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A CONFIDENCE LIMIT COMPUTER

by

Joseph O Muench

April 1960

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A CONFIDENCE-LIMIT COMPUTER

Introduction

Confidence limits as a mathematical tool are applicable to many development engineering and production engineering situations. In order to make confidence limits readily available to engineers, a computer was designed which will approximate certain confidence limits. One of these computers is included as part of this monograph and will be found inside the back cover.

Confidence Limits

The usual purpose of engineering analysis of data is to estimate unknown values. In general, there are two ways to express such estimates. These estimates may be expressed as points or as intervals. Traditionally, point estimates[*] have been used in most engineering work. However, more recently it has been recognized that such point estimates are not always the "best" estimates of the unknown values,[†] and a more useful estimate has been developed whereby an interval is presented and is accompanied by a statement of the assurance or confidence that the interval really does include the true (but unknown) value.

In statistical terms, the unknown value (parameter) is sometimes denoted by θ, a point estimate by $\hat{\theta}$, the interval limits by L and U (where $L \leq \hat{\theta} \leq U$), and the degree of assurance or confidence that $L \leq \theta \leq U$ by α (where $0 \leq \alpha \leq 1$).

1. Such an interval is called a <u>confidence interval</u>.

2. The limits of the interval are called <u>confidence limits</u>.

3. The degree of assurance is called the <u>confidence coefficient</u> or <u>confidence level</u>.

<u>Illustration</u>

Consider a very large lot of items of which 100p% are defective ($0 \leq p \leq 1$). The parameter p (that is, θ), the fraction defective in the lot, is unknown. A random sample of 50 items is selected and one defective item is observed. A point estimate of p is given by $\hat{p} = \hat{\theta} = 1/50 = 0.02$. The 90% upper confidence limit for

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*A point estimate is any single value used as an estimate of an unknown.

†For a discussion of point estimates and their limitations see Reference 1.

the true but unknown value of p is determined to be U = 0.076. That is, we are 90% confident that the true fraction defective in the sampled lot is no greater than 0.076. (The upper confidence limit, U, has been denoted by k on the computer.)

The computer which is provided with this report estimates upper confidence limits. In using it the lower limit is always 0. Thus the confidence interval given is always from 0 to k.

How to Use the Computer

Instructions for the use of the computer are printed on the back of the envelope in which the computer is packaged. These instructions are repeated here in the event that the envelope becomes separated from the computer.

Note that on the computer the sample size n is divided into three ranges. The range for small values of n (from 1 through 19) is given in a table of values on the right portion of one side of the computer. The range of n from 20 through 100 is given on a logarithmic scale near the bottom of the computer on the same side as the table previously referred to. The range of n from 100 through 10,000 is given on the side opposite to the table of values.

A. To determine the upper confidence limit from a sample of size n = 20 to 10,000:

A-1. Select the side of the slide rule that has an n scale in the desired range.

A-2. Decide what confidence level is desired. Locate this value on the confidence-level (vertical) scale.

A-3. Pick the x value that corresponds to the number of items in the sample containing the specified characteristic. (The x values are the series of curves that intersect the vertical confidence-level scale.) Move the slide until the x value coincides with the previously selected confidence level.

A-4. Locate the sample size on the n scale.

A-5. Read on the k scale, directly beneath the n value, the maximum fraction of items that may be expected to contain the specified characteristic. This is the upper confidence limit.

B. To determine the upper confidence limit from a sample of size n = 1 to 19:

B-1. Use the table of values printed on one side of the slide rule. (Note that the table contains two confidence levels--.50 and .90.) Decide which confidence level to use.

B-2. Read across the top of the table and select the sample size n.

B-3. Locate on the left side of the table the number of items x containing the specified characteristic.

B-4. Find the upper confidence limit at the point of intersection of the chosen row x and column n.

Example 1: If a sample of 150 items is tested (or inspected) and there are four failures (or defectives), what is the upper confidence limit at the .95 confidence level?

Figure 1 shows the computer set to solve this problem. Note that the x curve for 4 crosses the .95 confidence level. Read under n = 150 the upper confidence limit, .06.

Example 2: The sample size is 90. An upper confidence limit of .02 is desired. If there were no failures in the sample of 90, what is the confidence level? If there were one failure, what is the confidence level? (Note that there is a different unknown than in Example 1.)

Refer to Figure 2 which shows the computer set for the solution. An n value of 90 is set at a k value of .02. Read the confidence level for no failures where the curve for x = 0 crosses the confidence-level scale. The reading is approximately .84. For one failure, the reading is .53.

Variations in Use of the Computer

When using the computer there will always be three known quantities and one unknown. Any three of the following may be known and the computer will provide a solution for the fourth:

1. Upper confidence limit (k)

2. Confidence level (α)

3. Number of observations (n)

4. Number of units with a specified characteristic (x)

Let n become the unknown and the following is typical of the type of probability problem that can be solved:

Example 3: How many items should be tested if one failure will be accepted, and it is desired to say with 90% confidence that the failure rate is no greater than one in a thousand? Two in a thousand? One in a hundred?

To solve the problem, set the x curve of 1 at the .90 confidence level on the appropriate side of the computer. For one in a thousand, read on the k scale above .001 that the number to test is approximately 3850. Leave the same setting in the computer and for two in a thousand, read 1925 above a k of .002. For one in a hundred, again leave the original setting in the computer and read above a k of .01 that the number to test is 390.

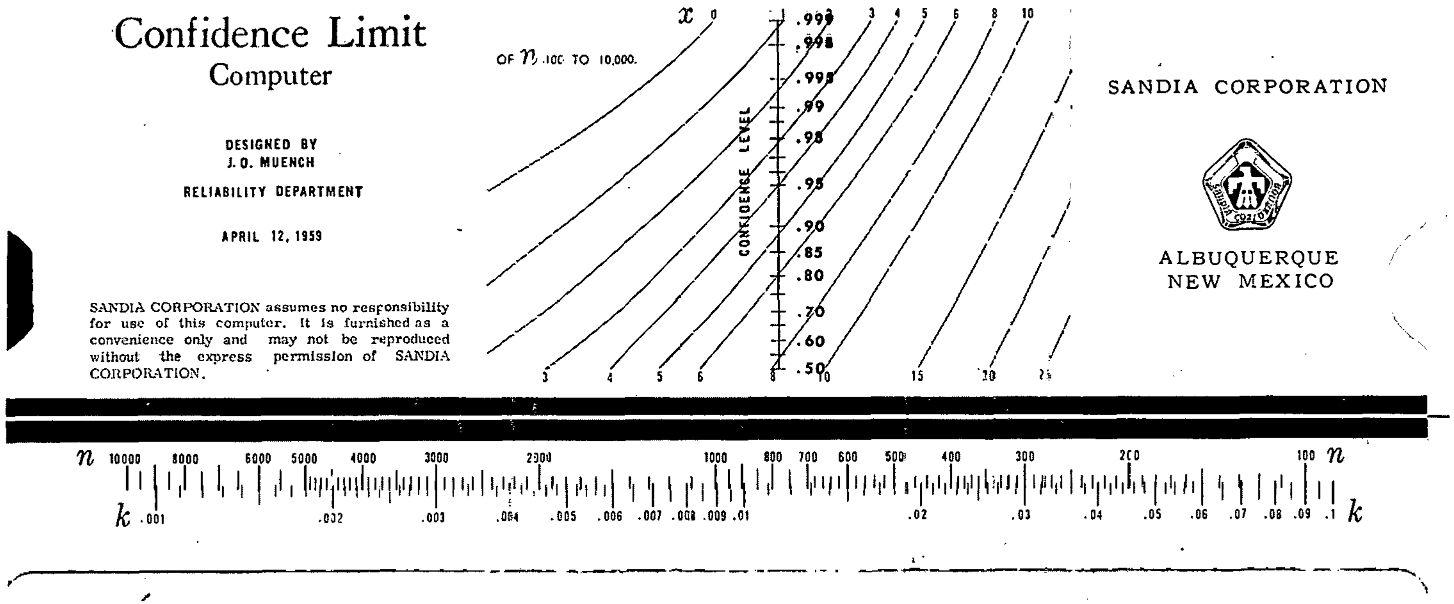
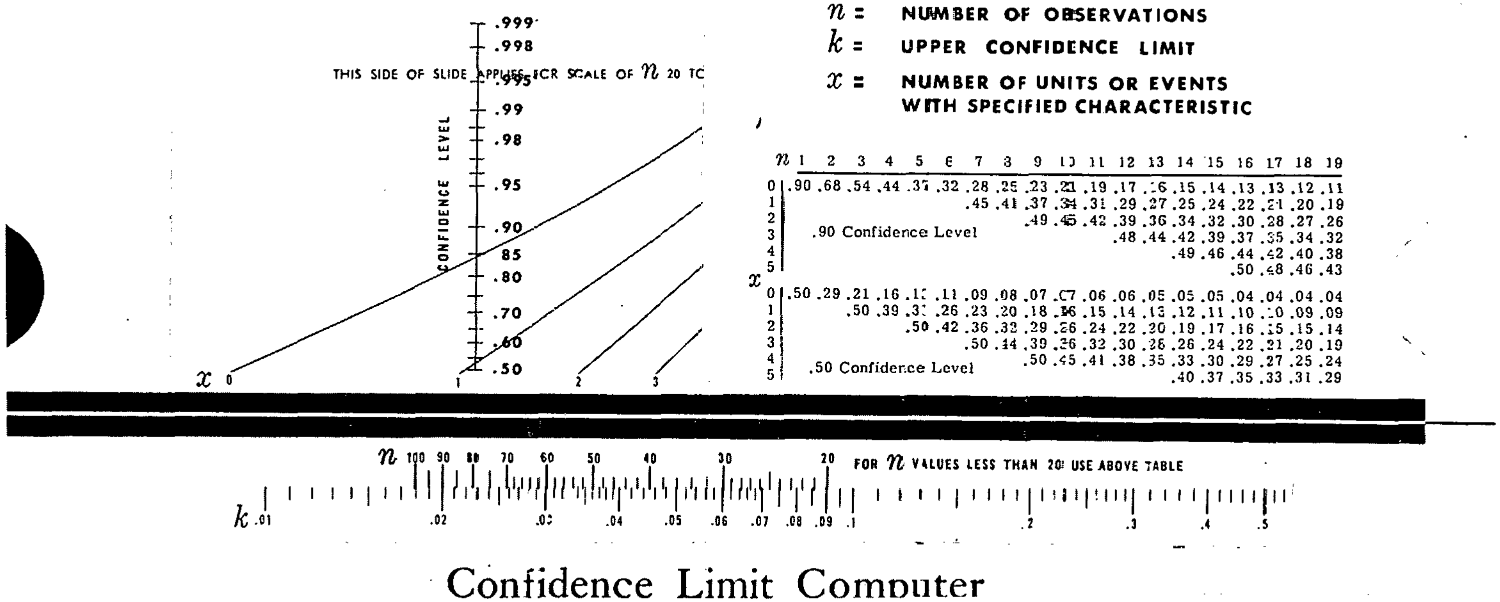


Figure 1. Computer Set for Solution of Example 1



x \ n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
.90 Confidence Level																			
0	.90	.68	.54	.44	.37	.32	.28	.25	.23	.21	.19	.17	.16	.15	.14	.13	.13	.12	.11
1							.45	.41	.37	.34	.31	.29	.27	.25	.24	.22	.21	.20	.19
2									.49	.45	.42	.39	.36	.34	.32	.30	.28	.27	.26
3												.48	.44	.42	.39	.37	.35	.34	.32
4														.49	.46	.44	.42	.40	.38
5																.50	.48	.46	.43
.50 Confidence Level																			
0	.50	.29	.21	.16	.13	.11	.09	.08	.07	.07	.06	.06	.05	.05	.05	.04	.04	.04	.04
1			.50	.39	.31	.26	.23	.20	.18	.16	.15	.14	.13	.12	.11	.10	.10	.09	.09
2					.50	.42	.36	.33	.29	.26	.24	.22	.20	.19	.17	.16	.15	.15	.14
3							.50	.44	.39	.36	.32	.30	.28	.26	.24	.22	.21	.20	.19
4									.50	.45	.41	.38	.35	.33	.30	.29	.27	.25	.24
5														.40	.37	.35	.33	.31	.29

Figure 2. Computer Set for Solution of Example 2

Construction of Scales

The k and n scales are logarithmic. The advantage of using logarithmic scales is that they permit a much greater range of values than a uniform scale, and the log scales have an accuracy of about the same number of significant figures throughout the range since scale lengths for 10 to 10^2, 10^2 to 10^3, and 10^3 to 10^4 are the same.

The vertical confidence-level scale on the computer is a normal probability scale. This scale was chosen because it expands as it approaches 1.00.

Basic Equation for the Confidence-Limit Computer

The confidence-limit computer provides an approximate solution to the equation (Reference 2):

$$\sum_{j=x+1}^{n} \frac{n!}{j!(n-j)!} k^j (1-k)^{n-j} = \alpha*$$

where:

x = number of units or events with specified characteristic
n = number of observations
k = upper confidence limit
α = confidence level

The curves on the computer were plotted by solving the above equation using the Harvard binomial tables (Reference 3) exclusively. The binomial distribution is based upon an infinite lot size, but gives good approximations to the true probabilities when the lot size is 10 times the sample size. The reason that the computer gives an approximation rather than an exact solution to the equation is that the basic equation plots as a curve and the computer uses a linear approximation. For most practical purposes, the approximation is as satisfactory as the exact solution.

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*The theory of confidence intervals was developed by Dr. Jerzy Neyman and introduced in 1937. For a development of the theory see Reference 4.

Accuracy of the Computer

The accuracy of this computer is determined by many factors, the most important being:

1. Closeness of the approximation of the computer's linear solution to the nonlinear equation.

2. Accuracy of the original drawings and layouts.

3. Faithfulness of the photoreproduction of original layout and the transfer of the reproduction to the printing plates.

4. Accuracy of the printing process.

5. Vertical orientation of confidence-level scale as determined by location of folds in the plastic jacket.

6. Fit of slide in jacket.

Some of the above-mentioned factors contribute less than others to possible errors. The original layout was made to a large scale and done with such care that little error contribution is from that source. The best all-inclusive measure of error in the computer is to take a production model and compare some points with the Harvard binomial tables. This was done, and hundreds of points were checked through all ranges of all scales. The most practical way to express the accuracy of the computer is to say that it is more accurate than the number of decimal places given on the k scale. That is, it is always safe to use one decimal place on the k scale in the range .1 to .6, use two decimal places in the range .01 to .09, and use three decimal places in the range .001 to .009.

Summary

The confidence-limit computer provides a rapid means of approximating upper confidence limits, confidence levels, sample sizes, or the number of units with a specified characteristic.

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