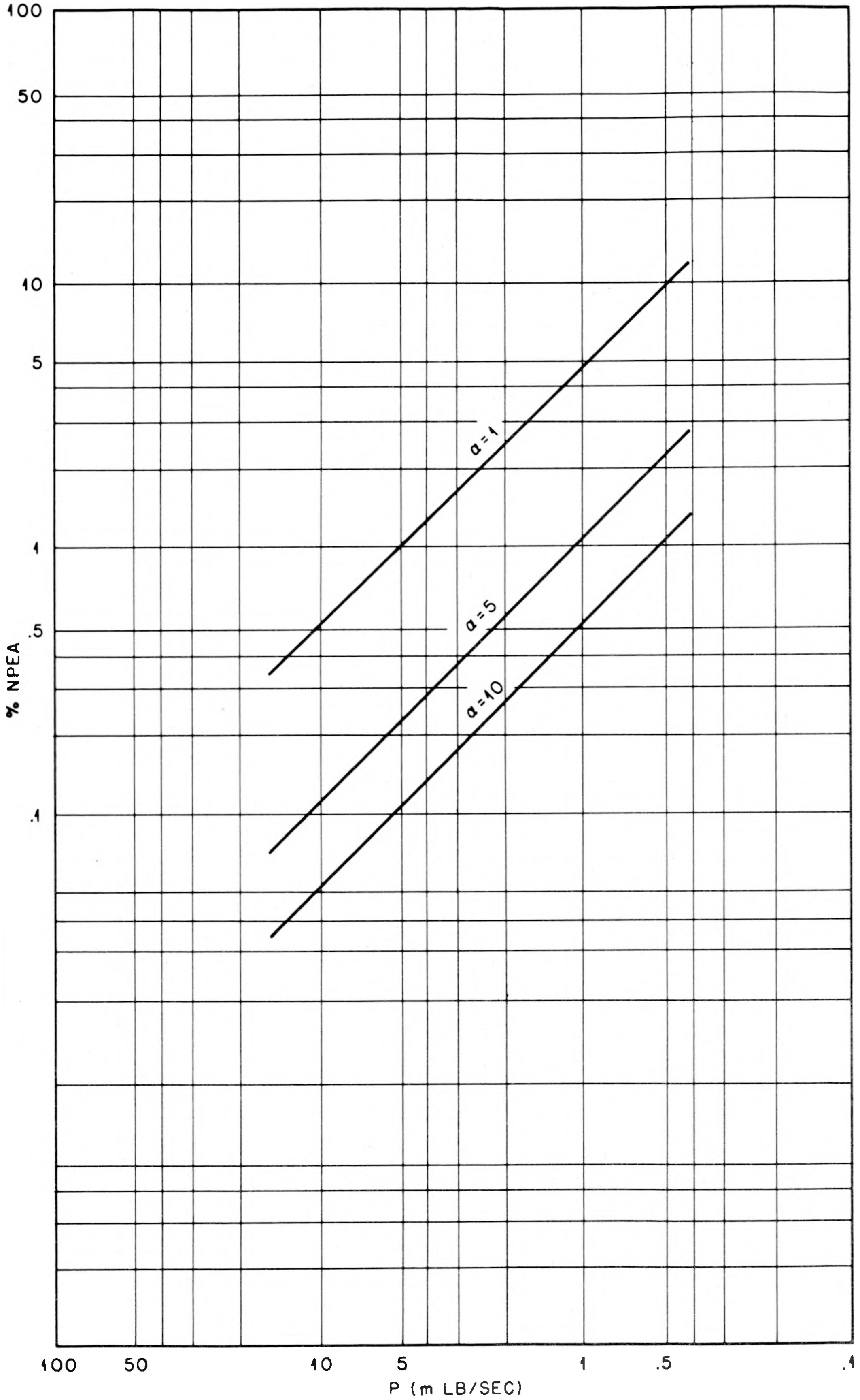


Fig. 3—% NPEA vs P

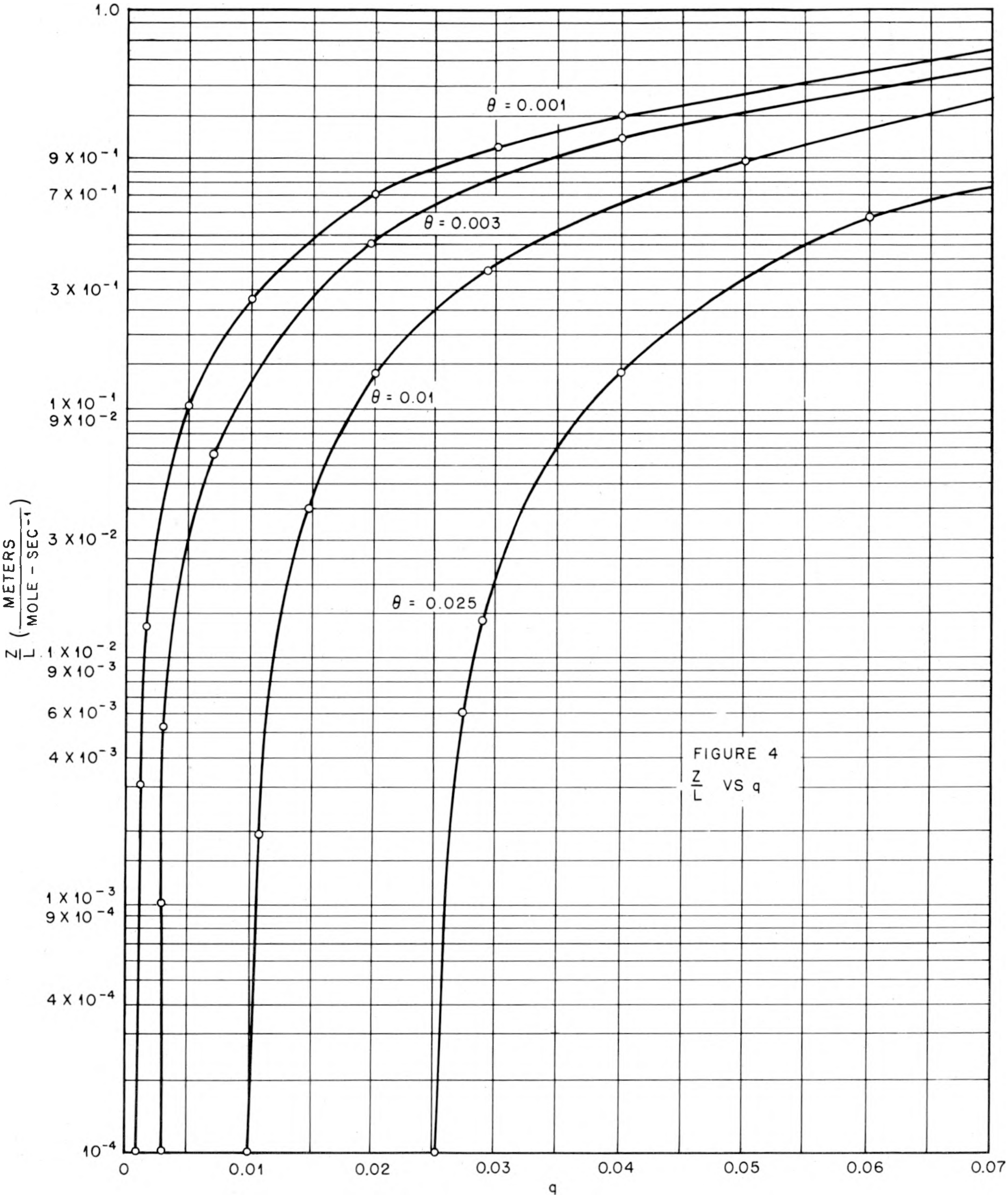


FIGURE 4

$\frac{Z}{L}$ VS q

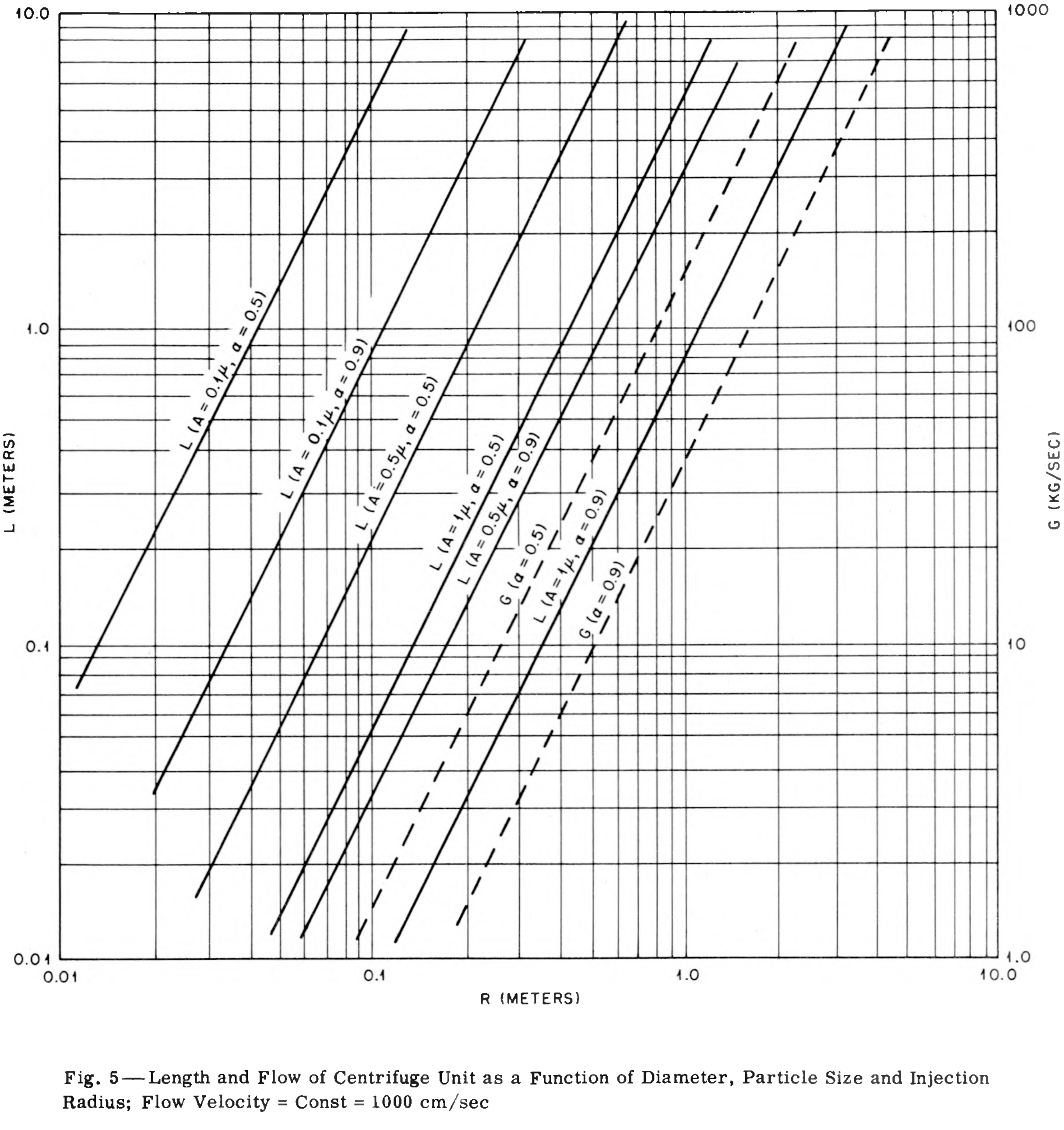


Fig. 5—Length and Flow of Centrifuge Unit as a Function of Diameter, Particle Size and Injection Radius; Flow Velocity = Const = 1000 cm/sec